Criteria of structural stability

Summarized:

This document presents the various criteria of stability, with the meaning buckling of structure, available in *Code_Aster*. One can classify them according to two categories:

- criterion of Eulerian on linearized problem,
- nonlinear criteria.

These criteria make it possible to detect the loss of unicity in solution of the quasistatic problem.
They are directly applicable to the frame of the dynamics, but as they take account neither of the mass matrix nor of that of damping, one cannot speak about dynamic criterion of stability to the classical meaning (for example, of negative or null damping becoming).

The nonlinear choice of criteria fulfills the requirements of:

- versatility (general method for any behavior model and being able to accept any strain tensor available in the code),
- minimization of cost CPU and the additional obstruction memory.

The criterion presented is a generalization of the criterion of Eulerian, based on the analysis of the reactualized total stiffness matrix. It is called within operators STAT_NON_LINE and DYNA_NON_LINE, to be able to be evaluated with each step of the nonlinear dynamic resolution incremental quasi-static or transitory.

Contents

Contents

1 Introduction

Code_Aster allows the search of buckling modes linear, qualified method of Eulerian. It is enough to solve a problem generalized with the eigenvalues (thanks to operator `MODE_ITER_INV` or `MODE_ITER_SIMULT` and key word `TYPE_RESU=' MODE_FLAMB'`). The two matrixes arguments of the generalized problem are the stiffness matrix and the geometrical stiffness matrix, resulting from a linear elastic preliminary computation (operator `MECA_STATIQUE`).

In all the cases where one cannot neglect nonthe linearities, which they, the approach Eulerian is geometrical or behavioral is not more valid.

We thus propose an ad hoc *criterion*, that one can regard as a generalization of the criterion of Eulerian on reactualized configuration. This criterion is built on the assembled tangent stiffness matrix, which is calculated in the algorithm of the Newton type to solve the nonlinear quasi-static problems (operator `STAT_NON_LINE`) or nonlinear transient dynamics (operator `DYNA_NON_LINE`).

This criterion, in nonlinear, makes it possible to treat the nonlinear elastic behavior models rigorously. On the other hand, the models which present a dissipative aspect are treated rigorously only if the loading, in any point of structure, follows a monotonous evolution (that corresponds to the assumption of Hill [bib4]).

2 Study of the stability of a structure

2.1 general Notion of buckling

buckling is a phenomenon of instability [bib6]. Its appearance can be observed in particular on slender elements of low flexural stiffness. Beyond of a certain level of loading, the structure undergoes an important change of configuration (which can appear by the sudden appearance of undulations, for example).
One distinguishes two types of buckling: buckling by bifurcation and buckling by boundary point ([bib1], [bib7], [bib8]). To describe the behavior of these two types of buckling, one considers a structure of which the parameter μ is characteristic of the loading and of which the parameter δ is characteristic of displacement.

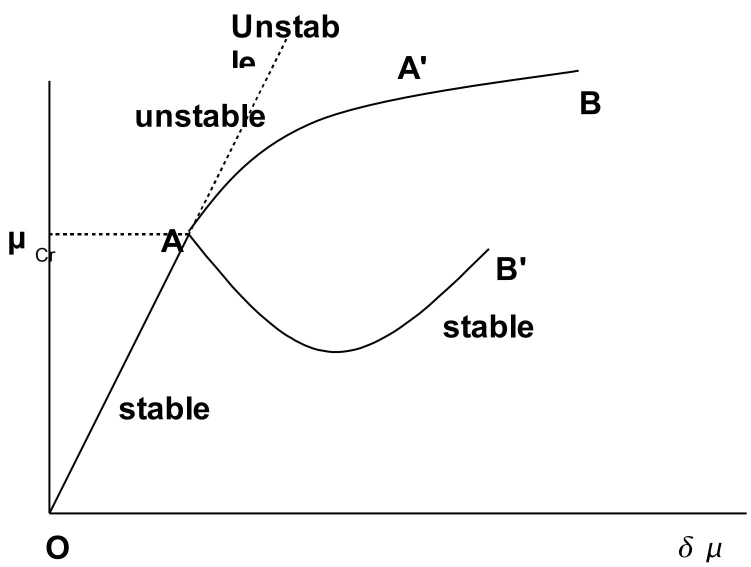


Figure 2.1-2.1-a2.1-a : Buckling by bifurcation

Between the point O and the point A, the structure admits only one family of curve (μ, δ), it can, for example to act of classical linear elasticity or of elastoplasticity, where if the problem is well posed (*cf* [§2.22.2]), there is result classical existence and of unicity of the solution.
On the other hand, beyond the point A, several families of curves are solution of the problem of equilibrium. This loss of unicity is accompanied by an instability of the initial branch (known as fundamental). The secondary branch can be stable (curved AB) or unstable (curve AB'). The load beyond which there is bifurcation calls the critical load μ_{cr}.
Buckling by bifurcation is characterized by the fact that the mode (or direction of buckling), which initiates the secondary branch, does not generate additional work in the loading applied: mode of buckling being orthogonal to him.
An example of buckling per bifurcation with instability of the secondary branch is in the case of a circular cylindrical shell under axial compression [bib10]. Examples of buckling per bifurcation with stability of the secondary branch are in elastic beams in axial compression, circular rings in radial compression and rectangular plates in longitudinal compression.

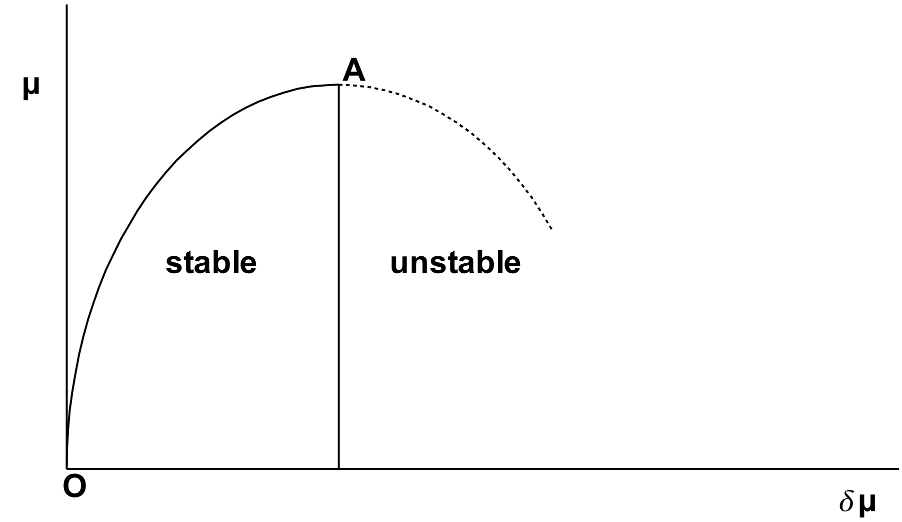
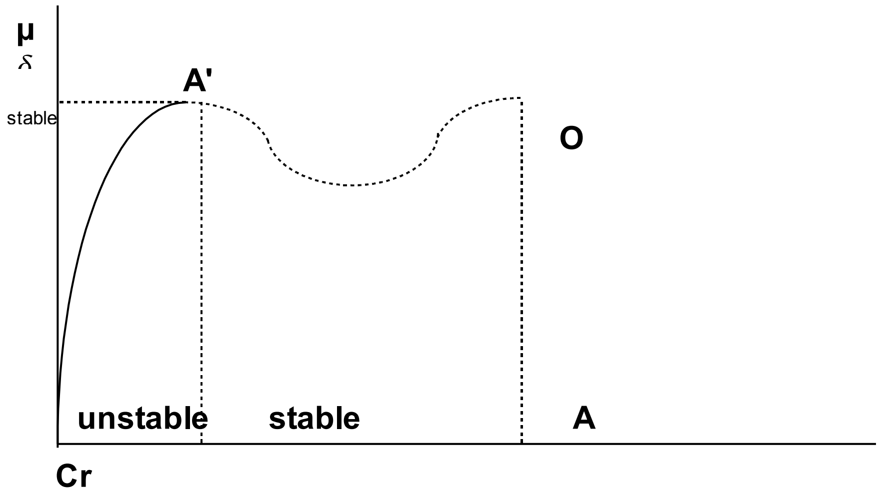


Figure 2.1-2.1-b2.1-b : Buckling by boundary point



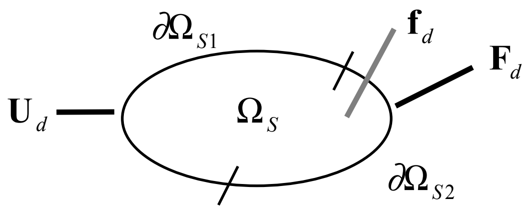
Appears 2.1-2.1-c2.1-c : Buckling by boundary point with breakdown

On figures [Figure 2.1-2.1-b2.1-b] and [Appears 2.1-2.1-c2.1-c], which illustrates buckling by boundary point, the structure does not admit that only one family (μ,δ) solution of the balance equations. At the point A , there is loss of stability of the solution with total loss of stiffness in the case of the figure [Figure 2.1-2.1-b2.1-b] and with a phenomenon of breakdown in the case of the figure [Appears 2.1-2.1-c2.1-c] (the solution becomes again stable after a discontinuity of displacement; case of a segment of a sphere under external pressure). The point A is then called boundary point.

