Multiaxial criteria of starting in fatigue

Summarized:

In this note we propose a formulation of the criteria of MATAKE, DANG VAN and FATEMI-SOCIE and criteria in formula adapted to the frame of the office plurality of damage under periodic and **non** periodic **multiaxial loading**. These criteria are usable in command `CALC_FATIGUE`.

The first part of this document is devoted to the criteria of MATAKE and DANG VAN adapted to the periodic multiaxial loadings. In this part after having approached the notions of endurance and office plurality of damage and the general form of the criteria of fatigue, we describe the two models of DANG VAN and MATAKE (Critical plane) designed to carry out computations of office plurality of damage under multiaxial loading. One details there the definition of the various planes of shears associated with Gauss points or the nodes, as well as the definition of an amplitude of loading through the circle circumscribed with the way of the loading in the plane of shears. Finally the criteria available in *Code_Aster* are presented.

In the second part we propose a formulation of the criteria of MATAKE, DANG VAN and FATEMI-SOCIE in the frame of the office plurality of damage under nonperiodic multiaxial loading. To define a cycle in the variable case amplitude, we reduce the history of the loading to a unidimensional function of time by projecting the point of the vector shears on an axis, and we use a method of counting of cycles. Here we choose method RAINFLOW. The criteria of MATAKE, DANG VAN and FATEMI-SOCIE adapted to the office plurality of damage under nonperiodic loading are established in *Code_Aster*.

Besides the well established criteria, one lays out in the Code_Aster **of the criteria in formula** allowing the user to build new preset criteria according to the quantities. This kind of criterion is detailed in third part of this document.

Lastly, option `VMIS_TRESCA` makes it possible to calculate the maximum variation, in the course of time, of a stress tensor according to the criteria of Von Mises and Tresca.

Contents

1 Introduction

the models of endurance in fatigue multiaxial under periodic loading are of the models of the following type:

$$VAR_{amplitude}+a\times VAR_{moyenne}<b \; ,$$

where b is the threshold of endurance in simple shears, and a positive constant without dimension has. $VAR_{amplitude}$ is a certain definition of the amplitude (half of the variation) of the cycle of loading and $VAR_{moyenne}$ is a variable in connection with the stress (or sometimes strain) or the stresses (or sometimes the strains) average. The models are characterized by definitions different from $VAR_{amplitude}$ and $VAR_{moyenne}$.
To pass from the endurance to the office plurality of the damage, one introduces an equivalent stress defined by:

$$\sigma_{eq}=VAR_{amplitude}+a\times VAR_{moyenne} \; .$$

This equivalent stress gives us a unit damage on the curve of fatigue. As the second member of the inequation b corresponds to the threshold in shears, one needs a curve of fatigue in shears. But the curves of fatigue in shears are rare since difficult to obtain, one thus tries to use the curves of fatigue in tension alternate compression. For that it is necessary to multiply the equivalent stress by a corrective coefficient about $\sqrt{3}$.

The macroscopic models of MATAKE (critical plane) and macro microphone of DANG-VAN are described. It is shown that under certain assumptions the model of DANG-VAN is similar to the macroscopic model of MATAKE. The only difference lies in the variable $VAR_{moyenne}$: DANG-VAN uses the hydrostatic pressure, while MATAKE employs the normal stress as regards maximum amplitude of shears.

After having defined the plane of shears, we express the shearing stress in this plane. The planes of shears are then explored according to a method described in the reference [bib4] which consists in cutting out the surface of a sphere into pieces of equal sizes.

The normal vectors being known we then determine for each plane the points which are most distant from/to each other. Among those we find the two points which are most distant one from the other. That being made we use, if necessary, the method of the circle passing by three points in order to obtain the circle circumscribed with the way of loading.

In the first part of this document we present the models of endurance in fatigue multiaxial under loading periodicals, as well as the notion of office plurality of damage. The transition of the endurance to the office plurality of damage is also approached.

In the second part the criteria of MATAKE and DANG-VAN are then presented under the aspects limiting of endurance and office plurality of damage under periodic loading.

The third part is devoted to the definition of the plane of shears, the statement of the shearing stresses in this plane and finally, in the manner of exploring the planes of shears.

The fourth part is dedicated to the determination of the circle circumscribed with the way of shears in the plane of the same name. Finally we describe the criteria and the quantities which are introduced into *Code_Aster*.

After having extended the models of MATAKE and DANG-VAN to the office plurality of damage under periodic loading, we present the adaptation of these models with the office plurality of damage under nonperiodic loading. Thus, the fifth part is devoted to the definition of the elementary equivalent stress. We describe also the criterion of modified FATEMI-SOCIE.

The sixth part is reserved in the manner of selecting the axis (or the two axes) on which is project the history of the cission.

The seventh part is dedicated to projection itself of the point of the vector cission on this axis or these two axes. Lastly, concerning the criteria of MATAKE and DANG-VAN formulated in office plurality of damage under nonperiodic loading, we describe the quantities which are introduced into *Code_Aster.*

2 Preliminaries

In this part we treat the notions of limit of endurance and office plurality of damage. We also present the general form of the criteria of fatigue.

2.1 Limit of endurance and the office plurality of damage, uniaxial case

In the uniaxial case, the rigorous definition of the threshold of endurance is the half-amplitude (half of the variation) of loading defined in stress below which the life duration is infinite. However, as in practice the life duration can never be infinite, one defines limits of endurance in 10^7 10^8 , etc cycles of loading. There exists another way of seeing the things: since in practice the infinite life duration does not exist, one uses the notion of office plurality of damage. The approach by the office plurality of damage consists in defining a limit of many cycles beyond which the cumulated damage is equal to one. Thus the limit with 10^7 wants to say that after 10^7 cycles the cumulated damage is equal to 1.

2.2 Criterion of fatigue, multiaxial case

In the literature a certain number of criteria was proposed to define **the threshold of endurance** under multiaxial cyclic loading. The general form of these criteria is:

$$VAR_{amplitude}+a\times VAR_{moyenne}<b \qquad \text{éq 2.2-1}$$

where b is the threshold of endurance in simple shears, a is a positive constant without dimension. $VAR_{amplitude}$ is a certain definition of the half-amplitude (half of the variation) of the cycle and $VAR_{moyenne}$ is a variable in connection with the stress (or sometimes strain) or the stresses (or sometimes the strains) average. Various models are characterized by definitions different from $VAR_{amplitude}$ and $VAR_{moyenne}$.

To pass from the endurance to **the office plurality of the damage**, one can define a stress (or a strain) equivalent:

$$\sigma_{eq}=VAR_{amplitude}+a\times VAR_{moyenne} \qquad \text{éq 2.2-2}$$

This equivalent stress gives us a unit damage on the curve of fatigue. As the second member of the inequation [éq 2.2-1] corresponds to the threshold in shears, one needs a curve of fatigue in shears. But the curves of fatigue in shears are rare since difficult to obtain, one thus tries to use the curves of fatigue in tension alternate compression. For that it is necessary to be coherent at least on the level of the threshold of endurance i.e. to multiply σ_{eq} by a constant about $\sqrt{3}$ being able to use the curve of fatigue in tension. The value $\sqrt{3}$ is the exact value for a criterion of the type Put, in experiments this coefficient is smaller than $\sqrt{3}$.

2.3 Definition of an amplitude of loading in the multiaxial case

In *Code_Aster*, there exist two definitions of amplitude of loading in the multiaxial case:

A : radius (half diameter) of the sphere circumscribed with the way of the loading;
B : half of the maximum of the distance between two unspecified points of the way.

It is clear that in the case of a loading being defined on a sphere, A and B give the same amplitude. On the other hand, if one takes a way (two-dimensional) in the form of an equilateral triangle of dimensioned l , the definition A gives us $l/\sqrt{3}$, while the definition B gives us $l/2$. To work in a conservative frame we take as definition of the amplitude (half-variation) of a way of loading the radius of the sphere (or rings for the case 2D) circumscribed.

2.4 Definition of the plane of shears

In a point M of a continuum we express the tensor of the stresses $\boldsymbol{\sigma}$ in an orthonormal reference $(O, \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$. With the unit norm $\boldsymbol{n}$ of components (n_x, n_y, n_z) in the orthonormal reference, we associate the vector forced $\boldsymbol{F}=\boldsymbol{\sigma}.\boldsymbol{n}$ of components (F_x, F_y, F_z). This vector $\boldsymbol{F}$ can break up into a normal vector with $\boldsymbol{n}$ and a scalar carried by $\boldsymbol{n}$, is:

$$\boldsymbol{F}=N\,\boldsymbol{n}+\boldsymbol{\tau} \qquad \text{éq 2.4-1}$$

where N represents the normal stress and the vector τ the shearing stress. In the reference $(O, \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$, the components of the vector τ are noted: (τ_x, τ_y, τ_z). The vector τ results directly from [éq 2.4-1] and the normal stress:

$$N=\boldsymbol{F}\,.\,\boldsymbol{n} \text{ from where } \boldsymbol{\tau}=\boldsymbol{F}-\boldsymbol{F}\,.\,\boldsymbol{n}\;\boldsymbol{n}. \qquad \text{éq 2.4-2}$$

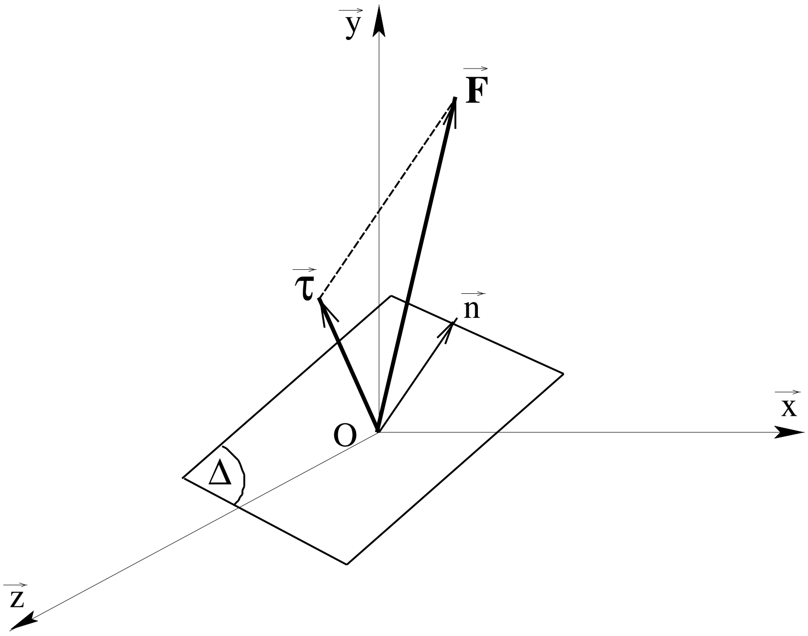


Figure 2.4-a: Representation of the stress vectors $\boldsymbol{F}$ and shearing stress $\boldsymbol{\tau}$

3 Criteria of MATAKE (critical plane) and DANG VAN

Here we clarify the criterion of MATAKE and DANG VAN at the same time from the limiting point of view of endurance and the point of view of the office plurality of damage.

3.1 Criterion of MATAKE

In this kind of criterion the computation of the stress fields and of strain is made under the assumption of elasticity, confer reference [bib1]. As it was known as in chapter 2, in the multiaxial case the criterion of endurance is generally written in the form:

$$VAR_{amplitude} + a \times VAR_{moyenne} < b \qquad \text{éq 3.1-1}$$

Amplitude of loading : In the case of the criterion of MATAKE at each point of structure (or Gauss point for a computation with the finite elements) to compute: $VAR_{amplitude}$ one proceeds in the following way:

[1] for each plane of norm $\boldsymbol{n}$ one calculates the amplitude of shears by determining the circle circumscribed with the way of shears in this plane;
[2] one seeks the norm $\boldsymbol{n}*$ for which the amplitude is maximum. This amplitude is indicated par. $\Delta\tau(\boldsymbol{n}*)$

Constraint average : For the computation of $VAR_{moyenne}$ one proceeds in the following way:
[1] as regards norm $\boldsymbol{n}*$ one calculates on a cycle the indicated maximum normal stress par. $N_{\max}(\boldsymbol{n}*)$

the criterion of endurance is written:

$$\frac{\Delta\tau(\boldsymbol{n}*)}{2} + a\,N_{\max}(\boldsymbol{n}*) \le b \ ,$$

where a and b are two positive constants and b represents the limit of endurance in simple shears.
Identification of the constants : to determine the constants a and b two simple tests should be used. Two possibilities exist:
A test of pure shears plus an alternate traction test compression. In this case the constants are given by: $b = \tau_0 \quad a = \left(\tau_0 - \frac{d_0}{2}\right) / \frac{d_0}{2}$, where τ_0 represents the limit of endurance in alternate pure shears and d_0 the limit of endurance in alternate pure traction and compression.
Two traction tests compression, alternated and the other not. The constants are given by:

$$a = \frac{(\Delta\sigma_2 - \Delta\sigma_1)}{(\Delta\sigma_1 - \Delta\sigma_2) - 2\,\sigma_m},$$

$$b = \frac{\sigma_m}{(\Delta\sigma_2 - \Delta\sigma_1) + 2\,\sigma_m} \times \frac{\Delta\sigma_1}{2}$$

where $\Delta\sigma_1$ is the amplitude of loading for the alternate case and $\Delta\sigma_2$ the case where the average constraint is non-zero.

3.2 Criterion of DANG VAN

One supposes that the material remains overall elastic while it is plasticized locally. The interesting assumption physical of the model is that the material adapts locally (it becomes elastic after being last

by plasticity) below the limit of endurance, which corresponds to nonthe initiation of crack. Above the limit of endurance there is locally accommodation plastic thus initiation of crack.
The basic assumptions of the microphone-macro interaction, Flax-Taylor, make it possible to write:

$$\sigma_{ij}^{Loc}(t)=\sigma_{ij}(t)+\rho_{ij}(t)$$
$$\rho_{ij}(t)=-2\mu\varepsilon_{ij}^{p}(t)$$

One indicates the local stress by $\sigma_{ij}^{Loc}(t)$, the total stress by $\sigma_{ij}(t)$, the local residual stress by $\rho_{ij}(t)$ and $\varepsilon_{ij}^{p}(t)$ the local plastic strain. As soon as there is adaptation the local plastic strain becomes constant and thus the local residual stress also.

