Examination of the random responses

Summarized :

The introduction of a “stochastic approach of seismic computation” to solve a vibratory problem of mechanics under random excitation requires a particular postprocessing.

Command POST_DYNA_ALEA [U4.76.02] allows, from the power spectral density of a interspectrum-response, to evaluate its standard deviation, its apparent frequency, the distribution of its peaks. It also allows, in a first approach, to calculate the useful function of Vanmarcke in the case of a seismic analysis.

NB:

This command also makes it possible to carry out the statistical estimates for any type of interspectrum of response to a random excitation not necessarily seismic (for example: effect of the swell or a turbulent flow).

Contents

1 Introduction

For a structure subjected to a random excitation of type swells, turbulent flow, or seisme… the loading is not known in a deterministic way, but is generally described by probabilistic or spectral information like the power spectral density. For linear structures it is possible to use a stochastic method of calculating which makes it possible to determine the power spectral densities of response to these random excitations.

Operator POST_DYNA_ALEA has as a function to carry out the statistical analyzes of the power spectral density of response. He thus provides probabilistic information of the response of structure. Statistical computations of parameters are carried out on the basis of computation of the spectral moments of the power spectral density considered.

These statistical parameters are: the standard deviation, the apparent frequency, distribution of the peaks. It is also possible in the operator to calculate, in a first approach, the function of useful VANMARCKE in the case of a seismic analysis.

Note:

Operator POST_DYNA_ALEA , designed initially for the seismic approach, after a computation with operator DYNA_ALEA_MODAL [U4.56.06] ([bib1], [bib2]) , p had also to carry out postprocessings of operator DYNA_SPEC_MODAL developed by department TTA in the frame of the resorption of FLUSTRU. This operator carries out the computation of the response of a structure of the type tubes GV uniformly excited by a transverse flow.

2 Spectrum - Interspectrum - Interspectral matrix

2.1 Processing of the signal - Conventions selected

2.1.1 Introduction

a signal can have two representations: a temporal representation of the form $x=f(t)$ or a frequential representation of the form $X=F(f)$. These two representations are connected between them by **the Transformation of Fourier.**

There exists in the numerical field and the experimental field various ways calculate the spectral quantities relative to a temporal signal $x(t)$ (dimensional representation or not, factor 1/2 π or not for the Transformation of Fourier).

However, if the various definitions of the DSP (cf [§2.2.2] and [Annexe1]) from the Transformation of Fourier of the signal do not change anything with the computation carried out by CALC_INTE_SPEC [U4.56.03], it is essential on the other hand, in computations carried out by the operator postprocessing POST_DYNA_ALEA, that the data are coherent so that the results produced by this operator are with the physical dimension of the starting signal.

It is also necessary to know, for a quantitative comparison between computation and experiment, which are the conventions adopted for the computation of the spectral quantities. The set of these conventions is recalled in [Annexe1] for each type of signals. We give again only the general formulas here.

2.1.2 Transformation of Fourier

For **the Transformation of Fourier in frequency** (f) **of a signal** (of unit U), expressed in u/Hz we adopt the following definition: $X(f)=\int_{-\infty}^{+\infty} x(t)\, e^{-2i\pi ft}\, dt$

The reverse transformation is expressed then by: $x(t)=\int_{-\infty}^{+\infty} X(f)\, e^{+2i\pi ft}\, df$

One can also express **the Transformation of Fourier in pulsation** $(\omega=2\pi f)$, by the following definition:

$$X^p(\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} x(t)\, e^{-i\omega t}\, dt$$

The reverse transformation is expressed by: $x(t)=\int_{-\infty}^{+\infty} X^p(\omega)\, e^{+i\omega t}\, d\omega$

What leads to equivalence: $X^p(\omega)=X^p(2\pi f)=\frac{1}{2\pi} X(f)$

2.2 Notion of Power - Power spectral density

2.2.1 Power of a signal - Spectrum of Power of a signal

Just like the signal itself, the power of the signal can be expressed according to time or frequency:

- the instantaneous temporal power is simply called power:

$$p(t)=x(t).\, \mathrm{x}^*(t)$$

where $\mathrm{x}^*(t)$ is the complex quantity combined of $x(t)$.

- the frequential power is commonly called power spectral density or spectrum:

$$\underline{S_{xx}(f)=X(f).\, \mathrm{X}^*(f)=|X(f)|^2}$$

This definition is possible only if when the transform of Fourier of the signal exists.

One can then express the total energy of the signal by $E=\int_{-\infty}^{+\infty} S_{xx}(f)\, df=\int_{-\infty}^{+\infty} |X(f)|^2\, df$

the statement of this DSP for the various types of signals is given in [Annexe1]. One will see later on [§3.3] another definition - equivalent according to the theorem of Wiener - Kinchine - but more general, of the power spectral density based on the statistical approach.

