# Estimate of the life duration in fatigue to a large number of cycles and oligocyclic

**Summarized:**

Most industrial structures are subjected to variable forces in the time which, repeated a large number of times can lead to their fracture by fatigue. One presents in this note the principal features of commands `POST_FATIGUE` [U4.83.01] and/or `CALC_FATIGUE` [U4.83.02] and/or `CALC_CHAMP` [U4.81.04] which make it possible to estimate the limit of endurance and the office plurality of damage of a part.

The various methods available are:

- linear office plurality: methods based on uniaxial tests (methods of Wöhler, Manson-Whetstone sheath and Taheri).
  These methods have as a common point to determine a value of damage from the evolution during the characterizing time of **a scalar** component, for the computation of the damage, the amplitude of stresses or structural deformations.
  With this intention, it is necessary to extract by a method of counting of cycles, the elementary cycles of loading undergone by structure, to determine the elementary damage associated with each cycle and to determine the total damage by a rule of linear office plurality;
- nonlinear office plurality: method of Lemaître and method of Lemaître-Sermage
  These methods make it possible to calculate the damage $\mathrm{D}$ at every moment $t$, from the data of the tensor of the stresses $\sigma(t)$ and the cumulated plastic strain $p(t)$ ;
- limit of endurance: criteria of Crossland and Dang Van Papadopoulos
  These criteria apply to uniaxial or multiaxial loadings in periodic **stresses**. They provide a value of criterion indicating if there is fatigue or not. The equivalent stresses defined for these criteria can also to compute: be used the office plurality as damage.

# Contents

# 1 Introduction

the industrial experiment shows that the fractures of structure or machine components under normal functioning are generally due to fatigue. Its masked progressive character very often leads to a brutal fracture.

One understands by fatigue the consecutive modification of the properties of the materials to the application of cycles of forces, cycles whose repetition can lead to the fracture of the parts made up with these materials [bib1].

Various methods are available for the evaluating of the damage. The second part of this document is devoted to the presentation of oldest which is methods based on uniaxial tests: method of Wöhler, method of Manson-Whetstone sheath and more recently methods suggested by S. Taheri (EDF-R&D/ AMA).

These methods have as a common point to determine a value of damage from the evolution during the characterizing time of a scalar component, for the computation of the damage, the stress state or of structural deformations.

The evaluating of the damage is based on the use of curves of fatigue of the material (Wöhler or Manson-Whetstone sheath), associating a variation of stress of amplitude given to a number of acceptable cycles.

To use these curves from a real uniaxial loading, it is necessary to treat the history of the stresses or the strains by identifying elementary cycles (cf [§2.2]).

The difficulty in defining a cycle for a complex signal explains the profusion of the methods of counting appeared in the literature [bib2].

Two methods among most usually used were introduced into *the Code_Aster* :

- counting of the extents in cascade or method RAINFLOW,
- rule RCC_M.

One adds to it a third method which we will call method of “natural” counting and which respects the order of application of the cycles of loading.

For each elementary cycle, one evaluates an elementary damage using methods founded on the curves of Wöhler, Manson-Whetstone sheath or both simultaneously.

For the method of Wöhler (cf [§2.3]) the user can correct the stress to be integrated in the curve of Wöhler by:

- a factor of stress concentration $K_T$, to take account of the geometry of the part,
- an elastoplastic coefficient of concentration $K_e$,
- a correction of Goodman or To stack in the diagram of Haigh to take account of the average constraint of the cycle.

In addition, one proposes to define the curve of Wöhler in three different forms, a point by point discretized form and two analytical forms.

The method of Manson-Whetstone sheath (cf [§2.4]) applies to loadings in strains. The curve of Manson-Whetstone sheath is defined in a single form, forms discretized point by point.

The methods of Taheri (cf [§2.5]) also apply to loadings in strains and require the data of the curve of Manson-Whetstone sheath and possibly of the curve of Wöhler. Their characteristic is to take account about application of the elementary cycles of loading with structure, contrary to the two other methods.

**Note:**

*Three methods of extraction of the elementary cycles are available: method of Rainflow, rule of RCC_M and "natural" counting.*

*The first two methods do not take account about application of the cycles what is of no importance for the computation of the damage by the methods of Wöhler or Manson-Whetstone sheath.*

*For the computation of the damage by the methods of Taheri, it is necessary to use the method of extraction of the cycles by "natural" counting [§2.2.3] which respects the order of application of the cycles.*

For the set of these methods computation of the total damage undergone by structure is determined by a method of office plurality, the rule To mine.

The third part of this document presents the methods of Lemaître and Lemaître-Sermage which are "analytical" methods making it possible to calculate the damage $D$ (in each time $t$) from the data of the stress tensor $\sigma(t)$ and the cumulated plastic strain $p(t)$. These two methods apply to loadings in unspecified stresses (uniaxial or multiaxial).

A linear rule of office plurality can be used to determine the total damage undergone by structure.

Lastly, the criteria of Crossland and Dang Van Papadopoulos are presented in fourth and last part of this document. They apply to unspecified loadings (uniaxial or multiaxial) in stresses and periodicals. They provide a value of criterion indicating if there is fatigue or not.

From the value of the criterion, one can specify a scalar component characterizing the state of structure for computation of the damage and determine a value of damage by means of the curve of Wöhler of the material.

# 2 Methods of Wöhler, Manson-Whetstone sheath and Taheri

## 2.1 Extraction of the peaks

the user provides to *Code_Aster* a function which defines the history (scalar) loading in a given point. For that, it has key word `HISTOIRE`.

On this history of the loading, which can be complex, a first operation of extraction of the peaks is carried out. This operation consists in reducing the load history to the only fundamental peaks.

**Note:**

*In fatigue, one names loading in a point given the value of the response of structure in this point.*
*In the use of the curves of Wöhler, it is about stress in this point.*
*In the use of the curves of Manson-Whetstone sheath, it is about strain in this point.*
*The load history is thus the evolution in the course of the time of a stress, or a strain.*

If the function remains increasing or decreasing on more than two consecutive points, one removes the intermediate points to keep only the two extreme points.

One also removes history of the loading the points for which the variation of the value of the stress or the strain is lower than a certain level chosen by the user. That amounts applying a filter to the history of the loading. The value of the level of the filter is introduced by the user under key word `DELTA_OSCI`.

For illustration let us consider the following load history:

| N° point | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Time | 0. | 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. | 9. | 10. | 11. | 12. | 13. |
| Loading | 4. | 7. | 2. | 10. | 9.6.9 .8 | | 5. | 9. | 3. | 4. | 2. | 2.4. 2.2 | | 12. |

| N° point | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Time | 14. | 15. | 16. | 17. | 18. | 19. | 20. | 21. | 22. | 23. | 24. | 25. | 26. | 27. | 28. |
| Loading | 5. | 11. | 1. | 4. | 3. | 10. | 6. | 8. | 12. | 4. | 8. | 1. | 9. | 4. | 6. |

The extraction of the peaks of this load history, with a value of delta of 0.9 conduit to destroy all the oscillations of amplitude lower than 0.9. What leads to the following load history:

| N° point | 1 | 2 | 3 | 4 | 7 | 8 | 9 | 10 | 11 | 14 | 15 | 16 | 17 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Time | 0. | 1. | 2. | 3. | 6. | 7. | 8. | 9. | 10. | 13. | 14. | 15. | 16. |
| Loading | 4. | 7. | 2. | 10. | 5. | 9. | 3. | 4. | 2. | 12. | 5. | 11. | 1. |

| N° point | 18 | 19 | 20 | 21 | 23 | 24 | 25 | 26 | 27 | 28 | 29 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Time | 17. | 18. | 19. | 20. | 22. | 23. | 24. | 25. | 26. | 27. | 28. |
| Loading | 4. | 3. | 10. | 6. | 12. | 4. | 8. | 1. | 9. | 4. | 6. |

**Note:**

*Let us note $ch$ the value of the loading; $ch$ can be a stress or a strain.*

Load history was removed:

- the point $5$ because $\Delta ch = | ch(5) - ch(4) | < 0.9$ ,
- the point $6$ because $\Delta ch = | ch(6) - ch(4) | < 0.9$ ,
- the point $12$ because $\Delta ch = | ch(12) - ch(11) | < 0.9$ ,
- the point $13$ because $\Delta ch = | ch(13) - ch(11) | < 0.9$ .

In the same way the point is removed $22$ because the load history is increasing between the points $21{,}22$ and $23$ . Thus only the extreme points are kept.

## 2.2 Methods of counting of cycles

During their life, the industrial structures are generally subjected to complex loadings whose levels of requests are variable.

The methods of counting of cycles make it possible to extract from the load history, of the elementary cycles according to various criteria.

*Code_Aster* proposes three distinct methods including two nonstatistical methods among the methods most usually used.

### 2.2.1 Method RAINFLOW

The counting method of the extents in cascade more often called method of RAINFLOW, defines cycles which physically correspond to loops of hysteresis in the stress-strains plane. In the literature, several alternatives of this method are counted.

The algorithm implemented in *Code_Aster* essentially that is proposed by recommendation AFNOR A 03-406 of November 1993 [bib3] (with characteristics which is specified during the presentation of the detail of the algorithm) and breaks up into three stages:

- **A first stage** which consists in rearranging the history of the loading $\sigma(t)$ or $\varepsilon(t)$ so that the loading begin with the maximum value, in absolute value, of the loading.

**Note:**

*In recommendation AFNOR A 03-406, it is not mentioned a rearrangement of the load history. This rearrangement is however carried out in software POSTDAM [bib2] and included in Code_Aster.*

- **The second stage** consists in extracting the elementary cycles from the load history thus rearranged.

  The method consists in leaning on four successive points of the load history $(ch(i),\ i = 1,\ \text{Nbpoint})$ .

  One notes:

$$X = |ch(i+1) - ch(i)| \text{ et } Y = |ch(i+2) - ch(i+1)|$$
$$\text{et } Z = |ch(i+3) - ch(i+2)|.$$

As long as $Y$ is strictly higher than $X$ or with $Z$, one traverses the history of the loading while moving of a point towards the line (what amounts incrementing the value of $i$ ).

As soon as $Y$ is lower or equal than $X$ and inferior or equal to $Z$, it is considered that one met an elementary cycle which is defined by the two points $(i+1)$ and $(i+2)$. The amplitude of the cycle is given par. $\Delta ch = |ch(i+1) - ch(i+2)|$

When the cycle is extracted one removes the two points of the load history and one continues the algorithm.

- **The third stage** consists in treating the residue, i.e. the remaining load history after the stage of extraction of the cycles.

  With this intention, one adds the same residue with his continuation realising possibly some care on the level of connection following the values of the extrema considered thus that value of the first and the last slope of the residue.

  The last point of the residue the first point of the cycle succeeds. So the points considered can not seem extrema more. If that occurs, it is advisable to eliminate them. Eight different cases are encountered. To treat them explicitly, and the $R_1$ let us call $R_2$ the first two points of the residue and $R_{n-1}$ $R_n$ its last two points.