Plasticity criterion:
In a point of the continuum (where there is a distribution of the crystallographic directions random of the grains), one supposes that there is only one grain which is plasticized and this, following only one system of sliding. This system of sliding will be that which will be most favorably directed, i.e., the grain in which the greatest scission will occur (the projection of the vector shears on a given direction). The sliding is done in the planes of norm $\boldsymbol{n}=(n_1,n_2,n_3)$ and the direction of sliding is defined by the vector $\boldsymbol{m}=(m_1,m_2,m_3)$. The two vectors $\boldsymbol{n}$ and $\boldsymbol{m}$ are orthogonal.
The model of **Schmid** says that so that there is no irreversible sliding (plastic strain) it is necessary that the scission, does not exceed a certain threshold, that is to say:

$$\forall \boldsymbol{m}\ \ \forall \boldsymbol{n}\ \ |\tau^{Loc}(\boldsymbol{n},\boldsymbol{m},t)|-\tau_y^{Loc}(t)\leq 0 \qquad \text{éq 3.2-1}$$

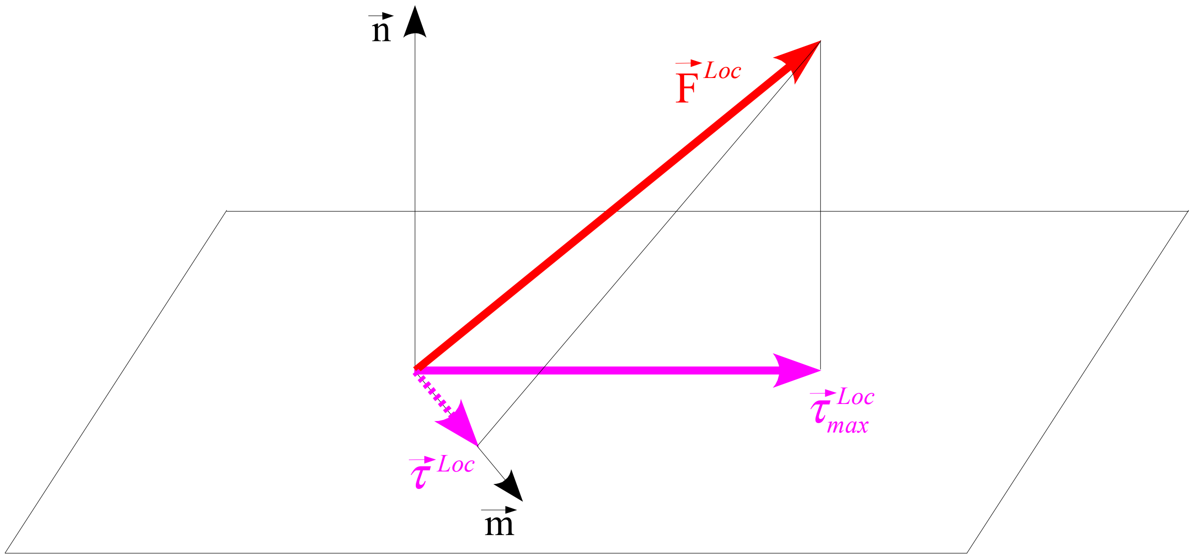
where

$$\tau^{Loc}(t)=a_{ij}\sigma_{ij}^{loc} \quad \text{et} \quad a_{ij}=\frac{1}{2}(m_i n_j+n_i m_j)$$

the drawing of [Figure 3.2-a] the watch that the maximum value of τ^{Loc}, indicated by $\tau_{\max}^{Loc}$, is obtained by the orthogonal projection of $F^{Loc}=\sigma_{ij}^{Loc}\,n_j$ as regards norm $\boldsymbol{n}$. The relation [éq 3.2-1] must in particular be checked if one replaces τ^{Loc} by its raising $\tau_{\max}^{Loc}$, this one is written then:

$$\forall \boldsymbol{n}\ \ |\tau_{\max}^{Loc}(\boldsymbol{n},t)|-\tau_y^{Loc}(t)\leq 0 \qquad \text{éq 3.2-2}$$

where $\tau_y^{Loc}(t)$ is the threshold of the microscopic or local scission. $\tau_y^{Loc}(t)$ depends on the variables of hardening.



Appear 3.2-a: Projection of F^{Loc} as regards norm n

One chooses a microscopic hardening of the linear isotropic type. That makes it possible to show the existence of a field of adaptation [bib2], [bib3].
At the state adapted by analogy with the formula:

$$\sigma_{ij}^{Loc}(t)=\sigma_{ij}(t)+\rho_{ij}^{*}$$

one has, if one places oneself in the plane $(\boldsymbol{n},\boldsymbol{m})$ in such a way that the scission is maximum, the following formula:

$$\tau_{\max}^{Loc}(\boldsymbol{n},t)=\tau(n,t)+\tau^{*}(n)$$

where $\tau(\boldsymbol{n},t)$ is the vector macroscopic shears defined in [the Figure 3.2-b] and where $\tau^{*}(\boldsymbol{n})$ is the microscopic vector residual shears (independent of time since we are in an adapted state).

Criterion of fatigue
Introduction of the maximum pressure: DANG VAN uses instead of the normal stress on a plane, as that is done in the model MATAKE, the maximum hydrostatic pressure on a cycle. The criterion is thus written:

$$\underset{\boldsymbol{n},t}{MAX}\left(\left|\tau_{\max}^{Loc}(\boldsymbol{n},t)\right|+a\ P_{\max}^{Loc}\right)\leq b$$

As the hydrostatic pressures local and total are identical the criterion becomes:

$$\underset{\boldsymbol{n},t}{MAX}\left(\left|\tau_{\max}^{Loc}(\boldsymbol{n},t)\right|+a\ P_{\max}\right)\leq b$$

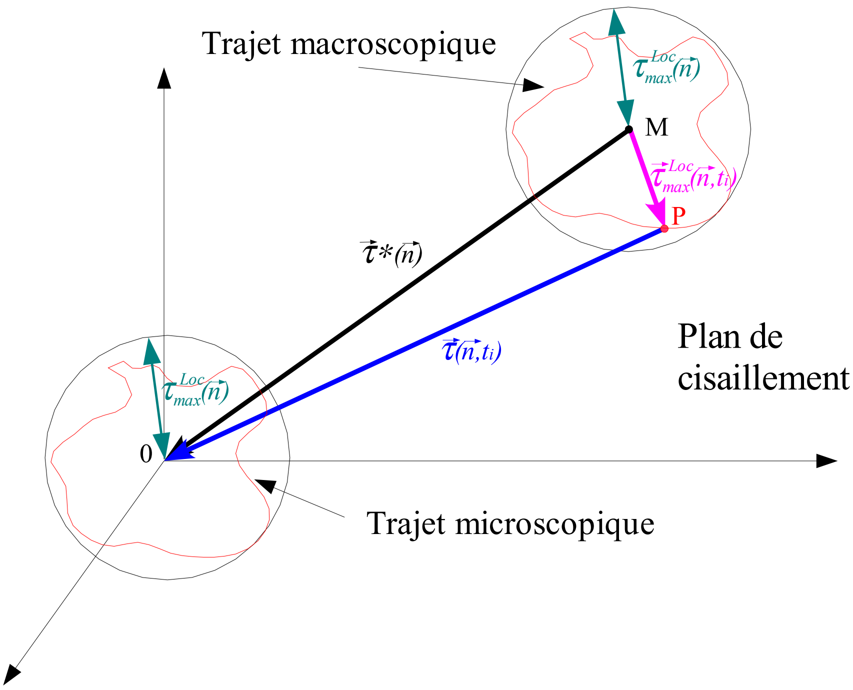
For a positive maximum pressure we have:

$$\underset{\boldsymbol{n},t}{MAX}\left(\left|\tau_{\max}^{Loc}(\boldsymbol{n},t)\right|\right)+a\ P_{\max}\leq b$$

For **an always negative pressure** one can take $P_{\max}=0$ to remain conservative.

Assumption on $\tau^*(\boldsymbol{n})$

In the radial case where the direction of the maximum shears is defined in advance one can calculate in an exact way $\tau^*(\boldsymbol{n})$. In general case DANG VAN proposes the following method to do a calculation simplified of $\tau^*(\boldsymbol{n})$. One gives for a plane n the macroscopic way of the vector shears defined previously. The vector residual shears taking into account the preceding assumption is defined by MO, where M is the center of the circle circumscribed with the way of the end of the vector shears in the plane of shears.



Appear 3.2-b: Ways macro microphone/in the plane of shears

Final Formulation: taking into account two formulas :

$$\tau_{\max}^{Loc}(\boldsymbol{n},t)=\tau(\boldsymbol{n},t)+\tau^*(\boldsymbol{n}) \text{ and } \underset{\boldsymbol{n},t}{MAX}\left(\left|\tau_{\max}^{Loc}(\boldsymbol{n},t)\right|\right)+a\ P_{\max}\le b$$

formulates finds oneself with

$$\underset{\boldsymbol{n},t}{MAX}\left(|\boldsymbol{MP}|\right)+a\ P_{\max}\le b$$

where P is a point running of the way of shears in the plane of norm $\boldsymbol{n}$.

Identification of the constants : to determine the constants a and b two simple tests should be used. Two possibilities exist:

- **A test of pure shears plus a traction test alternate compression**. In this case the constants are given by: $b=\tau_0$ $a=(\tau_0-d_0/2)/(d_0/3)$.
- **Two traction tests compression, alternated and the other not**. The constants are given by:

$$a=\frac{3}{2}\times\frac{(\Delta\sigma_2-\Delta\sigma_1)}{(\Delta\sigma_1-\Delta\sigma_2)-2\ \sigma_m} \quad b=\frac{\sigma_m}{(\Delta\sigma_2-\Delta\sigma_1)+2\ \sigma_m}\times\frac{\Delta\sigma_1}{2}.$$

with $\Delta\sigma_1$ the amplitude of loading for the alternate case and $\Delta\sigma_2$ the case where the average constraint is non-zero.

3.3 MATAKE and DANG VAN modified for the office plurality of damage

the models of MATAKE and DANG VAN were proposed initially for the study of the limit of endurance. As the infinite life duration does not exist one uses limits of endurance with 10^6 10^7, 10^n cycles of loading. Thus the initial criteria of MATAKE and DANG VAN are presented like criteria of going beyond a threshold and do not give an office plurality of damage. The use, in particular of the criterion of DANG VAN, in automotive industries is suitable since the sought purpose is nonthe going beyond a threshold of endurance contrary to the problems of EDF where one wishes to follow the damage.
Thus we use for the office plurality of damage an equivalent stress of MATAKE or DANG VAN defined by:

$$\text{MATAKE} \qquad \sigma_{\text{eq}}=\frac{\Delta\tau}{2}(n^*)+a\,N_{\max}(n^*).$$

$$\text{DANG VAN} \qquad \sigma_{\text{eq}}=\text{MAX}_{\boldsymbol{n},t}(|\boldsymbol{MP}|)+aP_{\max}$$

The taking into account of the surface treatment is summarized with the taking into account of the harmful effect of the pre - hardening over the life duration in controlled strain [bib5]. In the models of MATAKE and DANG VAN the effect of pre-hardening is taken into account by multiplying the half-amplitude of shearing stress by a corrective coefficient higher than the unit, noted c_p :

$$\text{MATAKE} \qquad \sigma_{\text{eq}}=c_p\frac{\Delta\tau}{2}(n^*)+a\,N_{\max}(n^*),$$

$$\text{DANG VAN} \qquad \sigma_{\text{eq}}=c_p\text{MAX}_{\boldsymbol{n},t}(|\boldsymbol{MP}|)+aP_{\max}$$

These equivalent stresses are to be used on a curve of fatigue in shears. For the use on a curve of fatigue in tension compression it is necessary to multiply these equivalent stresses by a corrective coefficient, noted here α :

$$\text{MATAKE} \qquad \sigma_{\text{eq}}=\alpha\left(c_p\frac{\Delta\tau}{2}(n^*)+a\,N_{\max}(n^*)\right),$$

$$\text{DANG VAN} \qquad \sigma_{\text{eq}}=\alpha\left(c^p\text{MAX}_{\boldsymbol{n},t}(|\boldsymbol{MP}|)+aP_{\max}\right)$$

4 Computation of the plane of maximum shears

We use here the definition of the plane of shears introduced in the paragraph [§2.4]. Practically, for us the point M of the continuum will be a Gauss point.

4.1 Statement of the shearing stresses in the plane Δ

For reasons of symmetry we vary the unit norm $\boldsymbol{n}$ according to a half-sphere using the angles γ and φ , cf [Figure 4.1-a].
In the reference $(O, \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$, the unit normal vector $\boldsymbol{n}$ is defined by:

$$n_x = \sin\gamma \cos\varphi \quad n_y = \sin\gamma \sin\varphi \quad n_z = \cos\gamma$$.éq 4.1-1

We introduce a new reference $(O, \boldsymbol{u}, \boldsymbol{v}, \boldsymbol{n})$ where $\boldsymbol{n}$ is perpendicular to the plane of shears Δ and where $\boldsymbol{u}$ and $\boldsymbol{v}$ are in this plane, cf [Figure 4.1-a]. In the reference $(O, \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ the unit vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are respectively defined by:

$$u_x = -\sin\varphi \quad u_y = \cos\varphi \quad u_z = 0 ,$$ éq 4.1-2

$$v_x = -\cos\gamma \cos\varphi \quad v_y = -\cos\gamma \sin\varphi \quad v_z = \sin\gamma$$.éq 4.1-3

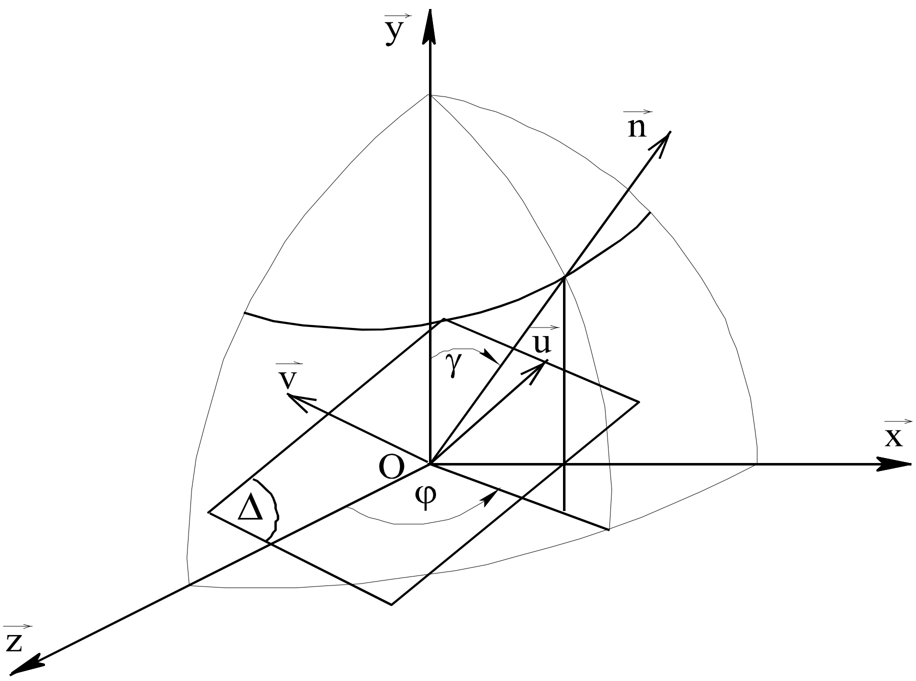


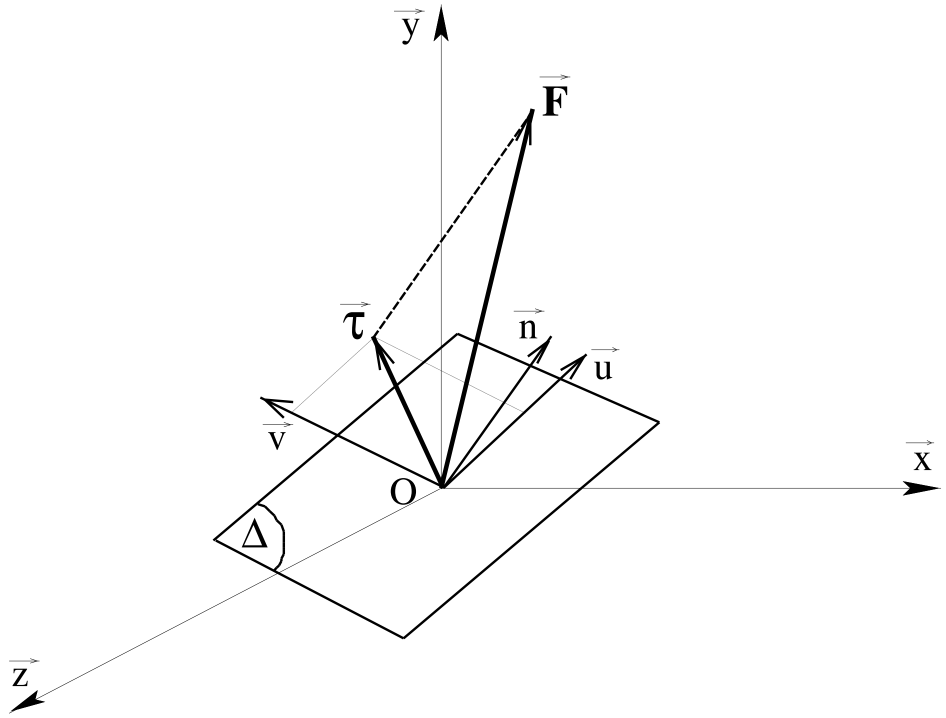
Figure 4.1-a: Location of the norm $\boldsymbol{n}$ to a plane by the angles γ and φ

In the plane Δ , the components τ_u and τ_v of the vector $\boldsymbol{\tau}$ representing the shearing stress are obtained by the relations:

$$\tau_u = \boldsymbol{u} \cdot \boldsymbol{\tau} = u_x \tau_x + u_y \tau_y + u_z \tau_z \text{ ,} \quad \text{éq 4.1-4}$$

$$\tau_v = \boldsymbol{v} \cdot \boldsymbol{\tau} = v_x \tau_x + v_y \tau_y + v_z \tau_z \text{ .éq} \quad \text{4.1-5}$$

On [Figure 4.1-b], we represented the shearing stresses in the plane Δ as well as the reference $(O, \boldsymbol{u}, \boldsymbol{v}, \boldsymbol{n})$.



