SDLD27 - Spring-mass system with 8 degrees of freedom with viscous damper non proportional (modal analysis)

Summarized:

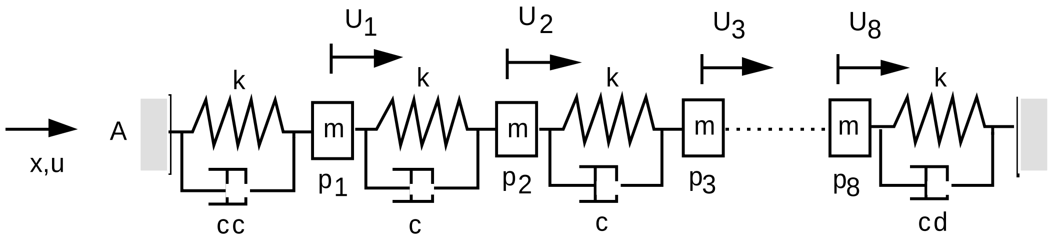
This two-dimensional problem consists in searching the frequencies, the modes of vibration and the depreciation of a mechanical structure made up of masses, springs and dampers viscous. This benchmark of Structural mechanics corresponds to a dynamic analysis of a discrete model having a linear behavior.

This test allows a complete validation of the options of discrete modelization of stiffness, viscous damping and mass (without finite elements) offered by the command AFFE_CARA_ELEM. Five different modelizations are proposed: the modelization of the discrete elements is either in translation, or in translation/rotation and is written or in total reference, or in local coordinate system. In addition, various features of commands MODE_ITER_INV (search for eigenvalues per inverse iteration), MODE_ITER_SIMULT (search for eigenvalues by the method of Lanczos) and NORME_MODE (definition of the norm of an eigenvector) are tested for this quadratic problem.

This test refers to a test VPCS, but it was modified. Indeed, the test directs the mechanical system towards an axis $3y=4x$, which makes it possible to validate the entry of the data in local coordinate system. The got results are in concord with the results of reference.

1 Problem of reference

1.1 Geometry



Point masses: $m_{P_1} = m_{P_2} = m_{P_3} = \ldots \ldots = m_{P_8} = m$

Stiffness of connection: $k_{AP1} = k_{P1P2} = k_{P2P3} = \ldots \ldots = k_{P8B} = k$

Viscous damping: $c_{P1P2} = c_{P2P3} = \ldots \ldots = c_{P7P8} = c$
$c_{AP1} = cc$
$c_{P8B} = cd$

1.2 Material properties

Comes out from elastic translation linear $k = 10^5 N/m$
Point mass $m = 10 kg$
one-way Boundary conditions and $c = 50 N/(m/s)$
$cc = 250 N/(m/s)$
$cd = 25 N/(m/s)$

1.3 loadings Points and

clamped A viscous B Dampers: $u = 0$.

1.4 Initial conditions

Without object for the modal analysis.

2 Reference solution

2.1 Method of calculating used for the reference solution

the reference solution is that given in file SDLD27 of guide VPCS.

The problem led to search the eigenvalues and eigenvectors of the following dissipative system:

$$\boldsymbol{M}\,\ddot{u}+\boldsymbol{C}\,\dot{u}+\boldsymbol{K}\,u=0$$

with $\boldsymbol{M}$ mass matrix, $\boldsymbol{C}$ damping matrix, $\boldsymbol{K}$ stiffness matrix.

One associates with this dissipative problem, the conservative problem: $\boldsymbol{K}\,u+\boldsymbol{M}\,\ddot{u}=0$. In harmonic form, he is written $\boldsymbol{K}-\omega^2\,\boldsymbol{M}=0$.

Are $\Lambda=[\omega_v^2]$ the spectral diagonal matrix of the eigenvalues of this conservative system and $\phi=[\varphi_v]$ the corresponding matrix of the eigenvectors.

φ_v Are standardized such as: $\phi^T\,M\,\phi=Id \quad \phi^T\,K\,\phi=\Lambda$.

