Energy the prediction of cleavage Gp Résumé

the parameter approaches G_p makes it possible to define a criterion of starting validates in the field of cleavage (brittle fracture in the presence of plasticity). The bases of this approach are first of all pointed out: modelization of crack by a notch, principle of minimization of energy, formulation, extension to 3D. One specifies then the essential components with the implementation of this approach in *Code_Aster*.

Contents

1 theoretical Bases

In this chapter one point out the interest of the approach, one describes the representation of crack by a notch then one presents the principle of minimization which leads to the definition of the parameter in 2D. One finishes by the extension of the definition of the parameter G_p in 3D.

1.1 And interest field of application of the Gp approach

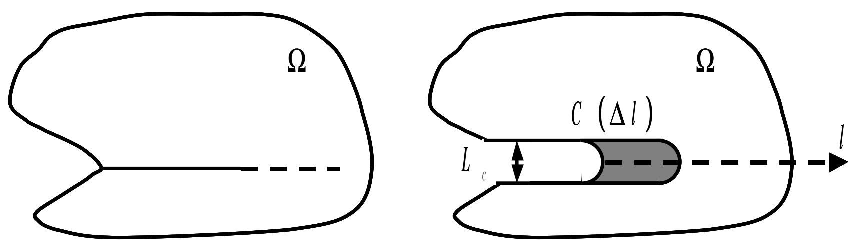
In fracture mechanics, the classical parameter making it possible to define a criterion of starting is the parameter G, rate of energy restitution elastic. One calculates it by the THETA method (command CALC_G) either in linear elasticity [R7.02.01], or in nonlinear elasticity [R7.02.03]. One can also calculate it for an elastoplastic behavior by supposing that the loading is proportional. This assumption implies that the equations of plasticity are equivalent to the equations of Hencky (which are those of nonlinear elasticity).

In the frame of incremental elastoplasticity, when the loading is nonproportional, one has the parameter GTP [R7.02.07], called mechanical rate of energy restitution total, which requires to net crack by a notch (calculated by the command CALC_G also). However this parameter does not make it possible, at the present time, to define a valid criterion in the field of the cleavage fracture (brittle fracture in the presence of plasticity).

The parameter G_p, on the other hand, is valid in incremental plasticity; it makes it possible to define a criterion of starting in the field of cleavage. Its definition, as one will see it, calls only on the free energy like with some geometrical parameters necessary to the definition of crack, modelled by a notch. With final, it represents an average elastic strain energy present in the adjacent zone at the front of notch. It will be noted that elastic strain energy is strongly related to the stress ($W_{elas}=\sigma^2/E$), which makes it possible to bring the energy approach closer to the criteria in stress such as Beremin, Bordet, or Corre.

1.2 Representation of crack by a notch

In the frame of the approach G_p, the real crack located in the field Ω is not modelled by a surface of discontinuity of the fields of displacements, but by a "notch" which corresponds, during the propagation, at a damaged zone of stiffness null (see the Figure 1.2-a).



Appear 1.2-a - Representation of crack by a notch

L_c is the thickness of this notch and $R=\frac{L_c}{2}$ is the radius of the circle representing the bottom of notch. The damaged zone or zones virtual propagation of the notch is called $C(\Delta l)$, Δl being the length of the zone $C(\Delta l)$, (or the distance to the bottom of notch). Thus, in any rigor, we are not any more in the context of the fracture mechanics but in that of the damage mechanics. A field of damage $\chi(M)$ is defined on Ω. By definition, $\chi(M)=0$ on Ω, before the propagation of the notch and, $\chi(M)=0$ on $\{\Omega-C(\Delta l)\}$, $\chi(M)=1$ on $C(\Delta l)$ after the propagation of the notch. The voluminal energy dissipated in the process of damage is noted: w_c.

Note:
One could have distinguished the thickness from the preexistent notch, of that of the damaged zone. This would have led to a model a little more complex (more parameters). By preoccupation with a simplicity one thus does not make this distinction but, one imposes the condition: $L_c \ll L$, L being the length of crack, so that the notch is quite representative of a true crack.