Appear 4.1-b: Representation of the vector shearing stresses $\boldsymbol{\tau}$ in the plane Δ

Now our problems is to determine for each Gauss point or each node of a mesh the plane of norm $\boldsymbol{n}$ such as $|\boldsymbol{\tau}|$ is maximum. With this intention we vary the unit norm $\boldsymbol{n}$.

4.2 Exploration of the planes of shears

the method which we present here is resulting from the reference [bib4]. Its principle is the following. As indicated in the paragraph [§4.1], for reasons of symmetry we vary the unit norm $\boldsymbol{n}$ according to a half-sphere using the angles γ and φ , cf [Figure 4.1-a]. Question which comes immediately is which must be the step of variation of the angles γ and φ . Indeed, it is necessary to find an optimum between the smoothness of exploration and a reasonable computing time insofar as it is necessary to make this operation at each Gauss point of the mesh. The author of the reference [bib4] proposes to divide the surface of the half sphere into facets of equal surfaces to the center of which the unit norm $\boldsymbol{n}$ is positioned, cf [Figure 4.2-a]. In practice surfaces are not strictly equal but of the same order of magnitude.

The step value of variation of γ , $\Delta\gamma$ is worth 10 degrees. The angle φ varies according to a step $\Delta\varphi$ which is function of the angle γ . The γ weaker is or close to 180 degrees and the more $\Delta\varphi$ must be large to preserve an area of about constant facet. It is in the vicinity of $\gamma = 90°$ $\Delta\varphi$ is smallest. [Table 4.2-1] the cutting summarizes which was retained.

With this method the number of facet thus the number of normal vectors to be explored is equal to 209 for a half sphere.

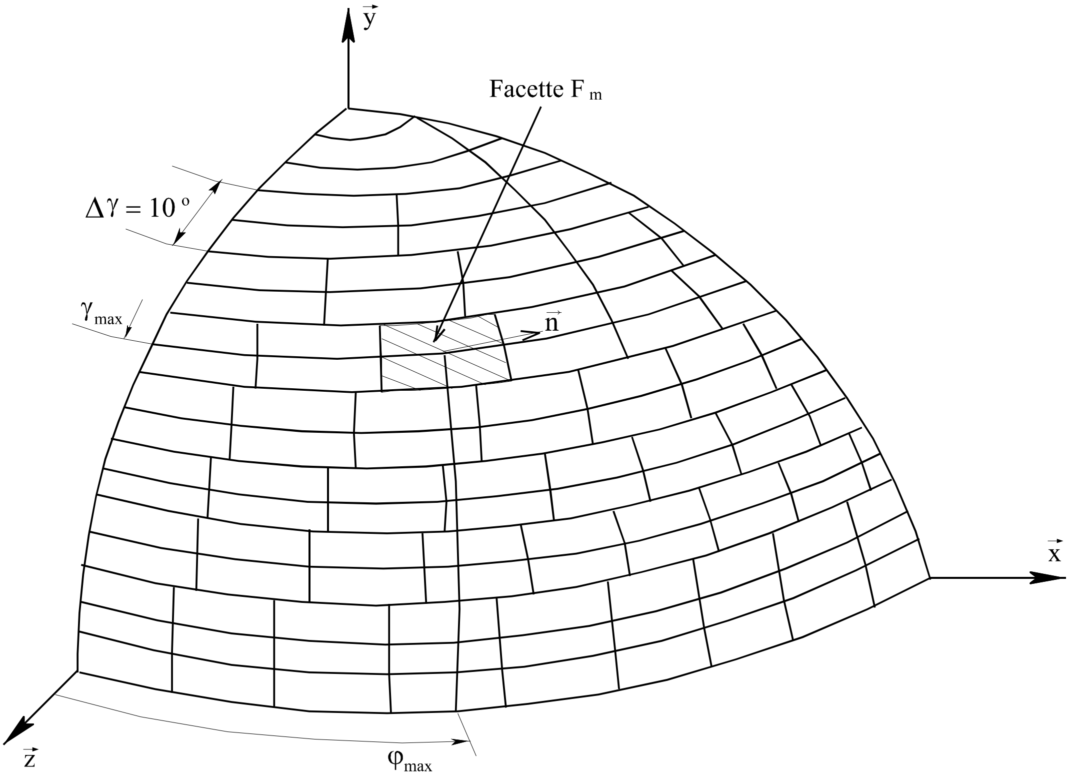


Figure 4.2-a : Division of the surface of the half sphere in facets

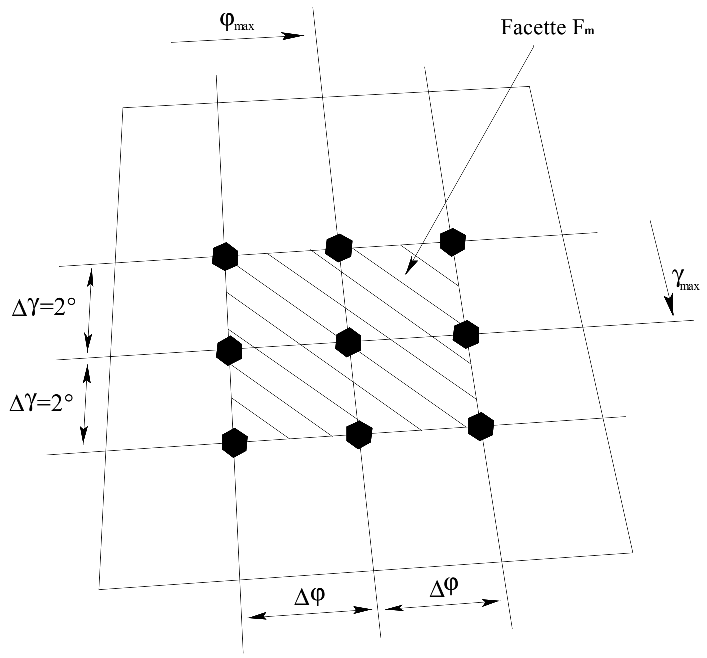
γ °	0°	10°	20°	30°	40°	50°	60°
$\Delta\Phi$ °	180°	60°	30°	20°	15°	$12,857$	$11,25$
Many facets	1	3	6	9	12	14	16

γ °	70°	80°	90°	100°	110°	120°	130°
$\Delta\Phi$ °	$10,588$	10°	10°	10°	$10,588$	$11,25$	$12,857$
Many facets	17	18	18	18	17	16	14

γ °	140°	150°	160°	170°	180°
$\Delta\Phi$ °	15°	20°	30°	60°	180°
Many facets	12	9	6	3	1

Table 4.2-1: Number of facet according to γ and $\Delta\Phi$

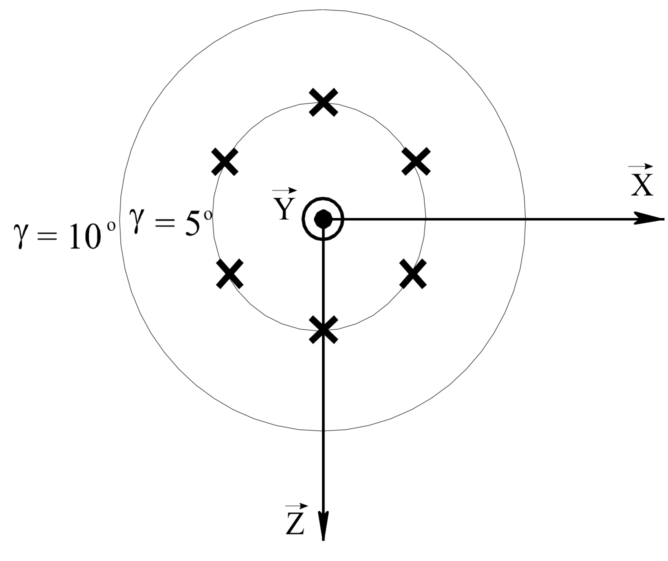
In order to determine the normal vector $\mathbf{n}$ which will give the plane of maximum shears with a good accuracy, the author recommends to resort to four additional successive refinings. The first consists in exploring eight new facets around the initial normal vector, like illustrates it [Figure 4.2-b].



Appear 4.2-b: Representation of the eight additional facets around $\mathbf{n}$

In this case $\Delta\gamma$ is equal to two degrees and for $\gamma\in]0°, 180°[$ $\Delta\varphi=\Delta\gamma/\sin\gamma$. For the last three refinings, $\Delta\gamma$ is equal to 1,0.5 and 0.25 degrees, respectively.

Typical case. If the facet F_m is perpendicular to $\mathbf{y}$, one considers the six facets all around it located at $\gamma=5°$ and respectively defined by $\Phi=0°$ $\Phi=60°$ $\Phi=120°$ $\Phi=180°$, $\Phi=240°$ and $\Phi=300°$, cf [Figure 4.2-c].



Appear 4.2-c: Localization of the explored facets when F_m is normal with $\mathbf{y}$

For each Gauss point or each node we let us explore the 209 normal vectors $\boldsymbol{n}$. A each normal vector is associated a history of the shears concretized by a certain number of points located in the plane with shears Δ of axes u and v . Now it is a question of finding the circle circumscribed at the points belonging to plane of shears so as to deduce to it half amplitude from it from shears.

5 Computation of the half amplitude of shears

the problems is thus to find the circle circumscribed at a certain number of points located in a plane. The half amplitude of shears will be equal to the radius of the circumscribed circle.

5.1 General presentation of the computation of the circle circumscribes

the method which we use is an exact method which breaks up into four stages.

Stage 1
We frame the points and we determine the coordinates of the four corners of the frame in the reference $(0, u, v)$, and the coordinated center of the frame O cf [Figure 5.1-a] and [Figure 5.1-c]. **In the typical case** where the frame summarizes himself with line horizontal or vertical it half length of line is equal to the half amplitude of shears.

Stage 2
the purpose of the second phase is to select the two most distant points. In order not to examine the distance between all the possible pairs of points, we build four sectors, cf [Figure 5.1-a] and [Figure 5.1-c]. These sectors are at the four corners of the frame and are delimited on the one hand, by the contour of the frame and on the other hand, by an arc of a circle whose center is the opposite corner and the radius the large side of the frame who in fact undervalues the distance between the two most distant points. Finally, we evaluate the distances between the points of the four sectors two to two:

distances between the points of sector 1 and the points of sector 2;
distances between the points of sector 1 and the points of sector 3;
distances between the points of sector 1 and the points of sector 4;
distances between the points of sector 2 and the points of sector 3;
distances between the points of sector 2 and the points of sector 4;
distances between the points of sector 3 and the points of sector 4.

In the typical case where the ratio on the small side of the frame on large on the east side strictly lower than $\sqrt{3/4}$ we do not evaluate the distances between the points belonging to sectors 1 and the 2 nor distances between the points of sectors 3 and 4, case of the example of [Figure 5.1-a].

Stage 3
In the third stage we build the fields 1 and 2 in which we will seek the points which are apart from the initial circumscribed circle, cf Stage 4. The purpose of the constitution of fields 1 and 2 is reducing the number of points to be explored at the time of stage 4. The principles of constructions of these two fields are the following.

- From the points mediums on the two large sides of the frame (Omi_1 and Omi_2 , cf [Figure 5.1-b] and [Figure 5.1-d]) we trace an arc of a circle whose radius corresponds to undervaluing value of the half amplitude of shears and is equal to the half length on the large side of the frame.
- Center of the frame O we trace four arcs of a circle whose radius is also undervaluing it of the value of the half amplitude of shears.

If O_i center of circle circumscribes initial has component according to axis u which places it between Omi_1 and O , then if there exists a point whose distance to O_i is higher than R_i the radius of the circle circumscribes initial, there can be only in field 1, cf [Figure 5.1-b].

