Calculation algorithm of the densities of Summarized

reinforcement:

One presents the calculation algorithm of the densities of reinforcement of plates and reinforced concrete shells established within command CALC_FERRAILLAGE [U4.81.42]. This algorithm was proposed in 1978 by Alain Capra and Jean-Francis Maury [bib1].

Contents

1 general Presentation of the method

This method [bib1], developed in 1978 by Alain Capra and Jean-Francis Maury (engineers at the technical Management of Socotec), refer on the matter and constitute the algorithm generally established in the computer codes of reinforcement of plates and reinforced concrete shells.

The method is founded on the principle of the equilibrium of each facet centered at the point of computation and whose norm turns in the tangent plane to the average average. It is about a postprocessing of a static computation of a reinforced concrete plate or shell, led from the state of forces generalized previously obtained.

For each position of the facet, one carries out a rectangular computation of section submitted to made up bending and one from of deduced the two sections from reinforcements higher and lower. These two sections are then readjusted in two sections (by unit of length) according to the axis X (AXS for the higher three-dimensions function and AXI the lower three-dimensions function) and two sections (by unit of length) along the axis Y (AYS for the higher three-dimensions function and AYI the lower three-dimensions function).

One proceeds then, using an adapted algorithm, in search of the optimum for each three-dimensions function (lower and higher) corresponding to the value minimum of the sum $AXS+AYS$ (respectively $AXI+AYI$).

The article of Capra and Maury also proposes a computation of transverse reinforcement partly current of plate, evaluated from the equivalent shearing stress which S `expresses according to the formula:

$$\tau=\frac{1}{\zeta}\sqrt{(T_{ZX}^2+T_{ZY}^2)}$$

where $\zeta=I/m(0)$ the lever arm of the elastic couple of the section represents and T_{ZX} T_{ZY} are the shears. The section of transverse reinforcement is simply obtained by dividing this stress by the acceptable ultimate stress of steel.

This method, at the cost of a cost of computation a little more extremely than in other methods, in particular that of Wood (rotation of the facet and search of the optimum) makes it possible to obtain an optimal steel distribution. The computation transverse reinforcement remains, him, rather summary. In particular, he does not take account of the contribution of the normal force as the BAEL requires it.

The stress analysis applied to a section is carried out, out of standard, according to the BAEL91. The user can however modify the value of the pivots in order to adapt to another regulation.

2 Detailed description of the algorithm

the object of this chapter is to describe the method of Capra and Maury in a detailed way and to also present the technique used to establish this method within *Code_Aster* as well as the additional operations necessary to its implementation.

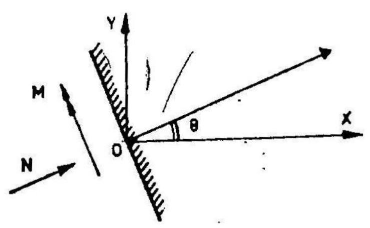
In a very total way, the algorithm used can be schematized in the following way:

- Acquisition of the parameters of configuration (type of computation ELS/ELU, values characteristic of the ultimate stresses for steel and concrete, value of coating etc)
- Acquisition of the characteristics of the elements (thickness, forces of membrane, bending moments)
- Determination of the lawful stresses according to the type of computation carried out (ELU/ELS)
- For each treated element, determination of reinforcement.

This last part, which is the heart of the method of Capra and Maury is presented in the paragraph according to

2.1 original Algorithm of Capra and Maury

One defines a set of facets, centered at the point of computation, whose norm turns in the tangent plane to the average average. The facet is located by the angle θ which its norm with the axis of OX the reference of the element forms (see figure 2.1-a). The angle θ is discretized regularly $-90°$ with $+90°$ (generally with a step of $10°$). The axes Ox and Oy are the axes of the three-dimensions functions of reinforcements.



Appear 2.1-a [bib1]

For each one of these facets, one evaluates the bending moment (M) and the tension of membrane (N) which apply to it according to the tensors of the forces using the equations:

$$M = M_{xx}\cos^2\theta + M_{yy}\sin^2\theta - 2\,M_{xy}\sin\theta\cos\theta$$
$$N = N_{xx}\cos^2\theta + N_{yy}\sin^2\theta - 2\,N_{xy}\sin\theta\cos\theta$$

By a computation in composed bending, one can determine the tensile forces $FI(\theta)$ and $FS(\theta)$, perpendicular to the section, which must be balanced by the lower and higher three-dimensions functions of reinforcement.

The resistant forces in the direction θ of the two three-dimensions functions can be evaluated using the statements $FRI = (AXI\cos^2\theta + AYI\sin^2\theta)\sigma$ and $FRS = (AXS\cos^2\theta + AYS\sin^2\theta)\sigma$ where σ the acceptable maximum stress of steel represents (identical in the two directions).

