Simplified harmfulness of default by the method Summarized K-beta

analyzes:

The method of analysis presented (method K-beta) is applied to the analysis of harmfulness of a default located under the coating of the tanks REFERENCE MARK. The purpose of it is codified in the RSE-M and is evaluating the factors of intensity of the stresses corrected plastically for the coating (in first point of the default) and for the base metal or of the welded joint (in second point of the default).

With this intention, one calculates the elastic stress intensity factors with the two points of the default, using the nodal stresses resulting from the mechanical resolution and the residual stresses given by the user. The ratios of critical tenacities on the stress intensity factors obtained determine the factors of margins.

The theoretical aspects of the method K-beta and its implementation data-processing are the objects of the following paragraphs.

This method corresponds to the Rupt1D approach in the nomenclature of the project EDF Epicure.

Contents

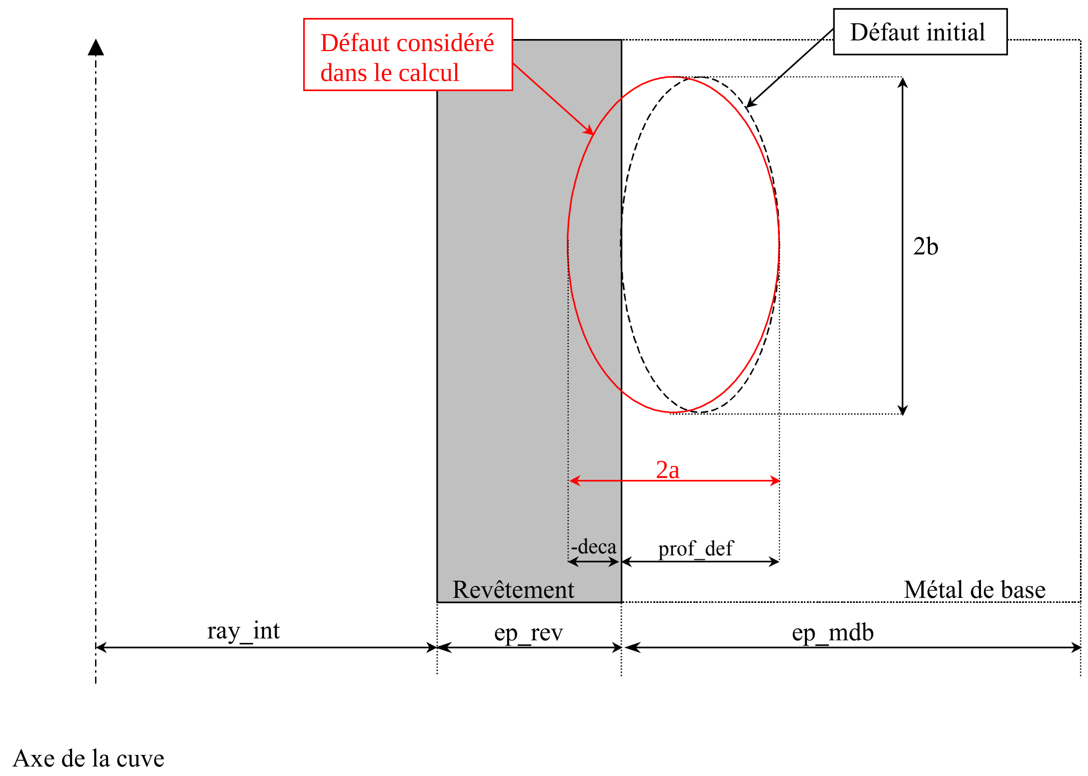
1 theoretical Aspects of the method K-beta

1.1 Validity of the method K_β

the method suggested applies to a default under coating located partly current of a ferritic steel tank covered by subjected austenitic stainless steel either:

- with a thermal transient applied on the surface possibly interns combined with a loading of pressure limited
- to a loading of direct compression.

The method β is valid only for defaults under coating of which the point, side coating, penetrate slightly in the coating. This is why for computation, to the initial size of the default considered **prof_def**, one adds the penetration in the coating **deca**. [Figure 1.1-a] the difference between the initial default specifies (dimensions sunken in `POST_K_BETA`) and the default considered in computation (default which takes account of the penetration in the coating) by the method β.



Appear 1.1-a: Diagram of the default under coating considered

Conditions of validity of the method:

•penetrating default in the coating,

• $\frac{|deca|}{ep_{rev}} \leq 0{,}2$ and $\frac{2a}{ep_{rev}} \leq 3$ $\frac{2a}{(ep_{rev}+ep_{mdb})} \leq \frac{1}{10}$.