**Note:**

*Recommendation AFNOR A 03-406 fact also state of possible a pre - processing of the signal, which would consist of a filtering of the signal (suppression of the parasites) and of a quantification of the load history.*

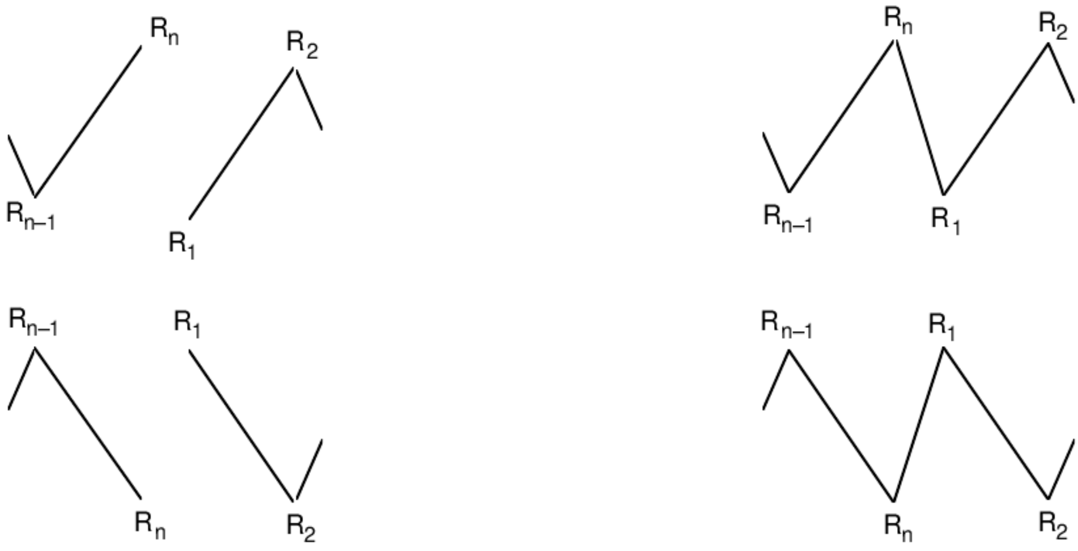
*The filtering of the signal is possible, at the request of the user (see [§2.1]. Extraction of the peaks).*

*The quantification of the signal can be useful for the speed of the analysis of results of the analysis of fatigue. Practically, the quantification of the signal consists to cut out the maximum extent of the signal in classes of intervals of constant width called not, and to bring back to a value representative of a given class (its mean value in general) all the values located in this class. This possibility of preprocessing of the signal as for it, is not available in Code_Aster.*

*In the special case where the load history is constant (for example, average loading applied), Code_Aster null will count the whole load history like a cycle of amplitude.*

Cas rencontré — Raccordement

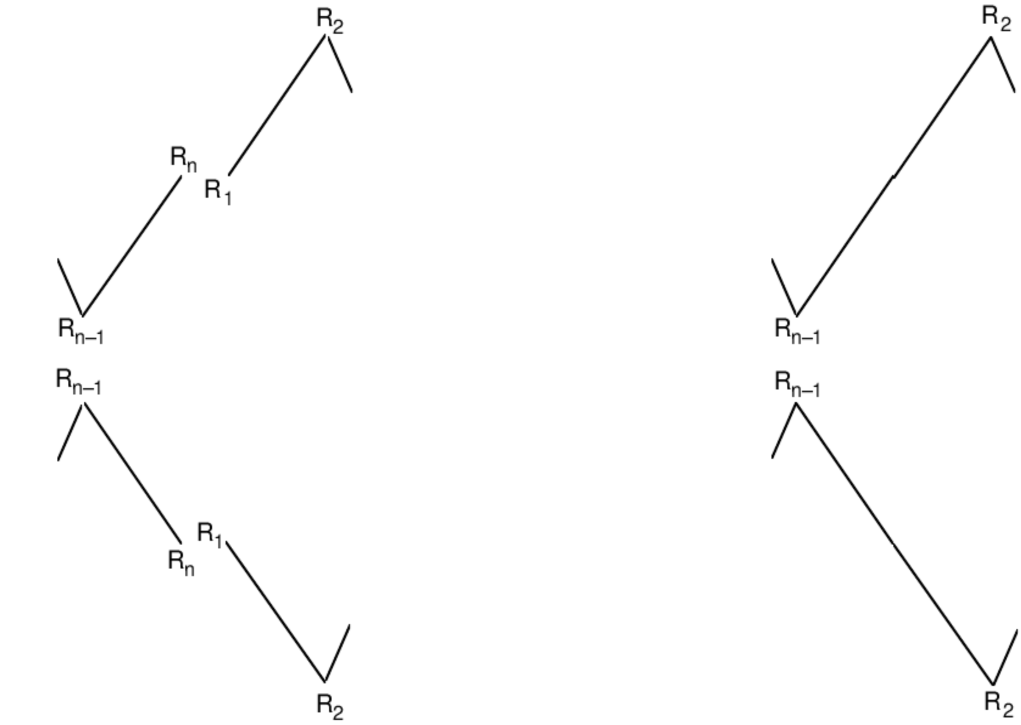
1) $(R_n - R_{n-1}).(R_2 - R_1) > 0$ et $(R_n - R_{n-1}).(R_1 - R_n) < 0$




a) Raccordement sans problème : transition $(R_n, R_1)$

Cas rencontré — Raccordement

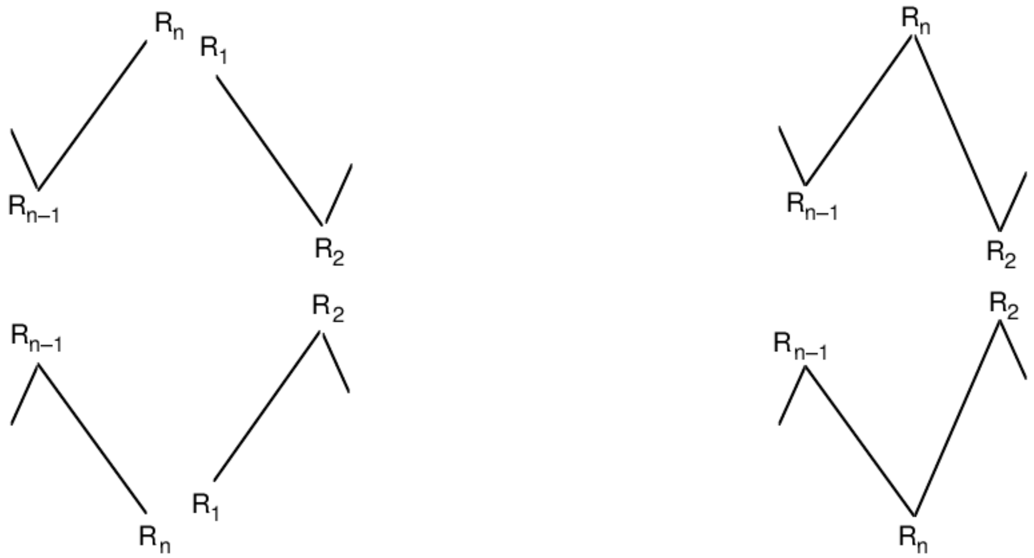
2) $(R_n - R_{n-1}).(R_2 - R_1) > 0$ et $(R_n - R_{n-1}).(R_1 - R_n) > 0$




b) Raccordement transition $(R_{n-1}, R_2)$, on élimine $R_1$ et $R_n$

Cas rencontré Raccordement

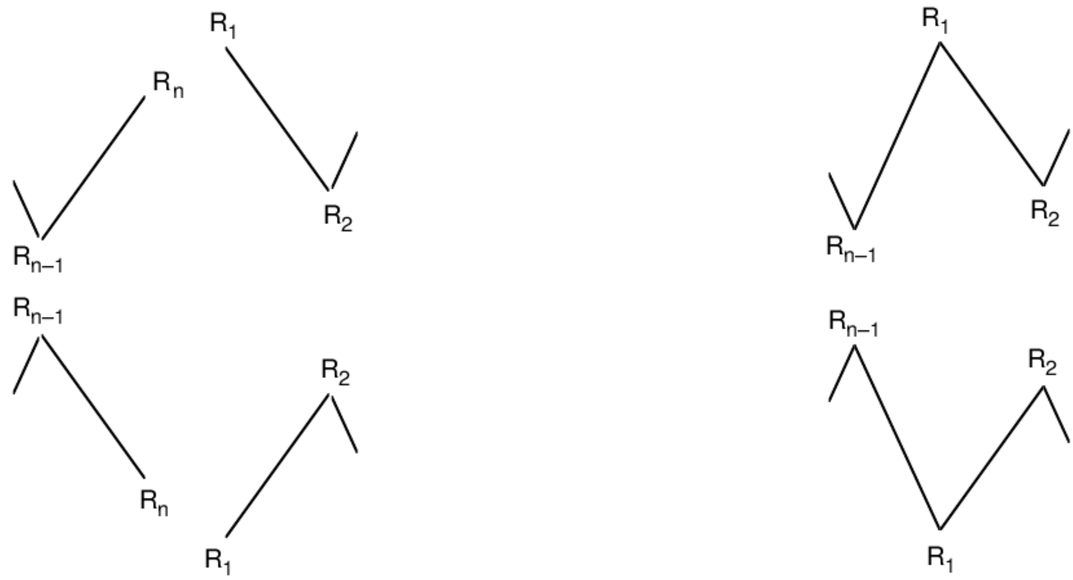
3) $(R_n - R_{n-1}).(R_2 - R_1) < 0$ et $(R_n - R_{n-1}).(R_1 - R_n) < 0$




c) Raccordement transition $(R_n, R_2)$, on élimine $R_1$

Cas rencontré Raccordement

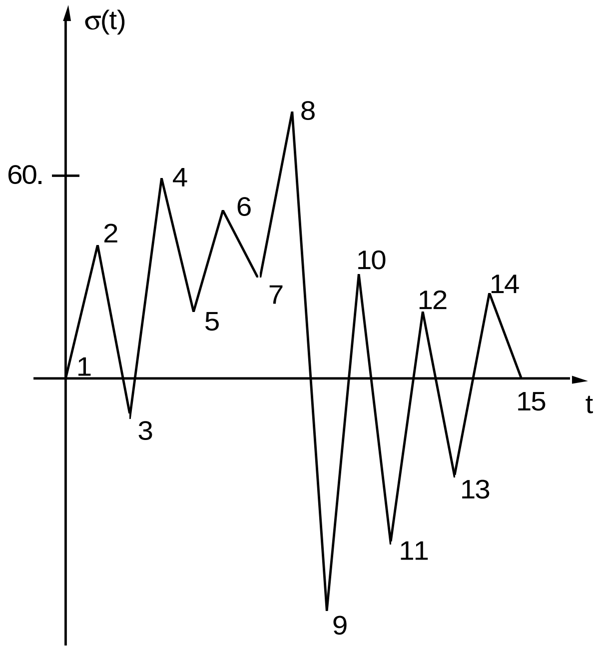
4) $(R_n - R_{n-1}).(R_2 - R_1) < 0$ et $(R_n - R_{n-1}).(R_1 - R_n) > 0$




d) Raccordement transition $(R_{n-1}, R_1)$, on élimine $R_n$

In order to illustrate the method and to clarify the points which would remain obscure, the following load history is considered (which for the example is considered of type forced):

| N° point | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Time | 0. | 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. | 9. | 10. | 11. | 12. | 13. | 14. |
| Loading | 0. | 40. | – 10. | 60. | 20. | 50. | 30. | 80. | – 70. | 30. | – 50. | 20. | – 30. | 25. | 0. |




The method of RAINFLOW thus leads, on this example, (see [§Annexe1], for detail of the stages of the algorithm) with the determination of 7 elementary cycles defined by the maximum value and the minimal value of the loading, for each cycle.

