Extrapolation of measurements on a digital model in dynamics

Summarized:

A approach of extrapolation of results of experimental measurements in dynamics (displacements, velocities, accelerations, strains, stresses) on a digital model is presented. Based on a representation of structure on a projection base chosen beforehand, it consists of the resolution of the inverse problems defined by the identification of the generalized coordinates relating to projection base. The resolution suggested uses a minimization, within the meaning of the least squares, by means of decomposition READ or decomposition in singular values (SVD), of a functional calculus possibly regularized via the addition of a criterion of proximity of a solution known a priori. In the case of a temporal identification, an explicit formulation of information a priori is proposed.

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One wishes to estimate numerically, the behavior in any point of a structure starting from the mesures taken in some points of structure. Taking into account the costs and stresses of accessibility, experimental measurements are generally of number restricted and located into cubes places which inevitably are not requested. Thus, in dynamics, the knowledge of the zones of stress concentration and the local values of stresses is crucial to check the mechanical resistance of the material. One is then brought to extrapolate results of measurement located, on the group of the numerical mesh of structure.
The approach of extrapolation suggested is based on a representation of structure on a judiciously selected projection base (eigen modes, static response,…). It consists of the determination of the generalized coordinates relative to this projection base. The resolution suggested uses a minimization, within the meaning of the least squares, by decomposition in LU or singular values (SVD), of a functional calculus possibly regularized via the addition of a criterion of proximity of a solution known a priori. In the case of a temporal identification, an explicit formulation of information a priori is proposed.

2 Notations

$q, \dot{q}, \ddot{q}$: vectors of displacements, velocities and accelerations in the physical reference

$\eta, \dot{\eta}, \ddot{\eta}$: vectors of displacements, velocities and accelerations generalized

Φ : stamp formed by the basic vectors of projection (displacements)

Φ_ϵ : stamp formed by the basic vectors of projection (strains)

Φ_σ : stamp formed by the basic vectors of projection (forced)

$\bar{\Phi}$: stamp basic vectors (displacements), restricted with the measured degrees of freedom

$\bar{\Phi}_\epsilon$: stamp basic vectors (strains), restricted with the measured degrees of freedom

$\bar{\Phi}_\sigma$: stamp basic vectors (forced), restricted with the measured degrees of freedom

I : stamp identity

N_{num} : many basic vectors of projection N_{exp} : many degrees of freedom measured

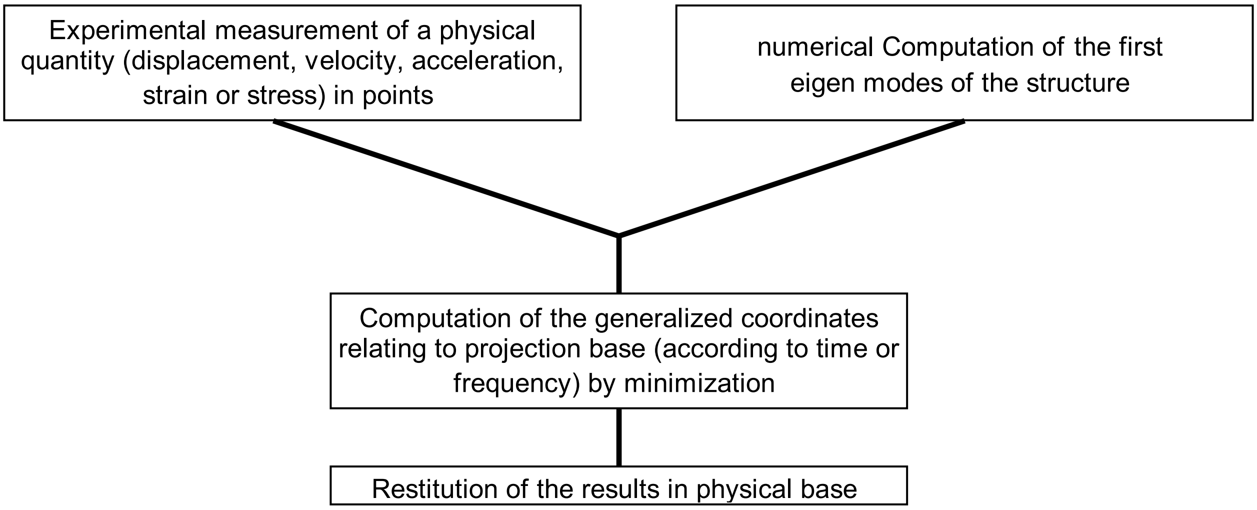
t : variable time ω : variable pulsation