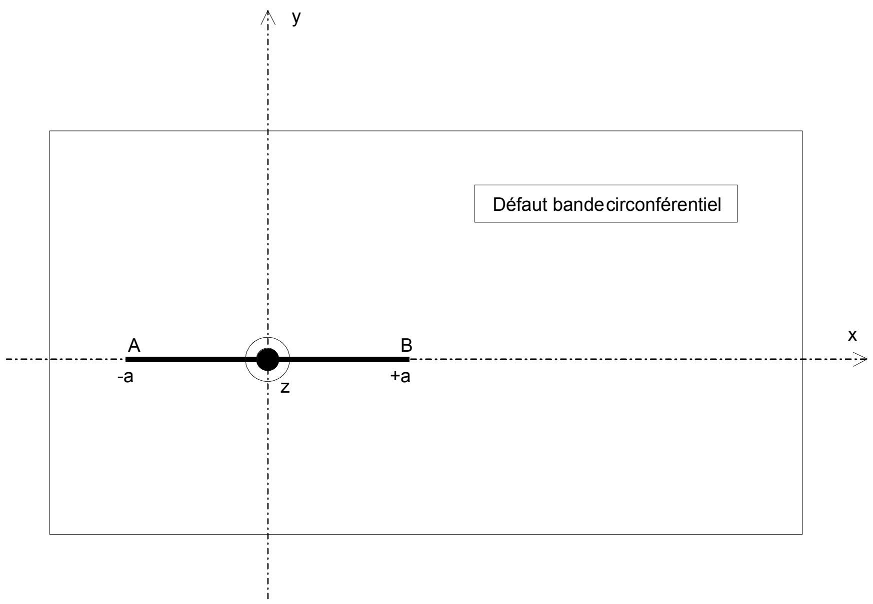
By convention in selected `command` POST_K_BETA one $deca \leq 0$. The value by default selected is $deca = -2.10^{-4}$.

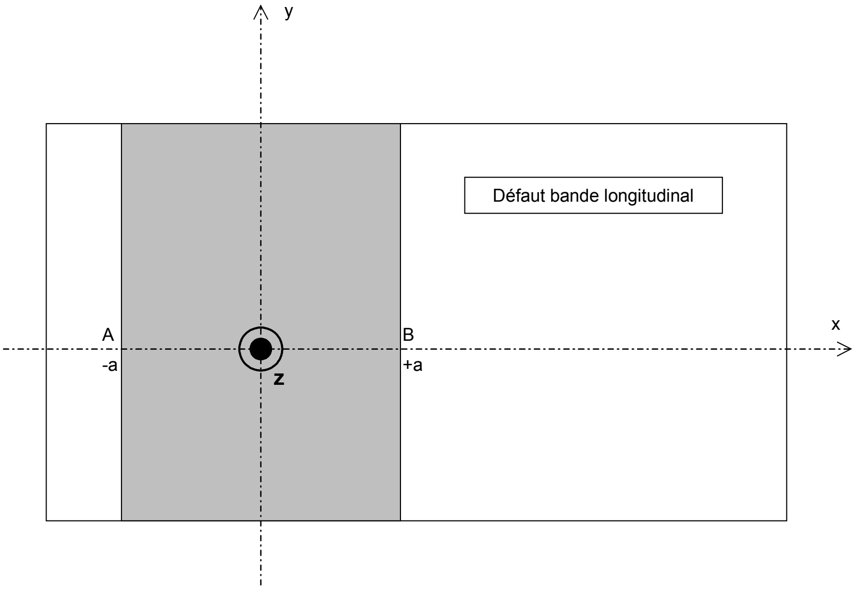
1.2 Stage n°1: Computation of the elastic stress intensity factors of a default bandages in a plate of infinite size

the elastic **stress intensity factors** of a default **bandages** in a plate of **infinite size** are given by the following relations:

$$K\beta 01 \ : \begin{cases} K_{IA\infty}=\int_{-a}^{+a}\frac{\sigma(x)}{\sqrt{\pi a}}\sqrt{\frac{a-x}{a+x}}dx \\ K_{IB\infty}=\int_{-a}^{+a}\frac{\sigma(x)}{\sqrt{\pi a}}\sqrt{\frac{a+x}{a-x}}dx \end{cases}$$

where $2\mathrm{a}$ (depth of the default is the bandwidth), A and B are the two ends (respectively in $-a$ and $+a$).
The stress $\sigma(x)$ is the normal stress useful for the plane of the crack (forced elastic added residual stress).
The configurations "default circumferential" and "longitudinal default" are defined by two sketches Ci - afterwards.