2.2.2 Power of interaction - Spectral concentration of interaction of two signals - Interspectrum

- One defines also **the instantaneous power of interaction of two signals** $x(t)$ et $y(t)$:

$$p_{xy}(t)=x(t).\mathrm{y}^*(t) \text{ et } p_{yx}(t)=\mathrm{x}^*(t).y(t)$$
$$\text{reliées par } p_{xy}(t)=p_{yx}*(t)$$

- If the two signals admit a transform of Fourier $X(f)$ et $Y(f)$, one can express **the frequential power of interaction** or **interspectrum** by $S_{XY}(f)=X(f).\mathrm{Y}^*(f)$

- If **the two signals are real** then the power of interaction $p_{xy}(t)=p_{yx}(t)=x(t).y(t)$ is real. But there is no reason so that $S_{XY}(f)$ is also real; on the other hand $S_{XY}(f)$ is complex with hermitian symmetry, namely:

  even real part and odd imaginary part or even modulus and odd phase

- If $X(f)=Y(f)$, one speaks then about **autospectrum.**

2.2.3 Interspectral matrix

an interspectral matrix of order N is a complex $N\times N$ matrix, whose each term depends on the frequency in the form of a function on f . The diagonal terms are the autospectrums, the extra-diagonal terms are the interspectrums between the points considered (each line or column representing a point in physical mesh or a mode in modal computation). The interspectrums handled in practice being hermitian, only $\frac{N(N+1)}{2}$ the terms of triangular higher (or lower) are sufficient to define the interspectral matrix completely.

2.3 Establishment in *Code_Aster*

interspectral matrixes handled by the operator `POST_DYNA_ALEA` consist of complex functions of the frequency: $S_{\mathbf{XY}}(f)$.

These matrixes are stored in arrays of concept `tabl_intsp`.

3 Recalls on the statistical models [bib4]

3.1 discrete

t or continuous $(t_n)_{n=1,\mathrm{N}}$ parameter Definitions (time or a variable of space).

$X(t)$ random process.

At every moment t_n is associated a random variable X_n, random variable of realization x_n.

Then $x(t)=(x_n=x(t_n))_{n=1,N}$ is a realization of the process $X(t)$, process made up of N random variables a priori independent.

Each variable X_n is characterized by its **function of distribution** $F_n(x,t_n)=\mathrm{Prob}(X_n \leq x)$ or its **density of probability** $p(x,t_n)=\frac{\partial F_n}{\partial x}(x,t_n)$.

The random process is also characterized by its **functions moments**, the first two moments have a particular importance. It acts of **the expectation** or **average** $m(t)$ also noted $E[X(t)]$ and for any couple (t_1,t_2) of **the function of autocorrelation** $R(t_1,t_2)$ or $R_{XX}(t_1,t_2)$ noted too $E[X(t_1)X(t_2)]$.

$$m(t)=E[X(t)]=\int xp(x,t)dx$$

$$R_{XX}(t_1,t_2)=E[X(t_1)X(t_2)]=\int\int x_1 x_2 p(x_1,t_1;x_2,t_2)dx_1 dx_2$$

One defines also **a function of intercorrelation** for two processes $X(t)$et $Y(t)$.

$$R_{XY}(t_1,t_2)=E[X(t_1)Y(t_2)]=\int\int x_1 y_2 p(x_1,t_1;y_2,t_2)dx_1 dy_2$$

The “spreading out” of the process is characterized by **the variance** :

$$\sigma^2(t)=E\left[(X(t)-\mu(t))^2\right]$$

For a process with average null (μ = 0.), **the variance** which then characterizes the “intensity of the phenomenon” (square of the standard deviation or average quadratic value) is equal to the function of autocorrelation at time $t=t_1=t_2$:

$$\sigma^2(t)=E[X(t)X(t)]=R_{XX}(t,t)=\int x^2 p(x,t)dx$$

3.2 Assumptions in random dynamics

Very classically several assumptions are posed in the frame of the random dynamics. One admits thus that the studied processes are **steady, with average null and ergodic**.

3.2.1 Steady processes with average null - variance

a process is known as **steady** if the set of its "probabilistic characteristics" is invariant during a translation t_0 of the parameter t. What implies:

$$\mu(t)=Cte$$
$$R_{XX}(t_1, t_2)=R_{XX}(t_2-t_1)=R_{XX}(\tau)=R_{XX}(-\tau)$$

For a process with average null $\sigma^2=R_{XX}(0)$.

3.2.2 Ergodicity

This notion comes from a reasoning of Gibbs (1839-1903) for whom time from observation from a physical phenomenon can be regarded as infinite in front of the scale of time at the molecular level. The system passes then by all the possible states while remaining more possible for a long time, or while generally passing, in the states which are most probable, so that **the temporal average** becomes **equal to the statistical average on the states**, i.e. the expectation. This is prolonged for the functions of correlation and intercorrelation.

$$\mu=\lim_{T\to+\infty}\frac{1}{T}\int_{-T/2}^{+T/2}x(t)\,dt$$
$$R_{\mathrm{XX}}(t)=\lim_{T\to+\infty}\frac{1}{T}\int_{-T/2}^{+T/2}x(t-\tau)\,x(t)\,dt$$

Note:

For the continuation of the document one will suppose that the random process is steady with average null and ergodic. All the developments carried out in the Code_Aster check these assumptions.

