# Estimate of fatigue under random request

**Abstract:**

This note presents two methods of counting of cycles of stresses which lead to an analytical statement of the mechanical damage generated by a random loading:

- method of counting of stresses peaks,
- method of counting of the goings beyond a given level.

The first method calls on the signal, its derivative first and its derivative second. The second requires only the knowledge of the signal and its derivative first.

The cycles of stresses being known by these methods, one determines the average damage over the period of the signal, using the method of Wöhler.

# Contents

# 1 Introduction

the evaluating of the damage is based on the use of curves of endurance of the material, associating a variation of stresses of amplitude given to a number of acceptable cycles defined by a curve of fatigue.

The curves of fatigue of the material are established by subjecting test-tubes to sinusoidal requests of constant amplitude since the beginning of the test until the fracture.

To use these curves from a real loading, it is necessary to identify cycles in the history of the stresses, which is done by methods of counting of cycles. Many methods exist: the document [R7.04.01] in the case of presents two methods of counting of cycles a deterministic real loading.

However, of many real mechanical loadings affecting the nuclear components present a randomness which results in privileging the use of statistical methods to evaluate the damage undergone by these structures.

Certain methods of counting of cycles of stresses were the object of a statistical interpretation:

- method of counting of stresses peaks
- the method of counting of goings beyond of a given level.

The field of application of these two methods [bib1] [bib2] is restricted with ergodic random loadings (the analysis of only one sample is enough to characterize the parameters of the process) and gaussian (the values of the measured signal are distributed according to a normal model).

In addition, the evolution of the signal is comparable to a random process characterized by its statistical parameters (spectral moments of order 0,2 and 4) [R7.10.01].

In both cases, the statistical event to take into account is simple to define:

- a peak of stresses is defined by a slope null and a negative acceleration for a positive peak, a positive acceleration for a negative peak,
- a going beyond level of stresses $S_0$ is characterized by a value of the signal equal to $S_0$ and by a positive slope.

The cycles of stresses being known by these methods, one passes then to computation amongst cycles to the fracture from a curve of fatigue. The curves of Wöhler which are established in experiments were approached by various mathematical statements characterizing more or less correctly the various zones of these curves.
Three mathematical statements are available in *Code_Aster* : a discretized form and two analytical forms.

Knowing the number of cycles of stresses (given by one of these two methods of counting of cycles) and the associated elementary damage (determined by interpolation on the curve of Wöhler of the material), one can calculate an average damage over the period of the signal.

# 2 Evaluating of the damage

For a structure without default geometrical and subjected to a pure alternate stress, the number of cycles to the fracture is given from a diagram of endurance, still called curve of Wöhler or curve $S-N$.

The number of cycles to the fracture is determined by interpolation of the curve of Wöhler of the material for a level of alternate stress (unidimensional) given (to each elementary cycle corresponds a level of amplitude of stress $Ds=|s_{max} - s_{min}|$ and an alternate stress $S_{alt}=1/2\mathrm{Ds}$ ).

The damage of an elementary cycle is equal contrary to many cycles to the fracture:

$$D=\frac{1}{N_{r(S_{alt})}}$$

## 2.1 Diagram of endurance

the diagram of endurance, also called curve of Wöhler or curve $S-N$ (force-NOMBRE curve of cycles to the fracture) is obtained in experiments by subjecting test-tubes to cycles of periodic forces (generally sinusoidal) of normal amplitude $\sigma$ and constant frequencies, and by noting the number of cycles $N_r$ to the end of which the fracture occurs [R7.04.01].

The various mathematical shapes of the curve of Wöhler are described in the document "Estimate of fatigue to great numbers of cycles", [R7.04.01] as well as the way of introducing them into *Code_Aster*.

## 2.2 Elastoplastic coefficient of concentration

It can also be necessary to balance the value of the stress determined by the method of counting by the elastoplastic coefficient of concentration $K_e$ .

The elastoplastic coefficient of concentration $K_e$ (aimed to the B3234.3 articles and B3234.5 of the RCC_M [bib4]) is defined as being the relationship between the amplitude of real strain and the amplitude of strain determined by the elastic analysis.

The value of the coefficient $K_e$ is given in the document [R7.04.01].