Strength is assured if the resistant force is higher than the force applied, which is written:

$FI < FRI$ and $FS < FRS$.

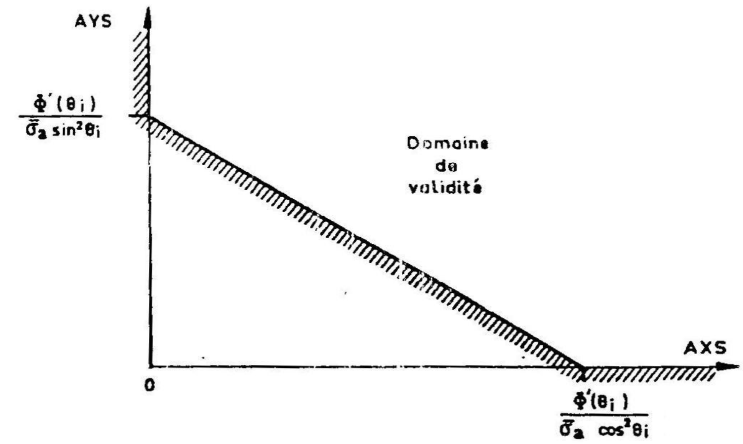
Thus, by considering an orthonormal reference comprising AXS in X-coordinate and AYS in Y-coordinate, one has to solve finally for higher reinforcement:

$$AXS\cos^2\theta + AYS\sin^2\theta \geq FS(\theta)/\sigma \text{ for all the angles } \theta \qquad (1)$$

and

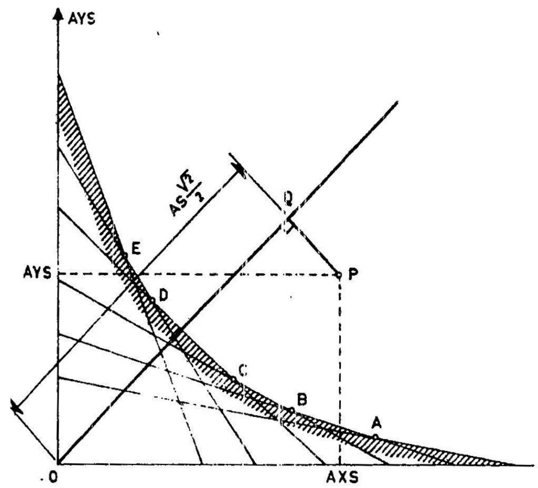
$AXS + AYS$ minimum.

The inequalities (1) define for each value of θ a half space limited by a line of negative slope which translates a field of validity (figure 2.1-b).



Appear 2.1-b [bib1]

By traversing all the values of θ, one obtains the field of validity indicated on the figure 2.1-c, delimited by broken line $A\,B\,C\,D\ldots$



the Figure 2.1-c [bib1]

For each point P of the field of validity, the total section of reinforcements can be obtained by projecting the point P in Q on the first bisectrix. The distance OQ represents the value then $(AS\sqrt{2})/2$ with $AS = AXS + AYS$.

It is thus noted that the optimum of reinforcement corresponds to the point of the edge $A\,B\,C\,D\ldots$ to which projection on the first bisectrix is closest to the origin of the axes. The search for this point is carried out by a method of type “dichotomy”.

2.2 Establishment retained in Code_Aster

the establishment selected is a little different in order to optimize the computing time. Moreover, during the computation of the section subjected to composed bending, one carries out a checking of nongoing beyond the stress concrete in the ELS, and of nongoing beyond the strain of the concrete with the ELU.

At the conclusion of computation in composed bending, one determines the total section of steels (higher and lower) necessary to the equilibrium of the forces. One thus has a series of triplets (AS AI , θ) representative of each facet (AS representing the higher section, AI the lower section and θ the angle characteristic of the facet).

One then carries out the optimization of reinforcement for the lower section, then higher using the original algorithm describes below.

It should be noted that all computations relate to only tended steels. It is not envisaged to be able to treat compressed steels.

2.2.1 Stress analysis with the ELU

the stress analysis leads to the determination of FI (lower force) and FS (higher force) from M (bending moments) and N (forces of membrane).

- For the forces of membrane (N) , a positive value means tractive effort
- For the bending moments (M) , positive value translates a tensile stress into bottom fiber of the element. This convention being opposite of that used within Code_Aster, an inversion of sign is carried out at the beginning of computation.

The goal of this algorithm is to determine the tractive effort to which each steel bed is subjected, in order to determine the section of reinforcements by simple division by the working stress of steel.

One calculates the significant height Hs (distance between useful steel and compressed fiber) and the useful height Hu (distance separating the two beds from steel).