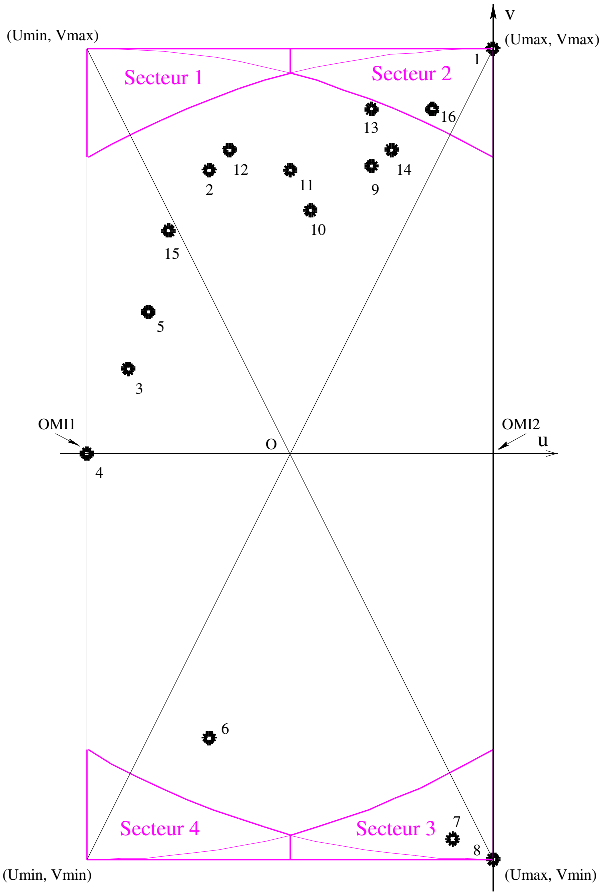
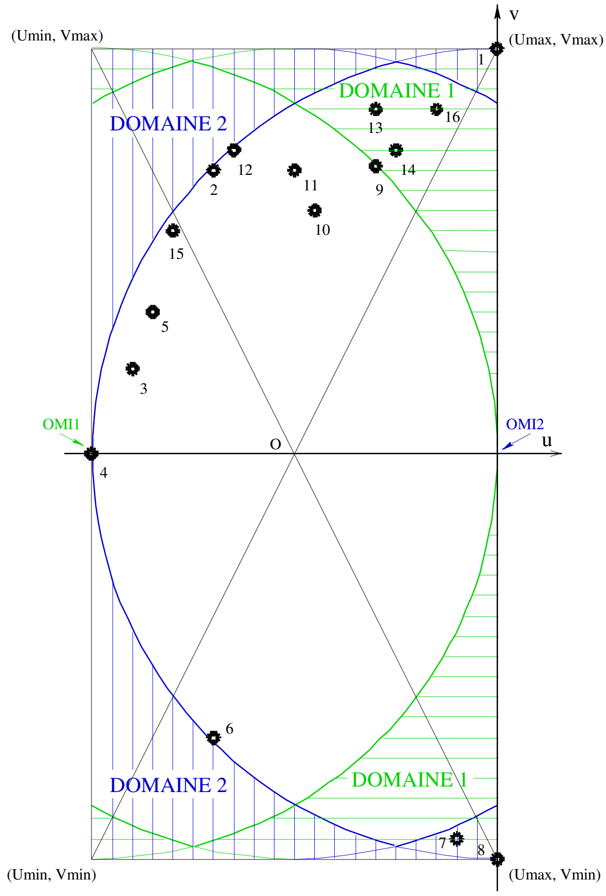
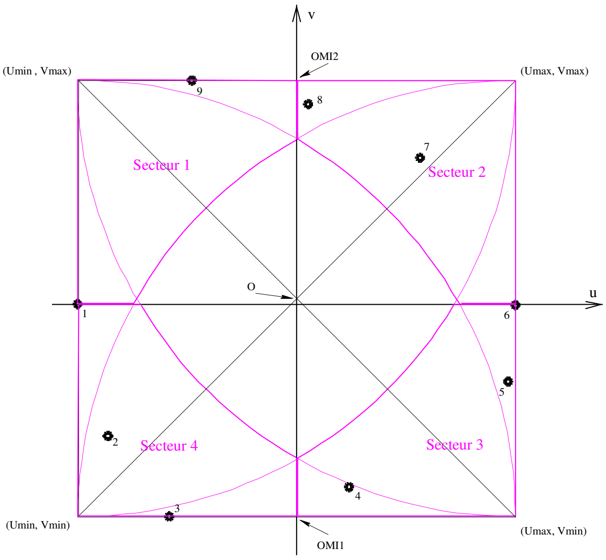


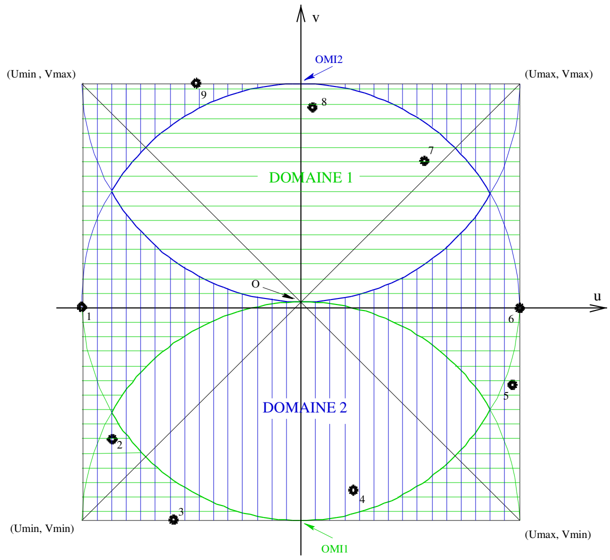
Figure 5.1-a : Exemple1, localization of the sectors



Appears 5.1-b: Exemple1, localization of the fields



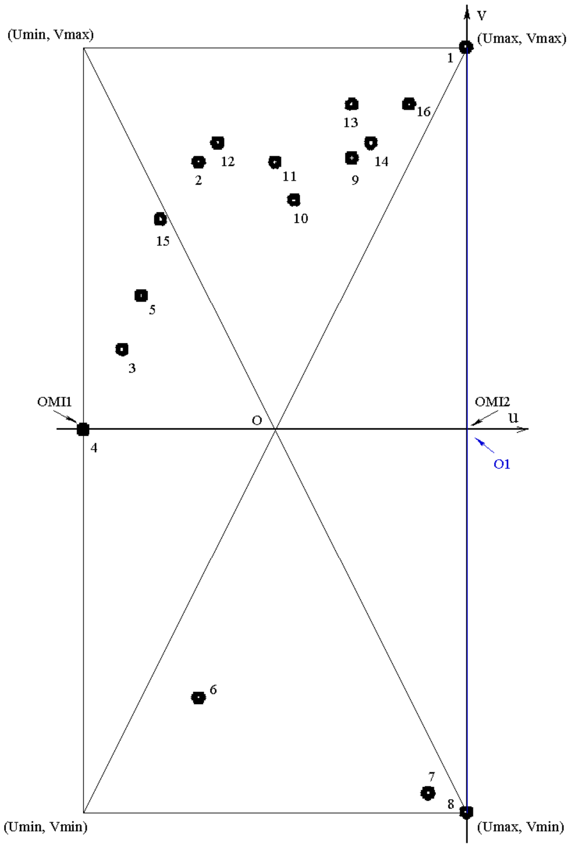
Appears 5.1-c: Exemple2, localization of the sectors



Appears 5.1-d: Exemple2, localization of the fields

Stage 4

the goal of the fourth stage is to find the circle circumscribed by the method of the circle passing by three points, cf [§5.2]. With this intention, we calculate the point medium O_1 associated with the two most distant points which we note P_1 and P_2 , we deduce the value from it from a first noted radius R_1 . According to the position from O_1 ratio with the main roads of the frame passing in his center, we seek either in field 1, or in field 2, if it has there a point located at a distance higher than the half outdistances measured between the two points most moved away P_1 and P_2 . Let us note P_3 such a point. If there is no point such as P_3 then it half amplitude of shears is equal to R_1, cf [Figure 5.1-c]. On the other hand, if P_3 exists we seek the coordinates of the point located at equal distance from P_1 , P_2 and P_3 ; we note this point O_2. We obtain a new radius thus, R_2 therefore new a half amplitude of shears. Again, according to the position from O_2 ratio with the main roads of the frame passing in his center, we seek either in field 1, or in field 2, if it has there a point located at a distance higher than R_2 O_2. Let us note P_4 such a point. If there is no point such as P_4 then it half amplitude of shears is equal to R_2 . On the other hand, if P_4 exists we seek the smallest circle circumscribed at the four points: P_1 P_2 , P_3 and P_4 by means of successively method of the circle passing by three points, cf [§5.2]. That provides us a new center O_3 and new R_3. As previously, according to the position from O_3 ratio with the main roads of the frame passing in his center, we seek either in field 1, or in field 2, if it has there a point located at a distance higher than R_3 O_3. Let us note P_5 such a point. If there is no point such as P_5 then it half amplitude of shears is equal to R_3. On the other hand if a point such that P_5 exists we have five points, if we the preceding method, where there wants to use are only four points concerned, it is necessary to eliminate one from the five points. That cannot be the last: P_5, therefore we preserve preceding iteration the three points which made it possible to determine O_3 and R_3 , i.e. the smallest circumscribed circle. Let us suppose that P_1 is thus eliminated. We thus seek the smallest circle circumscribed at the four points: P_2 P_3 , P_4 and P_5 by means of successively method of the circle passing by three points, cf [§5.2]. That provides us a new center O_4 and new R_4. According to the position from O_4 ratio with the main roads of the frame passing in his center, we seek either in field 1, or in field 2, if it has there a point located at a distance higher than R_4 O_4. If it is not the half case it amplitude of shears is equal to R_4 and the circumscribed circle has as a center O_4, cf [Figure 5.1-f]. Contrary, if such a point exists we remake an iteration identical to the preceding one.



Appear 5.1-e: Exemple1, search of the circle circumscribes

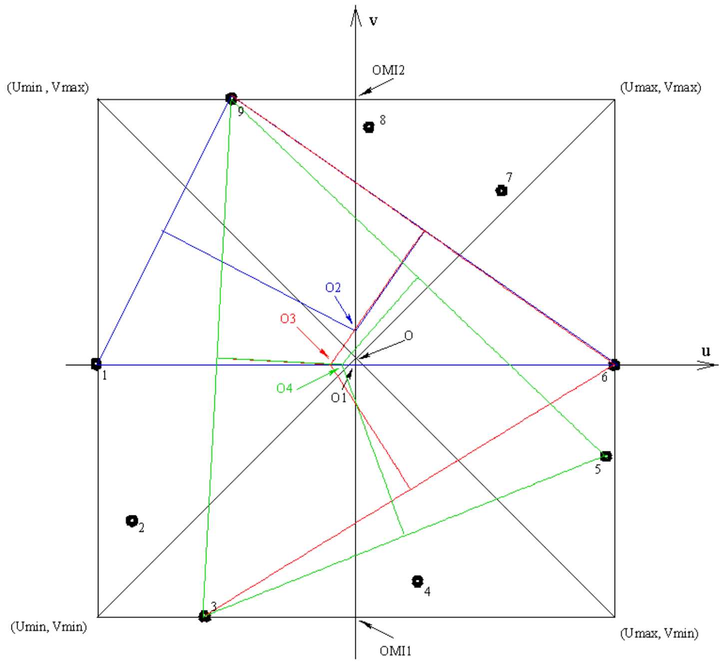


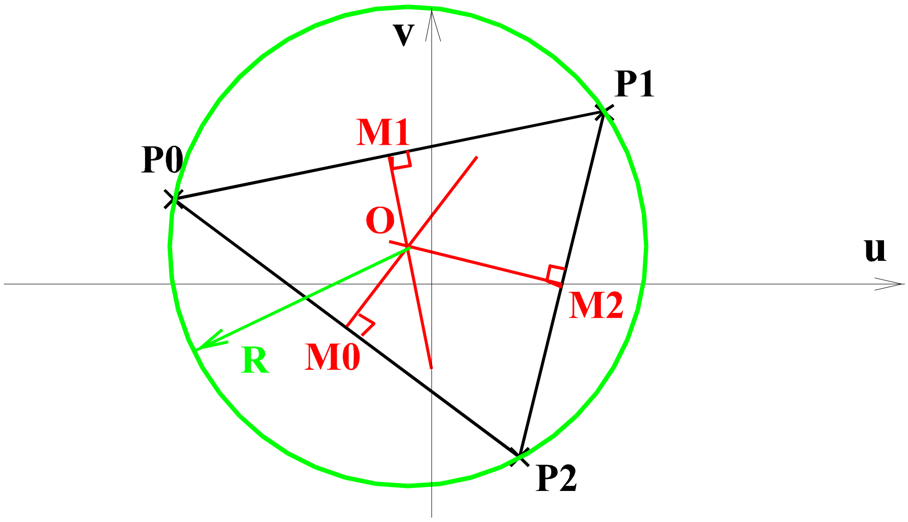
Figure 5.1-f: Exemple2, search of the circle circumscribes

5.2 Description of the method of the circle passing by three points

In this paragraph we will treat the general case, then the typical cases.

5.2.1 General case

to determine the circumscribed circle at three points P_0 , P_1 and P_2 , cf [Figure 5.2.1-a], we proceed in three stages.



Appear 5.2.1-a: Determination of the circle passing by three points

Stage 1
We calculate the coordinates of the three points mediums: M_0 , M_1 and M_2 , cf [Figure 5.2.1-a].

Stage 2
We determine the norms passing by the three points mediums: M_0 , M_1 and M_2 , cf [Figure 5.2.1-a]. These norms are of the rights of the type $v=a\,u+b$ where a and b are constants which it is possible to calculate with the coordinates of the points P_0 P_1 P_2 M_0 , M_1 and M_2 . Let us describe, now, the way determine these norms.

1) Norm at the segment P_0 P_1 passing through M_1
We let us determine the punctual coordinates $M_1^{'}$ per rotation of 90° of the segment P_0 M_1 :

$$U_{M1^{'}}=U_{M1}+\left(V_{M1}-V_{P0}\right)$$
$$V_{M1^{'}}=V_{M1}+\left(U_{P0}-U_{M1}\right) \quad \text{éq 5.2.1-1}$$

where U_k and V_k with $k=M1^{'},\ M1,\ P0$ the components and u of v the points represent $M_1^{'}$, M_1 and P_0 . We deduce the constants from them a_0 and b_0 from the right representing the norm at the segment P_0 P_1 passing by M_1:

$$a_0=(V_{M1'}-V_{M1})/(U_{M1'}-U_{M1})$$
$$b_0=(U_{M1'}V_{M1}-U_{M1}V_{M1'})/(U_{M1'}-U_{M1}) \quad \text{éq 5.2.1-2}$$

In the typical case where $(U_{M1'}-U_{M1})=0$, we force a_0 and b_0 with zero and we obtain the coordinates of the center O of the circle circumscribed at the points P_0 , P_1 and P_2 by a specific method described in the paragraph [§5.2.2].

2) Norm at the segment P_0 P_2 passing through M_0

We let us determine the punctual coordinates $M0'$ per rotation of 90° of the segment P_0 M_0 :

$$U_{M0'}=U_{M0}+(V_{M0}-V_{P0})$$
$$V_{M0'}=V_{M0}+(U_{P0}-U_{M0}) \quad \text{éq 5.2.1-3}$$

where U_k and V_k with $k=M0',\ M0,\ P0$ the components and u of v the points represent $M0'$, M_0 and P_0 . We deduce the constants from them a_1 and b_1 from the right representing the norm at the segment P_0 P_2 passing by M_0 :

$$a_1=(V_{M0'}-V_{M0})/(U_{M0'}-U_{M0})$$
$$b_1=(U_{M0'}V_{M0}-U_{M0}V_{M0'})/(U_{M0'}-U_{M0}) \quad \text{éq 5.2.1-4}$$

In the typical case where $(U_{M0'}-U_{M0})=0$, we force a_1 and b_1 with zero and we obtain the coordinates of the center O of the circle circumscribed at the points P_0 , P_1 and P_2 by a specific method described in the paragraph [§5.2.2].