1.3 Principle of minimization

In the elastic frame, the evolution of the damage is obtained by the minimization of the following total energy suggested by Frankfurt and Marigo 1 for its simplicity:

$$E_{tot}(u,\chi)=\int_{\Omega}\left[(1-\chi)\Phi_{el}+\Delta\chi\, w_C\right]d\Omega \qquad (1)$$

where Φ_{el} is the density of free energy, defined by $\Phi_{el}=\int_{\Omega}\frac{1}{2}[\boldsymbol{\sigma}:\boldsymbol{A}^{-1}:\boldsymbol{\sigma}]\,d\Omega$ (where A is the tangent matrix of the behavior), and Δ the variation of a quantity during the increment considered indicates. As the initial damage is null, and that one considers the brutal fracture here (that is to say $\chi=0$ or $\chi=1$), one has here: $\Delta\chi=\chi$. The equality of the energies dissipated during the propagation of crack G_C and the voluminal dissipated energy of the model of damage for the notch makes it possible to write:

$$G_C=w_c L_c \qquad (2$$

One can notice that G_C does not represent any more “energy of usual surface” but rather the breaking value of one density energy integrated on a volume. The mechanism of fracture is thus controlled by two material parameters, w_c and L_c , which is different from the criterion of Griffith who depends only on G_C .

In the case of an elastoplastic behavior the definition of total energy E_{tot} is wide, by supposing that the mechanisms of dissipation of fracture and plastic are independent. The following statement is considered (cf 3):

$$E_{tot}(u,\boldsymbol{\varepsilon}^p,\alpha,\chi)=\int_{\Omega}\left[(1-\chi)\Phi_{el}+\Delta\chi\, w_C\right]d\Omega+E_{bl}(\alpha)+D_{pl}(\Delta\boldsymbol{\varepsilon}^p,\Delta\alpha) \qquad (3)$$

$\boldsymbol{\varepsilon}^p$ α , E_{bl} and D_{pl} being respectively the plastic strain field, variables of hardening, free energy blocked by plasticity, and plastic dissipation. In addition, the energy dissipated during the propagation will be now noted G_{PC} (in plasticity) to differentiate it from G_C (in elasticity).

1.4 Definition of a criterion in elastoplasticity

In order to simplify the presentation one is placed in the frame 2D (the extension to 3D will be presented to the paragraph 1.5) like at ways of cracking preset and prone to a continuous evolution of crack (not of crack in dotted lines along the way). One can thus parameterize the position of crack, after propagation, by the parameter Δl , i.e. which one restricts the possible fields of damage χ to a family $\chi(\Delta l)$. Question of starting can then be formulated in the following way: in is a state given $(u,\boldsymbol{\varepsilon}^p,\alpha)$ corresponding to a quasi-static evolution without propagation of the notch, the solution without propagation $\Delta l=0$ always licit in comparison with the minimization of 3 ? In the contrary case, there is propagation. One can show that it is a question of determining the minimum (cf 3):

$$\min_{\Delta l \geqslant 0} E_{tot}\left(u, \boldsymbol{\varepsilon}^p, \alpha, \chi(\Delta l)\right) \quad (4)$$

the condition of propagation can be written:

$$\exists \Delta l > 0, G_P(\Delta l) \geqslant G_{PC}, \quad \text{with} \quad G_P(\Delta l) = \frac{[\int_{\Omega} \Phi_{el}\, d\Omega]}{\Delta l} \quad (5)$$

One defines the parameter then G_p as:

$$G_p = \max_{\Delta l} G_p(\Delta l) \quad (6)$$

the criterion of propagation can then be written:

$$G_p \geqslant G_{pc} \quad (7)$$

the criterion suggested implies the knowledge of 2 material parameters, L_c and G_{pc}, which thus require an identification (see paragraph 2.2).