The problem thus amounts in all the cases seeking the load from which the fundamental branch of equilibrium becomes unstable or of dubious stability. That generally mobilizes large displacements.
One can finally have the case of failure by yielding which is connected at the boundary point [Figure 2.1-b].

2.2 Writing of the mechanical problem

This chapter aims to introduce the general formalism of structural analysis adapted to the nonlinear mechanical problem which we wish to tackle.
To start, we thus briefly will point out the setting in equation of a problem type of computation of structure. To simplify, we place ourselves, all at least at the beginning, in the frame of the small disturbances.



Appear 2.2-2.2-a2.2-a : Representation of a problem of structural analysis

the structure Ω_S is subjected to imposed voluminal forces $\mathbf{f}_d$, surface forces $\mathbf{F}_d$ on edge $\partial\Omega_{S2}$ and of the displacements imposed $\mathbf{U}_d$ on the rest of edge of Ω_S, noted $\partial\Omega_{S1}$.
The unknowns of the problem of reference on solid are the field of displacement $\mathbf{u}$ and the stress field of Cauchy $\boldsymbol{\sigma}$. The solution ($\mathbf{u},\boldsymbol{\sigma}$) of the problem structure where the heating effects are neglected defines as:

To find $(\mathbf{u},\boldsymbol{\sigma})\in\mathbf{H_1}(\Omega_S)\times\mathbf{L^2}(\Omega_S)$ which checks:

•Equations of connections:

$$\mathbf{u}\,|_{\partial\Omega_{S1}}=\mathbf{U_d}$$ éq 2.2 -1

•Behavior model:

$$\boldsymbol{\sigma}=f(\boldsymbol{\varepsilon})$$ with $\boldsymbol{\varepsilon}$ which is the tensor of déformationéq 2.2 -2

$$\boldsymbol{\varepsilon}=\frac{1}{2}(\nabla\mathbf{u}+\nabla\mathbf{u}^{\mathrm{T}})$$ in assumption small perturbationséq 2.2 - 3

If one supposes a linear elastic behavior

$$\boldsymbol{\sigma}=\mathbf{C}:\boldsymbol{\varepsilon}$$ éq 2.2 - 4

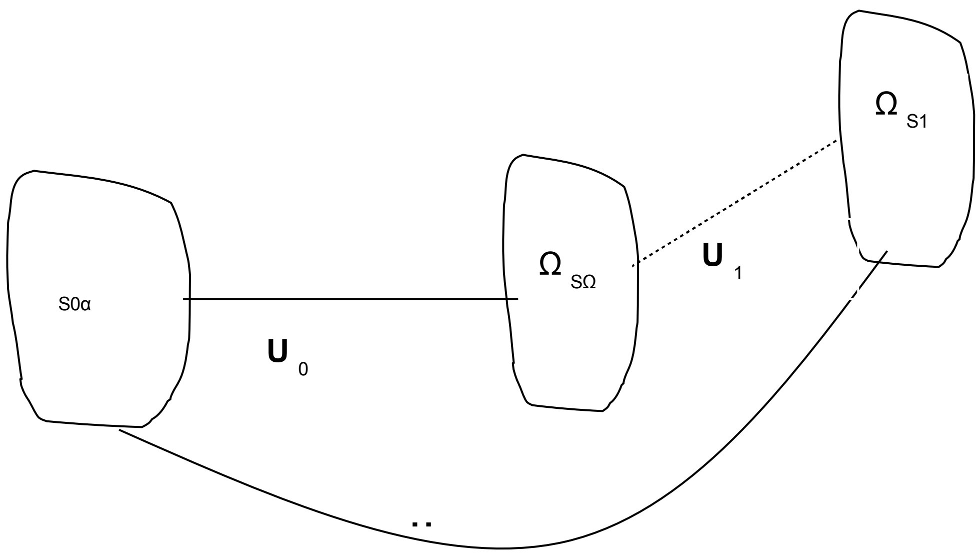
•Balance equations:

$$\begin{cases} \rho\,\boldsymbol{\gamma}=\nabla.\boldsymbol{\sigma}+\mathbf{f}_d \quad \text{avec}\,\boldsymbol{\gamma}=\dfrac{\mathrm{d}^2\mathbf{u}}{\mathrm{d}\,t^2} \\ \boldsymbol{\sigma}.\mathbf{n}\,|_{\partial\Omega_{S2}}=\mathbf{F_d} \end{cases} \qquad \text{éq 2.2 - 5}$$

2.3 Study of stability of the system

the object of this chapter is to present the methods making it possible to determine the stability of the nonlinear quasi-static equilibrium of a structure. To start, we are interested only in detection of instability, or more exactly in the loss of unicity of the solution [bib6]. Among recent works of synthesis, one can quote [bib9] or [bib7] and [bib8] which present very complete papers on the nonlinear analysis of stability of structures.
The computation of the post-critical solution will not be approached.

To analyze stability, we introduce an initial configuration of reference Ω_{S0}, a present configuration Ω_S and a disturbed configuration Ω_{S1} :



Appears 2.3-2.3-a2.3-a : Definition of the various configurations

Is $\mathbf{u}$ the field of displacement of the points of structure. The behavior is supposed, for the moment, linear elastic isotropic. The structure subjected to imposed displacements and forces will become deformed and become the structure located by the present configuration Ω_S. We seek to determine a state of equilibrium characterized by the field of displacement between the initial configuration Ω_{S0} and the current configuration Ω_S, as well as a stress field of Cauchy, noted $\boldsymbol{\sigma}$, or of Piola-Kirchhoff II, noted $\boldsymbol{\pi}$:

$$\boldsymbol{\pi}=\det\mathbf{F}\,.\,\mathbf{F}^{-1}\boldsymbol{\pi}_{\mathbf{I}} \text{ with } \begin{cases}\mathbf{F}=\nabla\,\mathbf{u}+\mathbf{I} : \text{tenseur gradient de la transformation}\\ \det\mathbf{F}=\dfrac{\rho_0}{\rho}\\ \boldsymbol{\pi}_{\mathbf{I}} : \text{tenseur de Piola - Kirchhoff I}\end{cases} \Rightarrow \boldsymbol{\pi}=\frac{\rho_0}{\rho}.\mathbf{F}^{-1}.\boldsymbol{\sigma}.\mathbf{F}^{-T}$$

éq 2.3 -3

In this statement, one sees appearing the relationship between the initial density ρ_0 and the current density ρ.
The following stage is the prediction of the stability of this equilibrium.

To this end, we will seek a criterion allowing to determine if there exists only one field of displacement balancing the forces applied. We will suppose that the forces increase gradually and we will seek to find from which moment there exist two configurations Ω_S and Ω_{S1} who respect the equations of problem: we seek a bifurcation point, it is - with-to say a loss of unicity of the solution. This time will be described as time of buckling.

2.3.1 Writing of the elastic geometrical nonlinear problem

the solution $\mathbf{u}\,,\boldsymbol{\pi}$ of the problem structure without heating effects checks ([bib1], [bib7], [bib2]):

- Equations of connections:

$$\mathbf{u}\,|_{\partial\Omega_{S0}}=\mathbf{U_d} \quad \text{éq 2.3.1-1}$$

- elastic Behavior model:

$$\boldsymbol{\pi}=\varphi_{,\varepsilon}(\boldsymbol{\varepsilon}) \quad \text{éq 2.3.1-2}$$

with $\boldsymbol{\varepsilon}$ which is the strain tensor. If a linear elastic behavior is supposed:

$$\boldsymbol{\pi}=\mathbf{C}\,\boldsymbol{\varepsilon} \quad \text{éq 2.3.1-3}$$