3) Norm at the segment P_1 P_2 passing through M_2

We let us determine the punctual coordinates M_2' per rotation of 90° of the segment P_1 M_2 :

$$U_{M2'}=U_{M2}+(V_{M2}-V_{P1})$$
$$V_{M2'}=V_{M2}+(U_{P1}-U_{M2}) \quad \text{éq 5.2.1-5}$$

where U_k and V_k with $k=M2',\ M2,\ P1$ the components and u of v the points represent M_2' , M_2 and P_1 . We deduce the constants from them a_2 and b_2 from the right representing the norm at the segment P_1 P_2 passing by M_2 :

$$a_2=(V_{M2'}-V_{M2})/(U_{M2'}-U_{M2})$$
$$b_2=(U_{M2'}V_{M2}-U_{M2}V_{M2'})/(U_{M2'}-U_{M2}) \quad \text{éq 5.2.1-6}$$

In the typical case where $(U_{M2'}-U_{M2})=0$, we force a_2 and b_2 with zero and we obtain the coordinates of the center O of the circle circumscribed at the points P_0 , P_1 and P_2 by a specific method described in the paragraph [§5.2.2].

Stage 3
In the general case, we deduce from the constants a_0 b_0 a_1 b_1 , a_2 and the b_2 coordinated center O of the circle circumscribed at the points P_0 , P_1 and P_2 three way different. Let us note O_0 , O_1 and O_2 the same center O , obtained in three different ways, and U_k V_k , where $k=O_0, O_1, O_2$, the components and u of v the points represent O_0 , O_1 and O_2 :

$$U_{O_0}=(b_1-b_0)/(a_0-a_1)$$
$$V_{O_0}=(a_0\ b_1-a_1\ b_0)/(a_0-a_1) \quad \text{éq 5.2.1-7}$$

$$U_{O_1}=(b_2-b_0)/(a_0-a_2)$$
$$V_{O_1}=(a_0\ b_2-a_2\ b_0)/(a_0-a_2) \quad \text{éq 5.2.1-8}$$

$$U_{O_2}=(b_2-b_1)/(a_1-a_2)$$
$$V_{O_2}=(a_1\ b_2-a_2\ b_1)/(a_1-a_2) \quad \text{éq 5.2.1-9}$$

After having checked that equalities: $U_{O_0}\equiv U_{O_1}\equiv U_{O_2}$ and $V_{O_0}\equiv V_{O_1}\equiv V_{O_2}$ we are satisfied determine the radius of the circle circumscribed by calculating the distance enters O and one of the three points P_0 , P_1 or P_2 .

5.2.2 Typical cases

In this paragraph we treat the three typical cases of stage 2 of the paragraph [§5.2.1].

Typical case where $(U_{M1'}-U_{M1})=0$
In this case we obtain the components immediately u and v of the center O by:

$$U_O=U_{M1}$$
$$V_O=(a_1\ b_2-a_2\ b_1)/(a_1-a_2) \quad \text{éq 5.2.2-1}$$

Typical case where $(U_{M0'}-U_{M0})=0$
Here the components u and v of the center O are given by:

$$U_0=U_{M0}$$
$$V_O=(a_0\ b_2-a_2\ b_0)/(a_0-a_2) \quad \text{éq 5.2.2-2}$$

Typical case where $(U_{M2'}-U_{M2})=0$
In this last case, them u and v of the center O are given by:

$$U_0=U_{M2}$$
$$V_O=(a_0\ b_1-a_1\ b_0)/(a_0-a_1) \quad \text{éq 5.2.2-3}$$

the value of the radius of the circumscribed circle is obtained same way as in the general case; i.e., while calculating the distance enters O and one of the three points P_0 , P_1 or P_2 .

5.3 Criteria with critical planes

In this paragraph we give the list of the criteria with critical planes, cf [bib3], which are programmed as well as a brief description.
Notation:

$\boldsymbol{n}*$	: norm with the plane in which the amplitude of shears is maximum;
$\Delta\tau(\boldsymbol{n})$	: amplitude of shears in a plane of norm $\vec{n}$;
$N_{\max}(\boldsymbol{n})$	: maximum normal stress as regards norm $\vec{n}$ during the cycle;
τ_0	: limit of endurance in alternate pure shears;
d_0	: limit of endurance in alternate pure traction and compression;
$N_m(\boldsymbol{n})$	: average normal stress as regards norm $\vec{n}$ during the cycle;
$\varepsilon_{\max}(\boldsymbol{n})$	: maximum normal strain as regards norm $\vec{n}$ during the cycle;
$\varepsilon_m(\boldsymbol{n})$	: average normal strain as regards norm $\vec{n}$ during the cycle;
P	: hydrostatic pressure;
c_p	: harmful effect of pre-hardening in controlled strain $c_p\geq 1$.

Criterion of MATAKE

$$\frac{\Delta\tau(\boldsymbol{n}*)}{2}+a\ N_{\max}(\boldsymbol{n}*)\leq b \qquad \text{éq 5.3-1}$$

where a and b are two constant data by the user, they depend on the characteristic materials and are worth:

$$a=\left(\tau_0-\frac{d_0}{2}\right)/\frac{d_0}{2} \qquad b=\tau_0.$$

Moreover, we define an equivalent stress within the meaning of MATAKE, noted $\sigma_{eq}(n*)$:

$$\sigma_{eq}(\boldsymbol{n}*)=\left(c_p\frac{\Delta\tau(\boldsymbol{n}*)}{2}+a\ N_{\max}(\boldsymbol{n}*)\right)\frac{f}{t},$$

where f/t the ratio of the limits of endurance in alternate bending and torsion represents.

Criterion of DANG VAN

$$\frac{\Delta\tau(\boldsymbol{n}*)}{2}+a\ P\leq b \qquad \text{éq 5.3-2}$$

where a and b are two constant data by the user, they depend on the characteristic materials and are worth:

$$a=\frac{3}{2}\times\frac{\left(\Delta\sigma_2-\Delta\sigma_1\right)}{\left(\Delta\sigma_1-\Delta\sigma_2\right)-2\ \sigma_m} \qquad b=\frac{\sigma_m}{\left(\Delta\sigma_2-\Delta\sigma_1\right)+2\ \sigma_m}\times\frac{\Delta\sigma_1}{2}.$$

Moreover, we define an equivalent stress within the meaning of DANG VAN, noted $\sigma_{eq}(\boldsymbol{n}*)$:

$$\sigma_{eq}(\boldsymbol{n}*)=\left(c_p\frac{\Delta\tau(\boldsymbol{n}*)}{2}+a\ P\right)\frac{c}{t}\ ,$$

where c/t the ratio of the limits of endurance in alternate shears and tension represents.

5.4 Many cycles to the fracture and damage

From $\sigma_{eq}(\boldsymbol{n}*)$ and of a curve of Wöhler we deduct the number of cycles to the fracture: $N(\boldsymbol{n}*)$, then the damage corresponding to a cycle: $D(\boldsymbol{n}*)=1/N(\boldsymbol{n}*)$.

6 Criteria with variable amplitude

the criteria with variable amplitude are implemented when the loading is not periodic. When the loading is not periodic it is necessary to break up the way of loading undergone by structure into elementary under-cycles using a method of counting of cycles. If the loading is nonradial there is no tested multiaxial method of counting. Consequently we choose, as in the literature, to use the method of counting RAINFLOW [bib7] which needs in entry for a scalar. This is why we reduce to a dimension the scission, which is the orthogonal projection of the vector forced on a plane, by projecting the point of the vector scission on one or two axes. Another important difference with the criteria with critical plane is that it is not the amplitude of shears which make it possible to select the critical plane but the office plurality of damage which results from the elementary under-cycles.

The method of projection that we use is clarified in chapters 7 and 8. In the continuation we describe the way in which we made evolve the criteria of MATAKE and DANG VAN to adapt them to the cases where the loading is not periodical.

6.1 Criterion of MATAKE modified

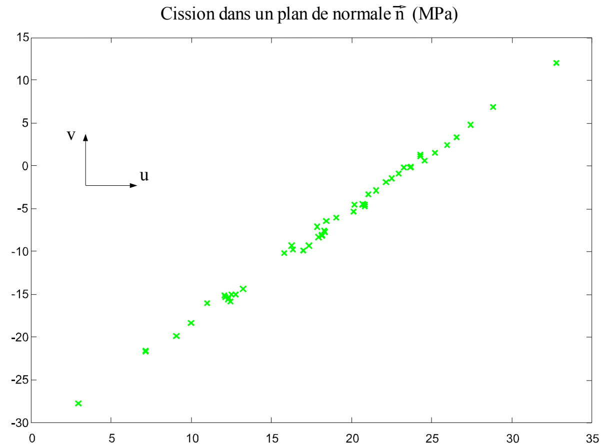
In the context of the office plurality of damage and a periodic loading, the criterion of MATAKE [bib6], is written in the following way:

$$\sigma_{eq}=c_p\frac{\Delta\tau\left(\vec{n}^*\right)}{2}+a\ N_{\max}\left(\vec{n}^*\right) \qquad \text{éq 6.1-1}$$

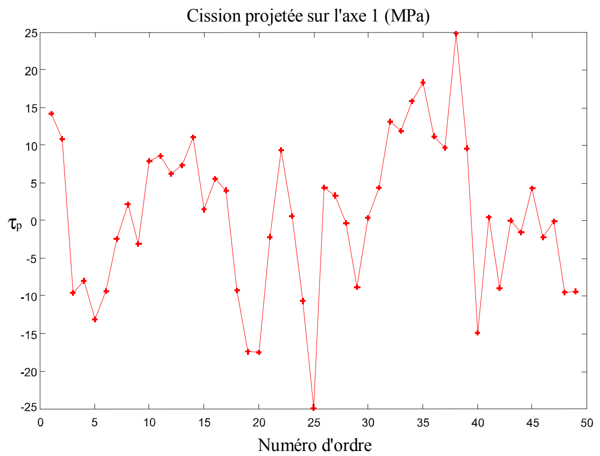
where σ_{eq} the equivalent stress within the meaning of the criterion of MATAKE represents and with:

$\vec{n}^*$	norm with the plane for which the amplitude of shears is maximum;
$\Delta\tau\left(\vec{n}^*\right)/2$	maximum half-amplitude of shears;
a	constant which perhaps defined by a test in alternate pure shears and tension - alternate compression or by a test in alternate traction and compression and nonalternate traction and compression;
$N_{\max}\left(\vec{n}^*\right)$	maximum normal stress as regards norm $\vec{n}^*$ during the cycle;
c_p	harmful effect of pre-hardening in controlled strain $c_p\geq 1$.

To compute: the cumulated damage if the loading is not periodical the first stage consists in determining the scission (vector shears) in a plane of norm $\vec{n}$ at all times of the loading. The technique which is used with this intention is described in the reference [bib6]. In the second stage we start by reducing the history of the scission to a unidimensional function of time by projecting the point of the vector scission on one or two axes defined in the plane of norm $\vec{n}$ considered, cf chapter 7 and 8. Thus the evolution of the projected scission is summarized with the relation: $\tau_p=f(t)$ what makes it possible to use the method of counting RAINFLOW. On the figure [Figure 6.1-a] we show the values reached by the end of the vector shears in a plane of norm $\vec{n}$ before projection on an axis or two axes and the figure [Figure 6.1-b] these same values after projection on an axis. At this stage we should introduce the notion of elementary equivalent stress σ_{eq}^i. Practically this notion has the same meaning as the notion of equivalent stress defined by the relation [éq 6.1-1], but it applies to the elementary under-cycles resulting from the method of counting RAINFLOW. Thus from the projected scission τ_p we calculate elementary equivalent stresses σ_{eq}^i.

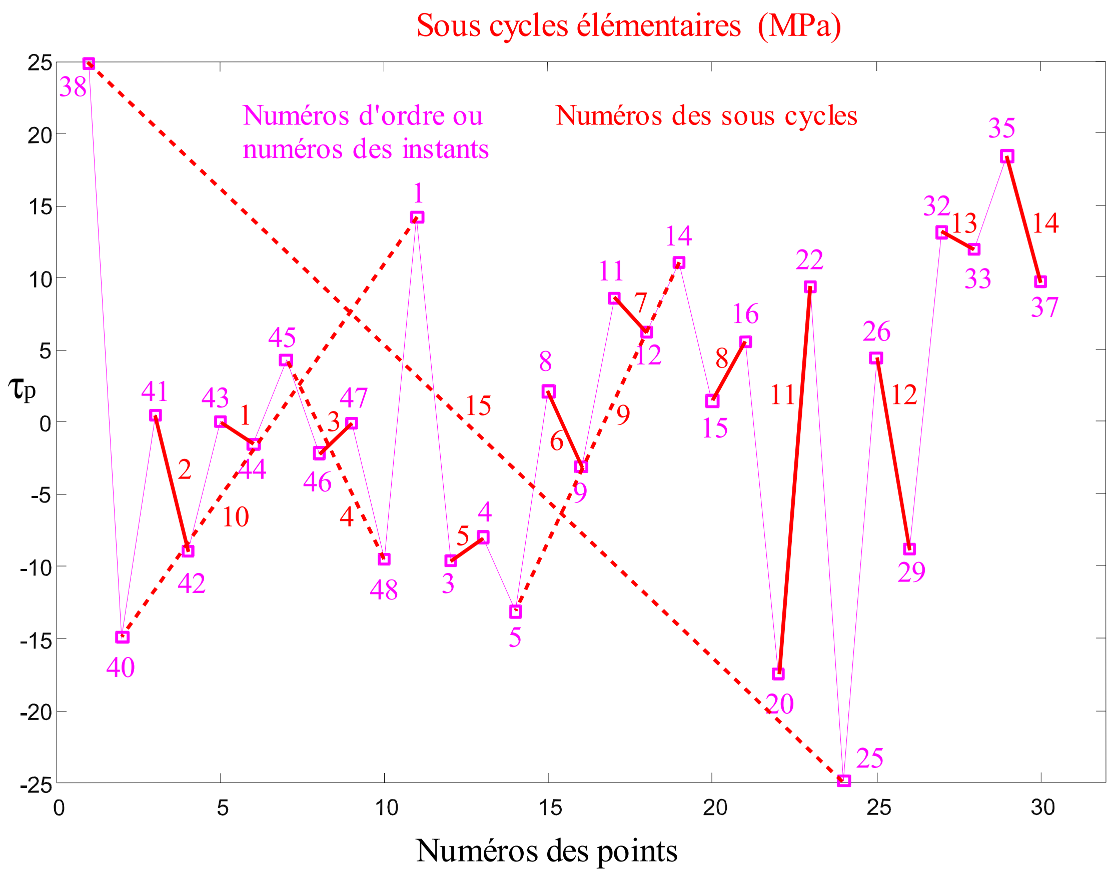


Appear 6.1-a: Points of the vector cission before projection



Appears 6.1-b: Points of the vector cission after projection on an axis

method RAINFLOW breaks up $\tau_p = f(t)$ into periodic elementary under-cycles and breeze the history of the loading, as we show it on the figure [Figure 6.1-c]. Thus, for a given $\vec{n}$ norm method RAINFLOW provides for each elementary under-cycle two values, points high and low, of the point of the vector cission $\tau^i_{p_1}(\vec{n})$ and $\tau^i_{p_2}(\vec{n})$ associated with two values of maximum normal stress $N^i_1(\vec{n})$ and $N^i_2(\vec{n})$.



