SDLD04 - Transient response of a system mass-springs subjected to an imposed acceleration

Summarized

This test consists in calculating the undamped transient response of a embed-free system mass-springs linear subjected to an imposed acceleration.

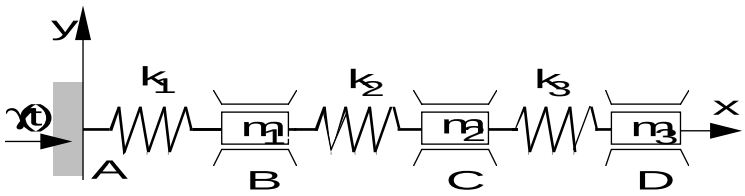
One tests the discrete element in traction and compression, the computation of the eigen modes, the static modes and the computation of the transient response of a system subjected to an imposed acceleration. One compares the direct computation of the response to his computation by modal recombination.

This case test is from guide VPCS. The reference solution is an analytical computation. The errors on the got results are normal taking into account time step selected for numerical integration.

1 Problem of reference

1.1 Geometry

One calculates the response of a linear system composed of three masses and three springs with an acceleration imposed on the level of its point of anchorage (A) :



1.2 Properties of the materials

- stiffness of connection: $k=k_1=k_2=k_3=1000\,N/m$;
- point masses: $m=m_1=m_2=m_3=1\,kg$.

1.3 Boundary conditions and loadings

Boundary conditions

only authorized displacements are the translations according to the axis x .
The point A is clamped: $dx=dy=dz=drx=dry=drz=0$.

Loading

the point of anchorage A is subjected to an acceleration, function increasing of time, according to the direction x : $\gamma(t)=2.10^5.t^2$ (t varies from 0 with $0{,}1\,s$).

1.4 Initial conditions

the system is initially at rest: at $t=0$, $dx(0)=0$ and $dx/dt(0)=0$ in any point.

2 Reference solution

2.1 Method of calculating used for the reference solution

One calculates the eigenfrequencies initially f_i and the associated ϕ_{Ni} eigenvectors standardized compared to the mass matrix. One calculates then the generalized response of the mono-excited system by solving analytically the integral of Duhamel [bib1]. Lastly, one restores on physical base displacement relating to the point D.

Computation of the eigenfrequencies

the mass matrixes and stiffness are the following ones:

$$M=\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix},\ K=k\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

The eigenfrequencies ω are solution of the equation $det[K-\lambda^2 M]=0$, that is to say $\lambda^3-5\lambda^2+6\lambda-1=0$ where $\lambda=\frac{\omega}{\omega_0}$ and $\omega=\sqrt{\frac{k}{m}}$.

Computation of the generalized response of the mono-excited system

$\gamma(t)=a.t^2$ with $a=2.10^5$.

In the absolute coordinate system, the fundamental equation of the dynamics of the undamped system mass-springs is written: $M\ddot{X}_a+K X_a=0$.

Absolute displacement X_a breaks up into a uniform displacement of training in translation X_e and a relative displacement X_r: $X_a=X_r+X_e$.
The equation of motion in the relative reference is written then: $M\ddot{X}_r+K X_r=-M\Psi\ddot{X}_s=Q$
with $\ddot{X}_s=\gamma(t)=a.t^2$ and $\Psi=\begin{bmatrix}1\\1\\1\end{bmatrix}$ thus $Q=a.t^2 m\begin{bmatrix}1\\1\\1\end{bmatrix}$.

The equation of motion projected on the basis of dynamic mode standardized compared to the mass matrix is written:

$$\ddot{\alpha}_i(t)+\omega_i^2\alpha_i(t)=\frac{\Phi_i^T.M.\Psi}{\Phi_i^T.M.\Phi_i}\gamma(t)=-p_i(t)\gamma(t).$$

The response of this linear system, to one time t is given by the integral of Duhamel:

$$\ddot{\alpha}_i(t)=\frac{1}{\omega_i}\int_0^t -p_i(t)\gamma(t).\sin\omega_i(t-\tau)\,d\tau=-\frac{p_i(t)}{\omega_i}\int_0^t a.t^2\sin\omega_i(t-\tau)\,d\tau.$$

However, according to [bib1] $\int_0^t a.t^2 \sin \omega_i (t-\tau)\, d\tau = \frac{\alpha}{\omega_i}\left[t^2 + \frac{2}{\omega_i}(\cos \omega_i t - 1)\right]$.

Thus $X_r = \Phi_i \cdot \alpha_i = -\sum_i \frac{a.p_i(t).\Phi_i}{\omega_i^2}\left[t^2 + \frac{2}{\omega_i}(\cos \omega_i t - 1)\right]$.