- Balance equations:

$$\begin{cases}\rho\,\boldsymbol{\gamma}=\nabla\,.\,\boldsymbol{\pi}+\mathbf{f}_d \quad \text{avec } \boldsymbol{\gamma}=\dfrac{\mathrm{d}^2\mathbf{u}}{\mathrm{d}\,t^2}\\ \mathbf{F}\,.\,\boldsymbol{\pi}\,.\,\mathbf{n_0}\,|_{\partial\Omega_{S0}}=\mathbf{F}_d\end{cases} \quad \text{éq 2.3.1-4}$$

the associated strain tensor is that of Green-Lagrange (referred with the initial configuration):

$$\boldsymbol{\varepsilon}(\mathbf{u})=\frac{1}{2}(\mathbf{F}^T\mathbf{F}-\mathbf{I})\quad \text{avec}\,\mathbf{F}=\nabla\mathbf{u}+\mathbf{I}$$

$$\Rightarrow\boldsymbol{\varepsilon}(\mathbf{u})=\boldsymbol{\varepsilon}^{\boldsymbol{L}}(\mathbf{u})+\frac{1}{2}\boldsymbol{\varepsilon}^{\boldsymbol{Q}}(\mathbf{u},\mathbf{u})$$

with: $$\begin{cases}\boldsymbol{\varepsilon}^{\boldsymbol{L}}(\mathbf{u})=\frac{1}{2}(\nabla\mathbf{u}+\nabla^T\mathbf{u})\text{: partie linéaire}\\ \boldsymbol{\varepsilon}^{\boldsymbol{Q}}(\mathbf{u},\mathbf{u})=\nabla^T\mathbf{u}.\nabla\mathbf{u}\text{: partie quadratique}\end{cases}$$ **éq 2.3.1-5**

We can now write the Principle of virtual power in geometrical nonlinear elasticity and quasi-static:

$$p^{intal\,\text{int}}-p^{ext}=0,\forall\,\mathbf{u}^*\text{C A 0}$$

$$\text{Avec :}\begin{cases}\text{p}^{\text{int}}=\int_{\Omega_{s0}}Tr(\boldsymbol{\pi}\,\boldsymbol{\varepsilon}^*)\,d\,\Omega=\int_{\Omega_{s0}}Tr\left[\left(\boldsymbol{\varepsilon}^{\boldsymbol{L}}(\mathbf{u})+\frac{1}{2}\boldsymbol{\varepsilon}^{\boldsymbol{Q}}(\mathbf{u},\mathbf{u})\right)\mathbf{C}(\varepsilon^L(\mathbf{u}^*)+\boldsymbol{\varepsilon}^{\boldsymbol{Q}}(\mathbf{u},\mathbf{u}^*))\right]d\,\Omega\\ \text{p}^{\text{ext}}=\int_{\partial\Omega_{s0}}\mathbf{F}_d.\mathbf{u}^*\,dS+\int_{\Omega_{s0}}\mathbf{f}_d.\mathbf{u}^*\,d\,\Omega\end{cases}$$

éq 2.3.1-6

In order to obtain a discretized formulation, one can rewrite the strain tensor:

$$\begin{cases}\boldsymbol{\varepsilon}(\mathbf{u})=\left[\mathbf{B}^{\mathbf{L}}+\frac{1}{2}\mathbf{B}^{\mathbf{NL}}(\mathbf{u})\right].\mathbf{u}\\ \boldsymbol{\pi}=\mathbf{C}\,\boldsymbol{\varepsilon}(\mathbf{u})\,\text{avec}\,\boldsymbol{\pi}\,\text{qui est le tenseur de Piola - Kirchhoff II}\end{cases}$$ éq 2.3.1-7

the power of the internal forces becomes:

$$\mathbf{P}^{\boldsymbol{int}}=\int_{\Omega_{SO}}Tr\left[\pi.[\mathbf{B}^{\mathbf{L}}+\mathbf{B}^{\mathbf{NL}}(\mathbf{u})]^T\mathbf{u}^*\right]d\,\Omega$$ éq 2.3.1-8

By taking account of the behavior model [éq 2.3.1-3]:

$$\mathbf{P}^{\boldsymbol{int}}=\int_{\Omega_{s0}}Tr\left[[\mathbf{B}^{\mathbf{L}}+\frac{1}{2}\mathbf{B}^{\mathbf{NL}}(\mathbf{u})]^T\mathbf{C}[\mathbf{B}^{\mathbf{L}}+\mathbf{B}^{\mathbf{NL}}(\mathbf{u})]\mathbf{u}.\mathbf{u}^*\right]d\,\Omega$$ éq 2.3.1-9

After discretization by the finite elements, one can put this equation in matric form:

$$\mathbf{u}^*.[\mathbf{K_0}+\mathbf{K^L}(\mathbf{u})+\mathbf{K^Q}(\mathbf{u})].\mathbf{u}=\mathbf{P^{ext}}$$ éq 2.3.1-10

the matrix $\mathbf{K^L}$ is symmetric and there are the following statements:

$$\begin{cases}\mathbf{K_0}=\int_{\Omega_{s0}}\mathbf{B}^{L^T}\mathbf{C}\mathbf{B}^L\,d\,\Omega\\ \mathbf{K}^L=\int_{\Omega_{s0}}\left[\frac{1}{2}\mathbf{B}^{NL}(\mathbf{u})^T\mathbf{C}\mathbf{B}^L+\mathbf{B}^{L^T}\mathbf{C}\mathbf{B}^L(\mathbf{u})\right]d\,\Omega\\ \mathbf{K}^Q=\frac{1}{2}\int_{\Omega_{s0}}\mathbf{B}^{NL}(\mathbf{u})^T\mathbf{C}\mathbf{B}^{NL}\,d\,\Omega\end{cases}$$ éq 2.3.1-11

One obtains directly what precedes the writing in matric form by the equilibrium:

$$[\mathbf{K_0}+\mathbf{K^L}(\mathbf{u})+\mathbf{K^Q}(\mathbf{u})].\mathbf{u}=\mathbf{F^{ext}}$$ éq 2.3.1-12

Is still, in an equivalent way:

$$\mathbf{F}^{int}=\mathbf{F^{ext}} \quad \text{avec} \quad \mathbf{F}^{int}=\int_{\Omega_{S0}}[\mathbf{B^L}+\mathbf{B^{NL}}(\mathbf{u})]^t \boldsymbol{\pi}\, d\, OMEGA$$ éq 2.3.1-13

We can just as easily formulate the Principle of virtual power from the stress state of Cauchy and the strain tensor of Almansi (thus on the current configuration). One obtains then:

$$\int_{\Omega_S} Tr(\boldsymbol{\sigma}\boldsymbol{\varepsilon}(\mathbf{u}^*))\,d\,\Omega=\int_{\partial\Omega_S}\mathbf{F}_d.\mathbf{u}^*\,dS+\int_{\Omega_S}\mathbf{f}_d.\mathbf{u}^*\,d\,\Omega$$ éq 2.3.1-14

That one can also put in the form, after discretization:

$$\int_{\Omega_S}\mathbf{B^T}\boldsymbol{\sigma}\,d\,\Omega=\mathbf{F}^{\text{int}}=\mathbf{F}^{ext}$$ éq 2.3.1-15

Is still, by supposing the elastic behavior model:

$$\mathbf{K}\,\mathbf{u}=\mathbf{F}^{ext} \quad \text{avec}\,\mathbf{K}=\int_{\Omega_S}\mathbf{B^T}\mathbf{C}\,\mathbf{B}\,d\,\Omega$$ éq the 2.3.1-16

integrals of these equations are calculated on the volume running Ω_S which depends, of course, of the field of solution displacement $\mathbf{u}$. In the same way, the operator $\mathbf{B}$ must be calculated on the present configuration Ω_S and not on the initial configuration Ω_{S0}, as it was the case previously.

2.3.2 Study of stability in nonlinear geometrical

One will seek if there exists a second kinematically admissible field of displacement which checks the balance equations: one thus seeks to know if there will be bifurcation.

This second field will be written as the sum of a disturbance added to the first solution, is: $\mathbf{u}=\alpha\,\mathbf{u_1}$, with α which is a very small reality and that one will make tend towards 0. The field $\mathbf{u_1}$ is selected kinematically admissible to 0.

The Principle of virtual power will be then written for this new field.

