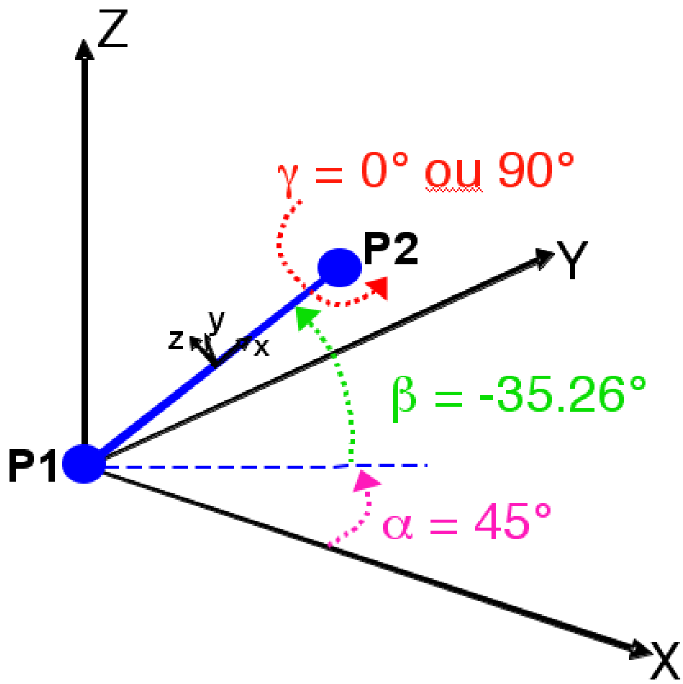
ZZZZ293 – Validation of the position of the subpoints of the multifibre beams

Summarized:
This test validates the computation of the position of the subpoints of integration in the total reference for modelization POU_D_EM and POU_D_TGM. An elementary mechanical computation is carried out in order to allow the creation of an array with CREA_TABLE from result. Only the coordinates of some subpoints are tested in the array.

1 Problem of reference

1.1 Geometry



Appears 1.1-a : directional sense of the element.

The beam is directed in space as indicated on the Figure 1.1-a.
Total coordinates of the points $P1$ and $P2$:

$X_{P1}=0.0;Y_{P1}=0.0;Z_{P1}=0.0$
$X_{P2}=2.0;Y_{P2}=2.0;Z_{P2}=2.0$

Length: $L=2\cdot\sqrt{3}m$

1.2 Properties of the materials

Concrete:

Young's modulus $E=3.7272^{10}Pa$
Poisson's ratio $\nu=0.0$

1.3 Boundary conditions and loadings

On the point $P1$ one blocks displacements according to X,Y,Z and rotation around the axes X,Y,Z :

$D_X^{P1}=0.0;D_Y^{P1}=0.0;D_Z^{P1}=0.0;DR_X^{P1}=0.0;DR_Y^{P1}=0.0;DR_Z^{P1}=0.0$

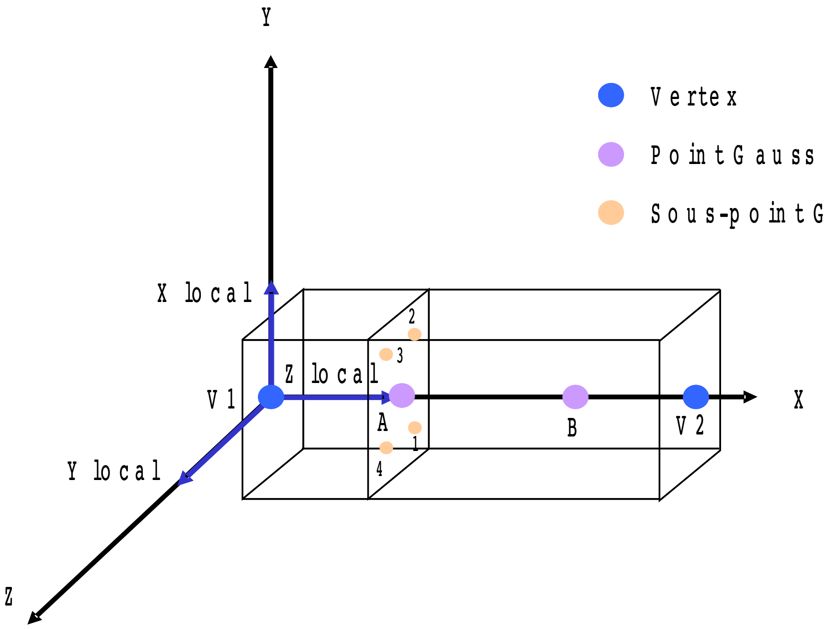
To the point $P2$ one applies a loading according to X,Y,Z:

