SDLD22 - Transient of a spring-mass system to 8 degrees of freedom with viscous damper

Abstract:

The mechanical structure considered made up of a linear one-way set of mass-springs with viscous dampers and is subjected to a transitory excitation of standard crenel.

Two modelizations are developed. The first retains only the degree of freedom in axial translation of the masses, the second considers the axial translation and rotation.

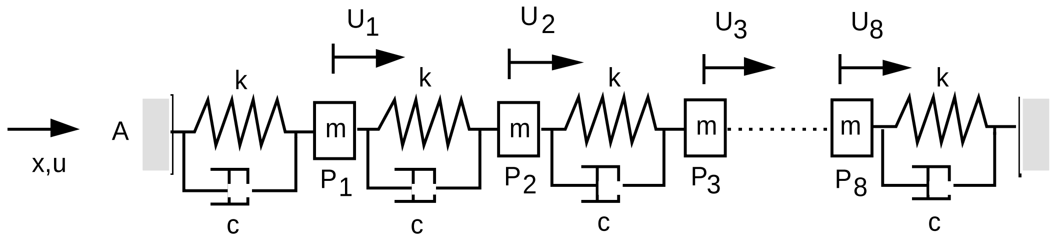
This problem makes it possible to test:

- the discrete elements (masses, springs, dampers) in translation-rotation,
- the definition of a force of specific excitation transitory,
- the computation of transient response by modal recombination as well as the recovery with initial conditions (modelization A),
- the computation of direct transient response with the diagram to time step adaptive (modelization B).

The got results (field of displacements, velocities) are in concord with the results of guide VPCS, taken for reference solution.

1 Problem of reference

1.1 Geometry



Point masses: $m_{P_1}=m_{P_2}=m_{P_3}=\ldots\ldots=m_{P_8}=m$

Stiffness of connection: $k_{AP_1}=k_{P_1P_2}=k_{P_2P_3}=\ldots\ldots=k_{P_8B}=k$

Viscous damping: $c_{AP_1}=c_{P_1P_2}=c_{P_2P_3}=\ldots\ldots=c_{P_8B}=c$

1.2 Material properties

Comes out from elastic translation linear $k=10^5 N/m$

Point mass $m=10 kg$

one-way Viscous damping $c=50 N/(m/s)$

1.3 Boundary conditions and loadings

Boundary conditions: points A and B clamped ($u=0$).

Loading: concentrated force at the point P_4 in the shape of crenel:

Not P_4	$F_{x_4}=F(t)$	$0\leq t\leq 1\text{s}$	$F(t)=1\text{N}$
		$t>1\text{s}$	$F(t)=0.$
Other points P_i	$F_{x_i}=0$		

1.4 Initial conditions

For $t=0$, in any point, $u=0$ and $\frac{du}{dt}=0$.

2 Reference solution

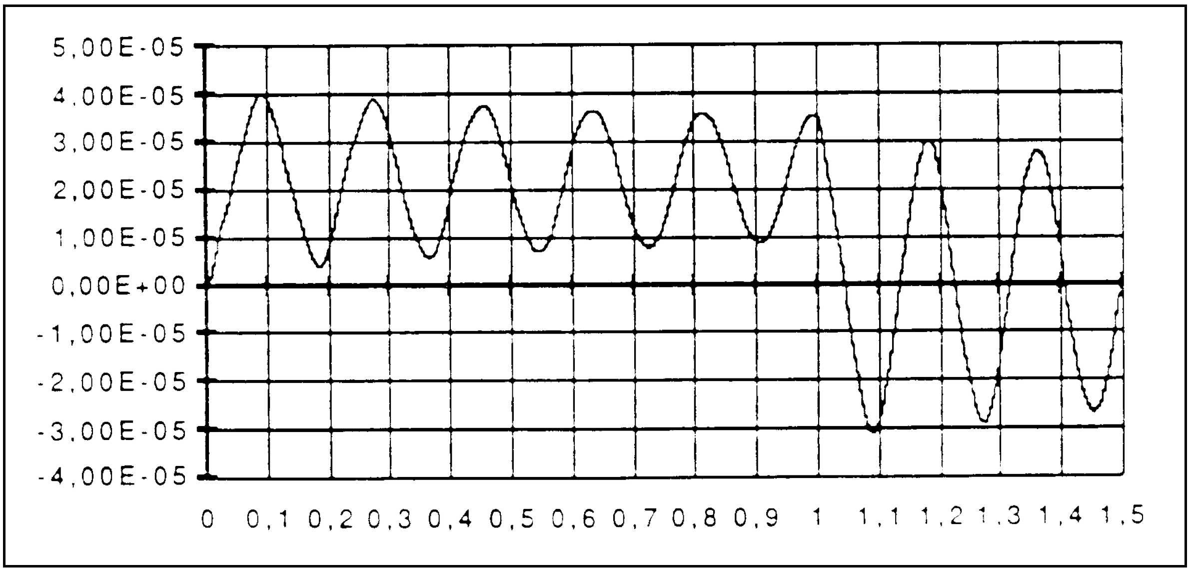
the reference solution is from guide VPCS.