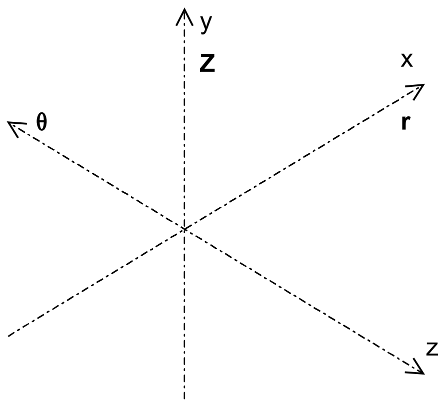


For the default bandages circumferential, one takes $\sigma(x)=\sigma_{yy}(x)$
For the default bandages longitudinal, one takes $\sigma(x)=\sigma_{zz}(x)$

1.2.1 Change of reference

1) Basic change

- **Case 1:** transition of the local Cartesian base (in the plane of cut of the axisymmetric model) at the cylindrical base



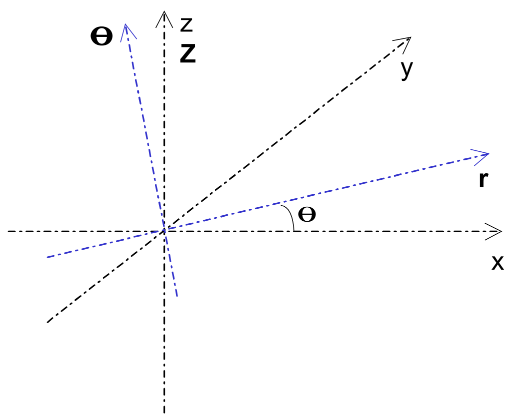
One a: $e_x=e_r$ $e_y=e_Z$ $e_z=-e_\theta$

the basic change for the tensor of the stresses is written:

$$\begin{bmatrix}\sigma_{rr} & \sigma_{r\theta} & \sigma_{rZ}\\ \sigma_{r\theta} & \sigma_{\theta\theta} & \sigma_{\theta Z}\\ \sigma_{rZ} & \sigma_{\theta Z} & \sigma_{ZZ}\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}\sigma_{xx} & \sigma_{xy} & \sigma_{xz}\\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz}\\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz}\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & -1 & 0\end{bmatrix}$$

One obtains finally: $\begin{cases}\sigma_{rr}=\sigma_{xx}\\ \sigma_{\theta\theta}=\sigma_{zz}\\ \sigma_{ZZ}=\sigma_{yy}\end{cases}$ et $\begin{cases}\sigma_{r\theta}=-\sigma_{xz}\\ \sigma_{rZ}=\sigma_{xy}\\ \sigma_{\theta Z}=-\sigma_{yz}\end{cases}$

- **Case 2:** transition of the total Cartesian base (models 3D) at the cylindrical base



One a: $\begin{cases}e_r=\cos\theta\, e_X+\sin\theta\, e_Y\\ e_\theta=-\sin\theta\, e_X+\cos\theta\, e_Y\\ e_Z=e_Z\end{cases}$ d'où $\begin{cases}e_X=\cos\theta\, e_r-\sin\theta\, e_\theta\\ e_Y=\sin\theta\, e_r+\cos\theta\, e_\theta\\ e_Z=e_Z\end{cases}$

the basic change for the tensor of the stresses is written:

$$\begin{bmatrix}\sigma_{rr} & \sigma_{r\theta} & \sigma_{rZ}\\ \sigma_{r\theta} & \sigma_{\theta\theta} & \sigma_{\theta Z}\\ \sigma_{rZ} & \sigma_{\theta Z} & \sigma_{ZZ}\end{bmatrix}=\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\sigma_{XX} & \sigma_{XY} & \sigma_{XZ}\\ \sigma_{XY} & \sigma_{YY} & \sigma_{YZ}\\ \sigma_{XZ} & \sigma_{YZ} & \sigma_{ZZ}\end{bmatrix}\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