Cycle 1: `VALMAX = 20.` `VALMIN = – 30.`
Cycle 2: `VALMAX = 25.` `VALMIN = 0.`
Cycle 3: `VALMAX = 30.` `VALMIN = – 50.`
Cycle 4: `VALMAX = 40.` `VALMIN = – 10.`
Cycle 5: `VALMAX = 50.` `VALMIN = 30.`
Cycle 6: `VALMAX = 60.` `VALMIN = 20.`
Cycle 7: `VALMAX = 80.` `VALMIN = – 70.`

**Note:**

- *The computation damage which does not take account about appearance of the elementary cycles of loading, it is without consequence to rearrange the history of the loading.*
- *For the methods of Taheri, the order of application of the elementary cycles of loading is taken into account, also is necessary it to be very vigilant with the use of such a method of counting of cycles. He is advised, for the computation of the damage by the methods of Taheri, to use the method of "natural" counting known as [§2.2.3].*

### 2.2.2 Method RCC_M

This method consists in forming the elementary cycles of request while starting with those which cause the greatest variations.

Thus for a load history comprising $N$ points, one determines $N/2$ elementary cycles if $N$ is even and $N/2+1$ if $N$ is odd.

The algorithm breaks up into two stages. The first stage consists in ordering the load history of smallest with the greatest value of the stress, or the strain.

The second stage consists, as for it, to form the elementary cycles with the greatest variation of the value of the stress, or the strain.

On the rearranged $ch(t)$ load history, the elementary cycles are defined by:

$$\begin{cases} \text{VALMAX} = ch_{N+1-i} & \text{pour } i = 1, \; N/2 \\ \text{VALMIN} = ch_i & \end{cases}$$

If $N$ is odd one determines a definite additional cycle by:

$$\begin{cases} \text{VALMAX} = ch_{N/2+1} & \text{si } ch_{N/2+1} > ch_m \\ \text{VALMIN} = -ch_{N/2+1} + 2 * ch_m & \end{cases}$$

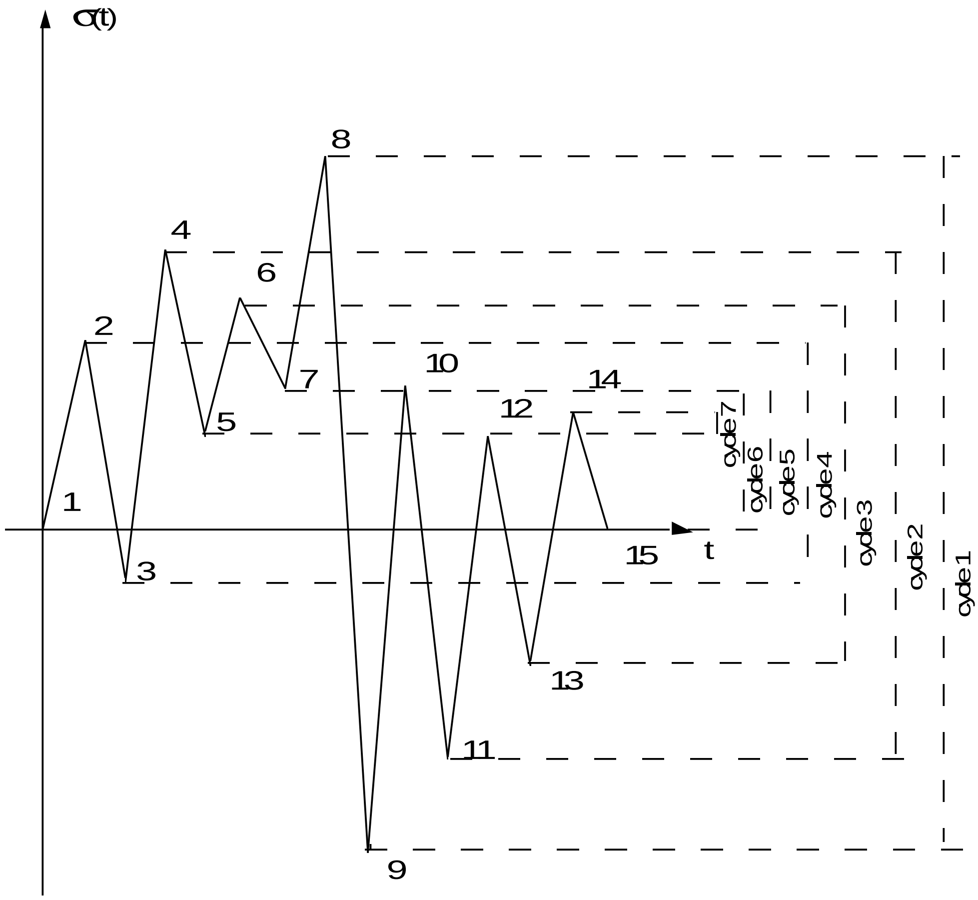
and

$$\begin{cases} \text{VALMAX} = ch_{N/2+1} & \text{sinon} \\ \text{VALMIN} = -ch_{N/2+1} + 2 * ch_m & \end{cases}$$

where $ch_m =$ constraint average or average strain of the loading = $\frac{1}{N}\sum_{1}^{N} ch_i$ .

To illustrate method RCC_M let us consider the same example as that used for the method RAINFLOW (of which the loading was considered of type forced).

| N° point | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Time | 0. | 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. | 9. | 10. | 11. | 12. | 13. | 14. |
| Loading | 0. | 40. | – 10. | 60. | 20. | 50. | 30. | 80. | – 70. | 30. | – 50. | 20. | – 30. | 25. | 0. |




The first stage which consists in ordering the history of the loading, of smallest with the greatest value of the loading, conduit to following storage:

| N° point | 9 | 11 | 13 | 3 | 1 | 15 | 5 | 12 | 14 | 7 | 10 | 2 | 6 | 4 | 8 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Loading | – 70. | – 50. | – 30. | – 10. | 0. | 0. | 20. | 20. | 25. | 30. | 30. | 40. | 50. | 60. | 80. |

The load history being composed of 15 points, method RCC_M determines 8 elementary cycles:

Cycle 1: `VALMAX = 80.` and `VALMIN = - 70.`
Cycle 2: `VALMAX = 60.` and `VALMIN = - 50.`
Cycle 3: `VALMAX = 50.` and `VALMIN = - 30.`
Cycle 4: `VALMAX = 40.` and `VALMIN = - 10.`
Cycle 5: `VALMAX = 30.` and `VALMIN = 0.`
Cycle 6: `VALMAX = 30.` and `VALMIN = 0.`
Cycle 7: `VALMAX = 25.` and `VALMIN = 20.`
Cycle 8: `VALMAX = 20.` and `VALMIN = 6.` because $\left(\sigma_m = \frac{1}{N}\sum_1^N \sigma_i = 6.\right)$

**Note::**

*This method of counting of cycles does not take absolutely account about appearance of the cycles, and systematically orders the elementary cycles by decreasing amplitude. This method must be used with vigilance for the computation of the damage by the methods of Taheri whose characteristic is to take account about application of the cycles of loading. For the computation of the damage by the methods of Taheri, it is strongly advised to use the method of "natural" counting of cycles known as [§2.2.3].*

### 2.2.3 Method "naturalness"

This method consists in generating the cycles in the order of their appearance in the load history.

Thus for a load history of $N+1$ points, one determines $N/2$ elementary cycles so $N$ even and $N/2+1$ elementary cycles so $N$ odd.

The method consists in leaning on three successive points of the load history.

One notes $X=|ch(i+1)-ch(i)|$ and $Y=|ch(i+2)-ch(i+1)|$ .

If $X \geq Y$ it is considered that one met an elementary cycle which is defined by the two points $(i)$ and $(i+1)$ .

The amplitude of the cycle is given par. $\Delta ch=|ch(i+1)-ch(i)|$

If $X<Y$ it is considered that one met an elementary cycle which is defined by the two points $(i+1)$ and $(i+2)$ .

The amplitude of the cycle is given par. $\Delta ch=|ch(i+2)-ch(i+1)|$

When the cycle is extracted one removes the two points $(i)$ and $(i+1)$ of the load history and one continues the algorithm.

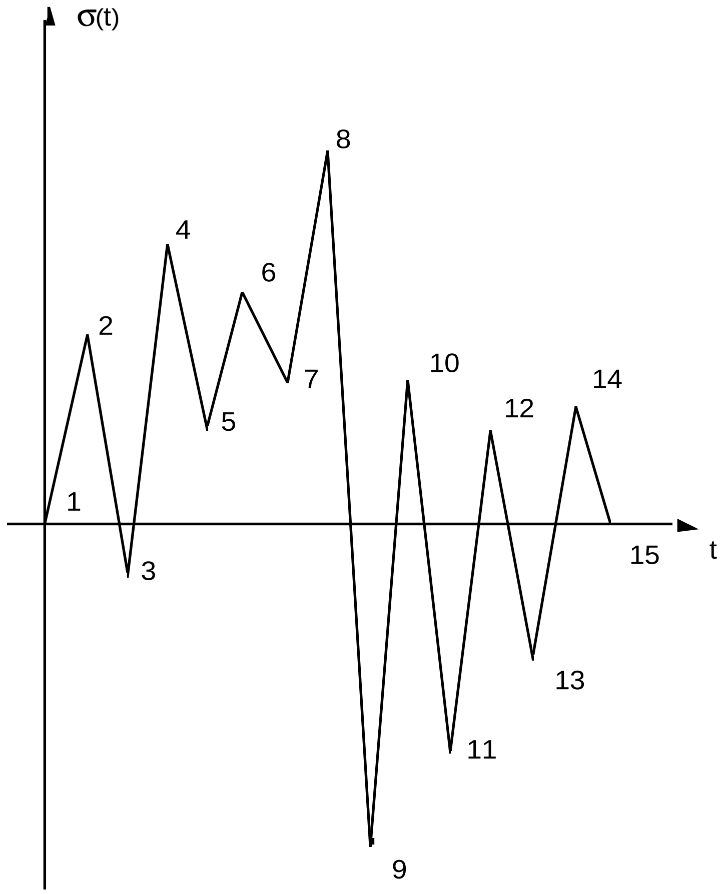
If the number of points $(N+1)$ of the load history is odd, the algorithm described previously makes it possible to discuss all items.

If the number of points $(N+1)$ of the load history is even, it remains to discuss the two remaining items.

It is considered that these two points form a cycle defines by the two points $N$ and $(N+1)$ . The amplitude of the cycle is given par. $\Delta ch=|ch(N+1)-ch(N)|$

to illustrate this method consider the same example as that used for methods RAINFLOW and RCC_M.

| N° point | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Time | 0. | 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. | 9. | 10. | 11. | 12. | 13. | 14. |
| Loading | 0. | 40. | – 10. | 60. | 20. | 50. | 30. | 80. | – 70. | 30. | – 50. | 20. | – 30. | 25. | 0. |




The load history being composed of 15 points, the method "naturalness" determines 7 elementary cycles:

| | | | |
|---|---|---|---|
| Cycle 1: | `VALMAX = 40.` | and | `VALMIN = - 10.` |
| Cycle 2: | `VALMAX = 60.` | and | `VALMIN = - 10.` |
| Cycle 3: | `VALMAX = 50.` | and | `VALMIN = 20.` |
| Cycle 4: | `VALMAX = 80.` | and | `VALMIN = - 70.` |
| Cycle 5: | `VALMAX = 30.` | and | `VALMIN = - 70.` |
| Cycle 6: | `VALMAX = 30.` | and | `VALMIN = - 50.` |
| Cycle 7: | `VALMAX = 25.` | and | `VALMIN = - 30.` |

**Note:**

*This method is that which it is strongly recommended to use in the case of the computation of the damage by the methods of Taheri.*

## 2.3 Computation of the damage: method of Wöhler

the number of cycles to the fracture is determined by interpolation of the curve of Wöhler of the material for a level of alternate stress given (to each elementary cycle corresponds a level of amplitude of stress $\Delta\sigma = |\sigma_{max} - \sigma_{min}|$ and an alternate stress $S_{alt} = 1/2\,\Delta\sigma$ ).