Appear 6.1-c: The fifteen elementary under-cycles after processing by method RAINFLOW

For the criterion of MATAKE we define the elementary equivalent stress in the following way:

$$\sigma^i_{eq}(\vec{n}) = c_p \frac{Max\left(\tau^i_{p_1}(\vec{n}),\tau^i_{p_2}(\vec{n})\right) - Min\left(\tau^i_{p_1}(\vec{n}),\tau^i_{p_2}(\vec{n})\right)}{2} + a\ Max\left(N^i_1(\vec{n}),N^i_2(\vec{n}),0\right) \quad \text{éq 6.1-2}$$

For the office plurality of damage, this elementary equivalent stress is to be used with a curve of fatigue in shears. If a curve of fatigue in tension compression is used it is necessary to multiply [éq 6.1-2] by a corrective coefficient which corresponds to the ratio of the limits of endurance in bending and alternate torsion and that we note α :

$$\sigma^i_{eq}(\vec{n}) = \alpha\left(c_p \frac{Max\left(\tau^i_{p_1}(\vec{n}),\tau^i_{p_2}(\vec{n})\right) - Min\left(\tau^i_{p_1}(\vec{n}),\tau^i_{p_2}(\vec{n})\right)}{2} + a\ Max\left(N^i_1(\vec{n}),N^i_2(\vec{n}),0\right)\right)$$

éq 6.1-3

From $\sigma^i_{eq}(\vec{n})$ and of a curve of Wöhler we deduct the number of cycles to the fracture $N^i(\vec{n})$ and the elementary damage $D^i(\vec{n})=1/N^i(\vec{n})$ corresponding to an elementary under-cycle. We use a linear office plurality of damage. That is to say k the number of elementary under-cycles, for a fixed $\vec{n}$ norm, the cumulated damage is equal to:

$$D(\vec{n})=\sum_{i=1}^{k} D^i(\vec{n}) \quad \text{éq 6.1-4}$$

to determine the normal vector $\vec{n}^*$ corresponding to the maximum cumulated damage it is enough to vary $\vec{n}$ and to calculate [éq 6.1-4]. The normal vector $\vec{n}^*$ corresponding to the maximum cumulated damage is then given by:

$$D(\vec{n}^*)=\underset{\vec{n}}{Max}\left(D(\vec{n})\right)$$

6.2 Criterion of DANG VAN modified

In the frame of the damage and a periodic loading, the criterion of DANG VAN is written:

$$\sigma_{eq}(\vec{n}^*)=c_p\frac{\Delta\tau(\vec{n}^*)}{2}+a\ P$$

where σ_{eq} the equivalent stress within the meaning of the criterion of DANG VAN represents and with:

$\vec{n}^*$	norm with the plane for which the amplitude of shears is maximum;
$\Delta\tau\left(\vec{n}^*\right)/2$	maximum half-amplitude of shears;
a	constant which perhaps defined by a test in alternate pure shears and tension - alternate compression or by a test in alternate traction and compression and nonalternate traction and compression;
P	maximum hydrostatic pressure during the cycle;
c_p	harmful effect of pre-hardening in controlled strain $c_p\geq 1$.

When the loading is not periodical, we calculate the damage by the same process as that used for the criterion of MATAKE. The only difference lies in the definition of the elementary equivalent stress:

$$\sigma^i_{eq}(\vec{n})=\frac{Max\left(\tau^i_{p_1}(\vec{n}),\tau^i_{p_2}(\vec{n})\right)-Min\left(\tau^i_{p_1}(\vec{n}),\tau^i_{p_2}(\vec{n})\right)}{2}+a\ Max\left(P^i_1(\vec{n}),P^i_2(\vec{n}),0\right) \quad \text{éq 6.2-1}$$

where P^i_1 and P^i_2 represents the two values of the hydrostatic pressure attached to each under - elementary cycle. This elementary equivalent stress is to be used with a curve of fatigue in shears. If one must employ a curve of fatigue in tension compression it is necessary to multiply [éq 6.2-1] by the corrective coefficient α :

$$\sigma^i_{eq}(\vec{n})=\alpha\left(c_p\frac{Max\left(\tau^i_{p_1}(\vec{n}),\tau^i_{p_2}(\vec{n})\right)-Min\left(\tau^i_{p_1}(\vec{n}),\tau^i_{p_2}(\vec{n})\right)}{2}+a\ Max\left(P^i_1(\vec{n}),P^i_2(\vec{n}),0\right)\right)$$

After having defined the criteria of MATAKE and DANG-VAN in the frame of the office plurality of damage and a nonperiodic loading, it remains us to specify the technique of projection which we propose.

6.3 Criterion of FATEMI-SOCIE modified

6.3.1 Description

the criterion of FATEMI and SOCIE is a criterion of type critical plane [9], [10]. Initially formulated for periodic loadings, we propose a version adapted to the nonperiodic loadings of it.
In this criterion the parameter a is defined as follows: $a=k/\sigma_y$ where σ_y is the elastic limit and k a coefficient which depends on the material. We will reconsider the way of calculating k. This criterion mixes the shears in strain and the maximum normal stress. We propose to define an equivalent strain "elementary" in the following way:

$$\varepsilon_{eq}^{i}(\vec{n})=\alpha\left(c_p\ \frac{Max\left(\gamma'^{i}_{p_1}(\vec{n}),\gamma'^{i}_{p_2}(\vec{n})\right)-Min\left(\gamma'^{i}_{p_1}(\vec{n}),\gamma'^{i}_{p_2}(\vec{n})\right)}{2}\left[1+a\ Max\left(N^{i}_{1}(\vec{n}),\ N^{i}_{2}(\vec{n}),\ 0\right)\right]\right)$$

where $\gamma'^{i}_{p_1}$ and $\gamma'^{i}_{p_2}$ the extreme shear strains of the under-cycle number represent i.

Except a definition different from the criterion, the approach used to compute: the damage is identical to the two preceding criteria. Lastly, it is also the maximum damage which makes it possible to select the critical plane.

It will be noted that the shear strains used in the criterion of FATEMI and SOCIE are distortions γ_{ij} ($i\neq j$). If one uses the shear strains of the tensorial type ϵ_{ij} ($i\neq j$), they should be multiplied by a factor 2 because $\gamma_{ij}=2\ \epsilon_{ij}$.

6.3.2 Identification of the coefficient *K*

the author proposes to identify the coefficient k from tests in pure traction and compression and pure alternate torsion on a thin tube [9], [10]. In order not to introduce skew, the two kinds of tests must be realized on the same type of test-tube. Before presenting the formula which defines k we let us introduce the following notations:

- ν : Poisson's ratio, (is generally worth $0{,}3$ for our materials);
- ν_p : coefficient of incompressibility of plastic strains (is worth $0{,}5$ in [9] and [10]);
- E : Elasticity modulus of Young;
- G : Elastic shear modulus;
- N_f : Many cycles to the fracture.

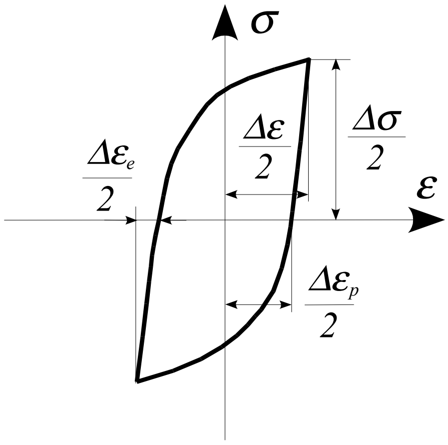
Contrary to the two preceding criteria this one can treat the cases where there remains elastoplastic zones in structure. The coefficient k is defined by the following relation:

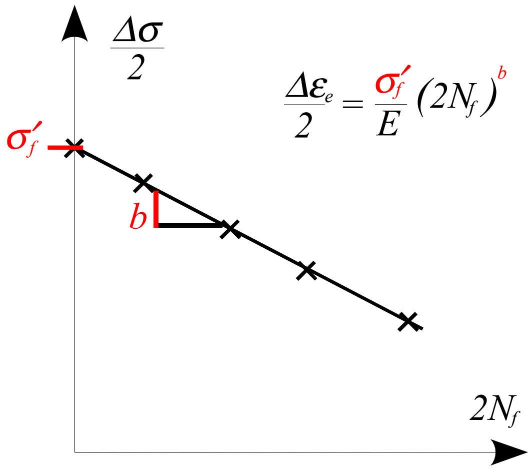
$$k=\left[\frac{\frac{\tau'_f}{G}\left(2\mathrm{N}_f\right)^{b_o}+\gamma'_f\left(2\mathrm{N}_f\right)^{c_o}}{\left(1+\nu\right)\frac{\sigma'_f}{E}\left(2\mathrm{N}_f\right)^{b}+\left(1+\nu_p\right)\varepsilon'_f\left(2\mathrm{N}_f\right)^{c}}-1\right]\frac{k'\left(0{,}002\right)^{n'}}{\sigma'_f\left(2\mathrm{N}_f\right)^{b}}\ ,$$

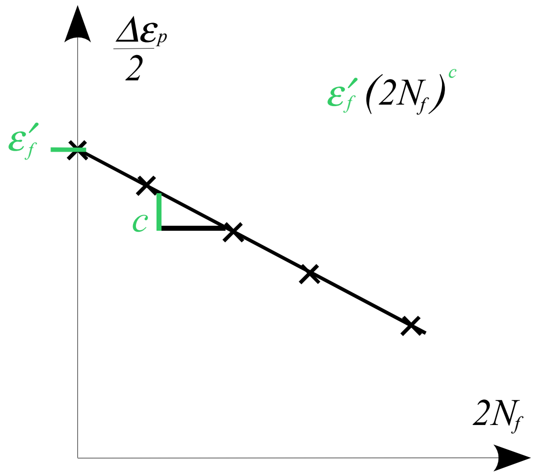
where terms: τ'_f b_o γ'_f c_o σ'_f b ε'_f c , k' and n' are defined by the means of tests.

Tests in pure traction and compression

the tests in pure traction and compression make it possible to identify the coefficients: σ'_f b ε'_f c , k' and n' .

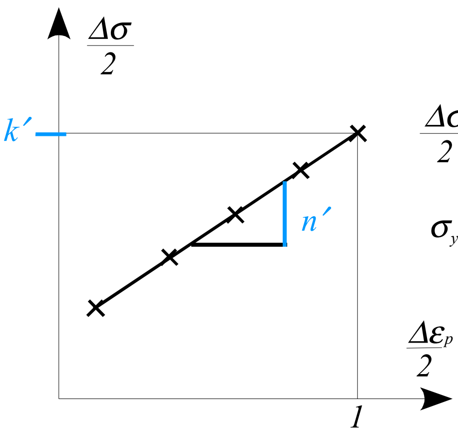






$$\varepsilon_a=\frac{\Delta\varepsilon}{2}=\frac{\sigma'_f}{E}\left(2N_f\right)^b+\varepsilon'_f\left(2N_f\right)^c$$

The curves opposite use scales Log-Log.

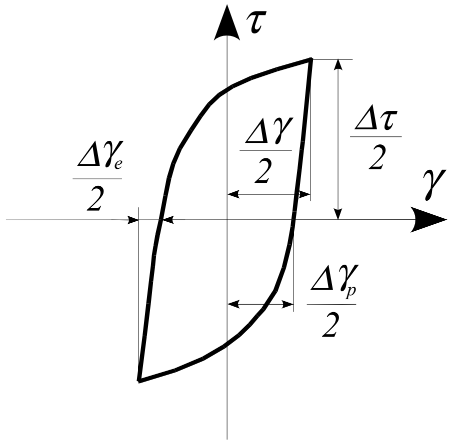


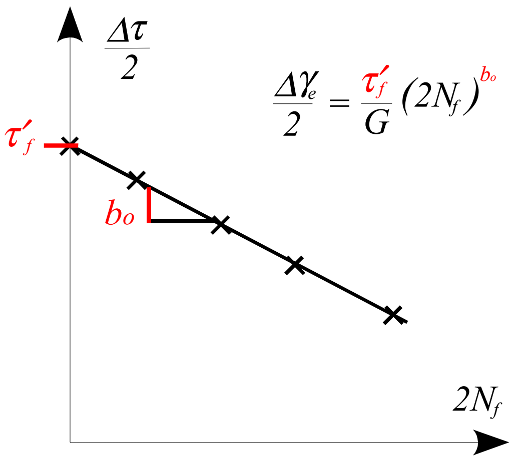
$$\frac{\Delta\sigma}{2}=k'\left(\frac{\Delta\varepsilon_p}{2}\right)^{n'}$$

$$\sigma_y=k'(0{,}002)^{n'}$$

Tests in pure alternate torsion

the tests in pure alternate torsion make it possible to identify the coefficients:
$\tau'_f \quad b_o \quad \gamma'_f \quad c_o$.





$$\gamma_a = \frac{\Delta\gamma}{2} = \frac{\tau'_f}{G}(2N_f)^{b_o} + \gamma'_f (2N_f)^{c_o}$$

$$\gamma_a = \frac{\Delta\gamma}{2} = \frac{\tau'_f}{G}(2n_f)^{b_0} + \gamma'_f(2N_f)^{c_0}$$

7 Choice of the axes of projection

With regard to the projection of the end of the vector cission we propose two options:

- a projection on an axis,
- a projection on two axes.

The axis of option 1 is in the same way given that the first axis of option 2. The second axis of option 2 is orthogonal with the first axis of this option.

7.1 Projection on an axis

We place ourselves in a plane of norm $\vec{n}$ given where each point represents the position of the point of the vector shears at one time, for more details to see the reference [bib6]. In this plane we build the smallest frame who contains all the points at every moment representing the end of the vector cission. The two diagonals of the frame enable us to define two axes: axis 1 corresponds at the segment $\overline{AC}$, and centers it 2 corresponds at the segment $\overline{DB}$, cf [Figure 7.1].

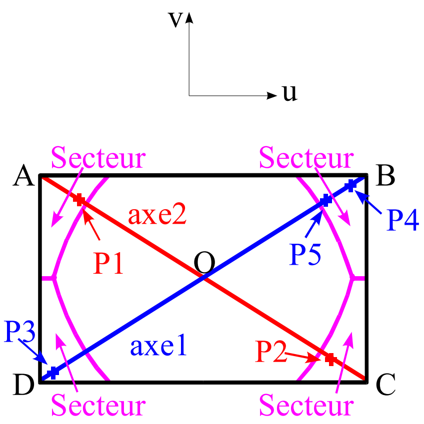


Figure 7.1-a

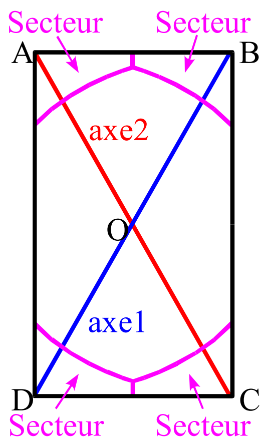


Figure 7.1-b

Figure 7.1: Definition of the axes of projection

We choose a priori the axis of projection among axes 1 and 2 because the diagonal of the frame is larger than the large side of the frame what has as a virtue to dilate a little the projected points. In addition the loadings which interest us are of thermal origin with the result that the points representing the evolution of the point of the vector cission, in the planes of norm $\vec{n}$, are generally aligned on an axis, as we show it on the figure [Figure 6.1-a].

Sectors 1,2,3 and 4 are built same way as in the reference [bib6]. Only the points which are in these sectors are projected orthogonally on axes 1 and 2.

We define the axis of projection as being the axis on which the distance between two projected points is largest.

For example, on [Figure 7.1-a] the axis of projection is axis 1 since the length of the segment $\overline{P_3 P_4}$ is higher than the length of the segment $\overline{P_1 P_2}$. This definition of the axis of projection makes it possible to make sure that the axis of projection retained will make it possible to give an account of the largest amplitude of shears projected.