TF : transform of Fourier TF^{-1} : transform of Fourier reverses

A+ : pseudo-opposite of the matrix A

3 Approach of extrapolation

the approach which one wishes to set up in order to extrapolate of the vibratory results of measurement on a digital model breaks up into 4 stages [bib1]:



This approach leans on the notion of projection of a field in a space of lower size corresponding to the space of the digital model and then extrapolation on the space of the digital model. The fact of projecting the field in a space of reduced size generates necessarily a loss of information during extrapolation. One thus sees here the importance of the choice of projection base. This base can be made of modal response and/or static response. It is supposed here that the model numerical is linear.

4 Relations between the physical quantities and the generalized quantities

One supposes that the intrinsic behavior of structure is represented in a space generated by N_{num} the basic vectors of projection. The transformation of Rayleigh-Ritz establishes the relation between the degrees of freedom of structure in the physical reference and its generalized coordinates:

$$\mathbf{q}=\boldsymbol{\Phi}\,\boldsymbol{\eta}$$

In this formulation, the matrix of the basic vectors contains all spatial information; the generalized coordinates, as for them, depend:

- time, in the case of a computation of temporal response: $\mathbf{q}(M,t)=\boldsymbol{\Phi}(M)\boldsymbol{\eta}(t)$
- pulsation, in the case of a harmonic computation of response: $\mathbf{q}(M,\omega)=\boldsymbol{\Phi}(M)\boldsymbol{\eta}(\omega)$

One can thus deduce from them very simply the following relations:

	Harmonic response	temporal Response
Displacement	$\mathbf{q}(M,\omega)=\boldsymbol{\Phi}(M)\boldsymbol{\eta}(\omega)$	$\mathbf{q}(M,t)=\boldsymbol{\Phi}(M)\boldsymbol{\eta}(t)$
Velocity	$\dot{\mathbf{q}}(M,w)=j\boldsymbol{\omega}\,\Phi(M)\boldsymbol{\eta}(\omega)$	$\dot{\mathbf{q}}(M,t)=\boldsymbol{\Phi}(M)\dot{\boldsymbol{\eta}}(t)$
Acceleration	$\ddot{\mathbf{q}}(M,w)=-\omega^2\boldsymbol{\Phi}(M)\boldsymbol{\eta}(\omega)$	$\ddot{\mathbf{q}}(M,t)=\boldsymbol{\Phi}(M)\ddot{\boldsymbol{\eta}}(t)$
Forced	$\boldsymbol{\epsilon}(M,\omega)=\boldsymbol{\Phi}_{\epsilon}(M)\boldsymbol{\eta}(\omega)$	$\boldsymbol{\epsilon}(M,t)=\boldsymbol{\Phi}_{\epsilon}(M)\boldsymbol{\eta}(t)$
Strains	$\boldsymbol{\sigma}(M,\omega)=\boldsymbol{\Phi}_{\sigma}(M)\boldsymbol{\eta}(\omega)$	$\boldsymbol{\sigma}(M,t)=\boldsymbol{\Phi}_{\sigma}(M)\boldsymbol{\eta}(t)$

All these formulations thus present an equivalent form: in the continuation of the document, we will treat primarily the case of temporal displacement, but the got results are applicable to all the other quantities: velocity, acceleration, strain and stress.
In the same way, the relations established according to time are applicable in the spectral field:

$$TF\left(\mathbf{q}(M,t)\right)=\boldsymbol{\Phi}(M)\,TF\left(\boldsymbol{\eta}(t)\right)=\boldsymbol{\Phi}(M)\boldsymbol{\eta}(\omega)$$
$$TF^{-1}\left(\mathbf{q}(M,\omega)\right)=\boldsymbol{\Phi}(M)\,TF^{-1}\left(\boldsymbol{\eta}(\omega)\right)=\boldsymbol{\Phi}(M)\boldsymbol{\eta}(t)$$

5 Computation of the generalized coordinates

5.1 Formulation of the problem

The computation of the generalized coordinates η is carried out on the matrix of displacements (respectively velocities, accelerations, strains, stresses) restricted to the measured degrees of freedom, by resolution of the matric system:

$$q_{\mathbf{exp}}=\bar{\Phi}\,\eta$$

Dimensions of the matrix $\bar{\boldsymbol{\Phi}}$ “to be reversed” are (N_{exp}, N_{num}).
It is seen here that the computation of the generalized coordinates is carried out in a restricted space: the dimension of the space generated by the basic vectors is lower corresponding to digital model, one exploits only information with the measured degrees of freedom.