The strain field is put in the form:

$$\boldsymbol{\varepsilon}(\mathbf{u}+\alpha\,\mathbf{u_1})=\boldsymbol{\varepsilon}(\mathbf{u})+\alpha\left[\boldsymbol{\varepsilon}^L(\mathbf{u_1})+\frac{1}{2}(\boldsymbol{\varepsilon}^Q(\mathbf{u_1},\mathbf{u}))\right]+\frac{\alpha^2}{2}\boldsymbol{\varepsilon}^q(\mathbf{u_1},\mathbf{u_1})$$ éq the 2.3.2-1

virtual strains are given by:

$$\boldsymbol{\varepsilon}_1^*=\boldsymbol{\varepsilon}^L(\mathbf{u}^*)+\boldsymbol{\varepsilon}^q(\mathbf{u},\mathbf{u}^*)+\alpha\,\boldsymbol{\varepsilon}^Q(\mathbf{u_1},\mathbf{u}^*)=\boldsymbol{\varepsilon}(\mathbf{u}^*)+\alpha\,\boldsymbol{\varepsilon}^Q(\mathbf{u_1},\mathbf{u})$$ éq 2.3.2-2

In the same way, if we choose Ω_{S0} like reference configuration, the stresses become:

$$\boldsymbol{\pi}_1=\boldsymbol{\pi}+\alpha\mathbf{C}\left[\boldsymbol{\varepsilon}^L(\mathbf{u_1})+\frac{1}{2}(\boldsymbol{\varepsilon}^Q(\mathbf{u},\mathbf{u_1})+\boldsymbol{\varepsilon}^Q(\mathbf{u_1},\mathbf{u}))\right]+\frac{\alpha^2}{2}\mathbf{C}\,\boldsymbol{\varepsilon}^Q(\mathbf{u_1},\mathbf{u_1})$$ éq 2.3.2-3

We can now express the Principle of virtual power for the field disturbed displacement. Let us take as assumptions that the imposed forces do not depend on displacement and that the initial configuration is selected like reference.

$$\begin{cases} P_1^{int}=P^{int} \\ +\alpha\left[\int_{\Omega_{S0}} Tr\left(\boldsymbol{\pi}\ \boldsymbol{\varepsilon}^{Q}(\mathbf{u_1},\mathbf{u}^*)\right)d\Omega+\int_{\Omega_{S0}} Tr\left[\boldsymbol{\varepsilon}\ (\mathbf{u}^*)\mathbf{C}\left(\boldsymbol{\varepsilon}^{L}(\mathbf{u_1})+\frac{1}{2}(\boldsymbol{\varepsilon}^{Q}(\mathbf{u},\mathbf{u_1})+\boldsymbol{\varepsilon}^{Q}(\mathbf{u_1},\mathbf{u}))\right)\right]d\Omega\right]+o(\alpha) \\ P_1^{ext}=P^{ext} \\ P_1^{int}-P_1^{ext}=0 \end{cases}$$

éq 2.3.2-4

For α sufficiently small, it will be enough that the term proportional to α in the statement [éq 2.3.2-4] is null so that the Principle of virtual power is checked for the field $\mathbf{u}=\alpha\mathbf{u_1}$. In this case, there will not be thus more unicity of the solution, which will translate the loss of stability of the system.

When the imposed forces do not depend on the geometrical configuration, the study of stability is thus stated like:

Knowing the actual position, i.e the kinematically admissible field of displacement $\mathbf{u}$ and the stress field, $\boldsymbol{\pi}$ if there exists a kinematically admissible field of displacement $\mathbf{u_1}$ with 0 and such as, for any kinematically admissible displacement $\mathbf{u}^*$ with 0, one has: éq

$$\int_{\Omega_{S0}} Tr(\boldsymbol{\pi}\ \boldsymbol{\varepsilon}^{Q}(\mathbf{u_1},\mathbf{u}^*))d\Omega$$
$$+\int_{\Omega_{S0}} Tr\left[\boldsymbol{\varepsilon}^{L}(\mathbf{u}^*)\mathbf{C}\boldsymbol{\varepsilon}^{L}(\mathbf{u_1})+\boldsymbol{\varepsilon}^{Q}(\mathbf{u},\mathbf{u}^*)\mathbf{C}\varepsilon^{L}(\mathbf{u_1})+\varepsilon^{L}(\mathbf{u}^*)\mathbf{C}\boldsymbol{\varepsilon}^{Q}(\mathbf{u},\mathbf{u_1})+\boldsymbol{\varepsilon}^{Q}(\mathbf{u},\mathbf{u}^*)\mathbf{C}\boldsymbol{\varepsilon}^{Q}(\mathbf{u},\mathbf{u_1})\right]d\Omega$$
$$=0$$

2.3.2-5 Then

the problem considered is unstable. One

can express this condition of bifurcation in matric form by introducing, moreover, the geometrical stiffness matrix which $\mathbf{K}(\boldsymbol{\pi})$ discretizes the first term of it: éq

$$\forall\ \mathbf{u}^*\mathrm{CA}\ 0, \mathbf{u}^{*T}\mathbf{K_t}\mathbf{u_1}=0$$
$$\text{Avec}\quad \mathbf{K}_T=\mathbf{K}_0+\mathbf{K}^{L}(\mathbf{u})+\mathbf{K}^{Q}(\mathbf{u})+\mathbf{K}(\boldsymbol{\pi})\quad \text{qui est la raideur tangente}$$

2.3.2-6 If

one writes the condition of bifurcation on the current configuration, $OMEGa_S$ then one a: éq

$$\forall\ \mathbf{u}^*\mathrm{CA}\ 0, \mathbf{u}^{*T}[\boldsymbol{K}+\boldsymbol{K}(\boldsymbol{\sigma})]\mathbf{u_1}=0$$

2.3.2-7

the stress to be considered is then the stress of Cauchy and all the integrals are evaluated on the current field. Ω_S Stability condition

2.3.2.1 of a nonlinear elastic equilibrium It

comes immediately, that if there exists a state such that the above definite tangent $\mathbf{K_T}$ matrix is singular, we will have displayed well a field of non-zero displacement $\mathbf{u}_1$ which shows the loss of unicity of the solution of the mechanical problem. This field of displacement is the mode of buckling.

One can notice that the condition of bifurcation is well checked, whatever the norm and the sign of: $\mathbf{u}_1$ in this meaning, one thus speaks about mode of buckling, like direction, because one limited oneself in [éq 2.3.2-4] to the first order in. α Case

2.3.2.2 of small displacements: charge with Eulerian When

displacements can be qualified the small ones before buckling, one can confuse the initial configuration with the current geometry. The matrixes and $\mathbf{K}^L$ can $\mathbf{K}^Q$ then be neglected. Moreover, the stress can $\boldsymbol{\pi}$ be confused with the usual stress; $\boldsymbol{\sigma}$ the equations of buckling are written then: éq

$$[\mathbf{K}_0+\mathbf{K}(\boldsymbol{\sigma})]\mathbf{u}_1=0 \qquad 2.3.2.2 - 1 \text{ It}$$

is appropriate to notice that the matrix is $\mathbf{K}(\boldsymbol{\sigma})$ proportional to and $\boldsymbol{\sigma}$ thus to the loading applied to structure. If one multiplies the stress by, λ one obtains: éq

$$[\mathbf{K}_0+\lambda\mathbf{K}(\boldsymbol{\sigma})]\mathbf{u}_1=0 \quad [\mathbf{K}_0+\lambda\,\mathbf{K}(\boldsymbol{\sigma})]\mathbf{u}_1=0 \qquad 2.3.2.2 - 2 \text{ This}$$

equation immediately makes think of a problem generalized with the eigenvalues, of the same type as in the case of the search of the modes of vibration, which is written: éq

$$[\mathbf{K}_0\text{-}\omega^2\mathbf{M}]\mathbf{v}_1=0 \qquad 2.3.2.2 - 3$$

the matrix $\mathbf{K}(\boldsymbol{\sigma})$ is replaced by the mass matrix, M and one sees appearing the own pulsation, ω whereas is $\mathbf{v}_1$ the associated mode of vibration. If

one wishes to study buckling under loading of which only a part is controlled (variable part of the loading), by a principle of superposition, the contribution, constant, loading not controlled must be added at the end and $\mathbf{K}_0$ only the stress generated by the loading controlled will be in the term in. λ Formally, the following problem is thus posed: éq

$$[\mathbf{K}_0+\mathbf{K}(\boldsymbol{\sigma}_{cte})+\lambda\,\mathbf{K}(\boldsymbol{\sigma}_{var})]\,\mathbf{u}_1$$

$$\text{Avec :}\begin{cases}\boldsymbol{\sigma}_{cte}\ : \text{contrainte générée par le chargement non piloté}\\ \boldsymbol{\sigma}_{var} : \text{contrainte générée par le chargement piloté}\end{cases} \qquad 2.3.2.2 - \text{the } 4$$

two stress fields are obtained by resolution of two linear problems, one for the loading not controlled, the other for the controlled part of the total loading (cf *[U* 2.08.04] and [bib17]). Case

2.3.2.3 of the imposed forces geometry dependant Example

of the following pressures: When

the external forces depend on the configuration, that involves that the work of the external forces intervenes under the stability condition. Let us take the example of a pressure applied on the structure. This pressure will be supposed to be constant during buckling: in other words, the value of pressure does not change during displacement. This

assumption corresponds to two types of real problems. The first type is that where the volume of the fluid imposing the pressure on the structure is very large in front of the variations of volume generated by the displacement of solid. The internal problems of pressure tanks where displacements of walls are considerable compared to dimensions of structure itself thus do not return in this frame.

The second case corresponds to the existence of a source of fluid which makes it possible to keep the pressure with a constant value. It is not then necessary any more to worry about the amplitude of the displacement of solid.