According to the presence or absence of points in sectors 1,2,3 and 4 the determination of the axis of projection can be immediate, it is then not necessary to implement the procedure of selection described above. For more details the reader will be able to refer to appendix 1.

A second axis is necessary to distinguish the case where the points representing the point of the vector cission are aligned on an axis of the case where these points describe a circle.

7.2 Construction of the second centers

the second axis of projection is orthogonal with the initial axis of projection and it passes by the point O.

Since we know the coordinates of the points A B, C and D, to characterize the second axis completely it is enough to determine the punctual coordinates M such as:

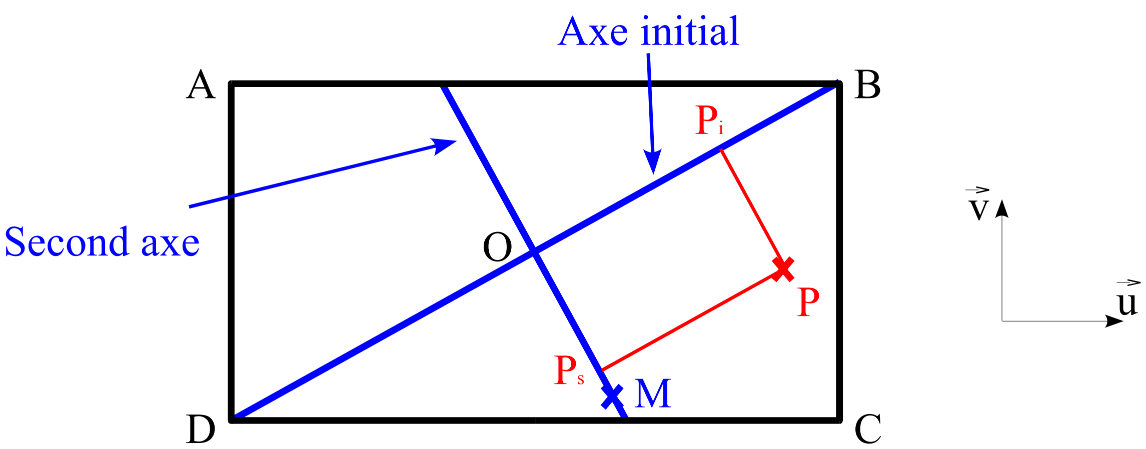
$\overrightarrow{DB} \cdot \overrightarrow{OM}=0$ if the initial axis is axis 1,
$\overrightarrow{AC} \cdot \overrightarrow{OM}=0$ if the initial axis is axis 2.

8 Projection of the shears

In this chapter we describe the process of projection on the initial axis, or first axis, and the second axis. We point out that projection on these two axes is orthogonal.

8.1 Case where axis 1 is the initial axis

This case is represented on [Figure 8.1-a]. We place in the reference $(O, \vec{u}, \vec{v}, \vec{n}$. The definitions of $\vec{u}$, $\vec{v}$ and $\vec{n}$ are given in the reference [bib6]. In the plane $(\vec{u}, \vec{v})$ of norm $\vec{n}$ the points A B C, D and O have respectively, for coordinates $(U_{\min}, V_{\max})$ $(U_{\max}, V_{\max})$ $(U_{\max}, V_{\min})$, $(U_{\min}, V_{\min})$ and (U_O, V_O) .



Appear 8.1-a: Projection if axis 1 is the initial axis

8.1.1 Determination of the second centers

Here to determine the second axis we solve the equation:

$$\overrightarrow{DB} \cdot \overrightarrow{OM}=0 \quad \text{éq 8.1.1-1}$$

where the coordinates U_M, V_M of the point M are the unknowns.
The equation [éq 8.1.1-1] is also written in the following form:

$$(U_{\max}-U_{\min})(U_M-U_O)+(V_{\max}-V_{\min})(V_M-V_O)=0$$

what leads to:

$$V_M = V_O - \frac{(U_{\max} - U_{\min})}{(V_{\max} - V_{\min})}(U_M - U_O)$$

By giving a value of U_M different from U_O we let us obtain immediately V_M .

8.1.2 Projection of an unspecified point on the initial axis

From a known P unspecified point, the first stage consists in calculating the punctual coordinates P such as:

$$\vec{DB}.\vec{PP'} = 0$$

While proceeding like previously, we obtain the relation:

$$V_{P'} = V_P \frac{(U_{\max} - U_{\min})}{(V_{\max} - V_{\min})}(U_{P'} - U_P)$$

where $V_{P'}$ results from a value from $U_{P'}$ different from U_P.

In the plane (u,v) the initial axis and the segment $\bar{PP}'$ are lines closely connected respectively described by $v = a_i u + b_i$ and $v = a_P u + b_P$, therefore to know the coordinates of the point project on the initial axis P_p we solve the equation:

$$a_i u + b_i = a_P u + b_P$$

where

$$a_i = \frac{(V_{\max} - V_{\min})}{(U_{\max} - U_{\min})} \qquad b_i = \frac{(U_{\max} V_{\min} - U_{\min} V_{\max}}{(U_{\max} - U_{\min})}$$

$$a_P = \frac{(V_{P'} - V_P)}{(U_{P'} - U_P)} \qquad b_P = \frac{(U_{P'} V_P - U_P V_{P'})}{(U_{P'} - U_P)} \; .$$

One obtains:

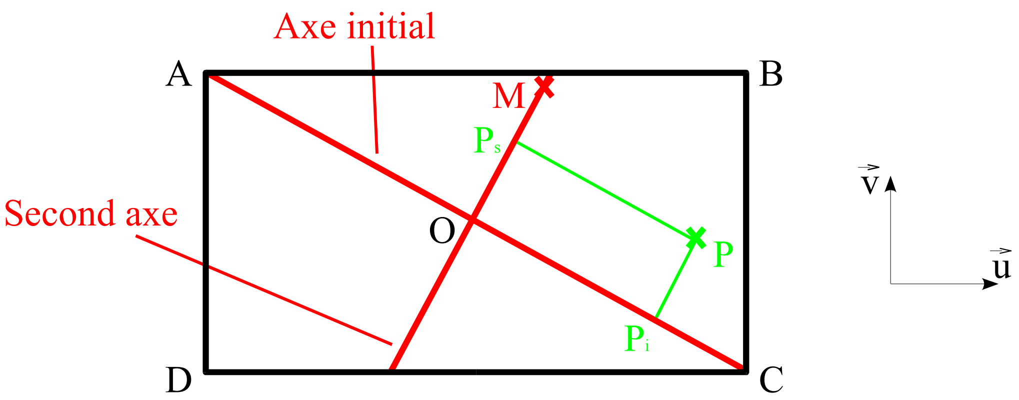
$$U_{P_i} = \frac{b_p - b_i}{a_i - a_p}$$

$$V_{P_i} = \frac{a_i b_P - a_P b_i}{a_i - a_P}$$

The projection of an unspecified point on the second axis is described in appendix 2.

8.2 Case where axis 2 is the initial axis

This case is represented on [Figure 8.2-a]. As previously, in the plane $(\vec{u}, \vec{v})$ the points A B C, D and O have respectively, for coordinates $(U_{\min}, V_{\max})$ $(U_{\max}, V_{\max})$ $(U_{\max}, V_{\min})$, (U_{min}, V_{min}) and $(U_{0,} V_0)$.



Appear 8.2-a: Projection if axis 2 is the initial axis

8.2.1 Determination of the second centers

Here to determine the second axis we solve the equation:

$$\vec{AC}\ .\ \vec{OM}=0 \quad \text{éq 8.2.1-1}$$

where the coordinates (U_M, V_M) of the point M are the unknowns.

The equation [éq 8.2.1-1] is also written in the following form:

$$\left(U_{\max}-U_{\min}\right)\left(U_{\mathrm{M}}-V_{\mathrm{O}}\right)\left(V_{\max}-V_{\min}\right)\left(V_{\mathrm{M}}-V_{\mathrm{O}}\right)=0$$

what leads to:

$$V_M=V_O+\frac{\left(U_{\max}-U_{\min}\right)}{\left(v_{\max}-V_{\min}\right)}\left(U_M-U_O\right)$$

By giving a value of U_M different from U_O we let us obtain immediately V_M.

8.2.2 Projection of an unspecified point on the initial axis

From a known P unspecified point, the first stage consists in calculating the punctual coordinates P' such as:

$$\vec{AC}\,.\,\vec{PP}'=0$$

While proceeding like previously, we obtain the relation:

$$V_{P'}=V_P\frac{\left(U_{\max}-U_{\min}\right)}{\left(V_{\max}-V_{\min}\right)}\left(U_{P'}-U_P\right)$$

where for a value of $U_{P'}$ different from U_P we let us calculate $V_{P'}$.

In the plane (u,v) the initial axis and the segment $\bar{PP}'$ are lines closely connected respectively described by $v=a_i u+b_i$ and $v=a_p u+b_p$, therefore to know the coordinates of the point project on the initial axis P_p we solve the equation:

$$a_i u+b_i=a_p u+b_p$$

where

$$a_i=-\frac{(V_{\max}-V_{\min})}{(U_{\max}-U_{\min})} \qquad b_i=\frac{(U_{\max}V_{\max}-U_{\min}V_{\min})}{(U_{\max}-U_{\min})}$$

$$a_p=\frac{(V_{P'}-V_P)}{(U_{P'}-U_P)} \qquad b_p=\frac{(U_{P'}V_p-U_P V_{P'})}{(U_{P'}-U_P)} .$$

One obtains:

$$U_{P_i}=\frac{b_P-b_i}{a_i-a_P} \qquad V_{P_i}=\frac{a_i\, b_P-a_P\, b_i}{a_i-a_P} .$$

The projection of an unspecified point on the second axis is described in appendix 2.

8.3 Definition of the modulus and directional sense of the axis of projection

We propose to define the sign of the modulus of the point project compared to the initial axis. That is to say the reference $(O, \vec{u}, \vec{v}, \vec{n})$ in which the scission evolves. In this reference if the component U_{P_i} of the point project is higher or equal to zero the sign of the modulus is positive, if not it is negative. In short the modulus and the sign of the modulus of the point project are in the following way defined:

$$P_{\text{mod}}=\sqrt{\overline{OP_i}^2+\overline{OP_s}^2} \quad si \quad U_{P_i}\geq 0 ,$$
$$P_{\text{mod}}=-\sqrt{\overline{OP_i}^2+\overline{OP_s}^2} \quad si \quad U_{P_i}<0 .$$

The definition of the modulus differentiates the loadings closely connected from the circular loadings. In accordance with the experiment a circular loading will be regarded as being more damaging that a loading refines [bib1].

9 Criteria formulates some

9.1 For the periodic loading

For the periodic loading, the computation of the damage is carried out only on the first complete cycle. The first part of the monotonic history of the loading corresponding to the loading is not taken into account because this one aims to impose a non-zero average loading. For the elastic behavior, computation is carried out between the maximum value and the minimal value of the cycle considered. For the elastoplastic behavior, computation is carried out between the first discharge and the second discharge.

The list of quantities available is in the following table:

TYPE_CHARGE = "PERIODIQUE", CRITERE = "FORMULE_CRITERE"
quantities available:
"DTAUMA": half-amplitude of shears in maximum stress ($\Delta\tau(\mathbf{n*})/2$) "PHYDRM": hydrostatic pressure (P) "NORMAX": normal maximum stress as regards norm ($N_{max}(\mathbf{n*})$) "NORMOY": average normal stress on critical plane ($N_{moy}(\mathbf{n*})$) "EPNMAX": maximum normal strain on critical plane ($\varepsilon_{Nmax}(\mathbf{n*})$)

"EPNMOY": average normal strain on critical plane ($\varepsilon_{Nmoy}(\mathbf{n*})$)

"DEPSPE": half-amplitude of equivalent plastic strain ($\Delta\varepsilon_{eq}^{p}/2$)

$$\Delta\varepsilon_{eq}^{p}/2=\frac{1}{2}max_{t1}\,max_{t2}\sqrt{\frac{2}{3}(\boldsymbol{\epsilon}^{p}(t_1)-\boldsymbol{\epsilon}^{p}(t_2)):(\boldsymbol{\epsilon}^{p}(t_1)-\boldsymbol{\epsilon}^{p}(t_2))}$$

"EPSPR1": half-amplitude of the first principal strain (with the taking into account of the sign)

$$\frac{\epsilon_{max}^{1}-\epsilon_{min}^{1}}{2}$$

"SIGNM1": maximum normal stress on the level associated with ε_1

$$max_t(\boldsymbol{\sigma}(t).\boldsymbol{n}_1(t).\boldsymbol{n}_1(t))$$

where $\boldsymbol{n}_1(t)$ is the normal vector of the plane associated with ε_1 .

"DENDIS": density of dissipated energy (W_{cy})

$$W_{cy}=\int_{cylce}\boldsymbol{\sigma}:\dot{\boldsymbol{\epsilon}}^{p}\,dt$$

where $\dot{\boldsymbol{\epsilon}}^{p}$ the rate of the plastic strain represents.

"DENDIE": density of energy of the elastic distortions (W_e)

$$W_e=\int_{cylce}\langle\boldsymbol{s}:\dot{\boldsymbol{e}}^{e}\rangle dt$$

where $\boldsymbol{s}$ the deviatoric part of the stress represents $\boldsymbol{\sigma}$, $\boldsymbol{e}^{e}$ represents the deviatoric part of the stress $\boldsymbol{\epsilon}^{e}$ and $\langle x\rangle$ gives x if $x\geq 0$ and gives 0 if $x<0$.

"APHYDR": half-amplitude of hydrostatic pressure (P_a)

$$P_a=\frac{P_{max}-P_{min}}{2}$$

"MPHYDR": average hydrostatic pressure (P_m)

$$P_m=\frac{P_{max}-P_{min}}{2}$$

"DSIGEQ": half-amplitude of equivalent stress ($\Delta\sigma_{eq}/2$)

$$\frac{\Delta\sigma_{eq}}{2}=\frac{1}{2}max_{t1}\,max_{t2}\sqrt{\frac{3}{2}(\boldsymbol{s}(t_1)-\boldsymbol{s}(t_2)):(\boldsymbol{s}(t_1)-\boldsymbol{s}(t_2))}$$

"SIGPR1": half-amplitude of first principal stress (with the catch in sign)

$$\frac{\sigma_{max}^{1}-\sigma_{min}^{1}}{2}$$

"EPSNM1": normal maximum strain on the level associated with σ_1

$$max_t(\boldsymbol{\epsilon}(t).\boldsymbol{n}_1(t).\boldsymbol{n}_1(t))$$

where $\boldsymbol{n}_1(t)$ is the normal vector of the plane associated with σ_1 .