5.2 Determination of a quasi-solution

For the resolution of the inverse problems, 3 cases can arise:

- $N_{exp}=N_{num}$: the number of measured degrees of freedom is equal to the number of basic vectors of projection which one wishes to identify the generalized coordinates.
In this case, there exists a single solution with the problem of inversion: $\boldsymbol{\eta}=\bar{\boldsymbol{\Phi}}^{-1}q_{\mathbf{exp}}$
- $N_{exp}>N_{num}$: the number of measured degrees of freedom is higher than many basic vectors of projection of the digital model which one wishes to identify the coordinates generalize.
In this case, it does not exist of exact solution to the problem of inversion. A quasi-solution can however be defined, which minimizes the distance: $\left| q_{\mathbf{exp}}-\bar{\boldsymbol{\Phi}}\boldsymbol{\eta}\right|$. The formula $\eta=\left[\bar{\boldsymbol{\Phi}}^T\bar{\boldsymbol{\Phi}}\right]^{+}\bar{\boldsymbol{\Phi}}^T q_{\mathbf{exp}}$ then provides the solution (single) within the meaning of the least squares. In this statement, the matrix $\left[\bar{\boldsymbol{\Phi}}^T\bar{\boldsymbol{\Phi}}\right]^{+}\bar{\boldsymbol{\Phi}}^T$ indicates the opposite matrix generalized of $\bar{\boldsymbol{\Phi}}$. The computation theopposite one can by means of be carried out decomposition READ or decomposition in singular values (SVD).

- $N_{exp} < N_{num}$: the number of measured degrees of freedom is lower than the number of basic vectors of projection which one wishes to identify the generalized coordinates (what corresponds to the case more running).
  In this case, there exist an infinity of solutions with the problem of inversion and the purpose is to determine an acceptable solution by introducing an additional condition (minimal norm of the solution or application of methods known as "of regularization").

5.3 Determination of a regularized opposite solution

5.3.1 Principles of the methods of regularization

the goal of the methods of regularization [bib4], [bib5] is to suggest an approximate and stable solution with respect to the variations of the data input. One does not seek any more to solve the equation of minimization resulting from the formulation: $q_{\mathbf{exp}} = \bar{\boldsymbol{\Phi}} \boldsymbol{\eta}$, but to determine a solution approximate (or regularized) answering two requirements:

- it satisfies a condition with proximity: one seeks $\boldsymbol{\eta}_{\delta}$ such as $| q_{\mathbf{exp}} - \boldsymbol{\Phi}_{num} \boldsymbol{\eta}_{\delta} | < \delta$,
- it answers additional a condition a priori known as "information".

The methods of regularization thus consist in supplementing the statement of the problem by introducing information a priori to extract, in the family of the solutions which are compatible with the experimental data, that which best corresponds to the problem. This is done by amalgamating in a single criterion a measurement of the fidelity of the solution compared to the experimental data and a measurement of its fidelity to information a priori [bib2].
An approach which can be easily put in work in finished dimension is the regularization by optimization. A to bring closer to the method of regularization of Tikhonov [bib3], it consists in considering a solution a priori $\boldsymbol{\eta}_{priori}$ problem of minimization and seeking the solution of the approximate system nearest to this solution. One then seeks to minimize the following functional calculus:

$$| \mathbf{q}_{\mathbf{exp}} - \bar{\boldsymbol{\Phi}} \boldsymbol{\eta} |^2 + a | \boldsymbol{\eta} - \boldsymbol{\eta}_{priori} |^2$$

The parameter a determines the affected weight with information a priori.
The solution of the equation of minimization is given by:

$$\boldsymbol{\eta} = \left[\bar{\boldsymbol{\Phi}}^T \bar{\boldsymbol{\Phi}} + a \mathbf{I} \right]^{-1} \left(\bar{\boldsymbol{\Phi}}^T \mathbf{q}_{\mathbf{exp}} + a \boldsymbol{\eta}_{priori} \right)$$

or, while revealing explicitly the variation compared to the solution a priori:

$$\boldsymbol{\eta} = \boldsymbol{\eta}_{priori} + \left[\bar{\boldsymbol{\Phi}}^T \bar{\boldsymbol{\Phi}} + a \mathbf{I} \right]^{-1} \bar{\boldsymbol{\Phi}}^T \left(\mathbf{q}_{\mathbf{exp}} - \bar{\boldsymbol{\Phi}} \boldsymbol{\eta}_{priori} \right)$$

