ZZZZ328 – Validation of CALC_CHAMP / Summarized

ENEL_ELEM:

The purpose of this test is to validate the elementary computation of elastic strain energy. For each modelization, an unit element is considered of which all the degrees of freedom are imposed, which make it possible to create strain fields and of stress homogeneous on the element. The results are compared with the analytical solution.

1 Problem of reference

1.1 Geometry

One considers an unit element of dimension 2 (quadrangle) or 3 (cubic) according to the modelization. The thickness of the element, when it is required, is worth 1m .

1.2 Properties of the material

the material is elastic isotropic, whose properties are:

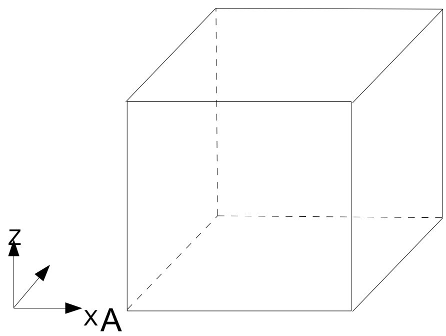
- $E=1000\,MPa$
- $\nu=0.2$

For the modelization E, the Poisson's ratio is null.

1.3 Boundary conditions and loadings

1.3.1 Modelization A

the cube of unit size is illustrated on Figure 1.3.1-1 . The displacement of the various nodes is imposed in accordance with Table 1.3.1-1 .



A
B
C
D
E
F
G
H
Y

Figure 1.3.1-1: Geometry of the modelization A

node	DX	DY	DZ
A	0,0.0, 0.0,0		
B	1,0E-3	0,0	-6,0E-4
C	2,0E-3	-2,0E-3	-3,0E-3
D	1,0E-3	-2,0E-3	-2,4E-3
E		0,0.2,2 E-3	3,0E-3
F	1,0E-3	2,2E-3	2,4E-3
G	2,0E-3	2,0E-4	
H	0,0.1, 0E-3	2,0E-4	6,0E-4

Table 1.3.1-1: Displacements of the nodes

1.3.2 Modelizations B, C E T D

the geometry of the quadrangle of unit size corresponds to the face $ABCD$ (Figure 1.3.1-1). The displacement of the four nodes A B , C and D are imposed in accordance with Table 1.3.1-1 for the components X and Y , the component Z being null. When degrees of freedom in rotation exist, those are blocked at the point A .

1.3.3 Modelization E

the geometry of the quadrangle of unit size corresponds to the face $ABCD$ (Figure 1.3.1-1). It is about an axisymmetric modelization, whose axis is distant of $1\mathrm{m}$ edge AD . Displacements of the nodes are null, except the component Y of the points C and D which is worth $1{,}0\mathrm{E}\text{-}4\,m$.

1.4 Initial conditions

Nothing

2 Reference solution

2.1 Method of calculating

the strain field is obtained starting from displacements and the stress field thanks to the Hooke's law. Elastic strain energy E_{elas} is calculated then by the following formula:

$$E_{elas}=\int_V \frac{1}{2}\sigma:\epsilon\, dV$$

2.1-1

2.1.1 Modelization A

the strain fields and of stress homogeneous on the element are indicated in Table 2.1.1-1 .

Component	X	Y	Z	XY	YZ	ZX
Strain	0,001	-0,002	0,003	0,0005	-0,0001	-0,0003
Stress (Pa)	1,3888889E6	-1,1111111E6	3,0555556E6	4,1666667E5	-8,3333333E4	-2,5E5

Table 2.1.1-1: Strain field and of stress

2.1.2 Modelization b: plane strains

the homogeneous strain fields and of stress on the element are indicated in Table 2.1.2-1 .

Component	X	Y	Z	XY	YZ	ZX
Strain	0,001	-0,002	0,0	0,0005	0,0.0,0	
Stress (Pa)	5,5555556 E 5	-1,9444444 E6	-2,7777778 E 5	4,1666667E5	-8,3333333E4	-2,5E5

Table 2.1.2-1: Strain field and of stress

2.1.3 Modelization C (plane stresses) and D (shell elements and of plate)

the homogeneous strain fields and of stress on the element are indicated in Table 2.1.3-1 . For the shell elements and plate, it is the membrane strain, the curvature being null.

Component	X	Y	Z	XY	YZ	ZX
Strain	0,001	-0,002	-0,00025	0,0005	0,0.0,0	
Stress (Pa)	6,25 E 5	-1,875 E6	0,0	4,1666667E5	0,0.0,0	

Table 2.1.3-1: Strain field and of stress

2.1.4 Modelization E : axisymmetric.

The strain fields and of stress homogeneous on the element are indicated in Table 2.1.4-1 .

Component	X	Y	Z	XY
Strain	0,0	0,0001	0,0.0,0	
Stress (Pa)		0,0.1,0E5	0,0.0,0	

Table 2.1.4-1 : Strain field and of stress

elastic strain energy is calculated according to 2.1-1 by expressing ground volume dV in the cylindrical coordinate system for a slice of infinitesimal thickness $d\theta$:

$$E_{elas}=\int_V \frac{1}{2}\sigma:\epsilon\, dV=\frac{1}{2}\sigma:\epsilon\int_{R_1}^{R_2} r dr\int_{H_1}^{H_2} dz$$

the limits of the integral are: $R_1=1\mathrm{m}$, $R_2=2\mathrm{m}$, $H_1=0\mathrm{m}$ and $H_2=1\mathrm{m}$.