3.3 Power spectral density

In the frame of this statistical approach, one can give a very general definition of **the power spectral density** or **DSP.** One will retain for *Code_Aster* the following definitions expressed in frequency or pulsation:

$$S_{\mathbf{XX}}(f)=\int_{-\infty}^{+\infty}R_{XX}(\tau)\,e^{-2i\pi f\tau}d\tau\,;\,G_{\mathbf{XX}}(f)=\int_{0.}^{+\infty}R_{XX}(\tau)\,e^{-2i\pi f\tau}d\tau$$
$$S^{p_{\mathrm{xx}}}(\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}R_{XX}(\tau)\,e^{-i\omega\tau}d\tau\,;\,G^{p_{\mathrm{xx}}}(\omega)=\frac{1}{2\pi}\int_{0.}^{+\infty}R_{XX}(\tau)\,e^{-i\omega\tau}d\tau$$

who lead to the following relations:

$$G^{p_{\mathrm{xx}}}(\omega)=\frac{1}{2\pi}G_{\mathbf{XX}}(f)$$
$$S_{\mathbf{XX}}(f)=2\,G_{\mathbf{XX}}(f)\,S_{\mathbf{XX}}^{p}(\omega)=2\,G^{p_{\mathrm{xx}}}(\omega)$$

One can show that $G_{\mathbf{XX}}(f)$, who is equal to the Transformation of Fourier of $R_{XX}(t)$, is real, positive. One will refer to [Annexe1] who contains all conventions adopted to ensure the coherence of the results.

3.4 Spectral moments

One calls spectral moments the following quantities (which one defined in pulsation):

$$\lambda_i=\int_{-\infty}^{+\infty}|\omega|^i S^{p_{xx}}(\omega)\,d\omega=\int_{-\infty}^{+\infty}|\omega|^i S_{XX}(f)\,df$$

One has in particular: $\lambda_0=\sigma^2\;\lambda_2=\sigma^2_{\dot{X}\dot{X}}\;\lambda_4=\sigma^2_{\ddot{X}\ddot{X}}$ who are the standard deviations of X and its first derived.

These moments are systematically calculated until order 4; using key word `MOMENT` it is possible to ask the computation of the higher modes. In *Code_Aster*, computation is carried out for a DSP expressed according to the frequency f.

The Code_Aster calculates the spectral moments on the basis of field of definition of the functions such as they are provided to him.

4 Measurements of going beyond threshold and reliability

the classical methods give access only the maximum of displacement (or acceleration) by summation "adapted" of the maxima on each mode. The essential interest of the stochastic approach of random vibratory computation lies in the statistical knowledge of the response of the structure which can thus be converted into statistical data of reliability. For this reason, two modes of failure can be taken into account:

- failure by going beyond threshold: this kind of failure occurs when the response of the system exceeds a limiting value. That amounts seeking the probability that the values of the process remain below an extreme value (**peak Factor** or **factor of peak**) during the period of observation T.
- failure by fatigue or accumulation of damages.

This second approach could also be treated starting from the first statistical elements calculated in `POST_DYNA_ALEA`. It is carried out in command `POST_FATI_ALEA` [U4.67.05] [R7.04.02].

In the frame of the studies under seismic excitations, we are interested primarily in the problem of going beyond of threshold. From where initially the computation of a certain number of statistical parameters which make it possible to characterize the signal to be studied (spectral moments and formulas of Rice [§4.1]), provided with these characteristics we will be able to then estimate the probabilities of going beyond threshold using classical models of probability [§4.2], as well as a criterion of reliability (model of Vanmarcke [§4.3]).

4.1 Spectral moments and characteristic parameters

the spectral moments are defined by: $\lambda_i=\int_{-\infty}^{+\infty}|\omega|^i S_{XX}(f)\,df$

The infinite set of these spectral moments characterize the interspectrum perfectly and thus make it possible to establish a certain number of numerical results. In the typical case of an oscillator with 1 d.o.f. or a signal with only one peak, the first three spectral moments are enough to find the autospectrum S_{XX}. It is the case which we retain in *Code_Aster* since it is supposed that the values are distributed according to a Gauss's law.

4.1.1 Formulas of Rice

For a random signal such as definite previously:: steady with **average null (centered)** and ergodic, one supposes moreover than the measured values are distributed according to **a normal model profile of the type Gauss** (cf [§4.2.1]).

The analysis of a steady gaussian random loading has the advantage of leading to simple analytical statements [bib8] - known under the name of formulas of Rice - and of representing many real phenomena.

The following statistical parameters are obtained as from the various spectral moments connected to various derivatives of X (cf [§3.4]):

- Standard deviation: $\sigma_X = \sqrt{\lambda_0}$

Note:

If only the positive part of the spectrum is provided, Code_Aster multiplies by 2 the 1st spectral moment $\sigma_X = \sqrt{2\lambda_0}$ *.*

A extremum (maximum or minimum) of amplitude X is defined by the probability of having a derivative null $\dot{X}=0$ associated with a derivative second $\ddot{X}$ unspecified.

- Median number of extrema a second: $N_e = \frac{1}{\pi}\frac{\sigma_{\ddot{X}}}{\sigma_{\dot{X}}} = \frac{1}{\pi}\sqrt{\frac{\lambda_4}{\lambda_2}}$

The going beyond a level X_0 is defined by the probability of having $X=X_0$ with an unspecified $\dot{X}$ slope: one thus counts the transitions of this level with the positive and **negative** slopes. Taking into account the assumptions of gaussian models, the number of transition by X_0 and a second is expressed by: $N_{X_0} = \frac{1}{\pi}\frac{\sigma_{\dot{X}}}{\sigma_X} e^{-\frac{X_0^2}{2\sigma_X^2}}$

What leads to the following statements:

- Many goings beyond level **with positive slope** a second: $N_{X_0^+} = \frac{1}{2} N_{X_0}$
- Many transitions **by zero** (X_0 = 0) a second: $N_0 = \frac{1}{\pi}\frac{\sigma_{\dot{X}}}{\sigma_X} = \frac{1}{\pi}\sqrt{\frac{\lambda_2}{\lambda_0}}$
- Many transitions **by zero with positive slope** a second:

$$N_{0^+} = \frac{1}{2} N_0 = \frac{1}{2\pi}\sqrt{\frac{\lambda_2}{\lambda_0}}$$

N_{0^+} represent an average statistical frequency of transition by zero with positive slope.