One evaluates then the ultimate moment in the reinforced concrete section which steels must take again and which takes account of the bending moment and the force of membrane.

If this moment is negative, one is in the situation of an entirely tended section. The computation is then particularly simple, each three-dimensions function taking again the tractive effort which is applied to him.
If not, one establishes the value of the reduced moment M_{ub} which, if it is higher than 0.48 conduit with an impossibility of computation (no tended steel).

One evaluates successively the value of the coefficient α , the stress concrete and the total force to which is subjected the bed more tended. If this force is negative, that translated an entirely compressed section for which no steel section is necessary. If this force is indeed positive, one it attribute with the actually tended section, provided that the compressive stress of the concrete did not exceed working stress what starts a stop and an error message.

The sections of reinforcement are finally drawn up by dividing the force by the working stress of steel.

2.2.2 Stress analysis in the ELS

the stress analysis leads to the determination of FI (lower force) and FS (higher force) from M (bending moment) and N (force of membrane)

- For the forces of membrane (N) , a positive value means tractive effort

- For the bending moments (M), positive value translates a tensile stress into bottom fiber of the element. This convention being opposite of that used within Code_Aster, an inversion of sign is carried out at the beginning of computation.

The goal of this algorithm is to determine the tractive effort to which each steel bed is subjected, in order to determine the section of reinforcements by simple division by the working stress of steel.

One calculates the significant height Hs (distance between useful steel and compressed fiber) and the useful height Hu (distance separating the two beds from steel).

One evaluates then the moment of service which steels must take again and which takes account of the bending moment and the force of membrane.

If this moment is negative, one is in the situation of an entirely tended section. The computation is then particularly simple, each three-dimensions function taking again the tractive effort which is applied to him.

If not, one calculates, using the method of Newton, the exact position of neutral fiber by solving an equation of the 3rd degree in b (in fact in the programming, this variable is called in fact alpha). Once this one known, one evaluates the value of the stress concrete and the force mentioned in the most tended section. If this force is negative, that translated an entirely compressed section for which no steel section is necessary; one then corrects simply the stress compressive of the concrete. If this force is indeed positive, one it attribute with the actually tended section.

A final test is carried out on the compressive stress of the concrete. If this one is beyond the acceptable limit, an error message is displayed and a positioned indicator.

The sections of reinforcement are finally drawn up by dividing the force by the working stress of steel.

2.2.3 Algorithm of optimization of the steel section

the algorithm used aims at geometrically searching the optimum of reinforcement in the orthonormal reference comprising AXS in X-coordinate and AYS Y-coordinate.

The discretization of the rotation of the facet equalizes with $10°$, led to a classification of these facets from 1 to 18.

In the orthonormal reference used, one positions 3 points $P0$, $P1$ and $P2$. The coordinates of the point $P1$ correspond respectively to the steel section of facet 9 (corresponding with a value zero of θ) and to the steel section of facet 1 (corresponding with a value of θ equal to $+\pi/2$). The point $P0$ is located at the same X-coordinate as $P1$ but with a very large Y-coordinate (thus defining a practically infinite vertical), and the point $P2$ is located at the same Y-coordinate as $P1$ but with a very large X-coordinate (thus defining horizontal practically infinite).

One then carries out a loop on all the facets of $-\pi/2$ with $+\pi/2$ by excluding the first as well as the facet corresponding to the value zero from θ, facets which were implicitly treated at the time of the definition of the initial points.

One then determines the points of intersection of the facet of reference with the other segments. In each case, a comparison of the position of the point representative of the facet treated compared to the field of validity of the facet of reference makes it possible to determine the directional sense of the segment to take into account for the search of the intersections.

At the conclusion of this processing, one has the points representative of the polyline defining the total field of validity.

At the end of the processing, one carries out a sweeping on the points selected, in order to determine that whose sum of the coordinates is minimum (sum of reinforcements minimum). In the event of identity (with 10^{-5} near into relative), the facet of smaller number is retained.

3 Establishment in *Code_Aster*

operator CALC_FERRAILLAGE [U4.81.42] makes it possible to determine the reinforcement of an element or a set of shell elements and shells of the type DKT.
Its implementation is illustrated by benchmarks SSLS134 [V3.03.134] and SSLS135 [V3.03.135].

4 Bibliographical references

[1] automatic Computation of the optimal reinforcement of the plates and reinforced concrete shells. Alain CAPRA and Jean-Francis MAURY Annals of the Technical Institute of the Building industry and public works, n°367, 12/1978.

5 Description of the versions of the document

Version Aster	Author (S)/Organization (S)	Description of modifications
10	*P. LANG (INGEROP IES)*	initial Text