The solutions of the dissipative system are form:

$$u=u_0\,e^{st} \text{ from where } (M\,s^2+C\,s+K)u_0=0 \text{ .}$$

One breaks up u_0 in the base of φ_v . There is then $u_0=\phi\,q$, from where:

$$(I\,s^2+\gamma\,s+\Lambda)q=0 \text{ with } \gamma=\phi^T\,C\,\phi \text{ (full matrix)}$$

This problem with the eigenvalues is solved by a method of power opposite while taking for initial estimate $s_v=j\,\omega_v$.

2.2 Results of reference

8 depreciation and eigenfrequencies of the system, as well as 1st and the 8th mode (complexes).

2.3 Uncertainty on the semi-analytical

solution Solution.

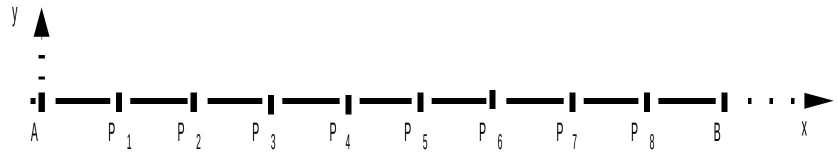
2.4 Bibliographical references

• J. PIRANDA - Note of use of the software of modal analysis MODAN - Version 0.2 (1990) Laboratory of Mechanics Applied - University of Honest - County - Besancon (France)
• Guides VPCS. Complement Groups Dynamic. September 94

3 Modelization A

3.1 Characteristic of the modelization

Discrete element of stiffness in translation DIS_T



Characteristics of the DISCRET

elements:

with nodal masses

in all the nodes	M_T_D_N	out of absolute coordinate system	$(m=10.)$
stiffness matrixes			
in all meshes	M_T_D_L	out of absolute coordinate system	$(K_x=1.10^5)$
damping matrixes			
meshes internal	A_T_D_L	out of absolute coordinate system	$(C_x=50.)$
nets initial	A_T_D_L	out of absolute coordinate system	$(C_x=250.)$
nets final	A_T_D_L	out of absolute coordinate system	$(C_x=25.)$

limiting Conditions:

DDL_IMPO:	(TOUT: “YES” DY: 0. , DZ: 0.)
with the nodes ends	(THE NODE IS OUTSIDE THE FIELD OF DEFINITION WITH A RIGHT PROFILE OF THE EXCLU TYPE NODE: (A B) DX: 0.)

Names of the nodes: $A, P_{1,} P_{2,} \ldots, P_{8,} B$

3.2 Characteristics of the mesh

Many nodes:	10
Number of meshes and types:	9 SEG2 and 8 POI1

3.3 Quantities tested and results

Frequency	Reference
Order of the eigen mode 1	5.53
Order of the eigen mode 2	10.90
Order of the eigen mode 3	15.93
Order of the eigen mode 4	20.45
Order of the eigen mode 5	24.34
Order of the eigen mode 6	27.49
Order of the eigen mode 7	29.84
Order of the eigen mode 8	31.29

Damping	Reference
Order of eigen mode 1	1.521e-2
Order of eigen mode 2	2.877e-2
Order of eigen mode 3	3.960e-2
Order of eigen mode 4	4.709e-2
Order of eigen mode 5	5.098e-2
Order of eigen mode 6	5.183e-2
Order of eigen mode 7	5.115e-2
Order of eigen mode 8	5.036e-2

Natural of the eigen mode	Not	Reference Eigen mode into 10^{-3} real Part imaginary Part
Translation 1 (Dy) Φ_1	P1	4.07, – 4.56
	P2	7.97, – 8.28
	P3	10.9, – 11.0
	P4	12.5, – 12.5
	P5	12.5, – 12.4
	P6	11.1, – 10.9
	P7	8.24, – 8.04
	P8	4.41, – 4.25
Translation 8 (Dy) Φ_8	P1	2.23, – 1.14
	P2	– 3.71, 2.98
	P3	4.75, – 4.41
	P4	– 5.25, 5.27
	P5	5.14, – 5.43
	P6	– 4.44, 4.88
	P7	3.23, – 3.69
	P8	– 1.66, 2.01

Eigen mode normalized with the unit modal mass: $\phi_i^t \boldsymbol{C}\, \phi_i + 2\lambda_i \phi_i^t \boldsymbol{M}\, \phi_i = 1$

λ : is the eigenvalue associated with damping and the eigenfrequency.

