SDLD21 - Spring-mass system with 8 degrees of freedom with viscous damper

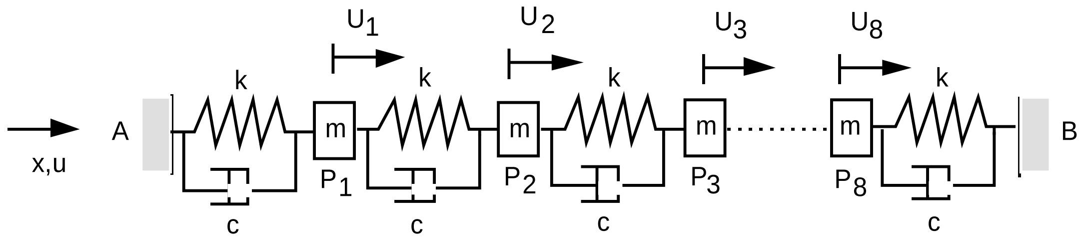
Abstract:

This one-way problem consists in carrying out a harmonic analysis of a mechanical structure made up of a set of mass-springs with viscous dampers and subjected to a sinewave excitation. This test of structural mechanics corresponds to a dynamic analysis of a discrete model having a linear behavior.

The got results (field of displacement, velocity and acceleration for various excitation frequencies) are in concord with the results of guide VPCS.

1 Problem of reference

1.1 Geometry



Point masses: $m_{P_1} = m_{P_2} = m_{P_3} = \ldots \ldots = m_{P_8} = m$

Stiffness of connection: $k_{AP1} = k_{P1P2} = k_{P2P3} = \ldots \ldots = k_{P8B} = k$

Viscous damping: $c_{AP1} = c_{P1P2} = c_{P2P3} = \ldots \ldots = c_{P8B} = c$

1.2 Material properties

Comes out from elastic translation linear $k = 10^5\, N/m$
Point mass $m = 10\, Kg$
one-way Viscous damping $c = 50\, N/(m/s)$

1.3 Boundary conditions and loadings

Boundary conditions:
Points A and B : embedded $(u=0)$.

Loading: Sinusoidal concentrated force of variable frequency at the point P_4

Not P_4 $F_{x_4} = F_0 \sin \Omega t$ $\Omega = 2\pi f$ $5\, Hz \le f \le 40\, Hz$

$F_0 = constante = 1\mathrm{N}$

Other points P_i $f_{x_i} = 0$

1.4 Initial conditions

Without object for the study of the permanent harmonic mode.

2 Reference solution

2.1 Method of calculating used for the reference solution

the system of equations differentials of the second order coupled is form:

$$M\ddot{u}+C\dot{u}+Ku=F$$

with $M=\begin{bmatrix} 10 & & & \\ & 10 & & \\ & & . & \\ & & & 10 \end{bmatrix}$ $C=50\begin{bmatrix} 2 & -1 & & & & \\ -1 & 2 & -1 & & & \\ & -1 & 2 & . & & \\ & & . & . & . & \\ & & & . & . & -1 \\ & & & & -1 & 2 \end{bmatrix}$

$$K=10^{+5}\begin{bmatrix} 2 & -1 & & & & \\ -1 & 2 & -1 & & & \\ & -1 & 2 & . & & \\ & & . & . & . & \\ & & & . & . & -1 \\ & & & & -1 & 2 \end{bmatrix}$$

the solution ω with a harmonic excitation $\boldsymbol{F}=\boldsymbol{F}_{\boldsymbol{0}}e^{j\omega t}(j^2=-1)$ is form $u=u_0 e^{j\omega t}$, which leads to: $(\boldsymbol{K}-\boldsymbol{M}\omega^2+j\omega\boldsymbol{C})u_0=\boldsymbol{F}_{\boldsymbol{0}}$

This system can be solved for all ω, either directly, or by means of the modal transformation starting from the real eigen modes obtained by the associated conservative system $(\boldsymbol{K}-\boldsymbol{M}\omega^2)\phi=0$.

He admits n clean solutions (8 in this case) ω_i^2 and associated vectors ϕ_i gathered in the spectral matrix $\Lambda=[\omega_i^2]$ and the modal matrix $\Phi=[\phi_i]$.

The modal transformation consists in writing: $\mathbf{u}_0=\mathbf{\Phi q}$ what leads to:

$$\left[\Lambda-\omega^2\boldsymbol{I}+j\omega\xi\right]q={}^t\Phi F_0$$

$\boldsymbol{I}$ is the identity,

here ξ is diagonal $\xi=\left[\xi_{ii}\right]$ because damping is proportional $(\boldsymbol{C}=\alpha\boldsymbol{K})$.

The response is written: $$u_0=\sum_{i=1}^{n}\frac{\phi_i^t\phi_i}{\omega_i^2-\omega^2+j\omega\xi_{ii}}F_0$$

One obtains the exact solution by taking all the eigen modes.