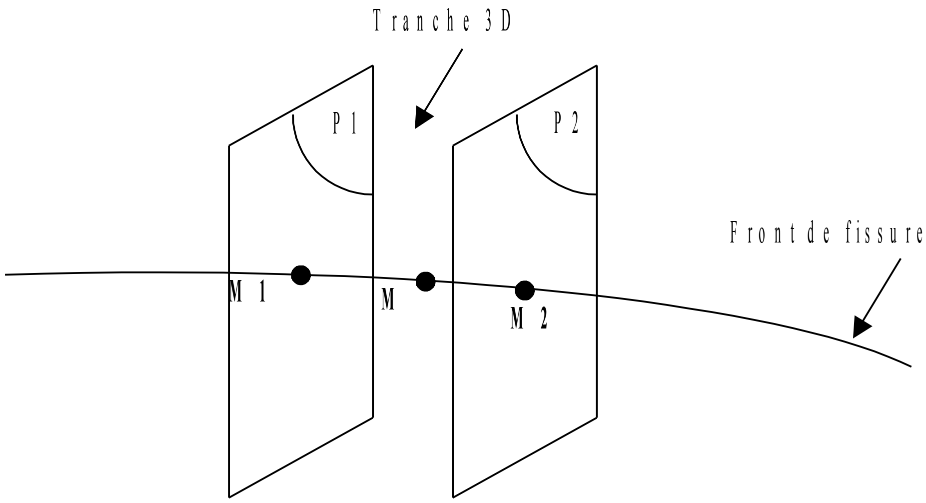
The parameter G_p is calculated like the maximum (compared to Δl, length of propagation) of the integral on the field $C(\Delta l)$ of the free energy Φ_{el}, divided par. Δl It is thus a density of average elastic strain energy (kJ/m^2). It is not a rate of energy restitution here because the energy considered for minimization is that of time running, and included by the rebalancing of the fields after propagation.
Following the example of $K_J(MPa\sqrt{m})$, definite from J via the relation of Irwin $\left(J = \left[\frac{(1-\nu^2)}{E}\right] K_J^2\right)$ - valid in 2D plane strain and 3D -, one can also define a parameter $K_{GP}(MPa\sqrt{m})$ from G_p using this same relation:

$$G_p = \left[\frac{(1-\nu^2)}{E}\right] K_{GP}^2 \quad (8)$$

1.5 Extension to 3D

the definition of the parameter G_p in 3D is come up against the following difficulty: how to define in a general way fields of damage according to “ Δl ” (virtual propagation measured according to the norm at the crack front outdistances) and of s (curvilinear abscisse measured along the crack front)? In front of this difficulty it seems reasonable to be limited to fields of damage defined “by extension” of those defined in 2D.

For a point M of the crack front of curvilinear abscisse s one will thus consider the segment $(M1 - M2)$ with the ends $M1$ and $M2$ of X-coordinates $[s - \delta s ; s + \delta s]$ (see Figure 1.5-a). It is supposed that δs is sufficiently small so that one can consider that the slice 3D ranging between the 2 normal planes at the crack front in $M1$ and $M2$ is in plane strain state. One thus excludes the points from the front too close to free edges if they exist. The computation parameter G_p at the point M is carried out then as in 2D, by replacing the plane 2D by the slice 3D corresponding.



Appear 1.5-a - Definition of the slices in 3D

Concretely the segment $(M1-M2)$ will be able to correspond to a finite element of the discretization of the crack front, and the plane normal could be those which one classically defines by building the mesh 3D by extrusion of a 2D mesh, locally around make of crack.

In 2D, it is thus necessary to define in 2D an area of thickness L_C representing the damaged zone; in 3D it is thus necessary moreover to define "slices" allowing the computation of G_p. However these two stresses necessarily do not result in defining particular meshes, even if it is this last solution which was implemented up to now in 3D. It is theoretically possible indeed, in a more general way, geometrically to define these zones on a free mesh (but sufficiently fine) and to select the finite elements belonging to these zones; it is what is made in 2D.

2 Implemented in *Code_Aster*

The computation mechanical is realized under the assumption of a thermoelastoplastic behavior associated with a criterion of Von Mises with isotropic or kinematical hardening linear (`VMIS_ISOT_TRAC`, `VMIS_ISOT_LINE`, `VMIS_CINE_LINE`). We will specify of what consists the computation of G_p and how the identification of (the radius $R=\frac{L_c}{2}$ of the notch is carried out materials parameters) and G_{PC} (rupture limit).
The method for calculation and identification with Code_Aster is presented in Doc. U2.05.08. The simple documentation of use is Doc. U4.82.31.