The damage of an elementary cycle is equal contrary to many cycles to the fracture $D = 1/N$ .

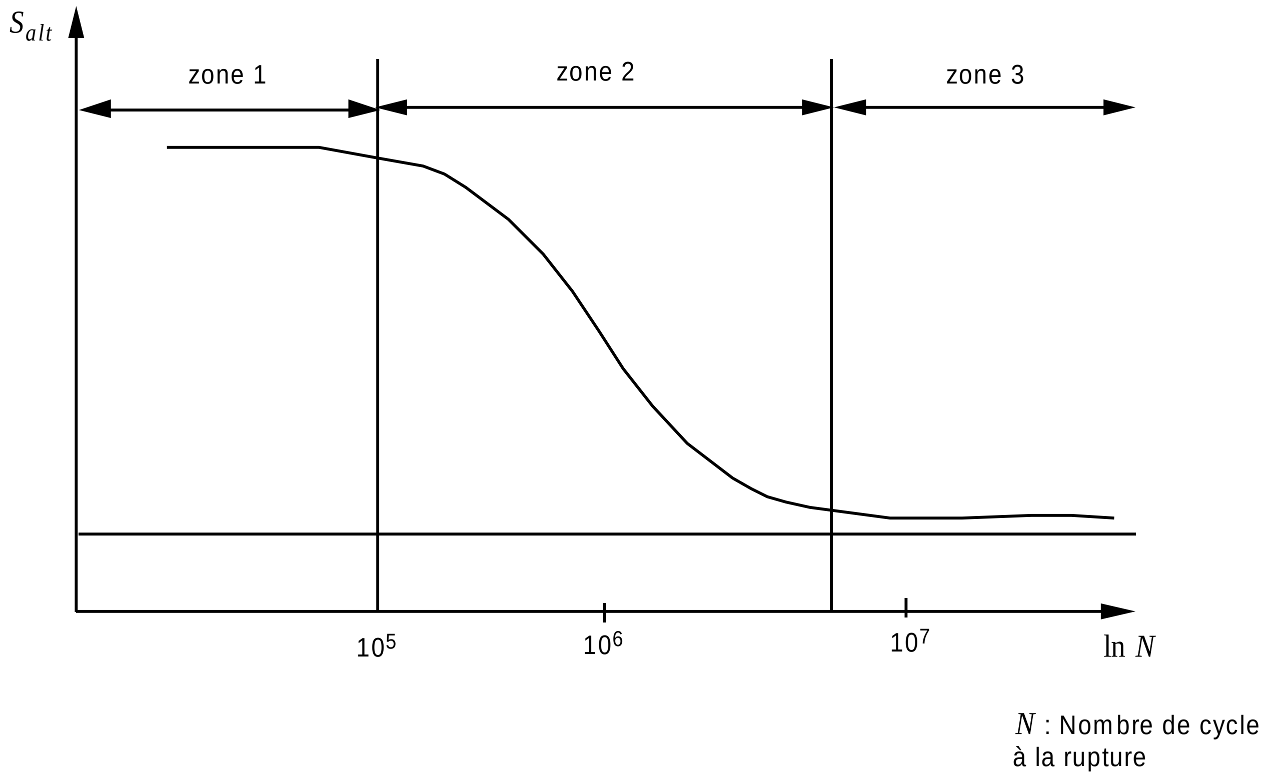
In the case of a uniaxial homogeneous test with an alternate stress pure (or symmetric), the number of cycles to the fracture is given from a diagram of endurance, still called curve of Wöhler or curve $S-N$.

In the case of geometrical defaults or of elementary cycles of non-zero average constraint, of the corrections of the curve of Wöhler are necessary before the determination amongst cycles for the fracture and thus to the elementary damage.

### 2.3.1 Diagram of endurance

the diagram of endurance, also called curve of Wöhler or curve $S-N$ (force-NOMBRE curve of cycles to the fracture) is obtained in experiments by subjecting test-tubes to cycles of periodic forces (generally sinusoidal) of normal amplitude $\sigma$ and constant frequencies, and by noting the number of cycles N to the end of which the fracture occurs.

The curve of Wöhler is thus defined for a given material and is presented in the form:




where $S_{alt}$ = the alternate stress of the cycle = $\frac{1}{2}|\sigma_{max}-\sigma_{min}|$

One distinguishes three zones on this curve:

- a zone of oligocyclic fatigue, under strong stress, where the fracture occurs after very a small number of alternations,
- a zone of fatigue or limited endurance, where the fracture is reached after a number of cycles which grows when the stress decrease,
- a zone of unlimited endurance or security zone, under low stress, for which the fracture does not occur before a number given of cycles superior to the life duration under consideration for the part.

There exist many statements of the diagram of endurance:

- Oldest is that of Wöhler:

$$\ln(N) = a - bS_{alt}$$ éq 2.3.1-1

where $N$ is the number of cycles to the fracture,
$S_{alt}$ the alternate stress applied,
$a$ and $b$ two characteristics of the material.

This analytical statement does not give an account well, of a branch horizontal or asymptotic of the curve SN supplements, but it gives a representation often very good of average part of the curve.

- By 1910, Basquin proposes the formula:

$$\ln(N) = a - b\ln(S_{alt})$$ éq 2.3.1-2

to take account of the curvature of the curve of Wöhler which connects the branch downward to the horizontal branch.

**D =** damage of an elementary cycle = $1/N = A\,S_{alt}^{\beta}$ where $A = e^{-a}$ and $\beta = b$ .

- Another analytical shape of the curve of Wöhler is proposed in POSTDAM to take account of the curve out of the singular zone:

$$S_{alt} = 1/2(E_C/E)\Delta\sigma$$ éq 2.3.1-3

$$\begin{aligned} &où\quad E_C = \text{Module d'Young associé à la courbe de fatigue du matériau,}\\ &E = \text{Module d'Young utilisé pour déterminer les contraintes.}\\ &X = \mathrm{LOG}_{10}(S_{alt})\\ &N = 10^{a0 + a1X + a2X^2 + a3X^3}\\ &D = \begin{cases} 1/N & \text{si } S_{alt} \geq S_l \quad \text{où } S_l \text{ est la limite d'endurance du matériau}\\ 0. & \text{sinon}\end{cases}\end{aligned}$$

**Note::**

*If one takes* $a2 = a3 = 0$ et $E_C/E = 1$ *one finds the formula of Basquin.*

The user can introduce the curve of Wöhler into operator `DEFI_MATERIAU` [U4.43.01] in three distinct forms:

- **a point by point discretized form** (key word `WOHLER` under the key word factor `TIRES` in `DEFI_MATERIAU`).

  The curve of Wöhler is in this case a function which gives the number of cycles to the fracture $N$ according to the alternate stress $S_{alt}$ and for which the user chooses the mode of interpolation:

- "LOG" ----> interpolation logarithmic curve on the number of cycles to the fracture and on the alternate stress (formula of Basquin per pieces),
- "LIN" ----> linear interpolation on the number of cycles to the fracture and on the alternate stress (this interpolation is disadvised because the curve of Wöhler is absolutely not linear in this reference).
- "LIN", "LOG" interpolation in logarithmic curve on the number of cycles to the fracture and into linear on the alternating load, which leads to the statement given by Wöhler.

The user must also choose the type of prolongation of the function on the right and on the left (if it is necessary to interpolate the function in an unauthorized point by the definition of the function there is program stop by fatal error).

- **an analytical form of Basquin** (key keys A_BASQUIN and BETA_BASQUIN under the key word factor TIRES in DEFI_MATERIAU)

  $D=A S_{alt}^{\beta}$ They are the constants $A$ and $\beta$ used in this formula which are to be introduced by the user (in accordance with code POSTDAM).

- an analytical form except singular zone

$$S_{alt}=\text{contrainte alternée}=1/2\left(E_C/E\right)\Delta\sigma$$
$$X=\text{LOG}_{10}\left(S_{alt}\right)$$
$$N=10^{a0+a1\,X+a2\,X^2+a3\,X^3}$$
$$D=\begin{cases}1/N & \text{si } S_{alt}\geq S_l \quad \text{où } S_l \text{ est la limite d'endurance du matériau}\\ 0. & \text{sinon}\end{cases}$$

the user must introduce:

$E_C$ = Young Modulus associated with the curve with fatigue with the material (key word E_REFE under the key word factor TIRES in DEFI_MATERIAU)
$E$ = Young Modulus used to determine the stresses (key word E under the key word factor ELAS in DEFI_MATERIAU),

the constants of the material $a0, a1, a2 \,\text{et}\, a3$ (key keys A0, A1, A2 and A3 under the key word factor TIRES in DEFI_MATERIAU)
and $S_l$ the limit of endurance of the material (key word SL under the key word factor TIRES in DEFI_MATERIAU).

**Note:**

*This statement of the damage is available in the same form in software POSTDAM.*

## 2.3.2 Influence geometrical parameters on the endurance

### 2.3.2.1 Coefficient of stress concentration

According to the geometry of the part, it can be necessary to balance the value of the pressure applied by the coefficient of stress concentration $K_T$. $K_T$ is a coefficient function of the geometry of the part, geometry of the default and type of loading.
This coefficient is given by the user under the key word $K_T$ of the key word factor COEF_MULT.

It is used to apply to the history of the loading, a homothety of ratio $K_T$, which amounts multiplying all the values of the load history by the coefficient $K_T$.

(The computation of the damage will be done on a load history $\sigma(t) = K_T \times \sigma(t)$ ).

#### 2.3.2.2 Elastoplastic coefficient of concentration

It can also be necessary to balance the value of the pressure applied by the elastoplastic coefficient of concentration $K_e$.

The elastoplastic coefficient of concentration $K_e$ (aimed to the B3234.3 articles and B3234.5 of THE RCC_M [bib4]) is defined as being the relationship between the amplitude of real strain and the amplitude of fictitious strain determined by the elastic analysis.

An acceptable value of the coefficient $K_e$ can be given by [bib4]:

$$\begin{cases} K_e = 1 & \text{si} & \Delta\sigma < 3S_m \\ K_e = 1+(1-n)(\Delta\sigma/3S_m-1)/(n(m-1)) & \text{si} & 3S_m < \Delta\sigma < 3mS_m \\ K_e = 1/n & \text{si} & 3mS_m < \Delta\sigma \end{cases}$$

where $S_m$ is the acceptable maximum stress,

and $n$ etdeux $m$ constants depending on the material.

The elastoplastic factor $K_e$ is a ratio of homothety of the loading. This factor depend on the amplitude of the loading. It is applied, cycle by cycle to the values of the maximum stress and minimal of each cycle.

The data $S_m$, $n$ etsont $m$ introduced under key keys `SM_KE_RCCM`, `N_KE_RCCM` and `M_KE_RCCM` under the key word factor `TIRES` in `DEFI_MATERIAU`.

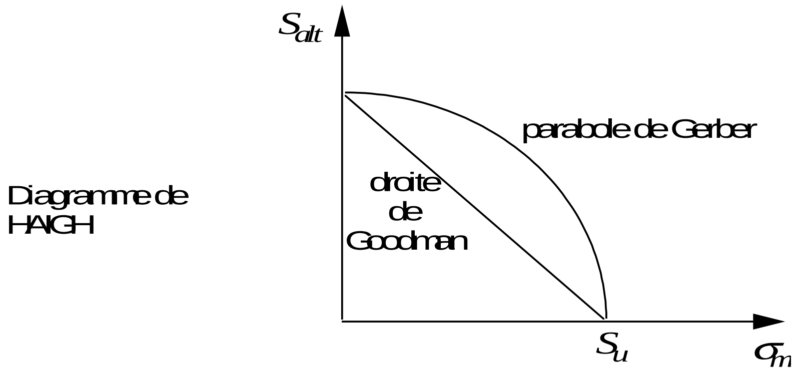
The user asks for the taking into account of the elastoplastic concentration factor by indicating `CORR_KE` : "`RCCM`" in `POST_FATIGUE` [U4.83.01].