2.2 Results of reference

One takes for results of reference the three eigenfrequencies of the system and displacement relating x_r to the point D, for various times understood enters 0 and $0,1\ s$.

2.3 Uncertainty on the solution

No if one calculates the integral of Duhamel analytically [bib1], [bib2].

2.4 Bibliographical references

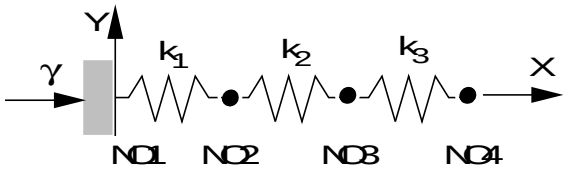
1) J.S. PRZEMIENIECKI: Theory of matrix structural analysis. New York, Mac Graw-Hill, 1968, p. 351-357

2) S.P. TIMOSHENKO, D.H. YOUNG and W. WEAVER: Vibrations problems in engineering 4th edition, New York, Wiley & Sounds, 1974, p. 284-321

3 Modelization A

3.1 Characteristic of the modelization

springs and point masses are modelled by discrete elements with 3 degrees of freedom DIS_T :



The node $NO1$ is embedded and subjected to an imposed acceleration $\gamma(t)$. One calculates the relative displacement of the node $NO4$.

Computations by modal synthesis
One considers the complete base of the eigen modes. Temporal integration is carried out with the algorithms of Newmark, Eulerian and Devogelaere with time step of $0,001\,s$. Computations are filed all time step.
One considers a null ξ_i reduced damping for all the calculated modes.

The loading is taken into account in the form of vector project on modal base EXCIT: (VECT_GENE) or in the form of modal component EXCIT: (NUME_MODE) or both at the same time.

Direct computations
temporal integration is carried out either with the algorithm of Newmark or with the explicit algorithm of the central differences with time step of $0,001\,s$. Computations are filed all the ten time step.

Note:

> *As the diagram of the central differences can be used only with one diagonal mass matrix, one calculates the elementary matrixes with option MASS_MECA_DIAG in operator CALC_MATR_ELEM .*

Taken into account of an initial state
In the two types of computation, one checks that the relative displacement obtained of a computation carried out in once is identical to that obtained in several times, i.e. while regarding as initial state, result the last time step calculated:

ETAT_INIT =_F (RESULTAT…) for a computation by modal synthesis;
ETAT_INIT =_F (DEPL…
QUICKLY.) for a direct computation.

Taking into account of the modes neglected by static correction:
One considers a modal base made up of the first two eigen modes and one has it complete by a mode corresponding to the static response of the studied system with a unit loading of type forces imposed in the direction $-x$ (key keys MODE_CORR and CORR_STAT in L `operator DYNA_VIBRA).

3.2 Characteristics of the mesh

Many nodes: 4
Number of meshes and types: 3 DIS_T

3.3 Functionalities tested

One test in particular the taking into account of an initial state and static corrections.

3.4 Quantities tested and Eigenfrequencies

results (in Hz) of the system:

Number of the Analytical	mode
1	2,239
2	6,275
3	9,069

Values of the relative displacement of the node $NO4$ for various times:

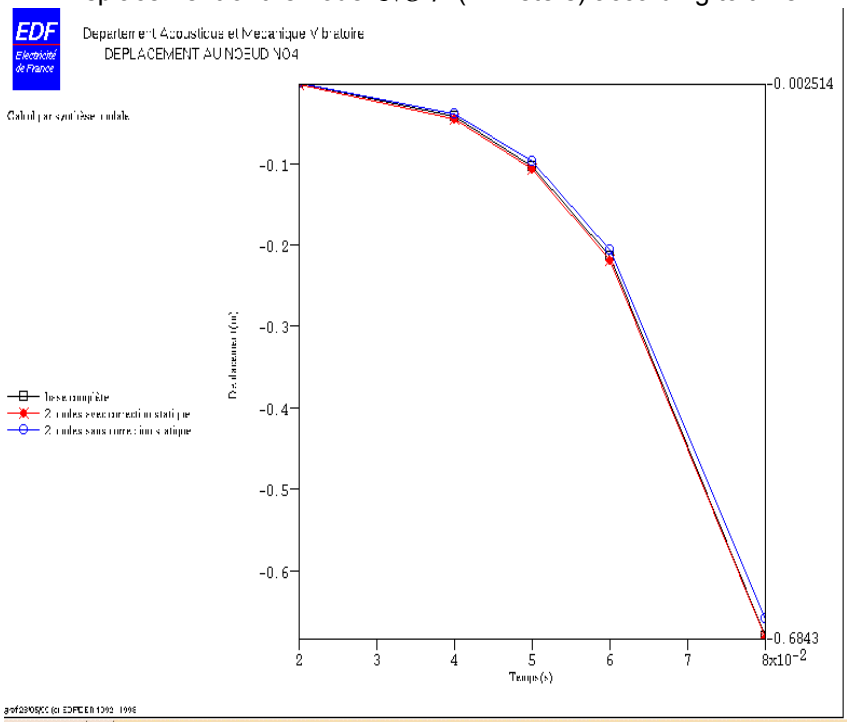
Transient computation by modal synthesis
One tests the taking into account of a loading in the form of vector project on modal base, modal component, vector project and of modal component simultaneously as well as the taking into account of the neglected modes.