2.2 Quantities and results of reference

calculated elastic strain energy in an analytical way for each modelization is indicated in Table 2.2-1.

Modelization	A	B	C	D	E
Elastic strain energy	6680,555556 J	2430,555556 J	2395,833333 J	2395,833333 J	$7{,}5\, J.rad^{1}$

Table 2.2-1: Elastic strain energy

2.3 Uncertainties on the solution

None. It is about an analytical solution.

2.4 Bibliographical references

3 Modelization A

3.1 Characteristic of the modelization

One uses successively the modelizations 3D , 3D_SI and 3D_GRAD_EPSI .

3.2 Characteristics of the mesh

The mesh contains 1 element of type HEXA8 for the modelization 3D and 1 element of type HEXA20 for the modelization S 3D_SI and 3D_GRAD_EPSI .

3.3 Quantities tested and results

One tests the elastic strain energy stored by the element.

Standard	identification of reference	Value of reference	Tolerance
3D - ENEL_ELEM	"ANALYTIQUE"	6680,555556	0.1%
3D_SI - ENEL_ELEM	"ANALYTIQUE"	6680,555556	0.1%
3D_GRAD_EPSI - ENEL_ELEM	"ANALYTIQUE"	6680,555556	0.1%

4 Modelization B

4.1 Characteristic of the modelization

One use successively modelizations D_PLAN , D_PLAN _S I , D_PLAN_GRAD_EPSI , D_PLAN_GRAD_SIGM and PLAN_ELDI .

4.2 Characteristics of the mesh

The mesh contains 1 element of type QUAD4 for modelizations D_PLAN, D_PLAN_SI and PLAN_ELDI and 1 element of type QUAD8 for modelizations D_PLAN_GRAD_EPSI and D_PLAN_GRAD_SIGM.

4.3 Quantities tested and results

One tests the elastic strain energy stored by the element.

Standard	identification of reference	Value of reference	Tolerance
D_PLAN – ENEL_ELEM	"ANALYTIQUE"	2430,555556	0.1%
D_PLAN_SI - ENEL_ELEM	"ANALYTIQUE"	2430,555556	0.1%
D_PLAN_GRAD_EPSI - ENEL_ELEM	"ANALYTIQUE"	2430,555556	0.1%
D_PLAN_GRAD_SIGM - ENEL_ELEM	"ANALYTIQUE"	2430,555556	0.1%
PLAN_ELDI - ENEL_ELEM	"ANALYTIQUE"	2430,555556	0.1%

5 Modelization C

5.1 Characteristic of the modelization

One use successively modelizations C_PLAN , C_PLAN _S I and C_PLAN_GRAD_EPSI .

5.2 Characteristics of the mesh

The mesh contains 1 element of type QUAD4 for modelizations C_PLAN and C_PLAN_SI and 1 element of type QUAD8 for modelization C_PLAN_GRAD_EPSI.

5.3 Quantities tested and results

One tests the elastic strain energy stored by the element.

Standard	identification of reference	Value of reference	Tolerance
C_PLAN - ENEL_ELEM	"ANALYTIQUE"	2395,833333	0.1%
C_PLAN_SI - ENEL_ELEM	"ANALYTIQUE"	2395,833333	0.1%
C_PLAN_GRAD_EPSI - ENEL_ELEM	"ANALYTIQUE"	2395,833333	0.1%

6 Modelization D

6.1 Characteristic of the modelization

One use successively modelizations DKT , DKTG and Q4GG .

6.2 Characteristics of the mesh

The mesh contains 1 element of type QUAD4.

6.3 Quantities tested and results

One tests the elastic strain energy stored by the element.

Standard	identification of reference	Value of reference	Tolerance
DKT - ENEL_ELEM	"ANALYTIQUE"	2395,833333	0.1%
DKTG - ENEL_ELEM	"ANALYTIQUE"	2395,833333	0.1%
Q4GG - ENEL_ELEM	"ANALYTIQUE"	2395,833333	0.1%

7 Modelization E

7.1 Characteristic of the modelization

One use successively modelizations AXIS , AXIS_SI and AXIS_ELDI .

7.2 Characteristics of the mesh

The mesh contains 1 element of type QUAD4 for modelizations AXIS and AXIS_ELDI and 1 element of type QUAD8 for modelization AXIS_SI.

7.3 Quantities tested and results

One tests the elastic strain energy stored by the element.

Standard	identification of reference	Value of reference	Tolerance
AXIS - ENEL_ELEM	“ANALYTIQUE”	7,5	0.1%
AXIS_SI - ENEL_ELEM	“ANALYTIQUE”	7,5	0.1%
AXIS_ELDI - ENEL_ELEM	“ANALYTIQUE”	7,5	0.1%

8 Summary of the results

all the modelizations provide results in conformity with the analytical solution. The elementary computation option of elastic strain energy ENEL_ELEM is thus validated.