### 2.3.3 Influence average constraint

If the part is not subjected to pure or symmetric alternate stresses, i.e. if the average constraint of the cycle is not null, strength with the dynamic stresses of the material (its limit of endurance) decreases.

One thus balances the curve of Wöhler to compute: the number of effective cycles to the fracture using various diagrams.

The diagram of Haigh makes it possible to determine the evolution of the limit of endurance according to the average constraint $\sigma_m$ and of the alternate stress $S_{alt}$.




From a cycle $(S_{alt}, \sigma_m)$ identified in the signal one calculates the value of the corrected alternate stress $S_{alt}^{'}$.

Si l'on utilise la droite de Goodman $S'_{alt} = \frac{S_{alt}}{1 - \frac{\sigma_m}{S_u}}$

Si l'on utilise la parabole de Gerber $S'_{alt} = \frac{S_{alt}}{1 - \left(\frac{\sigma_m}{S_u}\right)^2}$

If the line of Goodman is used: $S'_{alt} = \frac{S_{alt}}{1 - \frac{\sigma_m}{S_u}}$

If one uses the parabola To stack: $S'_{alt} = \frac{S_{alt}}{1 - \left(\frac{\sigma_m}{S_u}\right)^2}$

It is noticed that this last does not differentiate the average constraint in tension and compression.

where $S_u$ is the limit when the material breaks.

The influence of the average constraint is taken into account only on request of the user (key word `CORR_HAIG`).

**Note:**

*If the curve of Wöhler is defined by the analytical form except singular zone [éq 2.3.1 - 3], of the extents of variation of stresses being below the limit of endurance can find itself higher than this one. To avoid that, one corrects the limit of endurance $S_l$ by taking a limit of corrected endurance [bib5]:*

$$S'_l = \frac{S_l}{1 - \frac{\sigma_m}{S_u}}$$ for the line of Goodman

$$S'_l = \frac{S_l}{1 - \left(\frac{\sigma_m}{S_u}\right)^2}$$ for the parabola To stack

## 2.4 Computation of the damage: method of Manson-Whetstone sheath

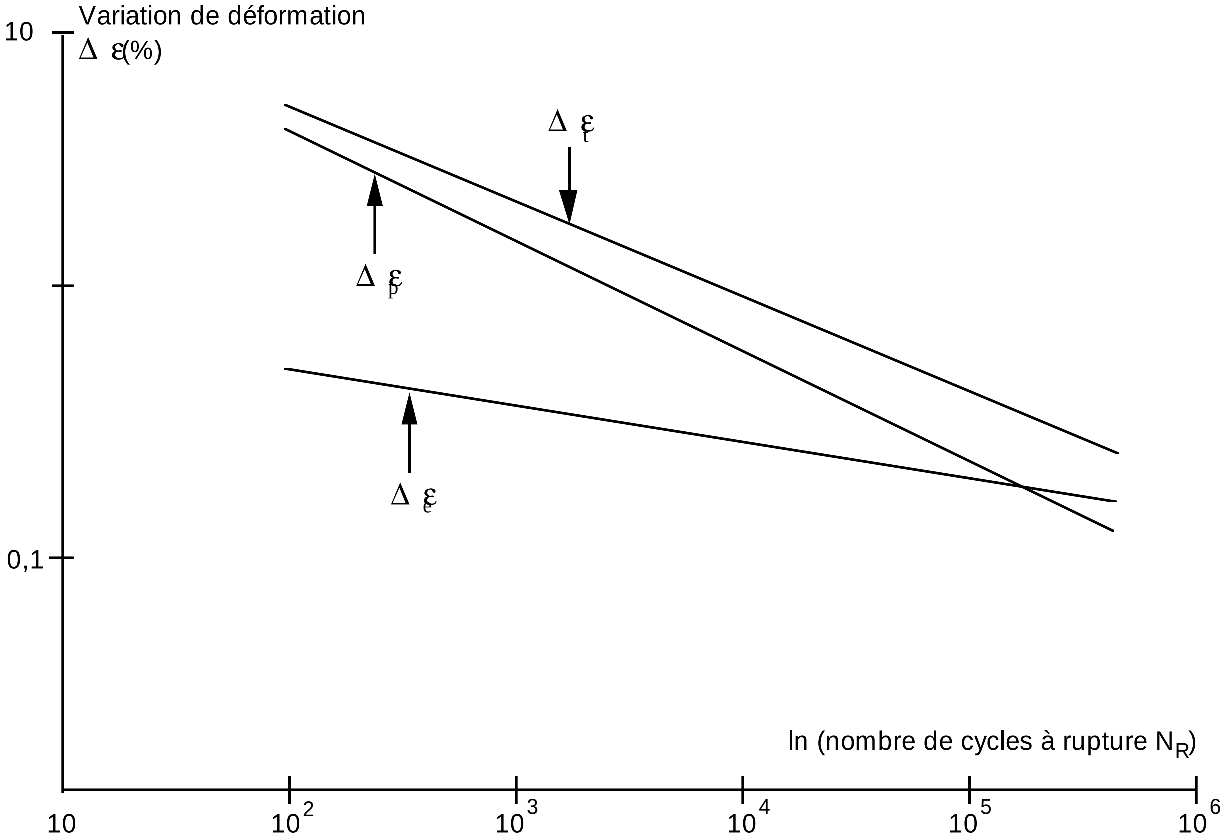
the scope of application of the method of Manson-Whetstone sheath [bib1] is the oligocyclic plastic fatigue, which as its name indicates it shows two fundamental characteristics:

- it is plastic, i.e. a significant plastic strain occurs with each cycle,
- it is oligocyclic, i.e. the materials have an endurance finished with this kind of request.

To describe the behavior of the materials in fatigue oligocyclic plastic, one uses tests with alternate imposed strain.

In the case, of a uniaxial homogeneous test with an alternated strain, the number of cycles to the fracture is given from a diagram of strength, which connects the variation of strain to the number of cycles involving the fracture.

In the diagram of strength, one separates the deflections total, elastic and plastic. These diagrams are still known under the name of Whetstone sheath-Manson which proposed them in 1950.




The relations $\frac{\Delta\varepsilon_e}{2} - \ln(N)$ and $\frac{\Delta\varepsilon_p}{2} - \ln(N)$ are lines. The relation $\frac{\Delta\varepsilon_t}{2} - \ln(N)$ has, as for it, a curvature towards the positive strains.

It was shown that a relation power connected the plastic strain $(\Delta\varepsilon_p)$ and the elastic strain $(\Delta\varepsilon_e)$ to the number of cycles to the fracture, which leads to the following relations:

$$\Delta\varepsilon_p = A\ N^{-a}$$
$$\Delta\varepsilon_e = B\ N^{-b}$$
$$\Delta\varepsilon_t = A\ N^{-a} + B\ N^{-b}$$

where $a$ and $b$ are two characteristics of the material (in general $a$ is close to $0{,}5$ and $b$ close to $0{,}12$ ); $A$ and $B$ , two constants of the material.

The user can introduce the curve of Manson-Whetstone sheath in a single mathematical form: form discretized point by point. It is a function which gives the number of cycles to the fracture $N$ according to the amplitude of strain $\left(\Delta \varepsilon_{t/2}\right)$.

As for the curve of Wöhler, the user can choose the mode of interpolation on the number of cycles to the fracture and on the amplitude of strain.

The type of prolongation of the function on the right and on the left is also with the choice of the user.

The damage of an elementary cycle is equal contrary to many cycles to the fracture $D=1/N$.

## 2.5 Computation of the damage: method of Taheri

the methods of calculating of the damage proposed by Taheri [bib12] are two: they will be named respectively Taheri-Manson and Taheri-mixed. These methods apply to loadings characterized by a scalar component of standard strain.

These methods have as a characteristic to take account about application of the elementary cycles of loading with structure. For this reason, it is advisable to be vigilant with the choice of the method of counting of the cycles. It is strongly advised to use the method of “natural” counting known as method [§2.2.3].

### 2.5.1 Taheri-Manson method

Are N cycles elementary of half-amplitude $\frac{\Delta\varepsilon_1}{2}, \cdots \frac{\Delta\varepsilon_n}{2}$.

The value of the elementary damage of the first cycle is determined by interpolation on the curve of Manson-Whetstone sheath of the material.

The computation elementary damage of the following cycles is carried out by the algorithm:

- if $\frac{\Delta\varepsilon_{i+1}}{2} \geq \frac{\Delta\varepsilon_i}{2}$

  the value of the elementary damage of the cycle $(i+1)$ is determined by interpolation on the curve of Manson-Whetstone sheath of the material.

- if $\frac{\Delta\varepsilon_{i+1}}{2} < \frac{\Delta\varepsilon_i}{2}$

  one determines:

$$\frac{\Delta\sigma_{i+1}}{2} = F_{NAPPE}\left(\frac{\Delta\varepsilon_{i+1}}{2},\ \underset{j<i}{\mathrm{Max}}\left(\frac{\Delta\varepsilon_j}{2}\right)\right)$$

  then

$$\frac{\Delta \varepsilon^*_{i+1}}{2} = F_{FONC}\left(\frac{\Delta \sigma_{i+1}}{2}\right).$$

$F_{NAPPE}$ is the cyclic curve of cyclic hardening with préécrouissage of the material.

$F_{FONC}$ is the cyclic curve of hardening of the material.

The value of the damage of the cycle $(i+1)$ is determined by interpolation of $\frac{\Delta\varepsilon^*_{i+1}}{2}$ on the curve of Manson-Whetstone sheath of the material.

**Note:**

*If all the cycles applied are arranged by ascending value of the amplitude of strain, this method is identical to the method of Manson-Whetstone sheath.*

### 2.5.2 Taheri-Mixed method

Are N cycles elementary, of half-amplitude $\frac{\Delta\varepsilon_1}{2}, \cdots \frac{\Delta\varepsilon_n}{2}$.

The value of the elementary damage of the first cycle is determined by interpolation on the curve of Manson-Whetstone sheath of the material.

The computation elementary damage of the following cycles is carried out by the algorithm:

- if $\frac{\Delta\varepsilon_{i+1}}{2} \geq \frac{\Delta\varepsilon_i}{2}$

  the value of the elementary damage of the cycle $(i+1)$ is determined by interpolation on the curve of Manson-Whetstone sheath of the material.

- if $\frac{\Delta\varepsilon_{i+1}}{2} < \frac{\Delta\varepsilon_i}{2}$

  one determines:

$$\frac{\Delta\sigma_{i+1}}{2} = F_{NAPPE}\left(\frac{\Delta\varepsilon_{i+1}}{2},\ \operatorname*{Max}_{j<i}\left(\frac{\Delta\varepsilon_j}{2}\right)\right)$$

  where $F_{NAPPE}$ is the cyclic curve of cyclic hardening with préécrouissage of the material.

  The value of the damage of the cycle $(i+1)$ is obtained by interpolation of $\frac{\Delta\sigma_{i+1}}{2}$ on the curve of Wöhler of the material.

**Note:**

*If all the cycles applied to structure are arranged by ascending value of the amplitude of strain, this method is identical to the method of Manson-Whetstone sheath.*

The damage of an elementary cycle is equal contrary to many cycles to the fracture $D = 1/N$.