One obtains finally:

$$\begin{cases}\sigma_{rr}=\cos^2\theta\,\sigma_{XX}+2\sin\theta\cos\theta\,\sigma_{XY}+\sin^2\theta\,\sigma_{YY}\\ \sigma_{r\theta}=-\sin\theta\cos\theta\,\sigma_{XX}+\left(\cos^2\theta-\sin^2\theta\right)\sigma_{XY}+\sin\theta\cos\theta\,\sigma_{YY}\\ \sigma_{rZ}=\cos\theta\,\sigma_{XZ}+\sin\theta\,\sigma_{YZ}\\ \sigma_{\theta\theta}=\sin^2\theta\,\sigma_{XX}-2\sin\theta\cos\theta\,\sigma_{XY}+\cos^2\theta\,\sigma_{YY}\\ \sigma_{\theta Z}=-\sin\theta\,\sigma_{XZ}+\cos\theta\,\sigma_{YZ}\\ \sigma_{ZZ}=\sigma_{ZZ}\end{cases}$$

- **Synthesis:** components used for the computation of the stress intensity factors

  circumferential Default : σ_{zz} in the cylindrical base is

  σ_{yy} with an axisymmetric model

  σ_{zz} with a model 3D

  longitudinal Default : $\sigma_{\theta\theta}$ in the cylindrical base is

  σ_{zz} with an axisymmetric model

  $\sin^2\theta\ \sigma_{xx}-2\ \sin\theta\ \cos\theta\ \sigma_{xy}+\cos^2\theta\ \sigma_{yy}$ with a model 3D

2) Translation of the origin

   the origin of the reference must be relocated radially to coincide with the point medium of the tape:
   $r\Leftrightarrow r-r_0$ with $r_0=(\text{ray\_int}+\text{ep\_rev}+\text{deca})+a$

   With: **ray_int** : radius interns tank
   ep_rev : thickness of the coating
   deca : penetration of the default in the coating
   a : half length of the default considered for computation

   All these quantities are schematized [Figure 1.1-a].

1.2.2 Method of calculating

the integrals giving $K_{IA\infty}$ and $K_{IB\infty}$ are calculated per pieces: decomposition comes from a subdivision of the interval $[-a/2\ ;+a/2]$ in N elementary subintervals on which the useful stress $\sigma(x)$ is linearized:

$$\sigma(x)=\alpha_i x+\beta_i \text{ for } x\in I_i=[a_i\,;a_{i+1}]$$

the meeting of N the subintervals I_i for $1\le i\le N$ reconstitutes the tape $[-a\,;+a]$.
The contributions of the subinterval $I_i=[a_i\,;a_{i+1}]$ to the computation of the FIC are given by:

$$K\beta 02\ :\begin{cases} K^i_{IA\infty}=\displaystyle\int_{a_i}^{a_{i+1}}\frac{\alpha_i x+\beta_i}{\sqrt{pia}}\sqrt{\frac{a-x}{a+x}}dx \\ K^i_{IB\infty}=\displaystyle\int_{a_i}^{a_{i+1}}\frac{\alpha_i x+\beta_i}{\sqrt{pia}}\sqrt{\frac{a+x}{a-x}}dx \end{cases}$$

These integrals can be calculated analytically. One obtains finally the relations $K\beta 03-a$:

$$\begin{cases} K_{IA\infty}=\sqrt{\dfrac{a}{\pi}}\times\displaystyle\sum_{i=1}^{N}\left[\alpha_i\left(-\frac{a}{2}Arc\sin\left(\frac{x}{a}\right)+\left(\frac{x}{2}-a\right)\sqrt{1-\left(\frac{x}{a}\right)^2}\right)+\beta_i\left(Arc\sin\left(\frac{x}{a}\right)+\sqrt{1-\left(\frac{x}{a}\right)^2}\right)\right]_{a_i}^{a_{i+1}} \\ K_{IB\infty}=\sqrt{\dfrac{a}{\pi}}\times\displaystyle\sum_{i=1}^{N}\left[\alpha_i\left(\frac{a}{2}Arc\sin\left(\frac{x}{a}\right)-\left(\frac{x}{2}+a\right)\sqrt{1-\left(\frac{x}{a}\right)^2}\right)+\beta_i\left(Arc\sin\left(\frac{x}{a}\right)-\sqrt{1-\left(\frac{x}{a}\right)^2}\right)\right]_{a_i}^{a_{i+1}} \end{cases}$$

N.B.