2.1 Method of calculating used for the reference solution

numerical integration selected to obtain this solution rests on a diagram of integration by finite differences, of the standard method β - Newmark improved, with time step of $0.001\mathrm{s}$ [bib2].

$$\left[\frac{1}{\Delta t^2}M+\frac{1}{2\Delta t}C+\frac{1}{3}K\right]u_{n+2}=\frac{1}{3}(F_{n+2}+F_{n+1}+F_n)+\left[\frac{2}{\Delta t^2}M-\frac{1}{3}K\right]u_{n+1}+\left[\frac{1}{\Delta t^2}M+\frac{1}{2\Delta t}C-\frac{1}{3}K\right]u_n$$

The displacement of the point 4 according to time takes the following form:



Appear 2.1-a: Point 4: displacement according to time

2.2 Results of reference

Displacement according to x point P_4 .

2.3 Uncertainty on the solution

Accuracy of the diagram of Newmark.

2.4 Bibliographical references

[1] Card-indexes SDLD22/90 of commission VPCS.

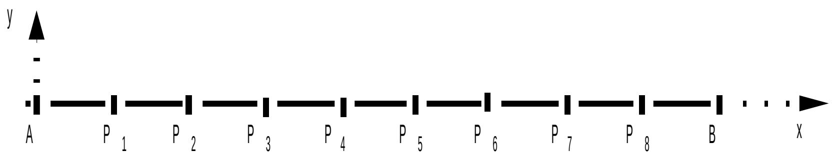
[2] NEWMARK N. MR.: “A method of computation for structural dynamics”, proceeding ASCE J. Eng. Mech. Div E-3, July 1959, pp 67-94.

3 Modelization A

3.1 Characteristic of the modelization

This modelization allows the validation of integration by modal recombination.

Discrete element of stiffness in translation



Characteristics of the DISCRET

`elements` with	nodal masses	`M_T_D_N`	`M_T_N`
	stiffness matrixes	`K_T_D_L`	`K_T_L`
	damping matrixes	`A_T_D_L`	`A_T_L`

Blocking of the degrees of freedom in Y and Z of all nodes

```
DDL_IMPO:    (TOUT: "YES"    DY: 0. , DZ: 0. )
```

Boundary conditions with the extreme nodes

```
(GROUP_NO: AB  DX: 0. )
```

Names of the nodes:

Not $A=N1$ $P_1=N2$
Not $B=N10$ $P_2=N3$
....
$P_8=N9$

modal Recombination with all the modes (that is to say 8),

diagram of EULER, resumption of the first computation with $t=0.455\,s$

time step used: $\Delta t=1.E-3\,s$.

3.2 Characteristics of the mesh

Many nodes: 10

Number of meshes and types: 9 `SEG2`

3.3 Quantities tested and results

Time	Reference
0.09	4.02 E-5
0.18	4.22 E-6
0.27	3.89 E-5
0.37	5.98 E-6
0.46	3.73 E-5
0.54	7.14 E-6
0.63	3.64 E-5
0.72	8.07 E-6
0.81	3.58 E-5
0.9	8.76 E-6
0.99	3.52 E-5
1.08	– 3.08 E-5
1.18	3.02 E-5
1.27	– 2.88 E-5
1.36	2.80 E-5
1.45	– 2.65 E-5

3.4 Remarks

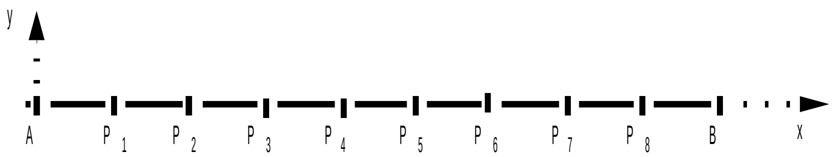
the relative minima ($t=0.18, 0.54, \ldots$) do not have a very good accuracy during the phase of excitation with a step $\Delta t=0.001$.