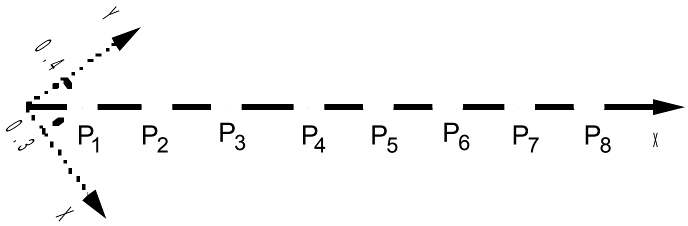
3.4 Contents of the file results

8 depreciation and eigenfrequencies, as well as the associated eigenvectors.

4 Modelization B

4.1 Characteristic of the modelization

Discrete element of stiffness in translation DIS_T



Characteristics of elements

ORIENTATION: angle:	in all the nodes	with a DISCRETE $\alpha = 53.130102°$	
with nodal masses			
in all the nodes	M_T_D_N	out of absolute coordinate system	$(m = 10.)$
stiffness matrixes			
in all meshes	K_T_D_L	in local coordinate system	$(K_x = 1.10^5)$
with the nodes ends	K_T_D_N	in local coordinate system	$(K_x = 1.10^5)$
damping matrixes			
meshes internal	A_T_D_L	in local coordinate system	$(C_x = 50.)$
initial mesh	A_T_D_N	in local coordinate system	$(C_x = 250.)$
final mesh	A_T_D_N	in local coordinate system	$(C_x = 25.)$

limiting Conditions:

DDL_IMPO:	(TOUT: "YES" DZ: 0.)
LIAISON_DDL:	(such as $3Dy = 4Dx$ in all the nodes)

Names of the nodes: $P_{1,} P_{2,} \ldots, P_8$

4.2 Characteristics of the mesh

Many nodes:	8
Number of meshes and types:	7 SEG2

the points P_1 and P_8 are connected to a fixed point fictitious by nodal springs (K_T_D_N, A_T_D_N) what makes it possible not to model the nodes A and B.

4.3 Quantities tested and results

Frequency	Reference
Order of the eigen mode 1	5.53
Order of the eigen mode 2	10.90
Order of the eigen mode 3	15.93
Order of the eigen mode 4	20.45
Order of the eigen mode 5	24.34
Order of the eigen mode 6	27.49
Order of the eigen mode 7	29.84
Order of the eigen mode 8	31.29

Damping	Reference
Order of eigen mode 1	1.521e-2
Order of eigen mode 2	2.877e-2
Order of eigen mode 3	3.960e-2
Order of eigen mode 4	4.709e-2
Order of eigen mode 5	5.098e-2
Order of eigen mode 6	5.183e-2
Order of eigen mode 7	5.115e-2
Order of eigen mode 8	5.036e-2

Natural of the eigen mode	Not	Reference Eigen mode into 10^{-3} real Part imaginary Part
Translation 1 (Dy) Φ_1	P1	– 2.442, 2.736
	P2	– 4.782, 4.968
	P3	– 6.54 , 6.6
	P4	– 7.5 , 7.5
	P5	– 7.5 , 7.44
	P6	– 6.66 , 6.54
	P7	– 4.944, 4.824
	P8	– 2.646, 2.55
Translation 8 (Dy) Φ_8	P1	– 1.338, 0.684
	P2	– 2.226, 1.788
	P3	– 2.85 , 2.646
	P4	– 3.15 , 3.162
	P5	– 3.084, 3.258
	P6	– 2.664, 2.928
	P7	– 1.938, 2.214
	P8	– 0.996, 1.206

Eigen mode normalized with the unit modal mass: $\phi_i^t \boldsymbol{C}\, \phi_i + 2\lambda_i \phi_i^t \boldsymbol{M}\, \phi_i = 1$

λ : is the eigenvalue associated with damping and the eigenfrequency.