"INVA2S": half-amplitude of the second invariant of strain ($J_2(\Delta\epsilon)$)

$$J_2(\Delta\epsilon)=\frac{1}{2}max_{t1}\,max_{t2}\sqrt{\frac{2}{3}(\boldsymbol{e}(t_1)-\boldsymbol{e}(t_2)):(\boldsymbol{e}(t_1)-\boldsymbol{e}(t_2))}$$

"DSITRE": half-amplitude of half-forced Tresca ($(\sigma_{max}^{Tresca}-\sigma_{min}^{Tresca})/4$) "DEPTRE": half-amplitude of the half-strain Tresca ($(\epsilon_{max}^{Tresca}-\epsilon_{min}^{Tresca})/4$) "EPSPAC": plastic strain accumulated p "RAYSPH": the radius of the smallest sphere circumscribed with the way of loading within the space of deviators of the stresses R . See the document [R7.04.01] for the definition of this parameter. "AMPCIS": amplitude of cission ($\tau_a = \frac{1}{2} \underset{0 \le t_0 \le T}{\mathrm{Max}} \underset{0 \le t_1 \le T}{\mathrm{Max}} \|\sigma^D_{(t_1)} - \sigma^D_{(t_0)}\|$)

By means of at least one of the first six quantities, one implicitly will build the criterion of standard "the critical plane". In this case there, one will find two planes different on which the shears are maximum.

It is noticed that the names of the quantities are identical to those used in the programming. The operators used in the formula must be in conformity with the syntax of Python as indicated in the note [U4.31.05].

It is noted that the equivalent quantity left for the periodic loading is under name "SIG1" as a result.

9.2 For the loading NON-periodical

the list of quantities available is in the following table:

TYPE_CHARGE = "NON-PERIODIQUE", CRITERE = "FORMULE_CRITERE"
quantities available: "TAUPR_1": shearing stress projected of the first top of the under-cycle ($\tau_{p1}(\mathbf{n})$) "TAUPR_2": shearing stress projected of the second top of the under-cycle ($\tau_{p2}(\mathbf{n})$) "SIGN_1" : normal stress of the first top of the under-cycle ($N_1(\mathbf{n})$) "SIGN_2" : normal stress of the second top of the under-cycle ($N_2(\mathbf{n})$) "PHYDR_1": hydrostatic pressure of the first top of under-cycle "PHYDR_2": hydrostatic pressure of the second top of under-cycle "EPSPR_1": shears in strain projected of the first top of the under-cycle ($\gamma_{p1}(\mathbf{n})$) "EPSPR_2": shears in strain projected of the second top of the under-cycle ($\gamma^i_{p2}(\mathbf{n})$) "SIPR1_1": first principal stress of the first top of the under-cycle ($\sigma_1(1)$) "SIPR1_2": first principal stress of the second top of the under-cycle ($\sigma_1(2)$) "EPSN1_1": normal strain on the level associated with $\sigma_1(1)$ with the first top with under-cycle "EPSN1_2": normal strain on the level associated with $\sigma_1(2)$ with the second top with under-cycle "ETPR1_1": first principal total deflection of the first top of the under-cycle ($\epsilon_1^{tot}(1)$) "ETPR1_2": first principal total deflection of the second top of the under-cycle ($\epsilon_1^{tot}(2)$) "SITN1_1": normal stress on the level associated with $\epsilon_1^{tot}(1)$ with the first top with under-cycle "SITN1_2": normal stress on the level associated with $\epsilon_1^{tot}(2)$ with the second top with under-cycle "EPPR1_1": first principal plastic strain of the first top of the under-cycle ($\epsilon_1^p(1)$) "EPPR1_2": first principal plastic strain of the second top of the under-cycle ($\epsilon_1^p(2)$) "SIPN1_1": normal stress on the level associated with $\epsilon_1^p(1)$ with the first top with under-cycle "SIPN1_2": normal stress on the level associated with $\epsilon_1^p(2)$ with the second top with under-cycle

"SIGEQ_1": equivalent stress of the first top of the under-cycle ($\sigma_{eq}(1)$)
"SIGEQ_2": equivalent stress of the second top of the under-cycle ($\sigma_{eq}(2)$)
"ETEQ_1": equivalent total deflection of the first top of the under-cycle ($\epsilon_{eq}^{tot}(1)$)
"ETEQ_2": equivalent total deflection of the second top of the under-cycle ($\epsilon_{eq}^{tot}(2)$)

For the loading NON-periodical, after having extracted the elementary under-cycles with method RAINFLOW, we calculate an elementary equivalent quantity by the formula of criterion for any elementary under-cycle. It is noted that under cycle is represented by two stress states or strain, noted by the first and the second tops of the under-cycle.

By means of the criterion in formula, one implicitly will build the criterion to determine the plane of the maximum damage with a linear office plurality of the damage.

It is noted that the use of quantities "TAUPR_1" and "TAUPR_2" exclude that from "EPSPR_1" and "EPSPR_2" because one can project either the shearing stress, or shears in strain. It is not possible to project all these two parameters simultaneously.

It is noticed that the names of the quantities are identical to those used in the programming. The operators used in the formula must respect the syntax of Python as indicated in the U4.31.05 note.

10 Quantity and components introduced into *Code_Aster*

10.1 Calculated by CALC_FATIGUE

the computed values are stored with Gauss points or the nodes according to the option selected. Quantity FACY_R (Cyclic Fatigue) was introduced into the catalog of quantities. The components of the field of this quantity, calculated by CALC_FATIGUE [U4.83.02] are described in the following tables.

For the periodic loading and the criteria of the type of maximum critical plane shears

DTAUM1	**first** value of the half amplitude max of the shears in component
critical plane	VNM1X x normal vector with the critical plane related to component
DTAUM1	VNM1Y y normal vector with the critical plane related to component
DTAUM1	VNM1Z z normal vector with the critical plane related to DTAUM1
SINMAX1	normal maximum stress with the critical plane corresponding to constraint
DTAUM1	SINMOY1 average norm with the critical plane corresponding to DTAUM1
EPNMAX1	normal maximum strain with the critical plane corresponding to DTAUM1
EPNMOY1	average maximum strain with the critical plane corresponding to DTAUM1
SIGEQ1	Equivalent stress within the meaning of the criterion selected corresponding with DTAUM1
NBRUP1	many cycles before fracture (function SIGEQ1 and of a curve of Wöhler)
ENDO1	damage associated with NBRUP1 (ENDO1=1/NBRUP1)
DTAUM2	**second** value with the half amplitude max with the shears in component
critical plane	VNM2X x normal vector with the critical plane related to component
DTAUM2	VNM2Y y normal vector with the critical plane related to component
DTAUM2	VNM2Z z normal vector with the critical plane related to DTAUM2
SINMAX2	normal maximum stress with the critical plane corresponding to constraint
DTAUM2	SINMOY2 average norm with the critical plane corresponding to DTAUM2
EPNMAX2	normal maximum strain with the critical plane corresponding to DTAUM2
EPNMOY2	average maximum strain with the critical plane corresponding to DTAUM2
SIGEQ2	Equivalent stress within the meaning of the criterion selected corresponding with DTAUM2
NBRUP2	many front cycles fracture (function of SIGEQ2 and a curve of Wöhler)
ENDO2	damage associates with NBRUP2 (ENDO2=1/NBRUP2)

Table 5.5-1: Components specific to multiaxial cyclic fatigue for the periodic loading

For the loading NON-periodical and the criteria of the type of critical plane of maximum damage

VNM1X	component x normal vector with the critical plane related to the damage component
max	VNM1Y y normal vector with the critical plane related to the damage component
max	VNM1Z z normal vector with the critical plane related to the damage max
ENDO1	damage associated with the block with component
loading	VNM2X x normal vector with the critical plane related to the damage component
max	VNM2Y y normal vector with the critical plane related to the damage component
max	VNM2Z z normal vector with the critical plane related to the damage max

Table 5.5-2: Components specific to multiaxial cyclic fatigue for the loading NON-periodical

For the loading NON-periodical, if there exist only one critical plane of the maximum damage, VNM2X, VNM2Y, VNM2Z are identical to the VNM1X, VNM1Y, VNM1Z. If several planes exist, one emits an alarm and leaves the two foregrounds.

10.2 Calculated by POST_FATIGUE

the computed values are stored with Gauss points or the nodes according to the option selected. Quantity FACY_R (Cyclic Fatigue) was introduced into the catalog of quantities. The components of the field of this quantity, calculated by POST_FATIGUE [U4.83.01] are described in the following tables.

For the periodic loading and the criteria of the type of maximum critical plane shears

DTAUM1	**first** value of the half amplitude max of the shears in component
critical plane	VNM1X x normal vector with the critical plane related to component
DTAUM1	VNM1Y y normal vector with the critical plane related to component
DTAUM1	VNM1Z z normal vector with the critical plane related to DTAUM1
SINMAX	normal maximum stress with the critical plane corresponding to constraint
DTAUM1	SINMOY average norm with the critical plane corresponding to DTAUM1
EPNMAX	normal maximum strain with the critical plane corresponding to DTAUM1
EPNMOY	average maximum strain with the critical plane corresponding to DTAUM1
SIGEQ	Equivalent stress within the meaning of the criterion selected corresponding with DTAUM1
NBRUP	many cycles before fracture (function SIGEQ1 and of a curve of Wöhler)
DOMMAGE	damage associated with NBRUP1 (ENDO1=1/NBRUP1)
VNM2X	component x normal vector with the critical plane related to component
DTAUM2	VNM2Y y normal vector with the critical plane related to component
DTAUM2	VNM2Z z normal vector with the critical plane related to DTAUM2

Table 5.5-1: Components specific to multiaxial cyclic fatigue for the periodic loading

For the loading NON-periodical and the criteria of the type of critical plane of maximum damage

VNM1X	component x normal vector with the critical plane related to the damage component
max	VNM1Y y normal vector with the critical plane related to the damage component
max	VNM1Z z normal vector with the critical plane related to the damage max
DOMMAGE	damage associated with the block with component
loading	VNM2X x normal vector with the critical plane related to the damage component
max	VNM2Y y normal vector with the critical plane related to the damage component
max	VNM2Z z normal vector with the critical plane related to the damage max

Table 5.5-2: Components specific to multiaxial cyclic fatigue for the loading NON-periodical

11 Other criteria

11.1 Criterion `VMIS_TRESCA`

In this part we describe the option `VMIS_TRESCA` which makes it possible to calculate the maximum variation, in the course of time, of a stress tensor according to the criteria of Von Mises and Tresca.

This computation can be carried out with the nodes or Gauss points according to the request of the user.

We give below the algorithm which is programmed in *Code_Aster*.

Notations:

N : Many times T_i : Tensor at time i
$DIFF_T$: difference between two tensors

$VAVMIS=0.0$

$VATRES=0.0$

For i 1 with $(N-1)$

For j $(i+1)$ with N

$DIFF_T=T_i-T_j$

$VMIS=$ Von-Put of $DIFF_T$

$TRES=$ Tresca de $DIFF_T$

Si $VMIS>VAVMIS$ then

$VAVMIS=VMIS$

So $TRES>VATRES$ then

$VATRES=TRES$

11.2 Component of *Code_Aster* used

the computed field by `CALC_FATIGUE` has as components:

`VAVMIS Von Mises`	maximum Amplitude of variation of the criterion of maximum VATRES Amplitude of variation of the criterion of Tresca

12 Conclusion

In this document we presented the criteria of MATAKE and DANG-VAN adapted to the office plurality of damage under periodic and nonperiodic loading.

When the loading is periodic the criteria of MATAKE and DANG-VAN are tested by the cases tests SSLV135a and SSLV135b. The cases tests SSLV135c and SSLV135d test these two criteria if the loading is not periodical.

The keywords which make it possible to use these two criteria are described in the document [U4.83.02] devoted to command CALC_FATIGUE. One will be able to also consult the key word factor CISA_PLAN_CRIT of the command DEFI_MATERIAU [U4.43.01].

13 Bibliography

•TAHERI S.: Bibliography on fatigue with a large number of cycles, Notes K

•HI-74/94/086/0 DANG VAN., GRIVEAU B., MESSAGE O.: There is new multiaxial tires limit criterion: theory and application. Biaxial and Multiaxial Tires, ED. Brown/Miller, 1989.

•MANDEL J., ZARKA J., HALPHEN B.: Adaptation of an elastoplastic structure with kinematic hardening. Mechanical research communications, vol. 4 (5), 1977.

•CLEMENT J.C.: Study and optimization of a method of calculating of fatigue under multiaxial requests of variable amplitude, Note HP-17/97/023/A, June 1997.

•TAHERI S.: The taking into account of a loading with variable amplitude for the computation of damage of fatigue in the zones of mixtures – Project FATMAV, Note HT-64/04/011, December 2004.

•ANGLES J.; Multiaxial criteria of starting in fatigue with a large number of cycles, critical plane, DANG VAN, Project FATMAV, Note HT-64/03/015/A.

•Estimate of the life duration in fatigue to a large number of cycles and oligocyclic, Manual of reference of the Code_Aster, Document [R7.04.01].

•PAPADOPOULOS INTER-F; Polycyclic fatigue of metals, a new approach, Thesis presented to the ENPC on December 18th, 1987.

•SOCIE D.F. and Multiaxial MARQUIS G.B. "Tires", SAE International, 2000.

•FATEMI A. and SOCIE D.F. "A critical planes approach to multiaxial tires ramming including OUT-of-phase loading", Fract Tires. MATER Struct. Vol. 11, NO3, pp. 149-165, 1988

14 Description of the versions of the document

Version Aster	**Author (S) Organization (S)**	**Description of the modifications**
8.4	J.ANGLE S EDF-R&D/AMA	initial Text

Annexe 1

the various situations are summarized in [A1-1 Table]. In [A1-1 Table], "0" and "1" mean respectively that there are no points and that there is at least a point in the indicated sectors.

Sector 1	Sector 3	Sector 2	Sector 4	Centers impossible
projection	0	0	0	0 Case.
0	0	0	1	impossible Case.
0	0	1	0	impossible Case.
0	0	1	1	Axis 1.
0	1	0	0	impossible Case.
0	1	0	1	Use of the procedure of selection.
0	1	1	0	Use of the procedure of selection.
0	1	1	1	Axis 1.
1	0	0	0	impossible Case.
1	0	0	1	Use of the procedure of selection.
1	0	1	0	Use of the procedure of selection.
1	0	1	1	Axis 1.
1	1	0	0	Axis 2.
1	1	0	1	Axis 2.
1	1	1	0	Axis 2.
1	1	1	1	Use of the procedure of selection.

A1-1 table: Summarized situations

the impossible cases result from the way in which are built the frame and the sectors. This construction makes impossible the presence of points in no or only one sector.

Annexe 2

The projection of an unspecified point on the second axis is quickly described in this appendix. From a known P unspecified point, we calculate the punctual coordinates P' such as:

$$\overrightarrow{OM}\,.\,\overrightarrow{PP'}=0$$

After simplification it comes the relation:

where a value of $U_{P'}$ different from U_P us gives $V_{P'}$.

In the plane (u,v) the second axis and the segment are lines closely connected respectively described by $v=a_s\,u+b_s$ and $v=a_P\,u+b_P$, therefore to know the coordinates of the point project on the second axis P_p we solve the equation:

$$a_s\,u+b_s=a_P\,u+b_P$$

where

$$a_s=\frac{(V_M-V_O)}{(U_M-U_O)} \qquad b_s=\frac{(U_M\,V_O-U_O\,V_M)}{(U_M-U_O)}$$

$$a_P=\frac{(V_{P'}-V_P)}{(U_{P'}-U_P)} \quad .$$

One obtains:

$$U_{P_s}=\frac{b_P-b_s}{a_s-a_P}$$

$$V_{P_s}=\frac{a_s\,b_P-a_P\,b_s}{a_s-a_p} \quad .$$