2.1 Computation of Gp

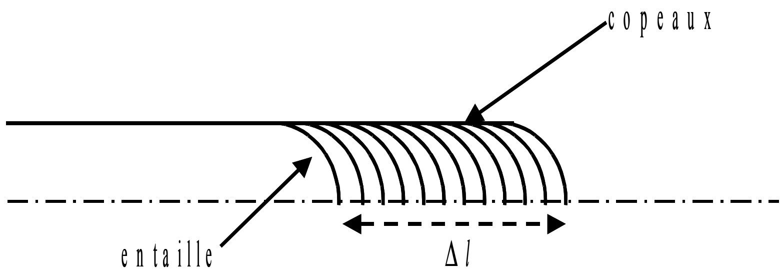
The computation of G_p, carried out using macro command `CALC_GP`, is based on the use of `POST_ELEM` which allows the computation of elastic strain energy on a mesh group. Modelizations (finite elements, small or large deformations, etc) and loadings usable are those of the command `POST_ELEM`, key word `ENER_ELAS`. More precisely it acts, for each time envisaged in the list of times of computation, to carry out the two following stages:

1 first of all to calculate the quantity $G_p(\Delta l)$ for ascending values from Δl :

$$G_P(\Delta l)=\frac{[\int_{\Omega}\Phi_{el}\,d\,\Omega]}{\Delta l}$$

In 2D it is thus necessary to identify the elements of the zone $C(\Delta l)$ by a mesh group defined in the level of the mesh as presented in Figure 2.1-a, or by a geometrical zone of Gauss points, then to calculate elastic strain energy on this zone then to divide it par. Δl Into 3D one carries out the same operation in each slice, and one divides by surface " ΔS " slice.

To identify the elements of the zone $C(\Delta l)$ one will operate as follows: the elements of the first chip will set up a first mesh group, the elements of chips 1 and 2 will set up a second group, the elements of the chips $1,2,3,\ldots,i$ will constitute i a ème group, etc It is necessary to envisage a sufficiently large number of chips to be able to find the maximum of $G_p(\Delta l)$, which is generally at a distance from approximately $3\mathrm{R}$ bottom of notch.



Appear 2.1-a - Definition of the chips in mesh

2 then to calculate the maximum of this function:

$$G_p=\max_{\Delta l} G_p(\Delta l)$$

what, in 3D thus led to as many values of G_p slices.

One will find examples and advice of use in the document [U2.05.01], in the tests ssnp131 [V6.03.131] and ssnv218 [V6.04.218].

2.2 Identification of the parameters

One supposes known, for the material considered:

1) the Young modulus E,
2) the critical stress σ_c,
3) the energy of surface G_c.

The parameter G_{PC} is determined by simulation of a test on test-tube CT where the crack is represented by a notch of radius R given. For each value of the loading, crescent of 0 up to a breaking value, one calculates on the one hand the parameter G and on the other hand the parameter G_p. For the breaking value of the loading corresponding to $G=G_c$, one obtains $G_p=G_{PC}$. However, this value depends directly on the value of the parameter R.

This parameter R is given via a traction test on a bar. During the fracture of the bar (in the field of cleavage) a simple assessment of energy makes it possible to obtain:

$$\frac{\sigma_c^2}{E}R=G_{PC}(R)$$

The solution of this equation for various values of R, which requires to determine the function $G_{PC}(R)$ for these same values, makes it possible to determine and the value of R and the value of G_{PC}.
However, in most case, one can go up that the function G_p depends explicitly on R and that it is then enough to consider only one value of this parameter under the only condition: $R \ll L$, length of crack. In most case, one takes $R=50\,\mu m$. There is then only one simulation of test on test-tube CT to carry out.

3 Bibliography

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The references [1] and [2] are relating to the principle of minimization of energy or with the energy formulations, the references [3] and [4] relate to the theoretical bases of the approach G_P and the references [5], [6] and [7] are relative to applications of the approach.

4 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
10.4	Y. WADIER EDF R & D AMA	initial Text