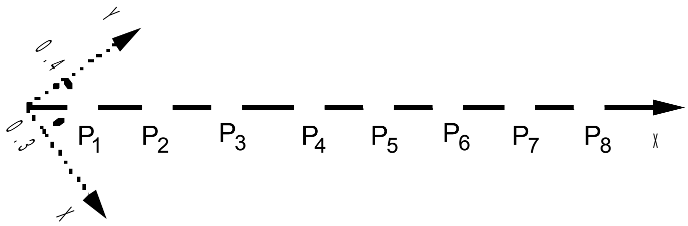
4.4 Contents of the file results

8 depreciation and eigenfrequencies, as well as the associated eigenvectors.

5 Modelization C

5.1 Characteristic of the modelization

Discrete element of stiffness in translation `DIS_T`



Characteristics of elements

`ORIENTATION:` `angle:`	in all the nodes		with a DISCRETE $\alpha=53.130102°$	
with nodal masses				
	in all the nodes	`M_T_N`	out of absolute coordinate system	$(m=10.)$
stiffness matrixes				
	in all meshes	`K_T_L`	in local coordinate system	$(K_x=1.10^5)$
	with the nodes ends	`K_T_N`	in local coordinate system	$(K_x=1.10^5)$
damping matrixes				
	in all meshes	`A_T_L`	in local coordinate system	$(C_x=50.)$
	with the initial node	`A_T_N`	in local coordinate system	$(C_x=250.)$
	with the final node	`A_T_N`	in local coordinate system	$(C_x=25.)$

limiting Conditions:

`DDL_IMPO:` `(TOUT: "YES" DZ: 0. )`
`LIAISON_DDL:` (such as $3Dy=4Dx$ in all the nodes)

Names of the nodes: $P_{1,} P_{2,} \ldots, P_8$

5.2 Characteristics of the mesh

Many nodes: 8
Number of meshes and types: 7 `SEG2`

the points P_1 and P_8 are connected to a fixed fictitious node by nodal springs (`K_T_N`, `A_T_N`).

5.3 Quantities tested and results

Frequency	Reference
Order of the eigen mode 1	5.53
Order of the eigen mode 2	10.90
Order of the eigen mode 3	15.93
Order of the eigen mode 4	20.45
Order of the eigen mode 5	24.34
Order of the eigen mode 6	27.49
Order of the eigen mode 7	29.84
Order of the eigen mode 8	31.29

Damping	Reference
Order of eigen mode 1	1.521e-2
Order of eigen mode 2	2.877e-2
Order of eigen mode 3	3.960e-2
Order of eigen mode 4	4.709e-2
Order of eigen mode 5	5.098e-2
Order of eigen mode 6	5.183e-2
Order of eigen mode 7	5.115e-2
Order of eigen mode 8	5.036e-2

Natural of the eigen mode	Not	Reference Eigen mode into 10^{-3} real Part imaginary Part
Translation 1 (Dy) Φ_1	P1	– 2.442, 2.736
	P2	– 4.782, 4.968
	P3	– 6.54 , 6.6
	P4	– 7.5 , 7.5
	P5	– 7.5 , 7.44
	P6	– 6.66 , 6.54
	P7	– 4.944, 4.824
	P8	– 2.646, 2.55
Translation 8 (Dy) Φ_8	P1	– 1.338, 0.684
	P2	– 2.226, 1.788
	P3	– 2.85 , 2.646
	P4	– 3.15 , 3.162
	P5	– 3.084, 3.258
	P6	– 2.664, 2.928
	P7	– 1.938, 2.214
	P8	– 0.996, 1.206

Eigen mode normalized with the unit modal mass: $\phi_i^t \boldsymbol{C}\,\phi_i + 2\lambda_i \phi_i^t \boldsymbol{M}\,\phi_i = 1$

λ is the eigenvalue associated with damping and the eigenfrequency.