The value of pressure being taken fixes, the variation of the norm in the course of time are to be taken into account. This variation is due to the field of displacement which modifies the surface of structure. In the same way, if one reasons in terms of resultant and thus of integral, the surface element can also change area. Consequently, the resultant of the compressive forces will vary and it is advisable to take account of it. We

see quickly that the power of the forces, expressed on the present configuration, associated with a pressure is given by the following equation (see for example [bib11]): éq

$$P^{ext}_{pression}=\int_{\partial\Omega_{SP}} p\left[\mathbf{n}+\alpha\frac{dS_1}{dS}\mathbf{n_1}\right].\mathbf{u}^*\,dS$$ 2.3.2.3 - 1 In

this equation, we notice that the power of the external forces is modified in displacement. $\alpha\,\mathbf{u_1}$ We will have then: éq

$$P^{ext}_1=P^{ext}+\int_{\partial\Omega_{SP}} p\,\alpha\,\mathbf{n_1}.\mathbf{u}^*\,dS_1$$ 2.3.2.3 - 2 Finally

, the matrix $\mathbf{K}_T$ is enriched by an additional term, function of the pressure: éq

$$\mathbf{K}_T=\mathbf{K}_0+\mathbf{K}^L(\mathbf{u})+\mathbf{K}^Q(\mathbf{u})+\mathbf{K}(\boldsymbol{\pi})+\mathbf{K}(p)$$ 2.3.2.3 - 3 If

one writes the operators on the current geometry, one leads to: éq

$$\mathbf{K}_T=\mathbf{K}+\mathbf{K}(\boldsymbol{\sigma})+\mathbf{K}(p)$$ 2.3.2.3 - 4 When

we are in the presence of follower forces of pressure, same methods that those presented previously will be able to apply the buckling loads to compute:: it will be enough to supplement the matrix with $\mathbf{K}_T$ the new term. $\mathbf{K}(p)$ One can show that the matrix is $\mathbf{K}(p)$ symmetric if the compressive forces do not work on "edge" of the model. Vibrations

2.3.2.4 under prestressed

same methodology can also under investigation apply vibrations of structure in the current configuration. Ω_S This structure is prestressed and deformed. It is enough to write the geometrical nonlinear Principle of virtual power [éq 2.3.1-6] by taking account of the effects of inertia and by injecting the assumption there that displacements are of the periodic functions of the type: éq

$$\mathbf{u}_1(t)=\mathbf{v}_1\sin(\omega t)$$ 2.3.2.4 - 1 It

results from this: éq

$$[\mathbf{K}_0+\mathbf{K}^L(\mathbf{u})+\mathbf{K}^Q(\mathbf{u})+\mathbf{K}(\boldsymbol{\pi})+\mathbf{K}(p)-\omega^2\mathbf{M}]\mathbf{v}_1=0$$ 2.3.2.4 - 2 First of all

, we notice, in this equation, that when we have a critical condition then the eigenfrequency of vibration of structure corresponding to the mode of buckling is null. Moreover

, we observe that the eigenfrequencies of structure charged are different from those of initial structure for two reasons:

The own pulsation ω is modified by prestressing formulates p it is the principal effect which is used, for example, to grant a violin. The tension of the rope exploits the height of the corresponding note, therefore on its eigenfrequency.

A second effect is the variation of the frequency by modification of the geometry: the geometrical starting stiffness matrix $\mathbf{K}_0$ is replaced by the stiffness matrix on the current geometry: . $\mathbf{K}_0+\mathbf{K}^L+\mathbf{K}^Q$ What causes to modify the vibratory equations.

Operator DYNA_NON_LINE `allows` to carry out vibratory analyses on the current nonlinear configuration (key word MODE_VIBR `),` but without taking into account of prestressing for time. Processing

2.3.2.5 of the elastoplastic behavior (plastic buckling) Far from

any exhaustiveness, we will present only the simplest approaches here, for their easy establishment in the code. When

the structure functions in an elastoplastic mode, buckling is affected by the loss of strength due to plasticity [bib2]. The modification comes from the behavior model during additional displacement. $\alpha\,\mathbf{u}_1$ The stress becomes, in incremental form: 2.3.2.5

$$\Delta\boldsymbol{\pi_1}=\Delta\boldsymbol{\pi}+\alpha\mathbf{C}_T[\Delta\boldsymbol{\varepsilon}^{\boldsymbol{L}}(\mathbf{u})+\Delta\varepsilon^Q(\mathbf{u},\mathbf{u})]+\frac{\alpha^2}{2}\mathbf{C}_T\Delta\boldsymbol{\varepsilon}^{\boldsymbol{Q}}(\mathbf{u}_1,\mathbf{u}_1)$$ - 1 In

this statement, the matrix of behavior is the tangent matrix. $\mathbf{C}_T$ The choice of this matrix is not immediate: indeed, the matrix depends on and $\alpha\mathbf{u}_1$ is thus not known as long as the mode is unknown. One can, for example, to discharge during buckling if the mode develops in a meaning and to charge if it develops in the opposite meaning. It is thus necessary to make an assumption for the behavior during plastic buckling. To start, we will apply the assumption of Hill [bib4] who leaves the principle that the structure continues to plastically charge during buckling. Let us consider
an elastoplastic model of type Von Mises. We define the three moduli: who E is the Young modulus, E_T the tangent modulus, and the secant modulus. These moduli are recalled on the following figure: E

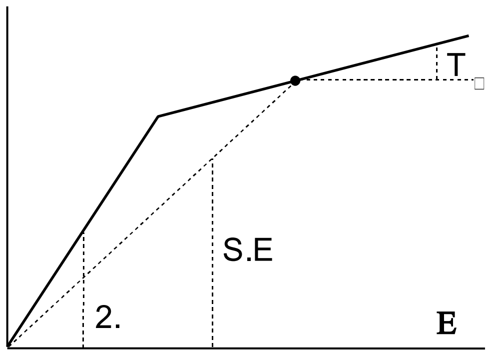


Figure - has: Representation of the various moduli on a curve of tension 1D Then

we propose three possible methodologies.
The assumption of the tangent modulus simply consists in replacing the Young modulus by the tangent modulus in the behavior model. One obtains then: 2.3.2.5

$$\mathbf{C}_T = \frac{E}{E_T}\mathbf{C}$$ - 2 This

method is very rudimentary, but it is always pessimistic, which can constitute an advantage, if one places oneself from the point of view of the design.

The method used usually consists in using the tangent matrix of incremental computation (operator STAT_NON_LINE `[feeding-bottle` 14]). We thus have the following equation in the case of the plasticity of Von Mises [bib15]: 2.3.2.5

$$\mathbf{C}_T = \mathbf{C}\left[\mathbf{I} - \frac{\mathbf{A}[\boldsymbol{\sigma}^{\boldsymbol{D}} \otimes \boldsymbol{\sigma}^{\boldsymbol{D}^T}]\mathbf{A}\,\mathbf{C}}{h + \frac{\boldsymbol{\sigma}^{\boldsymbol{D}^T}\mathbf{A}\,\mathbf{A}\boldsymbol{\sigma}^{\boldsymbol{D}}}{\|\boldsymbol{\sigma}^{\boldsymbol{D}}\|_{VM}}}\right]$$

$$\text{Avec}\begin{cases}\boldsymbol{\sigma}^{\boldsymbol{D}} : \text{vecteur déviateur des contraintes}\\ \mathbf{A} : \text{matrice intervenant dans la norme de VonMises}\left(\|\boldsymbol{\sigma}^{\boldsymbol{D}}\|_{VM} = \sqrt{\boldsymbol{\sigma}^{\boldsymbol{D}^T}\mathbf{A}\,\boldsymbol{\sigma}^{\boldsymbol{D}}}\right)\\ h : \text{pente plastique définie par } h = \dfrac{E\,.\,E^T}{E - E^T}\end{cases}$$ - 3 This

method is perfectly rigorous only in nonlinear elasticity or if the assumption of Hill is respected: it does not make it possible to predict the bifurcations in the ways of loading. As soon as the behavior model is dissipative, the calculated critical loads will not be exact that if one can check that the loading is monotonous, in any point of the structure (Hill [bib4]).