Time (s)	Reference	*Code_Aster* *Loading of type vector generalized* *Algorithm of Newmark*	relative Error %	*Code_Aster* *Loading of modal the component type* *Algorithm of Eulerian*	relative Error %
0,02	– 2,700E-03	– 2,680E-03	– 0,741	– 2,660E-03	– 1,481
0,04	– 4,260E-02	– 4,272E-02	0,279	– 4,264E-02	0,091
0,05	– 1,041E-01	– 1,042E-01	0,134	– 1,041E-01	0,015
0,06	– 2,158E-01	– 2,161E-01	0,121	– 2,159E-01	0,038
0,08	– 6,813E-01	– 6,819E-01	0,094	– 6,816E-01	0,049
0,10	– 1,658E+00	– 1,659E+00	0,082	– 1,659E+00	0,055

Type of loading	Time (s)	Reference	*Code_Aster*	relative error %
Vector generalized and modal component simultanéement (Eulerians)	0,02	– 5,400E-03	– 5,320E-03	– 1,482
	0,04	– 8,520E-02	– 8,528E-02	0,091
	0,05	– 2,082E-01	– 2,082E-01	0,015
	0,06	– 4,316E-01	– 4,318E-01	0,038
	0,08	– 1,363E+00	– 1,363E+00	0,049
	0,10	– 3,316E+00	– 3,318E+00	0,055
Vector generalized Devogelaere (more correction static)	0,02	– 4,000E-03	– 3,985E-03	– 0,373
	0,04	– 4,640E-02	– 4,640E-02	0,01
	0,05	– 1,085E-01	– 1,086E-01	0,084
	0,06	– 2,203E-01	– 2,204E-01	0,039
	0,08	– 6,842E-01	– 6,843E-01	0,021
	0,10	– 1,659E+00	– 1,659E+00	the 0,026

results with incomplete modal base without static correction are not tested. One illustrates Ci - below the interest of static correction:

Displacement of the node $NO4$ (in meters) according to time



Bases supplements — incomplete Base without static correction — Bases incomplete with static correction

direct Transient computation

One compares displacements calculated with the node $N04$ according to various diagrams of integration:

Time (s)	Reference	*Code_Aster* *Diagram of Newmark*	relative Error %	*Code_Aster* *Diagram of the central differences*	relative Error %
0,02	– 2,700E-03	– 2,680E-03	– 0,741	– 2,660E-03	– 1,482
0,04	– 4,260E-02	– 4,272E-02	0,279	– 4,264E-02	0,091
0,05	– 1,041E-01	– 1,042E-01	0,134	– 1,041E-01	0,015
0,06	– 2,158E-01	– 2,161E-01	0,121	– 2,159E-01	0,038
0,08	– 6,813E-01	– 6,819E-01	0,094	– 6,745E-01	– 1,004
0,10	– 1,658E+00	– 1,659E+00	0,082	– 1,645E+00	– 0,803

Taking into account of an initial state:
As expected, the relative displacements calculated in once are strictly identical to those obtained by regarding as initial state result the last time step calculated.

One also carries out tests of non regression on the energy balance.

Energy	Time (s)	*Code_Aster* *Diagram of Newmark*	*Code_Aster* *Diagram of central differences*
TRAV_EXT	0.06	2,9 7989 E+02	2,9 7.838 E+02
ENER_TOT	0.06	1,8 8902 E+01	1,8 8.688 E+01
ENER_CIN	0.06	2,79099E+02	2,79 132 E+02
TRAV_EXT	0,1 1	1,03435E+04	9,98152 E+0 3
ENER_TOT	0,1 1	1,71815E+03	1,67741 E+03

Energy	Time (s)	Code_Aster Diagram of Newmark	Code_Aster Diagram of central differences
ENER_CIN	0,1 1	8,62533E+03	8,30528 E+03

4 Summary of the results

the reference solution is an analytical computation. The errors on the got results are normal taking into account time step selected for numerical integration.