In the case of a “simple” signal, i.e. with **only one peak** N_{0^+}, many transitions by zero can be comparable to **such a noted** apparent frequency f_c. In the case much more general of an unspecified signal, the physical interpretation of the value N_{0^+} is more prone to guarantee!

The factor of irregularity translates the frequential pace of the signal. Ranging between 0 and 1, it tends towards 1 when the process is to narrow tape, on the other hand it tightens towards 0 for a broad band process. Its statement is:

$$I=\frac{N_0}{N_e}=\frac{\sigma_{\dot{X}}^2}{\sigma_X\sigma_{\ddot{X}}}=\sqrt{\frac{\lambda_2^2}{\lambda_0\lambda_4}}$$

The three parameters - N_0, N_e, I - characterize the signal entirely. One can, in particular, estimate the median number of positive peaks a second: $N_{pic^+}=1/4(1+I)N_e$.

The set of these parameters is calculated and stored in a "printable" array on the results file using command `IMPR_TABLE`.

4.2 Distributions of the positive peaks

One of principal knowledge interesting the structure originators from his response estimated at a random excitation is the determination of the goings beyond threshold and in particular **the probabilities of goings beyond** certain critical points.

The formulas of Rice (preceding paragraph) make it possible to know the average rate of crossings of certain levels. The following approach makes it possible to give a model of probability of presence of such or such peak. One is thus interested in maximum positive of the response.

A maximum occurs when $\dot{x}(t)=0\,\text{avec}\;\ddot{x}(t)<0$. One is thus interested in the density of joint probability $p(x,\dot{x}=0,\ddot{x},t)\,\text{de}\;X(t),\dot{X}(t),\ddot{X}($. (It is necessary thus that the process is twice differentiable, which is acquired when one admits a gaussian distribution of the signal.)

This density of probability of **the positive peaks** makes it possible for example to calculate the proportion of peaks ranging between has and B (or the probability that the next peak lies between has and b) which is worth:

$$\int_a^b p(x,0,\ddot{x},t)\,dx$$

The steady gaussian signal, being centered compared to its mean value (null in seismic analysis), the distribution of the peaks is symmetric compared to this average. One is thus interested in **the distribution of the positive peaks**. In the general case, the distribution of the peaks of positive X amplitude is written in the form [bib5]:

$$p_{\text{pic}}^{+}(\boldsymbol{X})=\frac{2}{\sqrt{2\pi}\,\sigma_{\boldsymbol{X}}(1+I)}\left[\sqrt{1-I^2}\,e^{\frac{\boldsymbol{X}^2}{2\sigma_X^2(1-I^2)}}+\frac{I\,\boldsymbol{X}}{\boldsymbol{\sigma}_X}\,e^{\frac{\boldsymbol{X}^2}{2\sigma_X^2}}\int_{-\infty}^{\alpha}e^{\frac{-t^2}{2}}\,dt\right]$$

$$\text{Si}\,X<0\,\text{alors}\,p_{\text{pic}}^{+}(X)=0 \qquad \text{avec}\;\begin{cases} I=\dfrac{\sigma_{\dot{X}}^2}{\sigma_X\sigma_{\ddot{X}}} \\ \alpha=\dfrac{X}{\sigma_X}\dfrac{I}{\sqrt{1-I^2}} \end{cases}$$

It is the so known formula under the name of LONGUET-HIGGINS [bib7], whose approach is also clarified in [bib11]. We present, Ci after, the chart of this formula for 4 values of I .