If one poses $\boldsymbol{\eta}_{priori} = 0$, this formulation consists in seeking the solution known as of "minimal norm" (or Tikhonov of order 0).
The regularizing addition of the term related to the matrix $a \mathbf{I}$ has as a role to shift the spectrum $\bar{\boldsymbol{\Phi}}^T \bar{\boldsymbol{\Phi}}$ in order to ensure the stage of matric inversion. This approach of computation thus makes it possible to implement a procedure of computation conditioned better, which softens the effects of the noise and which provides a physically acceptable solution.
In addition, the choice of the values of the matrix $a \mathbf{I}$ results from a compromise between the stability of the required solution and the confidence which one can grant to the solution a priori.

5.3.2 Choice of information a priori

In the case as of methods of regularization, the choice of information a priori constitutes a stage-word which determines the representativeness of the final results. This choice can lean on a physical

knowledge of the solution or on a knowledge of its evolution according to the parameter selected. We provide, in the continuation, an example applied to the determination by minimization of a temporal variable [bib1].
The minimization of a variable according to time can be realized with each time step independently of time step preceding. The introduction of information a priori however makes it possible to enrich the functional calculus by supposing a slow evolution by the given variables:

$$\eta_{priori}(t)=\eta(t-dt)$$

This assumption is acceptable only when the step of sampling is sufficiently weak. Indeed, the solution at a given time is approached by (development of Taylor):

$$\eta(t)=\eta(t-dt)+dt\,\dot{\eta}(t-dt)+o(dt)$$

The maximum frequency of response of structure is determined by the pulsation of the mode of the highest nature $\omega_{\mathbf{max}}$ taken in the modelization. One thus has:

$$\left|\frac{\eta(t)-\eta(t-dt)}{\eta(t-dt)}\right|<\omega_{\mathbf{max}}dt+|o(dt)|$$

So that term corrective is weak (and thus that information a priori constitutes an approximation with the first order of the required solution), the step of sampling must check:

$$dt<<\frac{1}{\omega_{\mathbf{max}}}$$

At initial time (t=0), since one does not have any information a priori on the solution, computation is carried out by seeking the solution of minimal norm. In order to avoid propagating the error which results from it, it can be necessary to assign a weak confidence to information a priori on the first time step (via the parameter α) and to exploit the results only from time when one can consider that the errors sufficiently attenuated. If necessary, complementary studies will be conducted in order to determine the optimal parameters of use of the functionality developed in *Code_Aster*.

In the frequential field, many opportunities are given to determine information a priori. They are based either on a physical knowledge of the solution (put in experimental obviousness of resonances or forced responses), or on a formulation of the displacements generalized according to the frequency (standard: functions gain), in which case minimization finally results in characterizing the dynamic stresses.

6 Put in work in *Code_Aster*

projection base is made up either of calculated dynamic modes by the command MODE_ITER_SIMULT [U4.52.03] stored in a concept of the mode_meca type, or of the dynamic modes and static modes calculated by the command DEFI_BASE_MODALE [U4.64.02] stored in a concept of the base_modale type.
The phase of computation of the generalized coordinates is treated by the command PROJ_MESU_MODAL [U4.73.01]. The data are gathered there under 4 key words factors.
The data relating to the digital model (projection base) are gathered under factor key word the MODELE_CALCUL. One specifies there the model numerical and projection base.
The data relating to measurements are gathered under factor key word the MODELE_MESURE. One specifies there in particular the model associated with structure and measurement read by the command LIRE_RESU.
The possible manual spatial association of the nodes is given under factor key word the CORR_MANU.
The data concerning the resolution of the inverse problems are gathered under factor key word the RESOLUTION. One specifies there the method of decomposition employed (READ, SVD) and the taking into account of term of regularization.
The restitution of the results in physical base can then be carried out by the command REST_GENE_PHYS [U4.63.31].

7 Bibliography

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4) Mr. BONNET: Digital processing of inverse problems of source in linear acoustics. EDF contract - Convention P55L08/1E5240

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8 Description of the versions of the document

Version Aster	**Author (S) Organization (S)**	**Description of modifications**
5		initial Text
6.4	*S. AUDEBERT,* *H. ANDRIAMBOLOLONA* EDF-R&D/AMA	