One from of deduced: $\dot{u}_0=j\omega u_0$ and $\ddot{u}_0=-\omega^2 u_0$

2.2 Results of reference

Displacement according to x point P_4 for certain frequencies.

2.3 Uncertainty on the semi-analytical

solution Solution.

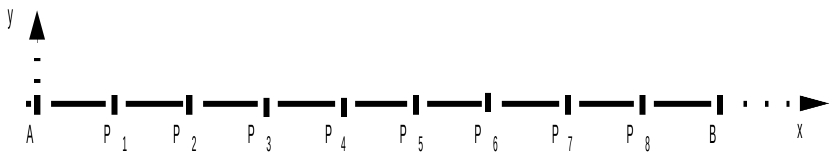
2.4 Bibliographical reference

[1] J. PIRANDA: Note of modal use of analysis software MODAN - Version 0.2 (1990). Laboratory of Mechanics Applied - University of Frank County - Besancon (France).

3 Modelization A

3.1 Characteristic of the modelization

Discrete element of stiffness in translation



Characteristics of the DISCRET
`elements`: with nodal masses `M_T_D_N`
and limiting `stiffness matrixes`
K_T_D_L and `damping matrixes`

A_T_D_L Conditions:
in all nodes `DDL_IMPO` `(TOUT=' OUI' DY= 0. , DZ= 0. )`
with the nodes ends `(GROUP_NO=AB DX= 0. )`

Names of the nodes:
$Point\ A = N1$ $P_1 = N2$
$Point\ B = N10$ $P_2 = N3$
....
$P_8 = N9$

3.2 Characteristic of the mesh

Many nodes: 10
Number of meshes and types: 9 SEG2

3.3 Quantities tested and results

Left real and imaginary the component DX of the displacement of the point P_4.

Frequency	Reference
5.00	1.0237 E-4 – 8.5187 E-6
5.50	4.5066 E-4 – 7.7914 E-4
6.00	– 9.4101 E-5 – 1.0585 E-5
10.00	8.4143 E-7 – 1.0335 E-6
15.00	1.2656 E-5 – 5.6652 E-6
20.00	2.9784 E-6 – 6.6970 E-6
25.00	– 1.2536 E-6 – 5.2703 E-6
30.00	– 2.0904 E-6 – 5.4821 E-6

35.00	– 4.5447 E-6 – 1.1190 E-6
39.50	– 2.6895 E-6 – 3.0505 E-7

Left real and imaginary the component DX velocity of the point P_4 .

Frequency	Reference
5.00	2.6762 E-4 3.2160 E-3
5.50	2.6925 E-2 1.5574 E-2
6.00	3.9904 E-4 – 3.5475 E-3
10.00	6.4937 E-5 5.2869 E-5
15.00	5.3393 E-4 1.1928 E-3
20.00	8.4157 E-4 3.7428 E-4
25.00	8.2786 E-4 – 1.9691 E-4
30.00	1.0333 E-3 – 3.9403 E-4
35.00	2.4608 E-4 – 9.9943 E-4
39.50	7.5709 E-5 – 6.6749 E-4

Left real and imaginary the component DX of the acceleration of the point P_4 .

Frequency	Reference
5.00	– 1.0103 E-1 8.4076 E-3
5.50	– 5.3819 E-1 9.3047 E-1
6.00	1.3374 E-1 1.5044 E-2
10.00	– 3.3218 E-3 4.0801 E-3
15.00	– 1.1242 E-1 5.0322 E-2
20.00	– 4.7033 E-2 1.0575 E-1
25.00	3.0931 E-2 1.3004 E-1
30.00	7.4273 E-2 1.9478 E-1
35.00	2.1979 E-1 5.4116 E-2
39.50	1.6566 E-1 1.8789 E-2

3.4 Remarks

Contained of the file results:

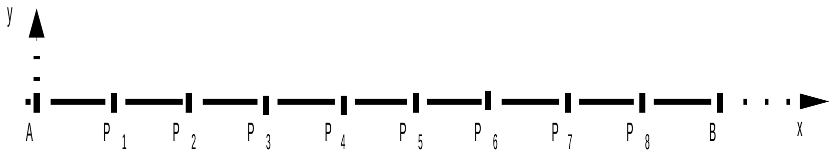
Values of the displacement of the component DX of the point P_4 for all the frequencies of 5 with $40\,Hz$ by step of 0.5 (Case initial test of VPCS).

Values the velocity and the acceleration of the component DX of the point P_4 for some frequencies of vibration.