$F_X=100.0N;F_Y=100.0N;F_Z=-100.0N$

2 Reference solution

2.1 Method of calculating

One calculates the position of the nodes, points of integration and subpoints of integration from their cordonnées in the local axes of the beam and the transition matrixes between the local axes and the total axes.
By defaults, the local axes and the total axes coincide (Figure 2.1-a).



Appear position by default

One applies two rotations (see Figure 1.1-a) to direct the axis of the beam, and the third rotation to position the cross-section:

$\alpha=45°$ around Z

$\beta=-35,26°$ around the new axis $Y1$

- $\gamma=0°$ or $90°$ around the new axis $X2$

Note:

one uses conventions of the nautical angles of Code_Aster (see key word `ORIENTATION` *of* `AFFE_CARA_ELEM`*)*

rotation around the axis Z (α) is made from the following matrix:

$$Tz(\alpha)=\begin{bmatrix}\cos(\alpha) & -\sin(\alpha) & 0\\ \sin(\alpha) & \cos(\alpha) & 0\\ 0 & 0 & 1\end{bmatrix}$$

Rotation around the new axis $Y1$ (β) is made from the following matrix:

$$Ty1(\beta)=\begin{bmatrix}\cos(\beta) & 0 & \sin(\beta)\\ 0 & 1 & 0\\ -\sin(\beta) & 0 & \cos(\beta)\end{bmatrix}$$

Rotation around the new axis $X2$ (γ) is made from the following matrix:

$$Tx2(\gamma)=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos(\gamma) & -\sin(\gamma)\\ 0 & \sin(\gamma) & \cos(\gamma)\end{bmatrix}$$

Therefore, for any point of coordinates (X,Y,Z) before rotations, one can calculate his coordinates (X',Y',Z') after rotations with the following transformation:

$$\begin{bmatrix}X'\\ Y'\\ Z'\end{bmatrix}=[Tx2(\gamma)][Tz(\alpha)][Ty(\beta)]\begin{bmatrix}X\\ Y\\ Z\end{bmatrix}$$

2.2 Quantities and results of reference

One calculates the position of some subpoints of integration in the total reference knowing their position in the local axes.

With the angles chosen, the numerical application gives:

$$Ty1(\beta)=\begin{bmatrix}0.8165 & 0 & -0.5774\\ 0 & 1 & 0\\ 0.5774 & 0 & 0.8165\end{bmatrix} \quad Tz(\alpha)=\begin{bmatrix}0.7071 & -0.7071 & 0\\ 0.7071 & 0.7071 & 0\\ 0 & 0 & 1\end{bmatrix}$$

and

$$Tx2(\gamma)=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \text{ or } Tx2(\gamma)=\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{bmatrix}$$

For an element length $L=2\cdot\sqrt{3}\,m$, the distance from the second Gauss point compared to the first node is:

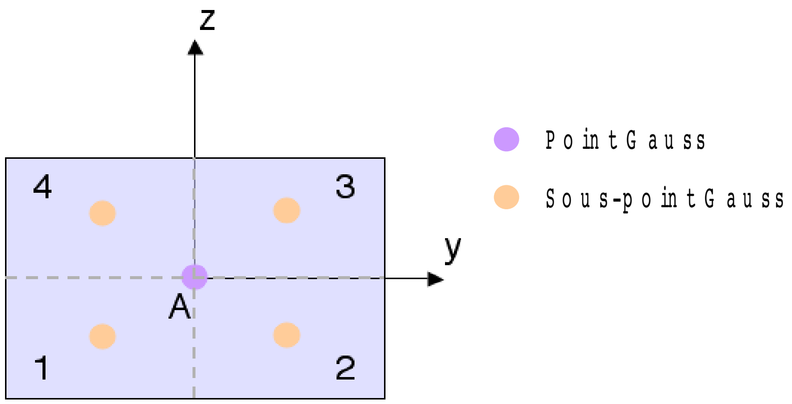
For the elements `POU_D_EM` (modelizations A and B) which have two Gauss points:

$$\left(\frac{1}{2}+\frac{1}{2\sqrt{3}}\right)L=1+\sqrt{3}=2.7320508075688772\,m$$

For the elements `POU_D_TGM` (modelizations C and D) which have three Gauss points:

$$\frac{L}{2}=\sqrt{3}=1.7320508075688772\,m$$

The cross-section of the beam $(0,2\,m\times0,1\,m)$ is discretized in 4 quadrilaterals (Figure 2.2-a).



Appear : position of the subpoint in the section

the position of the subpoints chosen in the initial reference for the modelizations A and B (POU_D_EM) is thus :

Not	Item	x	y	z
2	1	2.732050807568877	-0.05	-0.025
2	2	2.732050807568877	0.05	-0.025
2	3	2.732050807568877	0.05	0.025
2	4	2.732050807568877	-0.05	0.025

And the position of the subpoints chosen in the initial reference for the modelizations C and D (POU_D_TGM) is:

Not	Item	x	y	z
2	1	1.732050807568877	-0.05	-0.025
2	2	1.732050807568877	0.05	-0.025
2	3	1.732050807568877	0.05	0.025
2	4	1.732050807568877	-0.05	0.025

2.3 Uncertainties on the solution

No, exact solution.

3 Modelization A

3.1 Characteristic of the mesh

The mesh of the beam is composed of an element SEG2.
The mesh of the section is composed of 4 quadrilateral elements.
The nodes group ENC is composed of the node $P1$.
The nodes group CHA is composed of the node $P2$.

3.2 Characteristics of the modelization

It is about a modelization POU_D_EM (elements with 2 Gauss points). The angle of gimlet is equal to $0°$.

Modelization:

```
MOPOU=AFFE_MODELE (MAILLAGE=MAPOU,
    AFFE=_F (TOUT=' OUI', PHENOMENE=' MECANIQUE',
        MODELISATION=' POU_D_EM',),
)
```

Boundary conditions:

```
BLOCAGE=AFFE_CHAR_MECA (MODELE=MO,
    DDL_IMPO=_F (GROUP_NO=' ENC',
        DX=0.0, DY=0.0, DZ=0.0, DRX=0.0, DRY=0.0, DRZ=0.0,),
)
```

mechanical Loading:

```
CHARGE=AFFE_CHAR_MECA (MODELE=MO,
    FORCE_NODALE=_F (GROUP_NO=' CHA', FX = 100, FY = 100, FZ = 100.),
)
```

Assignment of the characteristics of the elements:

```
POUCA_0=AFFE_CARA_ELEM (MODELE=MOPOU, GEOM_FIBRE=GF,
    POUTRE=_F (GROUP_MA= ("POUTRE"),
        SECTION=' RECTANGLE', CARA= ("HY", "HZ"), VALE= (0.2, 0.1),
        PREC_AIRE=5., PREC_INERTIE=10.,),
    ORIENTATION=_F (GROUP_MA= ("POUTRE"), CARA=' ANGL_VRIL', VALE=0.0,),
    MULTIFIBRE=_F (GROUP_MA= ("POUTRE"), GROUP_FIBRE= ("SBET",),),
)
```

3.3 Values tested and results

Coordinated mesh SG01	Not of integration	Subpoint	Reference
COOR_X	2	1	0.377088184
COOR_X	2	2	0.447798863
COOR_X	2	3	0.468211277
COOR_X	2	4	0.397500599
COOR_Y	2	1	0.447798863
COOR_Y	2	2	0.377088184
COOR_Y	2	3	0.397500599
COOR_Y	2	4	0.468211277
COOR_Z	2	1	0.443062145
COOR_Z	2	2	0.443062145
COOR_Z	2	3	0.402237316
COOR_Z	2	4	the 0.402237316

All tolerance are those by default: $1.0\text{E}-3$

4 Modelization B

4.1 Characteristic of the mesh

The mesh of the beam is composed of an element SEG2.
The mesh of the section is composed of 4 quadrilateral elements.
The nodes group ENC is composed of the node $P1$.
The nodes group CHA is composed of the node $P2$.