## 2.6 Computation of the total damage

the simplest approach and most known to determine the total damage of a part subjected to $n_i$ cycles of alternate stress $S_{alt}$ or alternate strain $E_{alt}$ is the linear rule of the damage suggested by Mining:

$$Di=\frac{n_i}{N_i}$$

Under operation, the structures are subjected to various loadings of different amplitudes. Undergone fatigue is due to the accumulation of the elementary damages and the total damage is calculated using the rule of office plurality To mine [bib6]:

$$D_{total}=\sum_i \frac{n_i}{N_i}$$

In the case of Wöhler and Manson-Whetstone sheath, this model supposes that the damage increases linearly with the number of imposed cycles and that it is independent of the level of loading and about application of the levels of loading (whereas in experiments, it is shown that the order of application of the loading is a significant factor for the life duration of the material).

The computation total damage is required by the user with key word CUMUL.

The methods suggested by Taheri take account about application of the loading, in the computation of the elementary damages associated with each cycle.

## 2.7 Conclusion

For the methods based on uniaxial tests, the computation of the total damage undergone by a part subjected to a load history breaks up into several stages:

- extraction of the peaks of the load history, to lead to a simpler history,
- extraction of the elementary cycles of the load history by a method of counting of cycles,
- computation of the elementary damage associated with each elementary cycle resulting from the real history of the loading,
  - possibly (and for the method of Wöhler), correction of the loading by a coefficient of stress concentration $K_T$,
  - possibly (and for the method of Wöhler), correction of the loading by a coefficient of elastoplastic concentration $K_e$,
  - possibly (and for the method of Wöhler), correction of Haigh to take account of the non-zero value of the average constraint,
- computation of the total damage, by one regulate linear office plurality.

# 3 Computation of the damage of Lemaître generalized

This model of damage relates to the study of the starting of a macroscopic crack, using a post - processor of mechanics of damage based on a unified formulation of the laws of evolution of the damage. This one uses, on the one hand, of the laws of evolution of the damage specific to the various mechanisms considered, and, on the other hand, a more general model based on a micromechanical analysis of the phenomenon of starting.

This model offers a single formalism which supposes that the various damage mechanisms all are controlled by plastic strains, elastic strain energy and by a process of instability.

## 3.1 The model of Lemaître generalized

the model of Lemaître generalized consists of an enrichment of the method of calculating of damage of Lemaitre [bib7] by the introduction of a model in power (models of Lemaître-Sermage). She is written [bib14]:

$$\begin{cases} \dot{D} = \left[\frac{Y}{S}\right]^s \dot{p} & \text{si } p > p_D \\ D = 0 & \text{sinon} \end{cases}$$ éq 3.1-1

with:

$$Y = \frac{\sigma_{eq}^2}{2\,E\left(1-D\right)^2} R_\nu \quad \text{et} \quad R_\nu = \frac{2}{3}\left(1+\nu\right) + 3\left(1-2\,\nu\right)\left(\frac{\sigma_H}{\sigma_{eq}}\right)^2 .$$

$Y$ is the rate of refund of density of strain energy elastic.

$R_\nu$ is the function of triaxiality.

$\frac{\sigma_H}{\sigma_{eq}}$ is the rate of triaxiality.

$\sigma_{eq} = \sqrt{\frac{3}{2}\,\sigma_{ij}^D\,\sigma_{ij}^D}$ is the equivalent stress of von Mises.

$\sigma_{ij}^D = \sigma_{ij} - \frac{1}{3}\,\sigma_{kk}\,S_{ij}$ is the deviator of the stress.

$p_D$ is the threshold of damage.

$S$ and $s$ are characteristics material.

$p(t)$ is the cumulated plastic strain.

This model thus makes it possible to calculate the damage $D(t)$ from the data of the tensor of the stresses $\sigma(t)$ and the cumulated plastic strain $p(t)$.

The equation [éq 3.1-1] can be written:

$$\begin{cases} (1-D)^{2s} dD = \left[\frac{C}{S}\right]^s dp & \text{si } p > p_D \\ D = 0 & \text{sinon} \end{cases}$$ éq 3.1-2

with:

$$C = \frac{\sigma_{eq}^2}{2\,E} R_v$$

The integration of the equations [éq 3.1-2] enters $t_i$ and $t_{i+1}$ led to:

$$\begin{cases} \int_{D(t_i)}^{D(t_{i+1})} (1-D)^{2s} dD = \int_{p(t_i)}^{p(t_{i+1})} \left[\frac{C}{S}\right]^s dp & \text{si } p > p_D \\ D = 0 & \text{sinon} \end{cases}$$ éq 3.1-3

One notices that there exists a primitive for the term on the left of the equation [éq 3.1-3] but not for that on the right. A diagram of numerical integration is thus used to calculate the integral of it. The equation [éq 3.1-3] can be written:

$$\begin{cases} \frac{[1-D(t_i)]^{2s+1} - [1-D(t_{i+1})]^{2s+1}}{2s+1} = \frac{1}{2}(\left[\frac{C(t_i)}{S(t_i)}\right]^s + \left[\frac{C(t_{i+1})}{S(t_{i+1})}\right]^s)[p(t_{i+1}) - p(t_i)] & \text{si } p > p_D \\ D(t_{i+1}) = 0 & \text{sinon} \end{cases}$$ éq 3.1-4

One supposes that $D(t_o) = 0$ . The value of the damage $D(t_i),\ i = 0, n$ for each time $t_i$ can be given from the equation [éq 3.1-4]. In *the Code_Aster*, this quantity is named DOM_LEM.

The final damage with the fracture $D_r = D(t_r)$ is thus associated with the time-to-failure $t_r$ .

**Note:**

- *It is considered that the characteristics material $E$ (Young modulus), $\nu$ (Poisson's ratio) and $S$ (material parameter) depend on the temperature $T$ .*
- *The value of the Young modulus and the value of the Poisson's ratio are defined in `DEFI_MATERIAU` [U4.43.01] under the key word factor `ELAS_FO` .*
- *The values of $S$ , $p_D$ and of $s$ are defined in `DEFI_MATERIAU` under the key word factor `DOMMA_LEMAITRE` and the operands `S` , `ESPS_SEUIL` and `EXP_S` . The parameters $S$ and $p_D$ can depend on temperature `TEMP` .*
- *The model of Lemaître is obtained by assigning the value $s=1$*

## 3.2 Identification of the parameters of the generalized model of Lemaître

One notices that the equation [éq 3.1-4] is valid for fatigue and creep. For fatigue, $p$ is the cumulated plastic strain. For creep, $p$ is the instantaneous plastic strain if one neglects the elastic strain.

It is noted that materials parameters $S$ and $s$ depend strongly not only on the temperature $T$ but also on the stress $\sigma$ (via the statement of $C$ ). The process of determination of $S$ and $s$ starting from the fatigue tests was presented in [bib14].

This part aims at presenting a simple method to identify materials parameters $S$ and $s$ starting from the uniaxial tests of creep.

Initially, the temperature is fixed $T$. One will carry two creep tests to two levels of stress $\sigma_1$ and $\sigma_2$ sufficiently close relations so that the parameters $S$ and $s$ can be regarded as constant between these two levels of stress. One indicates $p_{r1}$ and the $p_{r1}$ plastic strains with the fracture, measured starting from the tests of creep associated with $\sigma_1$ and $\sigma_2$, respectively.

As the stress $\sigma$ is maintained constant during the creep test and that $D(t_o)=0 \quad D_r=D(t_r)=1$, the equation [éq 3.1-4] can be written for the levels of stress $\sigma_1$ and $\sigma_2$ like:

$$\frac{S^s}{2s+1}=[C(\sigma_1)]^s . p_{r1}$$ éq

3.2-1

$$\frac{S^s}{2s+1}=[C(\sigma_2)]^s . p_{r2}$$ éq

the 3.2-2

parameters $S$ and $s$ are the solution of the system of equations [éq 3.2-1] and [éq 3.2-2]. From these equations, the parameter $s$ is given like:

$$s=\frac{\log\frac{p_{r2}}{p_{r1}}}{\log\frac{C(\sigma_1)}{C(\sigma_2)}}$$ éq

3.2-3

Then, $S$ can be given either from the equation [éq 3.2-1], or from the equation [éq 3.2-2].

For $\sigma_1<\sigma_2$, from the equation [éq 3.1-7], the value of $s$ is positive (physically acceptable) if and only if $p_{r1}>p_{r2}$. In the contrary case, it does not exist of positive solution for the system of equations [éq 3.2-1] and [éq 3.2-2], i.e., the model of damage of Lemaitre is not applicable in this case. It

is noted that this simple method makes it possible to identify the parameters and $S$ with $s$ various temperatures and various levels of pressure applied starting from the experimental/numerical curves of creep. Criteria

# 4 of Crossland and Dang Van Papadopoulos

the criteria [bib9] and [bib13] allow for metal structures subjected to stresses forced following one a large number of cycles to distinguish the loadings damaging from the others. One

can classify the criteria in two categories according to nature of their approach: macroscopic

- approach: criterion of Crossland, microscopic
- approach: criterion of Dang Van Papadopoulos.

The criteria of Crossland and Dang Van Papadopoulos apply to uniaxial or multiaxial loadings periodic.

The goal of these criteria is not to determine a value of damage, but a value of criterion such as $R_{crit}$ : Criterion

$$\begin{cases} R_{crit} \leq 0 \ \ \text{pas de dommage} \\ R_{crit} > 0 \ \ \text{dommage possible (fatigue).} \end{cases}$$

## 4.1 of Crossland

the criterion of Crossland is empirical and is written only from macroscopic variables. In fact

, from trial runs, one could note that the amplitude of cission as well as the hydrostatic pressure played a fundamental role in the mechanisms of fatigue of structures. This is why

, Crossland applied the criterion: where

$$R_{crit} = \tau_a + a\ P_{max} - b$$

amplitude

$$\tau_a = \frac{1}{2} \underset{0 \leq t_0 \leq T}{\text{Max}} \ \underset{0 \leq t_1 \leq T}{\text{Max}} \ \| \sigma^D_{(t_1)} - \sigma^D_{(t_0)} \| =$$ of cission with

deviator $\sigma^D$ of the tensor of the stresses. maximum

$$P_{max} = \underset{0 \leq t \leq T}{\text{Max}} \left( \frac{1}{3} \ trace \ \sigma \right) =$$ hydrostatic pressure. with

$$a = \left( \tau_0 - \frac{d_0}{\sqrt{3}} \right) / \left( \frac{d_0}{\sqrt{3}} \right) \ \text{et} \ \ b = \tau_0$$

: limit

$\tau_0 =$ of endurance in alternated, limiting pure shears

$d_0 =$ of endurance in alternate pure traction and compression. Criterion

## 4.2 of Dang Van Papadopoulos It

appeared that the crack initiation of fatigue is a microscopic phenomenon occurring with a scale about the grain. This is why, of the criteria of fatigue, from local microscopic variables were applied.

The implemented criterion [bib8], [bib9] and [bib10] in Code_Aster *is* the criterion of Dang Van Papadopoulos, which is written in the form: where

$$R_{crit}=k^*+a\,P_{max}-b$$

: if

$k^*=\frac{R}{\sqrt{2}}$ so $R=\underset{0\le t\le T}{Max}\sqrt{\left(\sigma^D(t)-C^*\right):\left(\sigma^D(t)-C^*\right)}$

$k^*=R$ with $R=\underset{0\le t\le T}{Max}\sqrt{J_2(t)}=\underset{0\le t\le T}{Max}\sqrt{\frac{1}{2}\left(\sigma^D(t)-C^*\right):\left(\sigma^D(t)-C^*\right)}$

: ,

1) $R$ the radius of the smallest sphere circumscribed with the way of loading within the space of deviators of the stresses; ,
2) $J_2(t)$ the second invariant of the deviators of the stresses; ,
3) $C^*=\underset{C}{Min}\;\underset{t}{Max}\sqrt{\left(\sigma^D(t)-C\right):\left(\sigma^D(t)-C\right)}$ the center of L" hypersphère. Note:

**It**

*is the definition of which $R$ uses which $J_2(t)$ is programmed. maximum*

$$P_{max}=\text{ hydrostatic pressure and }=\underset{0\le t\le T}{\text{Max}}\left(\frac{1}{3}\;trace\;\sigma\right)$$

$$a=\left(\tau_0-\frac{d_0}{\sqrt{3}}\right)/\left(\frac{d_0}{3}\right)\text{ with }b=\tau_0$$

: limit

$\tau_0=$ of endurance in alternated, limiting pure shears

$d_0=$ of endurance in alternate pure traction and compression.