4 Modelization B

4.1 Characteristic of the modelization

This modelization allows, in addition to a new use of the modal recombination, the validation of direct integration with adaptive step.

Discrete element of stiffness in translation and rotation



Characteristics of the elements:

DISCRET:	with nodal masses	M_TR_D_N	M_TR_N
	and stiffness matrixes	K_TR_D_L	K_TR_L
	and damping matrixes	A_TR_D_L	A_TR_L

Boundary conditions and blocked directions:

in all nodes	DDL_IMPO:	(TOUT: "YES" DY: 0. , DZ: 0.)
		(TOUT: "YES" DRX: 0. DRY: 0 DRZ: 0)
with the nodes ends		(GROUP_NO: AB DX: 0.)

Diagrams of integration tested in this version:

- Integration by modal recombination with the diagram of Eulerian.
- Integration by integration direct with algorithm ADAPT_ORDRE2, time step maximum $10^{-3} s$.
- Integration by modal recombination with diagram RUNGE_KUTTA_32, a tolerance of relative error of $10^{-3} s$ and one time step maximum of $10^{-3} s$.
- Integration by modal recombination with diagram RUNGE_KUTTA_54, a tolerance of relative error of $10^{-3} s$ and one time step maximum of $10^{-3} s$.

4.2 Characteristics of the mesh

Many nodes: 10

Number of meshes and types: 9 SEG2

4.3 Quantities tested and Transient

results by modal recombination with algorithm EULER

Time	Reference
0.09	4.02 E-5
0.18	4.22 E-6
0.27	3.89 E-5
0.37	5.98 E-6
0.46	3.73 E-5
0.54	7.14 E-6
0.63	3.64 E-5
0.72	8.07 E-6
0.81	3.58 E-5
0.9	8.76 E-6
0.99	3.52 E-5
1.08	– 3.08 E-5
1.18	3.02 E-5
1.27	– 2.88 E-5
1.36	2.80 E-5
1.45	– 2.65 Transitory

E-5 by direct integration with algorithm `ADAPT_ORDRE2`

Time	Reference
0.09	4.02 E-5
0.18	4.22 E-6
0.27	3.89 E-5
0.37	5.98 E-6
0.46	3.73 E-5
0.54	7.14 E-6
0.63	3.64 E-5
0.72	8.07 E-6
0.81	3.58 E-5
0.9	8.76 E-6
0.99	3.52 E-5
1.08	– 3.08 E-5
1.18	3.02 E-5
1.27	– 2.88 E-5
1.36	2.80 E-5
1.45	– 2.65 Transitory

E-5 by modal recombination with algorithm `RUNGE_KUTTA_32`

Time	Reference
0.09	4.02 E-5
0.18	4.22 E-6
0.27	3.89 E-5
0.37	5.98 E-6
0.46	3.73 E-5
0.54	7.14 E-6
0.63	3.64 E-5
0.72	8.07 E-6
0.81	3.58 E-5
0.9	8.76 E-6
0.99	3.52 E-5
1.08	– 3.08 E-5
1.18	3.02 E-5
1.27	– 2.88 E-5
1.36	2.80 E-5
1.45	– 2.65 Transitory

E-5 by modal recombination with algorithm `RUNGE_KUTTA_54`

Time	Reference
0.09	4.02 E-5
0.18	4.22 E-6
0.27	3.89 E-5
0.37	5.98 E-6
0.46	3.73 E-5
0.54	7.14 E-6
0.63	3.64 E-5
0.72	8.07 E-6
0.81	3.58 E-5
0.9	8.76 E-6
0.99	3.52 E-5
1.08	– 3.08 E-5
1.18	3.02 E-5
1.27	– 2.88 E-5
1.36	2.80 E-5
1.45	– 2.65 E-5

4.4 Remarks

the modelizations A and B lead to the same results.

The relative minima ($t=0.18, 0.54, \ldots$) do not have a very good accuracy during the phase of excitation with a step $\Delta t=0.001$.

5 Summary of the results

This test is to be supplemented by means of:

- one time step $\Delta t=1.\mathrm{E}-4$,
- other diagrams of integration.