4.2 Characteristics of the modelization

It is about a modelization `POU_D_EM` (elements with 2 Gauss points). The angle of gimlet is of $90°$.

Modelization:
```
MOPOU=AFFE_MODELE (MAILLAGE=MAPOU,
   AFFE=_F (TOUT=' OUI', PHENOMENE=' MECANIQUE',
      MODELISATION=' POU_D_EM',),)
```

Boundary conditions:
```
BLOCAGE=AFFE_CHAR_MECA (MODELE=MO,
   DDL_IMPO=_F (GROUP_NO=' ENC',
      DX=0.0, DY=0.0, DZ=0.0, DRX=0.0, DRY=0.0, DRZ=0.0,),
)
```

mechanical Loading:
```
CHARGE=AFFE_CHAR_MECA (MODELE=MO,
      FORCE_NODALE=_F (GROUP_NO=' CHA', FX = 100, FY = 100, FZ = 100.),
)
```

Assignment of the characteristics of the elements:
```
POUCA_0=AFFE_CARA_ELEM (MODELE=MOPOU, GEOM_FIBRE=GF,
   POUTRE=_F (GROUP_MA= ("POUTRE"), SECTION=' RECTANGLE',
      CARA= ("HY", "HZ"), VALE= (0.2, 0.1),
      PREC_AIRE=5., PREC_INERTIE=10.,),
   ORIENTATION=_F (GROUP_MA= ("POUTRE"), CARA=' ANGL_VRIL', VALE=90.0,),
   MULTIFIBRE=_F (GROUP_MA= ("POUTRE"), GROUP_FIBRE= ("SBET",),),
)
```

4.3 Values tested and results

Coordinated mesh SG01	Not of integration	subpoint	Reference
COOR_X	2	1	0.419914986
COOR_X	2	2	0.460739815
COOR_X	2	3	0.425384476
COOR_X	2	4	0.384559647
COOR_Y	2	1	0.384559647
COOR_Y	2	2	0.425384476
COOR_Y	2	3	0.460739815
COOR_Y	2	4	0.419914986
COOR_Z	2	1	0.463474560
COOR_Z	2	2	0.381824902
COOR_Z	2	3	0.381824902
COOR_Z	2	4	the 0.463474560

All tolerance are those by default: $1.0E-3$.

5 Modelization C

5.1 Characteristic of the mesh

The mesh of the beam is composed of an element SEG2.
The mesh of the section is composed of 4 quadrilateral elements.
The nodes group ENC is composed of the node $P1$.
The nodes group CHA is composed of the node $P2$.

5.2 Characteristics of the modelization

It is about a modelization POU_D_TGM (elements with 3 Gauss points). The angle of gimlet is of $0°$.

Modelization:

```
MOPOU=AFFE_MODELE (MAILLAGE=MAPOU,
   AFFE=_F (TOUT=' OUI', PHENOMENE=' MECANIQUE',
      MODELISATION=' POU_D_TGM',),
)
```

Boundary conditions:

```
BLOCAGE=AFFE_CHAR_MECA (MODELE=MO,
   DDL_IMPO=_F (GROUP_NO=' ENC',
      DX=0.0, DY=0.0, DZ=0.0, DRX=0.0, DRY=0.0, DRZ=0.0,),
)
```

mechanical Loading:

```
CHARGE=AFFE_CHAR_MECA (MODELE=MO,
   FORCE_NODALE=_F (GROUP_NO=' CHA', FX = 100, FY = 100, FZ = 100.),
)
```