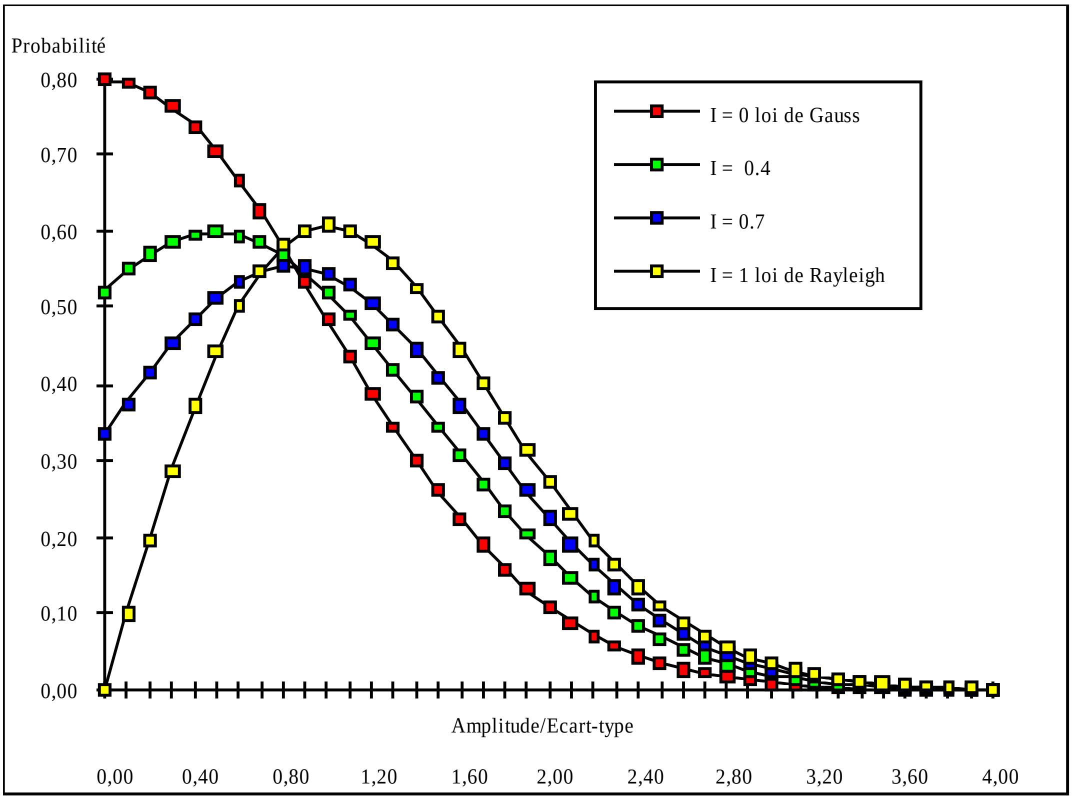


Figure 4.2-a : Distribution of peaks of positive amplitude standardized compared to the standard deviation of the signal

This distribution of the positive peaks is in the case of simplified the signals for which the factor of irregularity is worth $I=0 \text{ ou } I=1$.

4.2.1 $I=0$ Signal with broad band: Gauss's law or normal model

In the case of a broad band signal, the positive peaks is distributed according to a Gauss's law:

$$p_{pic}^{+}(X)=\frac{2}{\sqrt{2\pi\sigma_{X^2}}}e^{-\frac{X^2}{2\sigma_{X^2}}}$$

4.2.2 $I=1$ Signal with narrow tape: model of Rayleigh

In the case of a signal with narrow tape, the positive peaks are distributed according to a model of RAYLEIGH:

$$p_{pic}^{+}(X)=\frac{X}{\sigma_{X^2}}e^{-\frac{X^2}{2\sigma_{X^2}}}$$

4.2.3 Computation of the values in *Code_Aster*

the values of these two models are calculated in *Code_Aster* under the factor keywords RAYLEIGH or GAUSS.

From the standard deviation $\sigma_X=\sqrt{\lambda_0}$ calculated previously one calculates the values of probability of the peaks $p^+_{pic}(X)\,\text{pour}\,X\in[0{,}6\,\sigma_X]$ with a step by default of $\dfrac{6\sigma_X}{200}$.

If the user wishes to refine his analysis, it can provide values `VALMIN` and `VALMAX` of the field of variation of X. It can also provide the value of the computation step, if not this one will be taken with the 200ème of the tape selected.

The figure [Figure 4.2-a] shows that the selected field by default, until $6\sigma_X$, covers well the totality of the values of X to non-zero probabilities.

4.3 Seismic response: model of Vanmarcke [bib8]

In the case of the response to a seisme of a primary structure (i.e. excited at its base by the soil) having **a dominating mode**, IE which answers (taking into account the exiting frequencies) on only one mode, one uses the model of reliability of VANMARCKE [bib8] which makes it possible to estimate, over **an operation life** T **probability that the process exceeds the threshold of failure**.

The notion of dominating mode is very important here, if the structure answers on several modes the formula in its current statement is not appropriate more.

That is to say $X(t)$ the response with a gaussian white vibration, of a linear oscillator slightly damped. The probability is defined $W(T)$ that the process remains in the field of security. $W(T)$ represent the fraction of sample which did not cross the threshold of failure after a period T ; it is a measurement of reliability.

It can be written in the form $W(T)=\text{Prob}\{|X(t)<X_0;0\le t<T|\}:p_1(T)=-\dfrac{\text{dW}(T)}{dT}$ is the density of probability of crossing of the threshold.

For the high values of T one will take: $p_1(T)=A\alpha e^{-\alpha T}$ where A depends on the initial conditions and α is the limiting rate of the reduction.

4.3.1 Assumption of independent crossings

With the assumption that the goings beyond threshold with a positive slope are **independent events**, the number of crossings on $[0,T[$ constitutes a process of Fish of rate of arrival $N_{X_0}=2N^+_{X_0}$ (number of going beyond X_0 defined in [§4.1.1]). The probability that n transitions occur over the period T writes by application of the Fish model (see [§4] [bib8]):

$$P\{n\,\text{passages sur}\,[0,T[\}=e^{-N_{X_0}T\frac{(N_{X_0}T)^n}{n!}}$$

The structure is “reliable” if the threshold is not exceeded during the period T. Reliability $W(T)$ thus corresponds to $n=0$ transition from where $W(T)=e^{-N_{X_0}T}$.

The limiting rate of the reduction is thus worth here $\alpha=N_{X_0}=2N^+_{X_0}$.

4.3.2 Model of Vanmarcke

For a gaussian steady process, the probability of exceeding the value X is worth [§4.1.1]: $N_X = N_0 e^{-\frac{X^2}{2\sigma_{X^2}}}$; one from of deduced that the probability that the initial value of the envelope is lower than the threshold X is: $1-\frac{N_X}{N_0}=1-e^{-\frac{X^2}{2\sigma^2}}$.