There exist formulas equivalent to the relations, established above after the changes of variables.

$$\begin{cases} \gamma_i=Arc\sin\left(\dfrac{a_i}{a}\right) \\ \gamma_{i+1}=Arc\sin\left(\dfrac{a_{i+1}}{a}\right) \end{cases}$$

The FIC are then given by the new statements $K\beta 03-b$:

$$\begin{cases} K_{IA\infty}=\sqrt{\dfrac{a}{\pi}}\times\displaystyle\sum_{i=1}^{N}\left[\left(\beta_i-\frac{a\alpha_i}{2}\right)(\gamma_{i+1}-\gamma_i)+(\beta_i-a\alpha_i)(\cos\gamma_{i+1}-\cos\gamma_i)+\frac{a\alpha_i}{4}(\sin 2\gamma_{i+1}-\sin 2\gamma_i)\right] \\ K_{IB\infty}=\sqrt{\dfrac{a}{\pi}}\times\displaystyle\sum_{i=1}^{N}\left[\left(\beta_i+\frac{a\alpha_i}{2}\right)(\gamma_{i+1}-\gamma_i)-(\beta_i+a\alpha_i)(\cos\gamma_{i+1}-\cos\gamma_i)-\frac{a\alpha_i}{4}(\sin 2\gamma_{i+1}-\sin 2\gamma_i)\right] \end{cases}$$

Note:

In practice, the computation of $K_{IA\infty}$ and $K_{IB\infty}$ is carried out on the segment of bearing of the applied default. On this segment, the points A (side coating) and B (side base metal or welded joint) of crack necessarily do not coincide with nodes of the mesh.
A first stage thus consists in positioning A and B on the path of radial bearing on the basis of the external skin and finishing in external skin. This positioning compared to the takes account of the shift of the default localization of reference of a DSR, and also depth of the default.
A translation of the origin is then carried out, the new origin being located in the middle of the segment $[A,B]$ (cf preceding paragraph relating to the change of reference).
N The subintervals on which the computation of the FIC is broken up are defined by the succession $[A,NO_1],[NO_1,NO_2],\ldots,[NO_{N-2},NO_{N-1}],[NO_{N-1},B]$. The nodes of the mesh determine the limits of them. The linear interpolations of the useful stress $\sigma(x)$ are thus carried out on these under - intervals; for the first and the last, one respectively uses the interpolations on $[NO_0,NO_1]$ and $[NO_{N-1},NO_N]$, which will thus be used for computation of the FIC only on part of their field of definition (NO_0 is the immediate predecessor of A on the radial path, NO_N is the immediate successor of B).
The formulas $K\beta 03-a$ or $K\beta 03-b$ are then applied for the computation of $K_{IA\infty}$ and $K_{IB\infty}$.
*It is important to note that this computation uses **the nodal stresses of the mesh** , from which the linear interpolations per pieces are given.*

1.3 Stage n°2: The geometrical for a Default Under elliptic Coating

stress intensity factors and $K_{IA\infty}$ given $K_{IB\infty}$ corrections at the end of the stage n°1 relate to a default bandages in a plate of infinite size.
The applied default is a Default Under elliptic Coating of profile. The stress intensity factors determined for this kind of geometry are obtained by application of geometrical corrections on $K_{IA\infty}$ and $K_{IB\infty}$.

[Figure1.1-a] allows to define the geometry of the DSR considered for computation.

Certain conventions are built-in:

- The depth $2a$ of a longitudinal or circumferential DSR corresponds to its radial dimension, either according to the direction carried par. e_r
- the length $2b$ of a longitudinal DSR corresponds to its axial dimension, or according to the direction carried par. e_z

the presence of DSR of longitudinal directional sense is applied in the base metal. [Figure 1.1-a] thus precisely this configuration of default represents.

- The length $2b$ of a circumferential DSR corresponds to its dimension orthoradiale, that is to say according to the direction carried par. e_θ

the presence of DSR of circumferential directional sense is applied in the welded joint. Compared to the [Figure 1.1-a], this configuration of default would be obtained by carrying out a rotation of $90°$ crack front around the small axis of the ellipse.

1.3.1 Correction by the edge factors

This first correction takes account owing to the fact that the default is located in a noninfinite medium. The localization of the DSR defined by [Figure 1.1-a] coating side and base metal side. implies corrections in points of crack

One defines beforehand the variable of space reduced $z=a/\left(a+\left(ep_{rev}+\mathrm{deca}\right)\right)$, where $\mathrm{ep\_rev}$ is the thickness of the coating and deca is the penetration of the DSR in the coating (see [Figure 1.1-a]).