The most realistic method consists in using the finished theory of the strain only to compute: the load of plastic buckling. The tangent matrix of behavior is given by the equation below: 2.3.2.5

$$\mathbf{C}_T = \left[\left(\frac{1}{E_T} - \frac{1}{E_S}\right)\frac{\mathbf{A}[\boldsymbol{\sigma}^{\boldsymbol{D}} \otimes \boldsymbol{\sigma}^{\boldsymbol{D}^T}]\mathbf{A}}{\|\boldsymbol{\sigma}^{\boldsymbol{D}}\|_{VM}} + \mathbf{C}^{-1} + \left(\frac{1}{E_S} - \frac{1}{E}\right)\mathbf{A}\right]^{-1}$$ - 4 Compared to

the method based on the tangent stiffness matrix [éq 2.3.2.5 - 3], this criterion requires the construction and the assembly of a specific total matrix. This expensive operation comes to weigh down the incremental resolution. For
considerations of generality and minimization of the development cost and cost of computation (CPU and memory), we thus choose the criterion based on the tangent modulus [éq 2.3.2.5 - 3]. Establishment

3 in the code In

any rigor, in order to make sure the analysis of stability of a nonlinear quasi-static computation, it is necessary to use the criterion of ad hoc stability *to* each step of incremental computation. Any criterion of nonlinear stability must thus be intrinsically the least expensive possible in TEMPS CPU and core memory. Speaking
Algorithmiquement, it appears judicious to establish the call to the criterion inside even of the routine corresponding to operator STAT_NON_LINE `[feeding-bottle` 14]. Indeed, the principle of call to each step puts up badly with a completely outsourced operation of the incremental method of resolution of the nonlinear mechanical problem. Criterion

3.1 of Eulerian This

criterion (cf [§ 2.3.2.2]) requires only the resolution of a linear static problem, then the construction and the assembly of the geometrical stiffness matrix. This one and the assembled stiffness matrix is then to pass like argument of a solver [bib12] for the problem to the eigenvalues [éq 2.3.2.2 - 2]. In

output one thus recovers buckling modes and the corresponding critical loads. For more details, the user will be able usefully to consult the document [U2.08.04] [bib17]. Nonlinear

3.2 criterion Impact

3.2.1 on operator STAT_NON_LINE Let us start

by briefly pointing out the operation of the incremental method of resolution of the nonlinear structure problems [bib14]. Algorithm

3.2.1.1 of STAT_NON_LINE One

will use index I *(like* "time") to note the number of an increment of load and exhibitor N *(like* "Newton") to note the number of the iteration of Newton in progress. The algorithm used in operator STAT_NON_LINE can then be written schematically in the following way: and

$(\mathbf{u}_0, \boldsymbol{\lambda}_0)$ known $\boldsymbol{\sigma}_0$ Buckles

over times (or t_i increments of load): loading known $\mathbf{L}_i=\mathbf{L}(t_i)$

- $(\mathbf{u}_{i-1}, \boldsymbol{\lambda}_{i-1})$ Prediction
- : computation of and $\Delta\mathbf{u}_i^0$ Buckles $\Delta\boldsymbol{\lambda}_i^0$
- on iterations of Newton: computation of a known continuation $(\Delta\mathbf{u}_i^n, \Delta\boldsymbol{\lambda}_i^n)$
  - $(\mathbf{u}_i^n, \boldsymbol{\lambda}_i^n)$ and $(\Delta\mathbf{u}_i^n, \Delta\boldsymbol{\lambda}_i^n)$ Computation
  - of the matrixes and vectors associated with the following loads Form
  - of the behavior model computation
    - of the stresses and $\boldsymbol{\sigma}_i^n$ local variables from $\boldsymbol{\alpha}_i^n$ the values and $\boldsymbol{\sigma}_{I-1}$ with $\boldsymbol{\alpha}_{I-1}$ the preceding equilibrium () and t_{i-1} with the displacement increment since $\Delta\mathbf{u}_\mathbf{i}^\mathbf{n}=\mathbf{u}_i^n-\mathbf{u}_{i-1}$ this equilibrium computation
    - of the "nodal forces": possible $\mathbf{Q}^\mathbf{T}\boldsymbol{\sigma}_i^n+\mathbf{B}^T\boldsymbol{\lambda}_i^n$
    - computation of the tangent stiffness matrix: Computation $\mathbf{K}_i^n=\mathbf{K}(\mathbf{u}_i^n)$
  - of the direction of search per $(\Delta\mathbf{u}_i^{n+1}, \Delta\boldsymbol{\lambda}_i^{n+1})$ resolution of a linear system Iterations
  - of linear search: Actualization ρ
  - of the variables and their increments: Test

$$\begin{cases}\mathbf{u}_i^{n+1}=\mathbf{u}_i^n+\rho\boldsymbol{\delta}\mathbf{u}_i^{n+1}\\ \boldsymbol{\lambda}_i^{n+1}=\boldsymbol{\lambda}_i^n+\rho\boldsymbol{\delta}\boldsymbol{\lambda}_i^{n+1}\end{cases}\text{et}\begin{cases}\Delta\mathbf{u}_i^{n+1}=\Delta\mathbf{u}_i^n+\rho\boldsymbol{\delta}\mathbf{u}_i^{n+1}\\ \Delta\boldsymbol{\lambda}_i^{n+1}=\Delta\boldsymbol{\lambda}_i^n+\rho\boldsymbol{\delta}\boldsymbol{\lambda}_i^{n+1}\end{cases}$$

  - of convergence Archivage
- of the results at time One t_i

$$\begin{cases}\mathbf{u}_i=\mathbf{u}_{i-1}+\Delta\mathbf{u}_i\\ \boldsymbol{\lambda}_i=\boldsymbol{\lambda}_{i-1}+\Delta\boldsymbol{\lambda}_i\\ \boldsymbol{\sigma}_i\\ \boldsymbol{\alpha}_i\end{cases}$$

notices that there are three overlapping levels of loops: an external loop on time step, a loop of iterations (qualified the total ones) of Newton and possible subloops for the linear search (if she is asked by the user) and certain behavior models requiring of the iterations (known as interns), for example for elastoplasticity in plane stresses. If

one chooses the criterion based on the assembled tangent matrix [éq 2.3.2.5 - 3], it is necessary to have this matrix reactualized for each step where one wants to analyze stability. It is the case when one uses a method of the Newton type, and not a modified method of the Newton type. One

leads then to the following algorithm: and

$(\mathbf{u}_0, \boldsymbol{\lambda}_0)$ known $\boldsymbol{\sigma}_0$ Buckles

over times (or t_i increments of load): loading known $\mathbf{L}_i = \mathbf{L}(t_i)$

- $(\mathbf{u}_{i-1}, \boldsymbol{\lambda}_{i-1})$ Prediction
- : computation of and $\Delta\mathbf{u}_i^0$ Buckles $\Delta\boldsymbol{\lambda}_i^0$
- on iterations of Newton: computation of a known continuation $(\Delta\mathbf{u}_i^n, \Delta\boldsymbol{\lambda}_i^n)$
  - $(\mathbf{u}_i^n, \boldsymbol{\lambda}_i^n)$ and $(\Delta\mathbf{u}_i^n, \Delta\boldsymbol{\lambda}_i^n)$ Computation
  - of the matrixes and vectors associated with the following loads Form
  - of the behavior model computation
    - of the stresses and $\boldsymbol{\sigma}_i^n$ local variables from $\boldsymbol{\alpha}_i^n$ the values and $\boldsymbol{\sigma}_{l-1}$ with $\boldsymbol{\alpha}_{l-1}$ the preceding equilibrium () and t_{i-1} with the displacement increment since $\Delta\mathbf{u}_\mathbf{i}^\mathbf{n} = \mathbf{u}_i^n - \mathbf{u}_{i-1}$ this equilibrium computation
    - of the “nodal forces”: possible $\mathbf{Q}^\mathbf{T}\boldsymbol{\sigma}_i^n + \mathbf{B}^T\boldsymbol{\lambda}_i^{\boldsymbol{n}}$
    - computation of the tangent stiffness matrix: Computation $\mathbf{K}_i^n = \mathbf{K}(\mathbf{u}_i^n)$
  - of the direction of search per $(\Delta\mathbf{u}_i^{n+1}, \Delta\boldsymbol{\lambda}_i^{n+1})$ resolution of a linear system Iterations
  - of linear search: Actualization ρ
  - of the variables and their increments: Test

$$\begin{cases}\mathbf{u}_i^{n+1} = \mathbf{u}_i^n + \rho\,\boldsymbol{\delta}\,\mathbf{u}_i^{n+1}\\ \boldsymbol{\lambda}_i^{n+1} = \boldsymbol{\lambda}_i^n + \rho\,\boldsymbol{\delta}\,\boldsymbol{\lambda}_i^{n+1}\end{cases} \text{et} \begin{cases}\Delta\mathbf{u}_i^{n+1} = \Delta\mathbf{u}_i^n + \rho\,\boldsymbol{\delta}\,\mathbf{u}_i^{n+1}\\ \Delta\boldsymbol{\lambda}_i^{n+1} = \Delta\boldsymbol{\lambda}_i^n + \rho\,\boldsymbol{\delta}\,\boldsymbol{\lambda}_i^{n+1}\end{cases}$$