One then combines this statement with the limiting model of the reduction obtained with the assumption of independent crossings, which leads to **the statement of reliability** :

$$W(T)=Ae^{-\alpha T}=\left(1-e^{-s^2/2}\right)e^{-N_0 T\frac{\left(1-e^{-hs}\right)}{e^{\frac{s^2}{2}}-1}}$$

with $N_0=\frac{1}{\pi}\sqrt{\frac{\lambda_2}{\lambda_0}}$ rate of transition by 0 and T lasted of observation

$$\text{où} \quad s=\frac{X}{\sqrt{\lambda_0}} \quad h=d^{1.2}\sqrt{\frac{\pi}{2}} \quad \delta=\sqrt{1-\frac{\lambda_1^2}{\lambda_0\lambda_2}}$$

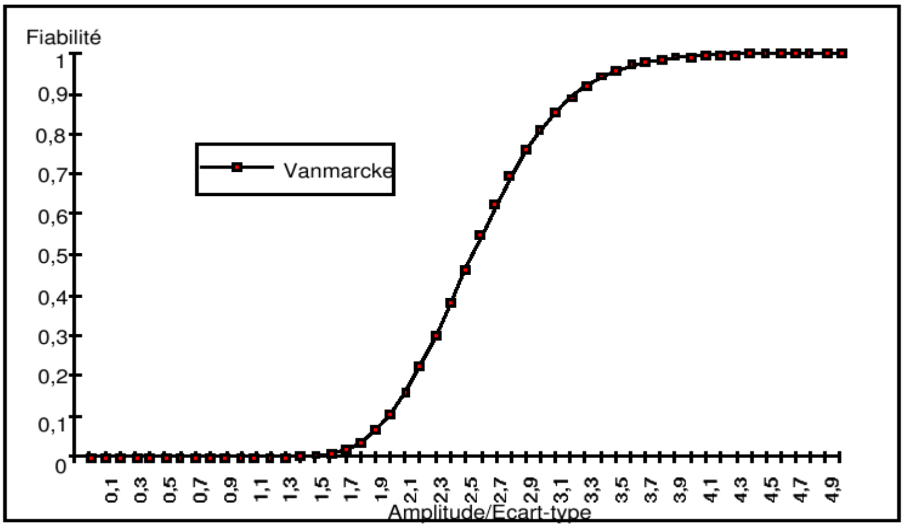
δ is an estimator of bandwidth of the DSP of X .

This relation has the immense advantage of providing an explicit estimator of reliability according to the reduced value of the threshold *S*, amongst equivalent semi-cycles N_0 , and of the parameter of bandwidth δ .

NB:

"The agreement between the estimator and the simulations can be improved if one replaces δ by $\delta^{1.2}$ " [bib8] "correction" introduced into the formula written above compared to the statement of the rate of the reduction limits given in the preceding paragraph.

The following chart is carried out in the case of a process of apparent frequency $15\,Hz$, that is to say $N_0=30\,Hz$, for a period of observation of $T=1\text{s}$. The estimator of bandwidth δ is taken equal to 0.30. As in the preceding illustrations, the amplitude is standardized compared to the standard deviation.



Appear 4.3.2-a: Evolution of reliability according to the model of Vanmarcke according to the amplitude of the process standardized compared to the standard deviation of the signal

Recall:

This statistical analysis is carried out from relatively restrictive assumptions, namely that the process must be "to narrow tape"; it will thus have to be checked that the factor of irregularity I is not too different from 1 and that the signal comprises only one principal peak.

4.3.3 Establishment in *the Code_Aster*

It is exactly the statement of $W(T)$ which is established in operator `POST_DYNA_ALEA`, under factor key word the VANMARCKE. The field of definition of the computation of the function of reliability is here also by default $[0,6\sigma_X]$ with a step of $\frac{6\sigma_X}{200}$. As for the Gauss's laws and of RAYLEIGH [§ 4.2.3], it can be restricted by the user.

The computation of reliability is made for a period T (in S) of operation: it is taken by defaults with T = 10 S what is appropriate well for a seismic computation.

The figure [Figure 4.3.2-a] shows well that for $6\sigma_X$, $W(T)$ tends towards asymptote 1, the process surely exceeded the threshold of failure.

5 Remarks

This postprocessing is carried out on interspectrums stored in `tabl_intsp`. It provides statistical elements of the response of structure which can thus be converted into statistical data of reliability, or to be useful then for computations of damage by fatigue (`POST_FATI_ALEA`).