# 3 Many cycles of stresses

## 3.1 Recalls: spectral moments and factor of irregularity

One calls spectral moment of order $i$ the following quantity [R7.10.01]:

$$\lambda_i = \int_{-\infty}^{+\infty} |\omega|^i G_{SS}(\omega) d\,\omega$$

where $\omega$ is the pulsation and $G_{SS}$ the power spectral density or DSP.

One has in particular: $\lambda_0 = s_S^2 \lambda_2 = s_{S'}^2 \lambda_4 = s_{S''}^2$ who are the standard deviations of $S$ and its first derived.

**The factor of irregularity** translates the frequential pace of the signal. Ranging between 0 and 1, it tends towards 1 when the process is to narrow tape, on the other hand it tightens towards 0 for a broad band process.

Its statement is:

$$I = \frac{s_{S'}^2}{s_S s_{S''}} = \sqrt{\frac{\lambda_2^2}{\lambda_0 \lambda_4}}$$

We point out these definitions because the evolution of the signal is comparable to a random process characterized by its statistical parameters (spectral moments of order 0,2 and 4).

For the method of counting of stresses peaks, the random signal is entirely characterized by the three spectral moments of order 0,2 and 4.

In the case of the method of counting of the goings beyond level, the spectral moments of order 0 and 2 are enough to characterize the random signal.

In a practical way, these values are given by the command `POST_DYNA_ALEA` [U4.76.02] which operates statistical processing on a random loading. The definition of the various parameters is given in the document [R7.10.01].

The operator of fatigue analysis in random field `POST_FATI_ALEA` [U4.67.05] uses the three spectral moments values calculated by `POST_DYNA_ALEA` and calculates the average damage and the standard deviation of the damage by the methods described in this document.

## 3.2 Method of counting of stresses peaks

the principle of this method consists in counting the local maxima (in absolute value) located on both sides of the mean value of the stresses.

The steady gaussian signal, being centered compared to its mean value, the distribution of the peaks is symmetric compared to this average. One is thus interested in **the distribution of the positive peaks**. In the general case, the distribution of the peaks of amplitude *S* positive is written in the form:

$$P^+_{pic}(S)=\frac{2}{\sqrt{2\pi}s_S(1+I)}\left[\sqrt{1-I^2}\,e^{\frac{S^2}{2s_S^2(1-I^2)}}+\frac{IS}{s_S}e^{\frac{S^2}{2s_S^2}}\int_{-\infty}^{\alpha}e^{\frac{-t^2}{2}}dt\right]$$

$$\text{avec}\begin{cases} I=\dfrac{\sigma^2_{S'}}{\sigma_S\ \sigma_{S''}} \\ \alpha=\dfrac{S}{\sigma_S}\dfrac{I}{\sqrt{1-I^2}} \end{cases}$$

This distribution of the positive peaks is in the case of simplified the signals for which the factor of irregularity is worth $I=0$ or $I=1$ .:

* Signal with broad band: Gauss's law or normal model $(I=0)$

$$P^+_{pic}(S)=\frac{2}{\sqrt{2\pi\sigma_{S^2}}}e^{-\frac{S^2}{2\sigma_{S^2}}}$$

* Signal with narrow tape: model of Rayleigh $(I=1)$

$$P^+_{pic}(S)=\frac{S}{\sigma_{S^2}}e^{-\frac{S^2}{2\sigma_{S^2}}}$$

The counting method of stresses peaks associates with each peak of positive amplitude $S$ a cycle of amplitude $\Delta S=2S$ (one thus has directly $S=Salt$ ).

The number of peaks of positive amplitude is written: $n^+_{pic}(S)=P^+_{pic}(S)\times N^+_{pic}$

where $N^+_{pic}=\frac{1}{4}(1+I)\times\frac{1}{\pi}\frac{\sigma S''}{\sigma_{S'}}$ = median number of positive peaks per unit of time.

From where the number $N$ of cycles to be taken into account is: $N(S)=\frac{1+I}{4\pi}\frac{\sigma_{S''}}{\sigma_{S'}}P^+_{pic}(S)$

**Note:**

*It is noticed well that the statement amongst $N$ cycles to be taken into account depends only on $s_S$ (for the computation of the factor of irregularity $I$ ), $\sigma_{S'}$ and $\sigma_{S''}$ .*

## 3.3 Method of counting of the goings beyond level

This method requires the variation division of stress in classes of amplitude.