  - of convergence Archivage
- of the results to time Criterion t_i

$$\begin{cases}\mathbf{u}_i = \mathbf{u}_{i-1} + \Delta\mathbf{u}_i\\ \boldsymbol{\lambda}_i = \boldsymbol{\lambda}_{i-1} + \Delta\boldsymbol{\lambda}_i\\ \boldsymbol{\sigma}_i\\ \boldsymbol{\alpha}_i\end{cases}$$

- of stability, function of the reactualized tangent stiffness: $\mathbf{K}_i^n = \mathbf{K}(\mathbf{u}_i^n)$

The criterion is calculated at the end of the step, just after the archivage. It thus has like arguments the quantities converged with the current step. Moreover, this choice of position of call makes it possible to take account correctly following loadings, since their computation is made the iterations of Newton during. The criterion could not thus be called before the end of these iterations. Impact

3.2.1.2 on the structure of data result of STAT_NON_LINE

the call of the nonlinear criterion of stability will induce the resolution of a problem to the eigenvalues. Result of this computation will be thus a set of couples critical load/mode of buckling.
The critical loads are scalars and the associated modes are fields of displacement, which will come to enrich data structure of STAT_NON_LINE result . `Characteristics`

3.2.2 related to the strain tensor In

the code, it is advisable to distinguish two large families from description of the strains. On the one hand

the linearized tensor corresponds to the case of the small disturbances (argument PETIT of key word DEFORMATION), but also to the case of the small disturbances reactualized (Lagrangian reactualized with each step of incremental computation: argument PETIT_REAC of key word DEFORMATION).

The strain tensor is written then (like [éq 2.2-3]): éq

$$\boldsymbol{\varepsilon}=\frac{1}{2}\left(\nabla\,\mathbf{u}+\nabla^{T}\,\mathbf{u}\right) \quad 3.2.2\text{-}1$$

the use of PETIT_REAC implies a resolution of the equilibrium of structure on its current geometry with a linearized strain tensor. One thus calculates the increment of strain compared to the position, $\mathbf{X}$ with displacement and $\mathbf{u}$ the displacement increment in the following way $\Delta\,\mathbf{u}$: éq

$$\Delta\,\varepsilon_{ij}=\frac{1}{2}\left(\frac{\partial\,\Delta\,u_i}{\partial(X+u)_j}+\frac{\partial\,\Delta\,u_j}{\partial(X+u)_i}\right) \quad 3.2.2\text{-}2$$

In addition, the code offers strain tensors of the type Green-Lagrange (GROT_GDEP) for the processing of large displacements (and of the rotations finished for certain structural elements) but under assumption of small strains. The tensor used is the following classical tensor [éq 2.3.1-5]: éq

$$\varepsilon_{ij}(u)=\frac{1}{2}\left(u_{i,j}+u_{j,i}=u_{k,i}u_{k,j}\right) \quad 3.2.2\text{-}3$$

key word GROT_GDEP applies to the modelizations beam, shell or 3D. Lastly,

the frame of modelization in great transformations most complete accessible in the Code_Aster *is* resulting from the theory of Simo and corresponds to the key word SIMO_MIEHE . He takes into account large rotations and the large deformations since the constitutive law is written in large deformations. For more precise details on the fundamental differences between the various types of strains, the documentation [bib16] of Code_Aster *presents* in detail modelization SIMO_MIEHE . Code_Aster

does not allow computations in eulerian configuration: as with the tensor of Almansi, for example. All the strain tensors available are of Lagrangian type.

The fundamental difference, as for the writing of the criterion, is between the linearized strains (PETIT and PETIT_REAC) and the strains GROT_GDEP and SIMO_MIEHE . Indeed

, the Code_Aster *needs* to make its search for equilibrium of the tangent matrix. This one is written according to the equation ([§ 2.2.2.1] of documentation on STAT_NON_LINE [feeding-bottle 14]): éq

$$\mathbf{K}_T=\mathbf{Q}^T:\frac{\partial\boldsymbol{\sigma}}{\partial\mathbf{u}}+\frac{\partial\,\mathbf{Q}^T}{\partial\,\mathbf{u}}:\boldsymbol{\sigma} \quad 3.2.2\text{-}4$$

Gold, corresponds $\mathbf{Q}^T:\frac{\partial\boldsymbol{\sigma}}{\partial\mathbf{u}}$ at the end classic of the material stiffness and corresponds $\frac{\partial\mathbf{Q}^T}{\partial\mathbf{u}}:\boldsymbol{\sigma}$ at the end of geometrical stiffness which is present only in large displacements. Thus when one wants to apply a criterion of buckling of the type (formally assimilable to [éq 2.3.2.2 - 2]):. $\left(\mathbf{K}+\lambda\,\mathbf{K}(\boldsymbol{\sigma})\right)\boldsymbol{\nu}=0$ This

criterion is valid only in small strains since the geometrical term of stiffness is regarded as negligible in the tangent matrix. One can thus, with reason, to make a classical search of the eigenvalues and eigenvectors of type buckling of Eulerian. On the other hand

in great transformations, the evaluating of this criterion by the same method is problematic for two reasons. On the one hand, in the tangent matrix, the geometrical term of stiffness is already calculated and, on the other hand, matrix qu $\mathbf{K}(\boldsymbol{\sigma})$ "it would possibly be necessary to add is obtained under Code_Aster *in* small strains. For these reasons, it is necessary D" to evaluate in a way different the criterion according to the type of strain tensor requested by the user. If

one made the choice of an eulerian description, the development of a criterion of the Eulerian type reactualized would be facilitated on the level of the computation of the term, $\mathbf{K}(\boldsymbol{\sigma})$ whatever the strain tensor. In

3.2.2.1 linearized strains: PETIT and PETIT_REAC As

we said previously, this case does not pose a major problem. It is enough to calculate the geometrical stiffness matrix and to make a classical search for modes and eigenvalues, of type Eulerian [éq 2.3.2.2 - 2]: éq

$$(\mathbf{K}+\lambda\,\mathbf{K}(\boldsymbol{\sigma}))\boldsymbol{\nu}=0$$ 3.2.2.1 - 1 is

$\mathbf{K}$ the tangent matrix reactualized at the end of time step. In this case,
one can thus speak indeed about criterion of the Eulerian type reactualized. As
one is in small strains, the matrix of the geometrical stiffness is proportional to the loading. Therefore, when the critical coefficient is obtained, it λ is enough to multiply it by the real load with time step current to obtain the critical load of buckling. The case thus $\lambda=1$ corresponds to the loss of stability. Certain
finite elements as shells DKT do not allow the computation of the geometrical stiffness matrix, contrary to the elements of the type COQUE_3D , for example. In

3.2.2.2 large displacements: GROT_GDEP and SIMO_MIEHE

the classical method does not apply any more in this case. Indeed, Code_Aster *calculates* like tangent matrix the material stiffness matrix plus the geometrical stiffness matrix (and possibly, the contribution due to the following pressures). One
of the ways check buckling then is to only make a search of the eigenvalues of the tangent matrix. If one of the eigenvalues is negative, it is that the matrix became singular and that an instability occurred between the moment when all its eigenvalues were positive and moment when one of it became negative.
The problem with treating is thus slightly different since in the case of small strains (PETIT and PETIT_REAC), one has the following system to solve [éq 3.2.2.1 - 1]: , whereas $(\mathbf{K}+\lambda\,\mathbf{I})\boldsymbol{\nu}=0$ in case GROT_GDEP and SIMO_MIEHE it is necessary to solve: formuleéq

$$(\mathbf{K}+\lambda\,\mathbf{I})\boldsymbol{\nu}=0$$ 3.2.2.2 - 1 With

which $\mathbf{I}$ is the matrix identity and is λ , this time, of physical size equivalent to, $\mathbf{K}$ whereas in the case of the small strains, the eigenvalue is λ adimensional (from where its direct interpretation as a multiplying coefficient of the loading). One

of the defaults inherent in this method compared to the more classical search higher explained [§2.3.22.3.2 why one can have forecasts of buckling only when one approaches "close" the critical load, even when one exceeds it. Far from this load, the first found eigenvalue does not have really physical meaning since nonlinearities can appear between the step running and the calculated critical load. The critical coefficient ratio on load at time is i thus different from that at time whereas $i+1$ in small strains this ratio remains constant. Moreover

, for all time step, all the eigenvalues and eigenvectors except lowest do not have any physical meaning since, for a couple eigenvector eigenvalue, one $(\mathbf{V}_i, \lambda_i)$ a: éq

$$(\mathbf{K}(\mathbf{u})+\mathbf{K}(\boldsymbol{\sigma}))\mathbf{V}_i=\lambda_i\mathbf{V}_i$$ 3.2.2.2 - 2 This

has clear meaning only as from the moment when, $\lambda_i \to 0$ in which case the critical load is found and the eigenvector criticizes associated. Always
compared to criterion of Eulerian (reactualized [éq 3.2.2.1 - 1] or not [éq 2.3.2.2 - 2]), one notices that the eigenvalue of the problem [éq 3.2.2.2 - 1]: $(\mathbf{K}+\lambda\mathbf{I})\boldsymbol{\nu}=0$ is not adimensionnée. It results from this a greater difficulty from interpretation as for knowing if the value is "small" or not. In other words, when can one say that one is close to a bifurcation?
To define a relevant interval and of general use, in order to limit the vicinity of an instability, it would be interesting of adimensionner the eigenvalues. Case