6 Bibliography

1) C. DUVAL: Specifications of a modulus of computation of dynamic response under random excitations for *the Code_Aster* - Ratio EDF HP61/91.177.
2) C. DUVAL: Dynamic response under random excitations in *the Code_Aster:* Theoretical principles and examples of use - Ratio EDF HP61/92.148.
3) J. MAX and saddle-point.: Methods and techniques of processing of the signal and applications to physical measurements.
4) Francoise BROUAYE: “The modelization of uncertainties” Chapter 5 - Collection of the Studies and Searches of EDF ESE n°8 – Eyrolles.
5) P. MORILHAT: “Thermal Faience manufacturing. Statistical estimate of the mechanical damage” - EDF July 90 Ratio HP 14/90.07.
6) P. LABBE and H. NOAH: “Stochastic approach for the seismic design of nuclear power seedling equipment”. Nuclear Engineering and Design 129 (1991) 367 - 379.
7) D.E. CARTWRIGHT & M.S. LONGUET-HIGGINS: The statistical distribution of the maximum of has random function - Proceedings of the Royal Society of London - Series A Flight 237 (1956).
8) A. PREUMONT: “Random Vibrations and analyzes spectral” - polytechnic Presses and French academics. Edition 1990 - In particular: Chapter 10.
9) R.J. GIBERT: “Vibrations of structures. Interactions with the fluids. Random sources of excitation” - CH 17: general notions on the random processes and the response of the linear systems - CH 20: Seismic excitations of structures. Collection of the Studies and Searches of EDF ESE n°69 - Eyrolles 1988.
10) R.W. CLOUGH and J. PENZIEN in “Dynamics of structures” 4th part: Random vibrations, 5th part: Structure response with the seismes - Mc Graw Hill 1975 analyzes.
11) Y.K. LIN & G.Q CAI: Structural Probabilistic dynamics (advanced theory and applications) Mc Graw Hill.

7 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
4	A .DUMOND-EDF/ R & D - AMV	initial Text
11.2	F.VOLDOIRE EDF/R & D /AMA	Some cosmetic corrections of formulas.

Annexe 1 Conventions for the Power spectral densities

A1.1 Introduction

In order to and the preserve coherence necessary for all computations comparisons with the experiment (cf [§2.3] and [§3.3]), we develop hereafter the two coherent sets of definitions with computations of random response and of postprocessing such as they were retained for *Aster*:

- the first from spectral data expressed according to the frequency. It is this group which is coherent with the computation carried out in operator CALC_INTE_SPEC [U4.56.03].
- the second from spectral data expressed according to the pulsation.

These two sets return validates postprocessing such as it is expressed in POST_DYNA_ALEA.

We will each time specify the unit in which are expressed the various quantities handled according to the unit U of the signal of reference. The explanations given are brief. One will be able for more details to refer to the reference [bib10].

A1.2 Types of signals and definition of the power

We consider four types of signals:

- signals of finished energy,
- periodic signals,
- signals of finished power and signals deterministic,
- the random satisfying assumption with ergodicity and steady signals.

In random dynamic computation the signals are random. For interpretation of results experimental, the signals are either periodic, or of finished power (deterministic).

We define for each type of signal an energy quantity which is either an energy, or a power and that we will indicate in the following paragraphs under the single term of power:

- **The signals of finished energy** are defined by their energy E expressed in $u^2 s$:

$$E=\int_{-\infty}^{+\infty} x(t)^2 dt<+\infty \qquad \text{éq An1.2-1}$$

- **the periodic signals** are defined by the power P of the signal expressed in u2 :

$$P=\frac{1}{T}\int_{[T]} |x(t)|^2 dt \qquad \text{éq An1.2-2}$$

  T indicates the period of the signal. $[T]$ is an interval length T .

- **The signals of finished power** are defined by the average power P of the signal expressed in u^2 :

$$P=\lim_{T\to+\infty}\left(\frac{1}{T}\int_{-T/2}^{+T/2}|x(t)|^2\,dt\right)<+\infty \quad \text{éq An1.2-3}$$

- **the random signals** are defined by the average power P of the signal expressed in u^2 :

$$P=E\left[|X(t)|^2\right]=\lim_{T\to+\infty}\left(\frac{1}{T}\int_{-T/2}^{+T/2}|x(t)|^2\,dt\right)<+\infty \quad \text{éq An1.2-4}$$

One makes use here of the assumption of ergodicity which underlies that the average statistics and temporal carried out on a realization of a process are identical.

A1.3 Autocorrelations

Taking into account the statistical recalls carried out in the body text one has for each type of signals previously definite:

- Autocorrelation of **the signals of finished energy**, expressed in u^2/Hz :

$$R_{XX}(\tau)=\int\overline{x(t)}\,x(t+\tau)\,dt \quad \text{éq An1.3-1}$$

- Autocorrelation of **the periodic signals**, expressed in u^2 :

$$R_{XX}(\tau)=\frac{1}{T}\int_{[T]}\overline{x(t)}\,x(t+\tau)\,dt \quad \text{éq An1.3-2}$$

- Autocorrelation of **the signals of finished power**, expressed in u^2 :

$$R_{XX}(\tau)=\lim_{T\to+\infty}\frac{1}{T}\int_{-T/2}^{+T/2}\overline{x(t)}\,x(t+\tau)\,dt \quad \text{éq An1.3-3}$$

- Autocorrelation of **the random signals**, expressed in u^2 :

$$R_{XX}(\tau)=E\left[\overline{X(t)}\,X(t+\tau)\right]=\lim_{T\to+\infty}\frac{1}{T}\int_{-T/2}^{+T/2}\overline{x(t)}\,x(t+\tau)\,dt \quad \text{éq An1.3-4}$$

A1.4 Definition of the power spectral density

A1.4.1 Statement in frequency

One defines the power spectral density by:

$$S_{XX}(f)=\int_{-\infty}^{+\infty} R_{XX}(\tau)\,e^{-2i\pi f\tau}\,d\tau \text{ ou } G_{XX}(f)=\int_{0}^{+\infty} R_{XX}(\tau)\,e^{-2i\pi f\tau}\,d\tau \qquad \text{éq An1.4.1-1}$$

the mechanic being interested only in the positive values of the frequency and time, the function G_{XX} is more often used.