The number of cycles $N(S)$ is obtained from the difference of the numbers of goings beyond of level with a positive slope between two successive classes, on the basis of the class of maximum amplitude.

For a gaussian process centered the statement of $N(S)$ is: $N(S)=\frac{1}{\pi}\frac{\sigma_{S'}}{\sigma_S}e^{-\frac{S^2}{2\sigma_S^2}}$

**Note:**

*The statement amongst $N$ cycles to be taken into account requires only the knowledge of $\sigma_S$ and $\sigma_{S'}$ (independence with respect to $\sigma_{S''}$ ).*

*In this method the coefficient of irregularity does not intervene $I$ .*

# 4 Statistical estimate of the damage

the mechanical damage is calculated by means of the linear rule of office plurality To mine.

The damage $D$ generated by $N$ cycles of half amplitude $S$ is expressed by $D=\frac{N}{Nr(S)}$

where $Nr(S)$ is the acceptable number of cycles determined by the curve of endurance of the material.

The mechanical damage is a random variable which one determines the average.

## 4.1 Average damage

the average damage is written in the form of the expectation:

$E(D)=T\int_{S_{min}}^{S_{max}}\frac{N(S)}{N_r(S)}dS$ where $T$ is the period of the signal

the two methods of counting suggested calculate the number of cycles $N(S)$ from stresses of positive amplitude from where $S_{\min}=0$ (except when the curve of Wöhler is given in the form “zones current”, in which case $S_{\min}=S_l$ with $S_l$ limit of endurance of the material).

In addition, models of distributions used being continuous $S_{\max}=\infty$ . However, the experiment shows that the statement to be integrated attenuates quickly and one thus takes $S_{\max}=10\sigma_S$ . where $\sigma_S$ is the standard deviation of the signal.

The computation of $E(D)$ is realized by numerical integration, by the method of the trapezoids while taking for step of integration $\frac{S_{\max}-S_{\min}}{300.}$ .

**Note:**

*In the case of the method of counting of goings beyond level and for a curve of Wöhler expressed in the mathematical form suggested by Basquin, average damage per unit of time to an analytical statement (this statement is not used in command `POST_FATI_ALEA`).*

# 5 Conclusion

In the case of a gaussian, ergodic and steady loading, two methods of counting of cycles find an interpretation statistical and provide an analytical statement of the average damage, utilizing only the formulation of the curve of endurance of the material and the standard variation - of the signal and of its derivatives first and seconds.

In *Code_Aster*, the computation of the damage under random request is carried out by the command `POST_FATI_ALEA` [U4.67.05].

The user can determine the damage by the method of counting of stresses peaks (`COMPTAGE = "PIC"`) or by the method of counting of goings beyond of a given level (`COMPTAGE = "NIVEAU"`).

According to the adopted method, the random signal will have to be introduced by the data of the spectral moments of order 0 and 2 or by the data of the spectral moments of order 0,2 and 4 (key words `MOMENT_SPEC_0`, `MOMENT_SPEC_2` and `MOMENT_SPEC_4`). The values of the spectral moments can also be recovered in an array created by `POST_DYNA_ALEA` [U4.76.02] (key word `COUNTS`).

The curve of Wöhler of the material can be introduced in three distinct forms (in accordance with command `POST_FATIGUE` [U4.67.01] (computation of the damage to great numbers of cycles) and software POSTDAM).

The given quantity is the average damage over the period of the signal which is stored in an array of the type `POST_FATI_ALEA`.

# 6 Bibliography

1) P. MORILHAT: Thermal faience manufacturing: Estimate of the mechanical damage. Note HP - 14/90.07

2) P. MORILHAT: Random mechanics: Statistical estimate of the mechanical damage generated by nonsteady loadings. Note HP - 14/91.19A

3) A. DUMOND: Operator POST_DYNA_ALEA [R7.10.01]

4) RCC_M Edition 1983

# 7 Description of the versions of the document

| Version Aster | Author (S) Organization (S) | Description of the modifications |
|---|---|---|
| 3 | A.M.DONORE, P.MORILHAT EDF-R&D/MMN - EDF-R&D/AMV initial Text | |