Assignment of the characteristics of the elements:

```
POUCA_0=AFFE_CARA_ELEM (MODELE=MOPOU, GEOM_FIBRE=GF,
   POUTRE=_F (GROUP_MA= ("POUTRE"),
      SECTION=' RECTANGLE', CARA= ("HY", "HZ"), VALE= (0.2, 0.1),
      PREC_AIRE=5., PREC_INERTIE=10.,),
   ORIENTATION=_F (GROUP_MA= ("POUTRE"), CARA=' ANGL_VRIL', VALE=0.0,),
   MULTIFIBRE=_F (GROUP_MA= ("POUTRE"), GROUP_FIBRE= ("SBET",),
)
```

5.3 Values tested and results

Coordinated mesh SG01	Not of integration	Subpoint	Reference
COOR_X	2	1	0.954438454
COOR_X	2	2	1.025149132
COOR_X	2	3	1.045561546
COOR_X	2	4	0.974850868
COOR_Y	2	1	1.025149132
COOR_Y	2	2	0.954438454
COOR_Y	2	3	0.974850868
COOR_Y	2	4	1.045561546
COOR_Z	2	1	1.020412415
COOR_Z	2	2	1.020412415
COOR_Z	2	3	0.979587585
COOR_Z	2	4	the 0.979587585

All tolerance are those by default: $1.0E-3$

6 Modelization D

6.1 Characteristic of the mesh

The mesh of the beam is composed of an element SEG2.
The mesh of the section is composed of 4 quadrilateral elements.
The nodes group ENC is composed of the node $P1$.
The nodes group CHA is composed of the node $P2$.

6.2 Characteristics of the modelization

It is about a modelization POU_D_TGM (elements with 3 Gauss points). The angle of gimlet is of $90°$.

Modelization:
```
MOPOU=AFFE_MODELE (MAILLAGE=MAPOU,
    AFFE=_F (TOUT=' OUI', PHENOMENE=' MECANIQUE',
        MODELISATION=' POU_D_TGM',),
)
```

Boundary conditions:
```
BLOCAGE=AFFE_CHAR_MECA (MODELE=MO,
    DDL_IMPO=_F (GROUP_NO=' ENC',
        DX=0.0, DY=0.0, DZ=0.0, DRX=0.0, DRY=0.0, DRZ=0.0,),
)
```

mechanical Loading:
```
CHARGE=AFFE_CHAR_MECA (MODELE=MO,
    FORCE_NODALE=_F (GROUP_NO=' CHA', FX = 100, FY = 100, FZ = 100.),
)
```

Assignment of the characteristics of the elements:
```
POUCA_0=AFFE_CARA_ELEM (MODELE=MOPOU, GEOM_FIBRE=GF
    POUTRE=_F (GROUP_MA= ("POUTRE"),
        SECTION=' RECTANGLE',
        CARA= ("HY", "HZ"), VALE= (0.2, 0.1),
        PREC_AIRE=5., PREC_INERTIE=10.,),
    ORIENTATION=_F (GROUP_MA= ("POUTRE"), CARA=' ANGL_VRIL', VALE=0.0,),
    MULTIFIBRE=_F (GROUP_MA= ("POUTRE"), GROUP_FIBRE= ("SBET",),)
)
```

6.3 Values tested and results

Coordinated mesh SG01	Not of integration	Subpoint	Reference
COOR_X	2	1	0.997265255
COOR_X	2	2	1.038090084
COOR_X	2	3	1.002734745
COOR_X	2	4	0.961909916
COOR_Y	2	1	0.961909916
COOR_Y	2	2	1.002734745
COOR_Y	2	3	1.038090084
COOR_Y	2	4	0.997265255
COOR_Z	2	1	1.040824829
COOR_Z	2	2	0.959175171
COOR_Z	2	3	0.959175171
COOR_Z	2	4	1.040824829

All the tolerance are those by default: $1.0E-3$.

7 The purpose of summary of the results

This test is principal to check if the positions of under points of integration of modelizations POU_D_EM and POU_D_TGM is well calculated.

For these modelizations, the maximum error found is of $1.3\mathrm{E}-07\%$.