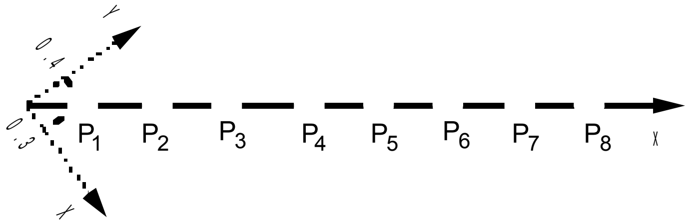
5.4 Contents of the file results

8 depreciation and eigenfrequencies, as well as the associated eigenvectors.

6 Modelization D

6.1 Characteristic of the modelization

Transposition of the test of reference to the case of the degrees of freedom of rotation (torsion spring + inertia) by means of the discrete element of stiffness in translation/rotation.



Characteristics of elements

ORIENTATION:	in all the nodes	with a DISCRETE $\alpha=53.130102°$	
angle:			
with nodal masses			
in all the nodes	M_TR_D_N	in local coordinate system	$(m=10.)$
stiffness matrixes			
in all meshes	K_TR_D_L	in local coordinate system	$(KR_x=1.10^5)$
with the nodes ends	K_TR_D_N	in local coordinate system	$(KR_x=1.10^5)$
damping matrixes			
in all meshes	A_TR_D_L	in local coordinate system	$(CR_x=50.)$
with the initial node	A_TR_D_N	in local coordinate system	$(CR_x=250.)$
with the final node	A_TR_D_N	in local coordinate system	$(CR_x=25.)$

limiting Conditions:

DDL_IMPO: (TOUT: "YES" DX: 0. , DY: 0. , DZ: 0. , DRZ: 0.)
LIAISON_DDL: (such as $3DRy=4DRx$ in all the nodes)

Names of the nodes: $P_{1,} P_{2,} \ldots, P_8$

6.2 Characteristics of the mesh

Many nodes: 8
Number of meshes and types: 7 SEG2

the nodes P_1 and P_8 are connected to a fixed fictitious node by nodal springs (K_TR_N, A_TR_N).

6.3 Contents of the file Results

results got with:

```
CALC_FREQ: (LIST_FREQ: (6. , 10. , 15. , 19. , 24. , 29. , 29. , 31.))
CALC_MODE: (NMAX_MODE: 75)
```

6.4 Quantities tested and results

Frequency	Reference
Order of the eigen mode 1	5.53
Order of the eigen mode 2	10.90
Order of the eigen mode 3	15.93
Order of the eigen mode 4	20.45
Order of the eigen mode 5	24.34
Order of the eigen mode 6	27.49
Order of the eigen mode 7	29.84
Order of the eigen mode 8	31.29

Damping	Reference
Order of eigen mode 1	1.521e-2
Order of eigen mode 2	2.877e-2
Order of eigen mode 3	3.960e-2
Order of eigen mode 4	4.709e-2
Order of eigen mode 5	5.098e-2
Order of eigen mode 6	5.183e-2
Order of eigen mode 7	5.115e-2
Order of eigen mode 8	5.036e-2

Natural of the eigen mode	Not	Reference Eigen mode into 10^{-3} real Part Imaginary part
Rotation 1 (DRx) Φ_1	P1	– 2.442, 2.736
	P2	– 4.782, 4.968
	P3	– 6.54 , 6.6
	P4	– 7.5 , 7.5
	P5	– 7.5 , 7.44
	P6	– 6.66 , 6.54
	P7	– 4.944, 4.824
	P8	– 2.646, 2.55
Rotation 8 (DRx) Φ_8	P1	– 1.338, 0.684
	P2	– 2.226, 1.788
	P3	– 2.85 , 2.646
	P4	– 3.15 , 3.162
	P5	– 3.084, 3.258
	P6	– 2.664, 2.928
	P7	– 1.938, 2.214
	P8	– 0.996, 1.206

Eigen mode normalized with the unit modal mass: $\phi_i^t \boldsymbol{C}\, \phi_i + 2\lambda_i \phi_i^t \boldsymbol{M}\, \phi_i = 1$

λ is the eigenvalue associated with damping and the eigenfrequency.