The basic idea of Papadopoulos is to write that the grain obeys a plasticity criterion of the type von Mises instead of the plasticity criterion of the Tresca type used by Dang Van. Papadopoulos

conducted a campaign of comparisons between the results provided by its criterion and of the experimental results, which shows that the predictions of the criterion of Papadopoulos are excellent for the loadings closely connected; they are a little less precise for the ways nonclosely connected. In

its thesis [bib10] Papadopoulos shows that the criterion of Crossland and the criterion of Dang Van Papadopoulos give the same results for radial loadings.

The algorithm employed for the computation of the radius of the smallest sphere circumscribed with the way of loading within the space of stress deviators, is that proposed in [bib11]. It is about a recurring algorithm which rests on the second invariant of the deviators of the stresses. Let us note

$S_i$ the value of the deviator of the stresses at time, $t_i$ $O_n$ the center of the hypersphère to the iteration $n$, $R_n$ the radius of the hypersphère to the iteration and $n$ $x$ the “isotropic hardening parameter” of the algorithm. Phase

- of initialization of the algorithm: Iteration

$$O_1 = \frac{1}{N}\sum_{i=1}^{N} S_i$$
$$R_1 = 0.$$

- of the stage at $n$ the stage: $n+1$ one

  supposes and $O_n$ known $R_n$ . One calculates then: If

$$D = \|S_{i+1} - O_n\|$$
$$P = D - R_n$$

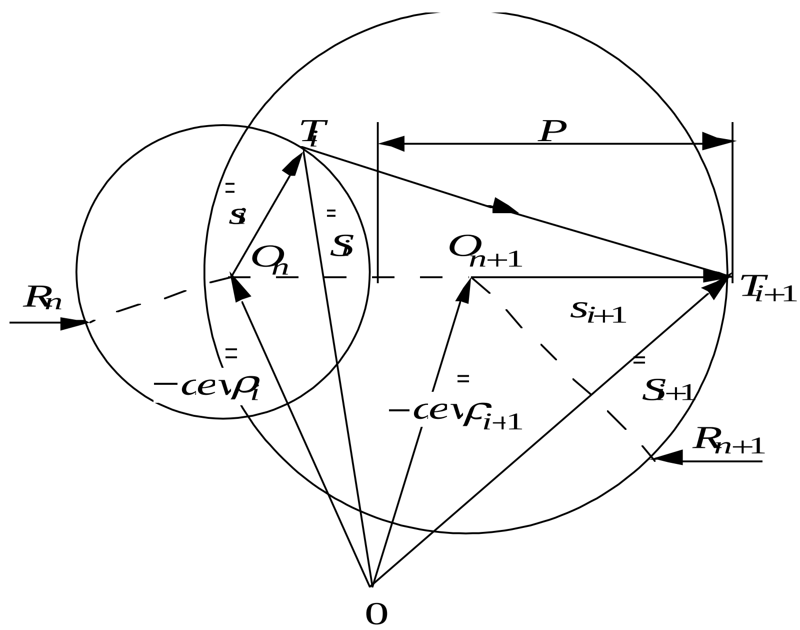
  - If $P > 0$

$$R_{n+1} = R_n + x\ .\ P$$
$$O_{n+1} = S_{i+1} + R_{n+1}\ \frac{O_n - S_i}{\|O_n - S_{i+1}\|}$$

  - $P < 0$

$$R_{n+1} = R_n$$
$$O_{n+1} = O_n$$

the algorithm ends when all the points are $S_i$ in the hypersphère of center and $O_n$ . $R_n$ Computation



## 4.3 of a value of damage These

two criteria applicable to multiaxial periodic loadings make it possible to say if there is damage or not: These

$$\begin{cases} R_{crit} \leq 0 & \text{pas de dommage} \\ R_{crit} > 0 & \text{dommage possible (fatigue).} \end{cases}$$

criteria do not provide a value of damage. It can however be interesting to calculate a value of damage by means of the curves of Wöhler of the material. With this intention, it is necessary to define an equivalent stress, $\sigma^*$ value to be interpolated on the curve of Wöhler.

The curves of Wöhler can be built from tests shear in which case the limit of endurance is, $\tau_0$ but are more generally built from traction tests - compression for which the limit of endurance is. $d_0 (d_0 < \tau_0)$ So that

there is coherence between the criterion and the curve of Wöhler it is necessary that: It

$$\begin{Bmatrix} \sigma^* \leq \tau_0 & \text{pas de dommage} \\ \sigma^* > \tau_0 & \text{dommage} \end{Bmatrix} \text{pour une courbe de Wöhler définie en cisaillement,}$$

$$\begin{Bmatrix} \sigma^* \leq d_0 & \text{pas de dommage} \\ \sigma^* > d_0 & \text{dommage} \end{Bmatrix} \text{pour une courbe de Wöhler définie en traction-compression.}$$

thus seems possible to us to take: In

$$\sigma^* = R_{crit} + \tau_0 \text{ pour une courbe de Wöhler en cisaillement (ce qui est assez rare),}$$

$$\sigma^* = (R_{crit} + \tau_0) * (d_0 / \tau_0) \text{ pour une courbe de Wöhler en traction-compression.}$$

a general way, the user can take where $\sigma^* = (R_{crit} + \tau_0) * corr$ is $corr$ a coefficient of correction introduced by the user. By default

, this coefficient $corr$ is taken equal to (case $(d_0 / \tau_0)$ of the curve of Wöhler introduced in traction and compression). Note:

**In**

*the literature, one to compute: does not find presentation of a approach of use of a criterion a value of damage. It is known however that certain industrialists use such a approach, but without knowing the adopted form of it.*

*The approach implemented in Code_Aster is proposed by department AMA. Conclusion*

# 5 In

this note are exposed different the méthodes de calcul from the damage available either in operator POST _FATIGUE OR in operator CALC _FATIGUE, OR in the two commands simultaneously. One

can classify these methods in two big classes: estimate

- of the damage to great numbers of cycles, estimate
- of the damage in fatigue oligocyclic plastic. In

the first class of problems, one finds the method of Wöhler, based on uniaxial tests, and which applies to loadings in stress. One also finds in this class, the criterion of Crossland, which is an empirical criterion leaning on macroscopic quantities and the criterion of Dang Van Papadopoulos which is based on microscopic phenomena.
The two criteria are addressed to loadings in stresses which can be uniaxial or multiaxial but periodic. In

the second class of problems, one finds the method of Manson-Whetstone sheath and the methods of Taheri, which apply to loading in strains.

All the methods based on uniaxial tests (method of Wöhler, method of Manson - Whetstone sheath and methods of Taheri) are available in two operators POST _FATIGUE AND CALC _FATIGUE.

The criteria, as for them, are only available in POST _FATIGUE. BIBLIOGRAPHY

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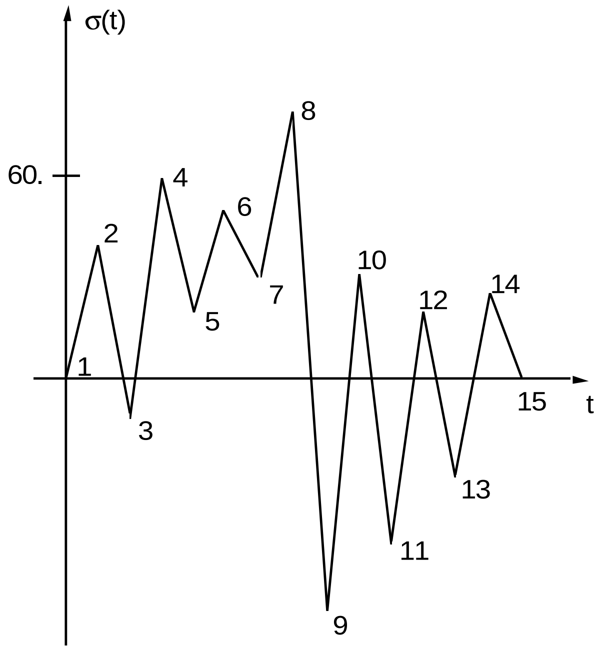
# 7 of the versions of the document Version

| **Aster Author** | **(S) Organization (S) Description** | **of modifications 6** |
|---|---|---|
| 6 | *M DONORE, F. MEISSONNIER EDF - R&D/AMA initial* | text 7,4 |
| A. | *M DONORE, F. MEISSONNIER EDF - R&D/AMA One* | |

## Annexe 1

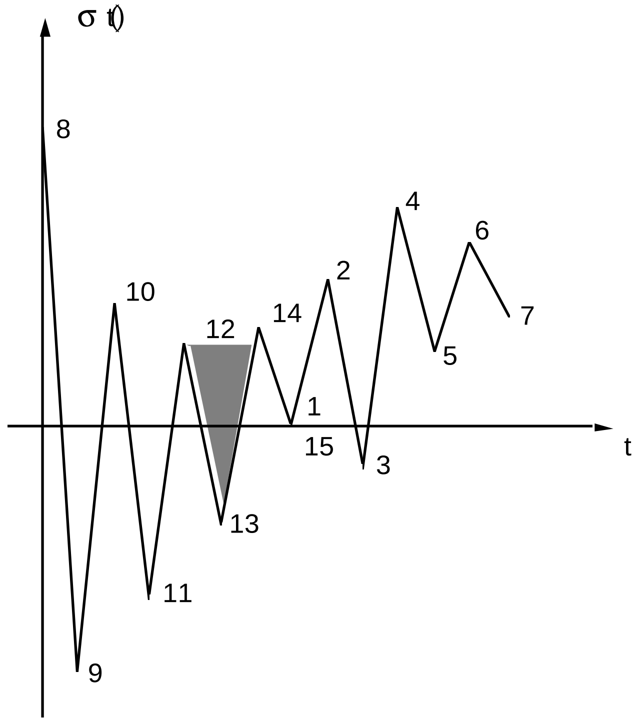
considers the following load history (which for the example is considered of type forced): N°

| point 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | Time |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0. | 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. | 9. | 10 | . 11 | . 12 | . 13 | . 14 | . Load ing |
| 0. | 40 | . – | 10. 60 | . 20 | . 50 | . 30 | . 80 | . – | 70. 30 | . – | 50. 20 | . – | 30. 25 | . 0. | |




The stage of rearrangement of the load history leads to the following loading: N°

| point 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 2 | 3 | 4 | 5 | 6 | 7 | Load ing |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 80 | . – | 70. 30 | . – | 50. 20 | . – | 30. 25 | . 0. | 40 | . – | 10. 60 | . 20 | . 50 | . 30 | . |