One can show, if the Transformations of Fourier of the signals exist, that this definition is equivalent (theorem of Wiener-Kinchine) to the following definitions of the power spectral density.

- For the signals of finished energy:

$$G_{XX}(f)=|X(f)|^2 \text{ expressed in } u^2/Hz^2 \qquad \text{éq An1.4.1-2}$$

- For the periodic signals:

$$\text{So } X(f)=\sum_{n=-\infty}^{n=+\infty} C_n\,\delta(f-nf_0) \text{ then } G_{XX}(f)=\sum_{n=-\infty}^{n=+\infty} C_n^2\,\delta(f-nf_0) \qquad \text{éq An1.4.1-3}$$

$G_{XX}(f)$ is expressed in u^2/Hz.
f_0 is the reverse of the period of the signal.
C_n coefficient of the Dirac functions.

- For the signals of finished power:

$$G_{XX}(f)=\lim_{T\to+\infty}\left(\frac{1}{T}|X_{[T]}(f)|^2\right) \text{ en } u^2/Hz \qquad \text{éq An1.4.1-4}$$

where $X_{[T]}$ the restriction indicates of $x(t)$ à $[-T/2\,;T/2]$.

- For the random signals:

$$G_{XX}(f)=\lim_{T\to+\infty} E\left[\frac{1}{T}|X_{[T]}(f)|^2\right] \text{ en } u^2/Hz \qquad \text{éq An1.4.1-5}$$

where $X_{[T]}$ the restriction indicates of $x(t)$ on $[-T/2\,;T/2]$.

Restrain between the DSP and the power.

With the definitions given above for the power spectral densities, one has for all the signals, the relation:

$$P=\int_{-\infty}^{+\infty} G_{XX}(f)\,df \qquad \text{éq An1.4.1-6}$$

This relation is established by means of the theorem of PARSEVAL.

A1.4.2 Statement in pulsation

In pulsation, one defines the power spectral density by:

$$G'_{XX}(\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} R_{XX}(\tau)e^{-i\omega\tau}\,d\tau \qquad \text{éq An1.4.2-1}$$

Just as for the statement in frequency, one can show, if the Transformations of Fourier of the signals exist, that this definition is equivalent (theorem of Wiener-Kinchine) to the following definitions of the power spectral density

- For the signals of finished energy:

$$G'_{XX}(\omega)=2\pi|X'(\omega)|^2 \text{ expressed in } u^2/Hz^2 \qquad \text{éq An1.4.2-2}$$

- For the periodic signals:

$$\text{Si } X'(\omega)=\sum_{n=-\infty}^{n=+\infty} C_n\delta(\omega-n\omega_0) \text{ alors } G'_{XX}(\omega)=\sum_{n=-\infty}^{n=+\infty} C_n^2\delta(\omega-n\omega_0) \qquad \text{éq An1.4.2-3}$$

  $G'_{XX}(\omega)$ is expressed in u^2/Hz, and $\omega_0=\frac{2\pi}{T}$ where T is the period of the signal.

  C_n coefficient of the Dirac functions.

- For the signals of finished power:

$$G'_{XX}(\omega)=\lim_{T\to+\infty}\left(\frac{2\pi}{T}|X'_{[T]}(\omega)|^2\right) \text{ en } u^2/Hz \qquad \text{éq An1.4.2-4}$$

$X'_{[T]}$ indicates the restriction of $x(t)$ on $[-T/2;T/2]$.

- For the random signals:

$$G'_{XX}(\omega)=\lim_{T\to+\infty} E\left[\frac{2\pi}{T}|X'_{[T]}(\omega)|^2\right] \quad \text{in } u^2/Hz \qquad \text{éq An1.4.2-5}$$

$X'_{[T]}$ indicates the restriction of $x(t)$ on $[-T/2;T/2]$.

Restrain between the DSP and the power.

In the same way, there is for all the signals the relation - which rises from the theorem of PARSEVAL -:

$$P=\int_{-\infty}^{+\infty} G'_{XX}(\omega)\,d\omega \qquad \text{éq An1.4.2-6}$$

A1.4.1 Relation between DSP in frequency and DSP in pulsation

For the four types of signals:

$$G'_{XX}(\omega)=\frac{1}{2\pi}G_{XX}(f) \qquad \text{éq An1.4.3-1}$$

Annexe 2Transformation of Hilbert

Is $X(t)$ a real signal of transform of Fourier $X(\omega)$.

That is to say $H(\omega)$ the Transfer transfer function: $H(\omega)=j\,\text{sign}(\omega)=\begin{cases} j & \omega>0 \\ -j & \omega<0 \\ 0 & \omega=0 \end{cases}$

$H(\omega)$ transform $X(t)$ into its transform of Hilbert noted $\hat{X}(t)$. The system of transfer transfer function $H(\omega)$ produces a phase shift of $+90°$ for the positive frequencies and of $-90°$ the negative frequencies. It follows theorem of convolution which $\hat{X}(t)$ can also be defined like the convolution from $X(t)$ the corresponding impulse response, that is to say $h(t)=1/\pi t$.

$\hat{X}(t)$ is also real, one second application of the transform of Hilbert restores the initial signal, changed of sign and amputee of his possible continuous component.

Example: $x(t)=A\cos\omega t\rightarrow\hat{x}(t)=\text{-}A\sin\omega t$

This property is at the base of the use of the transform of Hilbert to define the envelope of a process as a narrow tape.