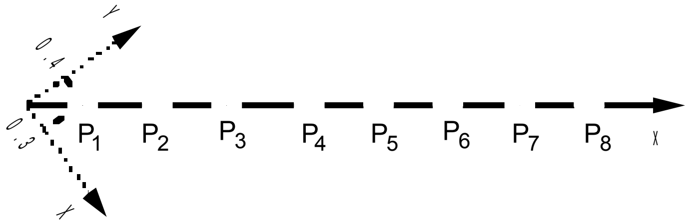
6.5 Contents of the file results

8 depreciation and eigenfrequencies, as well as the associated eigenvectors.

7 Modelization E

7.1 Characteristic of the modelization

Transposition of the test of reference to the case of the degrees of freedom of rotation (torsion spring + inertia) by means of the discrete element of stiffness in translation/rotation: `DIS_TR`



Characteristics of elements

`ORIENTATION:` angle:	in all the nodes		with a DISCRETE $\alpha = 53.130102°$
with nodal masses			
in all nodes	`M_TR_N`	in local coordinate system	$(I_{xx} = 10.)$
stiffness matrixes			
in all meshes	`K_TR_L`	in local coordinate system	$(KR_x = 1.10^5)$
with the nodes ends	`K_TR_N`	in local coordinate system	$(KR_x = 1.10^5)$
damping matrixes			
in all meshes	`A_TR_L`	in local coordinate system	$(CR_x = 50.)$
with initial node	`A_TR_N`	in local coordinate system	$(CR_x = 250.)$
with final node	`A_TR_N`	in local coordinate system	$(CR_x = 25.)$

limiting Conditions:

`DDL_IMPO:` `(TOUT: "YES" DX: 0. , DY: 0. , DZ: 0. , DRZ: 0. )`
`LIAISON_DDL:` (such as $3DRy = 4DRx$ in all the nodes)

Names of the nodes: $P_{1,} P_{2,} \dots, P_8$

7.2 Characteristics of the mesh

Many nodes:	8
Number of meshes and types:	7 `SEG2`

the nodes P_1 and P_8 are connected to a fixed fictitious node by nodal springs (`K_TR_N`, `A_TR_N`).

7.3 Quantities tested and results

Frequency	Reference
Order of the eigen mode 1	5.53
Order of the eigen mode 2	10.90
Order of the eigen mode 3	15.93
Order of the eigen mode 4	20.45
Order of the eigen mode 5	24.34
Order of the eigen mode 6	27.49
Order of the eigen mode 7	29.84
Order of the eigen mode 8	31.29

Damping	Reference
Order of eigen mode 1	1.521e-2
Order of eigen mode 2	2.877e-2
Order of eigen mode 3	3.960e-2
Order of eigen mode 4	4.709e-2
Order of eigen mode 5	5.098e-2
Order of eigen mode 6	5.183e-2
Order of eigen mode 7	5.115e-2
Order of eigen mode 8	5.036e-2

Natural of the eigen mode	Not	Reference Eigen mode into 10^{-3} real Part imaginary Part
Rotation 1 (DRx) Φ_1	P1	– 2.442, 2.736
	P2	– 4.782, 4.968
	P3	– 6.54 , 6.6
	P4	– 7.5 , 7.5
	P5	– 7.5 , 7.44
	P6	– 6.66 , 6.54
	P7	– 4.944, 4.824
	P8	– 2.646, 2.55
Rotation 8 (DRx) Φ_8	P1	– 1.338, 0.684
	P2	– 2.226, 1.788
	P3	– 2.85 , 2.646
	P4	– 3.15 , 3.162
	P5	– 3.084, 3.258
	P6	– 2.664, 2.928
	P7	– 1.938, 2.214
	P8	– 0.996, 1.206

Eigen mode normalized with the unit modal mass: $\phi_i^t \boldsymbol{C}\, \phi_i + 2\lambda_i \phi_i^t \boldsymbol{M}\, \phi_i = 1$

λ : is the eigenvalue associated with damping and the eigenfrequency.

7.4 Contents of the file results

8 depreciation and eigenfrequencies, as well as the associated eigenvectors.

8 Summary of the results

For all the options of modelization of the discrete elements of stiffness, mass and damping offered by AFFE_CARA_ELEM the solutions obtained are those of the reference solution (frequencies and eigen modes).