3.2.2.3 of the mixed modelizations As

Code_Aster *makes it possible* to assign several types of strains to same structure, the case should be considered where one uses several types of strain tensors in same computation.
The differentiation of the various elementary matrixes being of no utility, it is appropriate to be solved to slice at the total level between a method or the other. One chose to extract the values and eigenvectors from the tangent matrix without adding geometrical stiffness matrixes. All occurs as if the structure were in strain of the Green-Lagrange type from the point of view of the criterion. Indeed, I let us consider an unspecified solid made up of two parts and II. On the part I, the strain tensor which was adopted is the tensor linearized PETIT `and` on part II that of Green - Lagrange. The tangent matrix from the assembly of the two submatrices becomes: éq

$$\begin{bmatrix} \mathbf{K}_I & * & 0 \\ * & * & * \\ 0 & * & \mathbf{K}_{II}+\mathbf{K}_{II}(\boldsymbol{\sigma}) \end{bmatrix}$$ 3.2.2.3 - 1

the spangled terms represent the nodes common to both parts and are thus a linear combination of the values of the two matrixes. In this configuration, it appears that none the solutions is satisfactory but that less penalizing is to make a search for "type Green - Lagrange" [§ 3.2.2.23.2.2.2 use [éq $(\mathbf{K}+\lambda\mathbf{I})\boldsymbol{\nu}=0$ 3.2.2.2 - 1]. This

solution not being exact but nevertheless the only able one to be carried out simply, it is envisaged to add an alarm message informing the user whom the got results are not guaranteed due to the juxtaposition of several types of strain tensors. Improvement

3.2.3 of performances of criterion During

resolution incremental of problem quasi-static nonlinear, in ideal and if it is admitted that the discretization in time is sufficiently fine, he would be necessary to make an analysis of stability to each computation step. A each step, that induces the resolution of a problem to the eigenvalues, certainly limited in search of some modes. The analysis of stability thus brings an important overcost CPU, with a nonlinear computation already being able to be long.

The idea is to call on the resolution of a problem to the eigenvalues only when it is really necessary, therefore when the current configuration is "close" to an instability. If one can define this vicinity by a preset interval, then one can call on a test of Sturm type [bib12]. This
test makes it possible to know if there exists at least an eigenvalue on the interval of search. In the affirmative, one will be able to then carry out the modal search. In the contrary case, one continues the quasi-static incremental resolution, without solving problem with the eigenvalues.
The cost of a test of Sturm type is notably lower than the cost of search of the critical loads.
The interval of search for the test of Sturm type can, either to be given by the user, or to have a value by default in the code. In the case of

a criterion of Eulerian reactualized (case of the small strains [§ 3.2.2.13.2.2.1 where the problem to be solved is written: [éq $(\mathbf{K}+\lambda\,\mathbf{K}(\boldsymbol{\sigma}))\boldsymbol{\nu}=0$ 3.2.2.1 - 1], the interval of search must be centered on the eigenvalue (which $\lambda=1$ corresponds to value -1 for the algorithm of MODE_ITER_SIMULT , because he solves in fact a problem of the type:)) $\mathbf{K}\,\boldsymbol{\nu}=\mu\,\mathbf{K}(\boldsymbol{\sigma})\,\boldsymbol{\nu}$.
The limits of the interval are the limits of the multiplying coefficient of the loading, therefore adimensional quantities, which are function of the safety coefficients and the evaluating of uncertainties for the problem given. The test of Sturm type is implemented in this frame. In
the specific case adapted to the tensor of Green-Lagrange [§ 3.2.2.23.2.2.2 where one solves: [éq $(\mathbf{K}+\lambda\,\mathbf{I})\,\boldsymbol{\nu}=0$ 3.2.2.2 - 1], the interval is centered on 0. Moreover, the limits of the interval of test, contrary to the preceding case, are not adimensionnées [§ 3.2.2.23.2.2.2It is thus more difficult to identify relevant and general values (for the case of the default values). The test of Sturm type is not currently established for this case. Generalization

3.3 with the dynamics We

will not approach here the frame of the criteria of dynamic instability (negative damping…). It is just a question of announcing that the nonlinear criterion presented here can completely apply directly in nonlinear dynamics. It will then detect any potential buckling of structure, within the meaning of the singularity of the total matrix of reactualized tangent stiffness. In order to
be exhaustive in terms of analysis of stability on a nonlinear dynamic study, the user should use two criteria:

- •a criterion of buckling (criterion on the stiffness),
- •a dynamic criterion (criterion on damping or the total quadratic linearized problem [bib13], for example). For

time, the criterion of buckling on the stiffness (identical to that of STAT_NON_LINE) is only available in DYNA_NON_LINE . The modelization

coupled fluid-structure (U, p, F) [R4.02.02], which is available in DYNA_NON_LINE , requires some adaptations of use of the nonlinear criterion of stability. Indeed, this coupled formulation generates an intrinsically singular stiffness matrix on all the total assembled fluid degrees of freedom, which makes it incompatible with the research method of eigenvalues used for the analysis of stability. One can however circumvent the problem by correcting the problem assembled (stiffness matrix and of geometrical stiffness if need be) thanks to the use of two specific key words. The analysis of stability relates then to the structures degrees of freedom alone. Syntax

3.4 of call of the criterion In

operators STAT_NON_LINE or DYNA_NON_LINE , the call to the criterion is done in the following way: CRIT

```
_STAB = _F (CHAR_CRIT       = (-1.1, 0. ), NB_FREQ
                    = 3. ),
```

key word CHAR_CRIT defines the field on which one will make the test of Sturm type, in small linearized strains. If one finds at least an eigenvalue on the interval, then, one leads the resolution of the corresponding problem to the eigenvalues, if not, one does nothing and incremental computation can continue. If

one uses modelizations GROT_GDEP or SIMO_MIEHE , the modal resolution takes place inevitably, and one searches the smallest eigenvalues.
Key word NB_FREQ makes it possible to specify the number of modes which one searches (default value: 3). It can be useful to search more than one mode, mainly to be able to detect the "pathological" cases such as multiple or very close modes.

The mode of buckling corresponding to the smallest eigenvalue (in absolute value) is stored in the data structure RESULTAT (eigenvalues λ named CHAR_CRIT , fields of displacement named MODE_FLAMB , which one can visualize via IMPR_RESU). For the analyses of stability on transitory problems fluid-structure coupled (U, p, F) it is necessary to add two specific key words MODI _RIGI and DDL_EXCLUS under CRIT _STAB. U2.08.04 documentation gives all the details on their use and benchmark FDNV100 gives of it an example of implementation. Validation

3.5 of the developments

the cases tests of validation are: SSNL126 and SSLL105D.
More precisely, the cases tests SSNL126 treat the case of a beam fixed at an end and subjected to a compression at the other end. The modelization is three-dimensional, with elastoplastic behavior model with linear isotropic hardening. Two kinematical representations are presented: modelization

•a: linearized strains, modelization
•b: strains of Green-Lagrange.

The case test SSLL105D is based on a problem of beam in, L which one studies elastic buckling. The finite elements are of standard beam. Code_Aster

4 conclusions

offers two criteria of stability, within the meaning of buckling, for the structural analyzes. On the one hand

, whenever a linearized approach is enough, one can apply a criterion of the Eulerian type ([bib12] and [bib17]), by call to an operator of resolution of the problem to the eigenvalues generalized (for example MODE_ITER_SIMULT with key word TYPE _RESU=' MODE_FLAMB'). In addition

, for all the cases where it is essential to take account of nonthe linearities, which they to the behavior model or the great transformations, the user is due can employ an adapted criterion, of Eulerian type generalized. The call of this criterion is done during the incremental resolution of the quasi-static problem (operator STAT_NON_LINE [feeding-bottle 14]). A
each time step, the criterion is based on the resolution of a problem to the eigenvalues [bib12] on the brought up to date total stiffness matrixes. This
criterion, which is declined in two different forms, according to the strain tensor chosen, is based on a linearization around the current computation step. It accepts any type of strain tensor, as any type of behavior model for which one is able to build the total stiffness matrix, at every moment. Moreover, the selected criterion is perfectly rigorous in the case of the nonlinear elastic behavior models, and in the case of elastoplasticity associated with the assumption with Hill [§ 2.3.2.52.3.2.5Bibliography

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