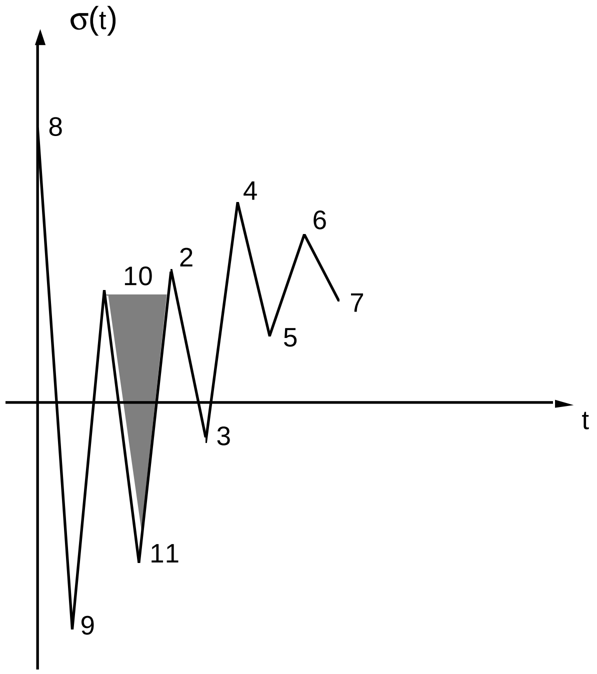



The second stage consists in extracting the elementary cycles. The first extracted cycle is the cycle defined by items 12 and 13 since is $|\sigma(12)-\sigma(13)|$ lower than and $|\sigma(14)-\sigma(13)|$ is $|\sigma(12)-\sigma(13)|$ lower than. $|\sigma(12)-\sigma(11)|$ Cycle

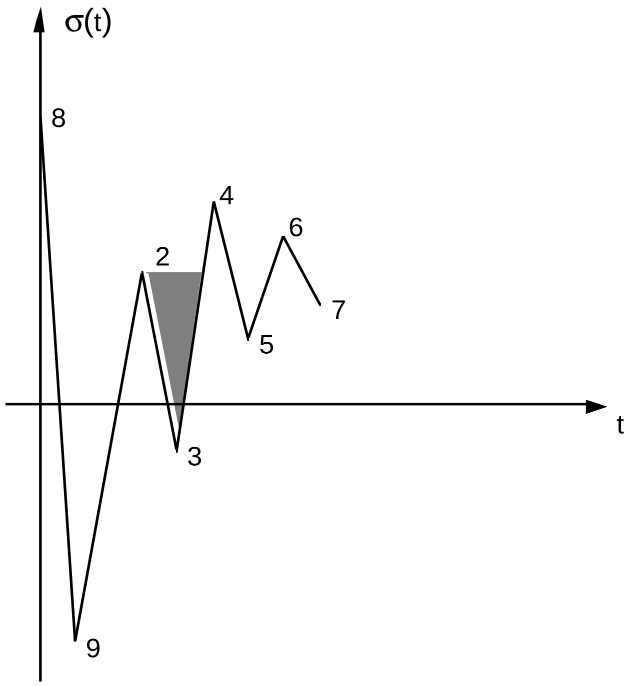
1: and $\mathrm{VALMAX}=20$ . $\mathrm{VALMIN}=-30$

The cycle having been extracted one removes these two points of the history of the loading, and one starts again on the remaining history.

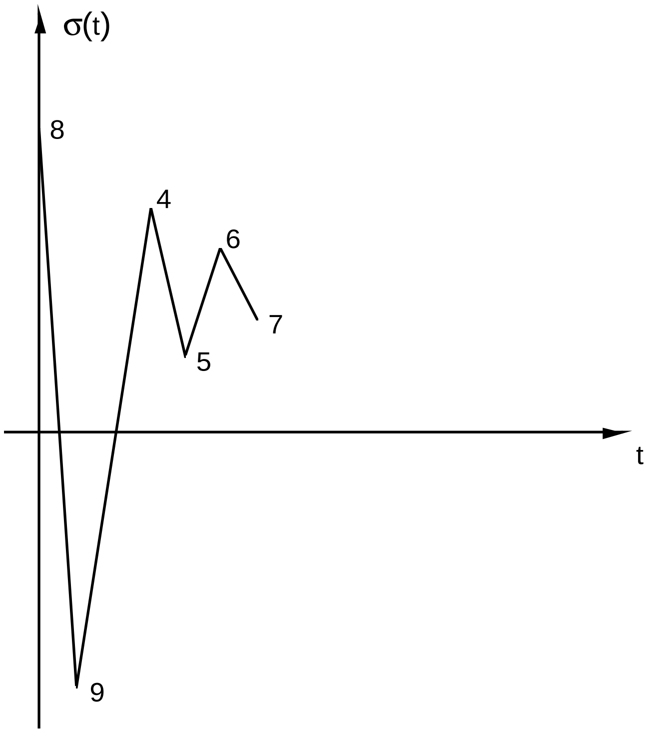
The following cycle extract is the cycle defined by items 14 and 15. Cycle
2: and $\mathrm{VALMAX}=25$ . $\mathrm{VALMIN}=0$
The remaining history, after suppression of these two points is: One



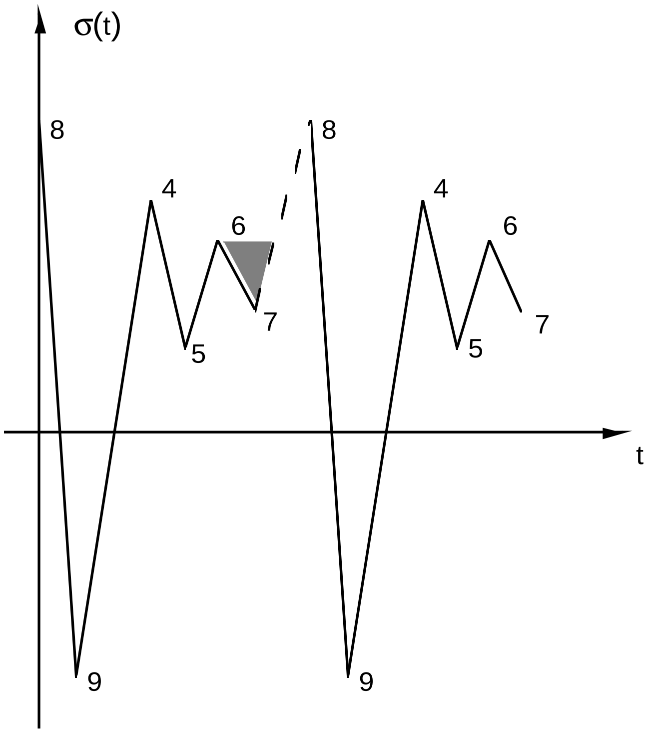

extracts then the cycle defined by items 10 and 11. Cycle
3: and $VALMAX=30$ . $VALMIN=-50$ One
sets out again on the following load history:




The following cycle extract is defined by items 2 and 3. Cycle
4: and $VALMAX=40$ . $VALMIN=-10$
The remaining load history is (it is the residue of the history of the loading): One

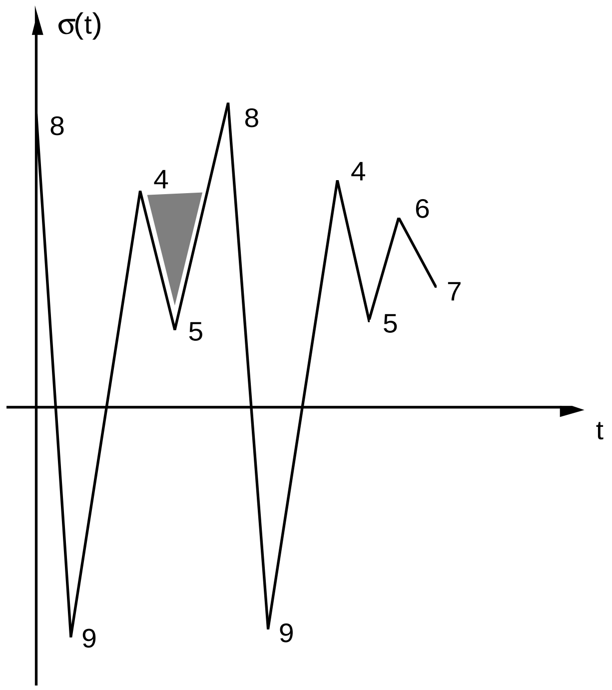



cannot extract any more from cycles, because all the history of the loading was traversed. One thus passes at the third stage, which consists in treating the residue: One

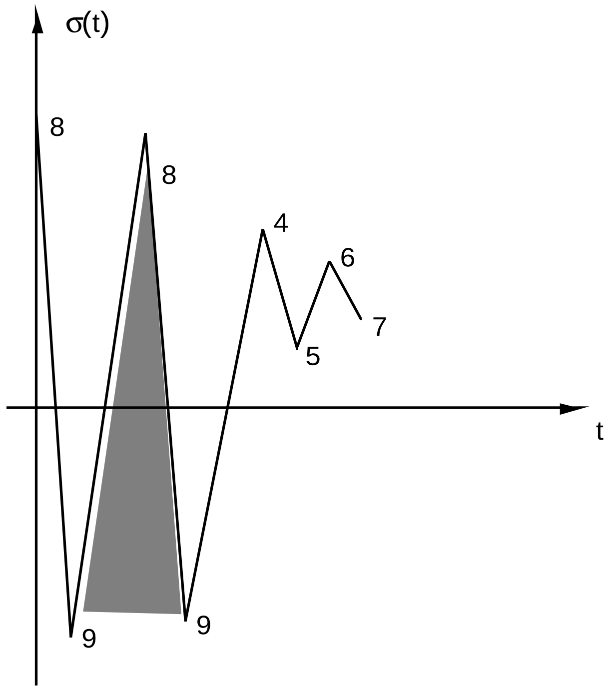



adds the same residue with his continuation, and one starts again the second phase on this loading.

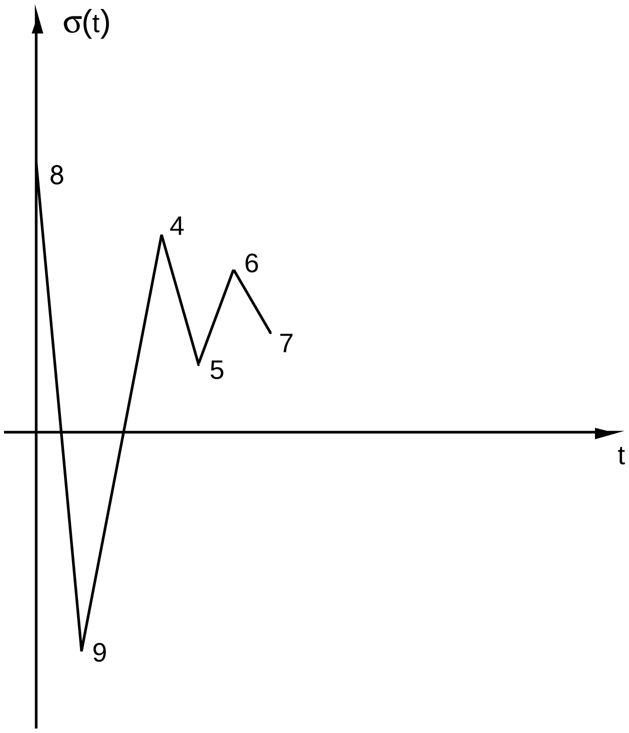
The following cycle extract is defined by items 6 and 7. Cycle
5: and $VALMAX=50$ . $VALMIN=30$
The remaining load history is:




The following cycle extract is defined by items 4 and 5. Cycle
6: VALMAX = 60. and VALMIN = 20.
The remaining load history is:



The last extracted cycle is a cycle defined by items 8 and 9. Cycle
7: and $\mathrm{VALMAX}=80$ . $\mathrm{VALMIN}=-70$ It




is noticed well that when one applies counting RAINFLOW to the whole made up of the two residues, one obtains in the end counting again the initial residue.