4 Modelization B

4.1 Characteristic of the modelization

Discrete element of stiffness in translation



Characteristics of the DISCRET

`elements`:	with nodal masses	`M_T_D_N`
	and limiting	`stiffness matrixes`
	K_T_D_L and	`damping matrixes`

A_T_D_L Conditions:

in all nodes	`DDL_IMPO`	`(TOUT=' OUI'   DY= 0. , DZ= 0. )`
with the nodes ends		`(GROUP_NO=AB  DX= 0. )`

Names of the nodes:

$Point\ A = N1$ $P_1 = N2$

$Point\ B = N10$ $P_2 = N3$

....

$P_8 = N9$

4.2 Characteristic of the mesh

Many nodes: 10
Number of meshes and types: 9 SEG2

to test `REST_SPEC_TEMP`, one will compare several approaches (on modal base and physics) by testing the component DX at the point P_4 displacement, velocity and acceleration.
Two methods thus are tested:

- computation on modal base, then after `REST_SPEC_TEMP`, return on physical base with `RECU_FONCTION` on `the RESU_GENE`,
- computation on physical base directly.

A each time one tests option `TOUT_CHAM=' OUI'` or by separately calculating the three kinematical fields with `NOM_CHAM = "DEPL"`, `"QUICKLY"` or `"ACCE"`.

By these various paths one must find the same results because modal base is complete (it is not problematic because there are a small number of physical degrees of freedom).
Rather than to test only in particular times, the comparisons (on physical base) are done by analyzing the sum on all the urgent of the absolute values of the maximum (with each step) of the differences between the temporal solutions (obtained by opposite FFT with `REST_SPEC_TEMP`). This norm must each time be strictly null.

Being given the number of additional operations related to the tests of `REST_SPEC_TEMP`, the TEMPS CPU of the modelization B is clearly increased, compared to other modelizations which do not comprise these tests.

4.3 Quantities tested and results

Left real and imaginary the component DX of the displacement of the point P_4 .

Frequency	Reference
5.00	1.0237 E-4 – 8.5187 E-6
5.50	4.5066 E-4 – 7.7914 E-4
6.00	– 9.4101 E-5 – 1.0585 E-5
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30.00	– 2.0904 E-6 – 5.4821 E-6
35.00	– 4.5447 E-6 – 1.1190 E-6
39.50	– 2.6895 E-6 – 3.0505 E-7

For the tests on REST_SPEC_TEMP, all the norms on the maximum values of the differences between the solutions calculated for the fields of displacement, velocity and acceleration are strictly null: there are thus the same results on modal base or physics and whatever the mode of use of REST_SPEC_TEMP.

4.4 Remarks

Contained of the file results:

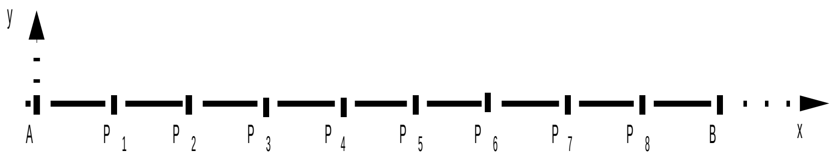
Values of the displacement of the component DX of the point P_4 for all the frequencies of 5 with $40\,Hz$ by step of 0.5 (Case initial test of VPCS).

Values the velocity and the acceleration of the component DX of the point P_4 for some frequencies of vibration.

5 Modelization C

5.1 Characteristic of the modelization

Discrete element of stiffness in translation



Characteristics of the DISCRET `elements`: with nodal masses and limiting K_T_D_L and `M_T_D_N stiffness matrixes damping matrixes`

A_T_D_L Conditions:
- in all nodes `DDL_IMPO (TOUT=' OUI' DY= 0. , DZ= 0. )`
- with the nodes ends `(GROUP_NO=AB DX= 0. )`

Names of the nodes:

$Point\ A = N1$ $P_1 = N2$

$Point\ B = N10$ $P_2 = N3$

....

$P_8 = N9$

5.2 Characteristic of the mesh

Many nodes: 10
Number of meshes and types: 9 SEG2

5.3 Quantities tested and Eigenfrequencies

results of structure for the sequence numbers from 1 to 5.

Sequence number	Reference
1	5.5271
2	10.8868
3	15.9155
4	20.4606
5	24.384

Damping reduce structure for the sequence numbers from 1 to 5.

Sequence number	Reference
1	0.00868241
2	0.017101
3	0.025
4	0.0321394
5	0.0383022

6 Modelization D

6.1 Characteristic of the modelization

This modelization is identical to the modelization A, the only difference is on the level of the solver employed: MUMPS here is used.

6.2 Characteristics of the mesh

Many nodes: 10
Number of meshes and types: 9 SEG2

6.3 Quantities tested and results

As for the modelization A, one tests on the parts real and imaginary component DX displacement, velocity and acceleration of the point P_4. The results are equal to those obtained with the modelization A, until less the eleventh decimal included.

7 Summary of the results

the got results are excellent, which is normal for a direct integration.