Point A **side coating** : formulate $K\beta 04$

$$F_{bA}=0,998742+0,142801\, z-1,133379\, z^2+5,491256\, z^3-8,981896\, z^4+5,765252\, z^5$$

Point B **side base metal (or welded joint)** : formulas $K\beta 05$

$$F_{bB}=\begin{cases} 1-0,012328\, z+0,395205\, z^2-0,527964\, z^3+0,432714\, z^4 & \text{si} \quad 0\le z\le 0,92 \\ -414,20286+1336,75998\, z-1436,11970\, z^2+515,14949\, z^3 & \text{si} \quad 0,92<z\le 1 \end{cases}$$

1.3.2 Correction by the factors of ellipticity

This second correction takes account owing to the fact that the defect found an elliptic profile. It must be applied to the estimates determined for a default bandages.
Two cases are distinguished, according to the preponderance of one or the other of two dimensions of the elliptic profile.

First case: $a\le b$ Depth of the default $\le$ Length

$$K\beta 06 \ : \quad f_A=f_B=\frac{1}{\sqrt{1+1,464\left(\frac{a}{b}\right)^{1,65}}}$$

Second case: $b\le a$ Length of the default $\le$ Depth

$$K\beta 07 \ : \quad f_A=f_B=\frac{b}{a}\times\frac{1}{\sqrt{1+1,464\left(\frac{b}{a}\right)^{1,65}}}$$

1.3.3 Stress intensity factors of an elliptic DSR

the stress intensity factors of a Default Under elliptic Coating, obtained by correction of the FIC of a default bandages in a plate of infinite size, are given by the relations
Points A **side coating** :
$K\beta 08-a \ : \ K_{IA}=f_A\times F_{bA}\times K_{IA\infty}$
Point B **side base metal (or welded joint)** : $K\beta 08-b \ : \ K_{IB}=f_B\times F_{bB}\times K_{IB\infty}$

1.4 Stage n°3: Plastic correction known as “correction β ”

1.4.1 Formulation of the correction β

the stress intensity factors determined by the relations $K\beta 08-a$ and $K\beta 08-b$ are those of an elliptic DSR, under the assumption of an elastic behavior of the materials.
The correction β , specific with the DSR stuck to the interface, makes it possible to take account of plasticization with the two points of the crack side coating (point A) and side base metal or welded joint (point B).
The corrective factors are defined by the following relations:

For a longitudinal default:

$$K\beta 09\ :\ \begin{cases}\beta_A = 1+0{,}165\times\ln(prof_{déf})\times\tanh\left(\dfrac{36\,r_{yA}}{ep_{rev}}\right)\\ \beta_B = 1+0{,}465\times(1+prof_{déf}/100)\times\tanh\left(\dfrac{36\,r_{yA}}{ep_{rev}}\right)\end{cases}\quad \text{where}\quad r_{yA}=\frac{1}{6\pi}\left(\frac{K_{IA}}{\sigma_{yA}}\right)^2$$

For a circumferential default:

$$K\beta 09\ :\ \begin{cases}\beta_A = 1+0.5\times\tanh\left(\dfrac{36\,r_{yA}}{ep_{rev}}\right)\\ \beta_B = 1+0.5\times\tanh\left(\dfrac{36\,r_{yA}}{ep_{rev}}\right)\end{cases}\quad \text{where}\quad r_{yA}=\frac{1}{6\pi}\left(\frac{K_{IA}}{\sigma_{yA}}\right)^2$$

$\mathrm{ep\_rev}$ is the thickness of the coating, σ_{yA} is the yield stress of the coating to the temperature of the point A.
From where FIC corrected with the two points of crack:

$$K\beta 10\ :\ \begin{cases}K_{bêtaA} = \beta_A\times K_{IA}\\ K_{bêtaB} = \beta_B\times K_{IB}\end{cases}$$

1.4.2 Plastic correction as the history of the loading

the plastic correction is calculated according to the formulas $K\beta 09$ and $K\beta 10$ above for a phase of load considered separately in the history of the loading.
To evaluate the plastic correction as the history of the loading, one must retain at a time given the maximum correction obtained on all the preceding phases of load.

Principle
A each new phase of load, one revalues a plastic correction
$\boldsymbol{K\beta 11}: \Delta K = K_\beta - K_I = (\beta-1)\times K_I$
(even computation with the two points A and B crack, from where the omission of the indices). If this plastic new correction is higher than the maximum correction $\Delta K_{\max}$ obtained hitherto, one updates $\Delta K_{\max}$. The correction finally applied is written
$\boldsymbol{K\beta 12}: K_{\mathrm{CP}} = K_I + \Delta K_{\max}$
In phase of discharge, the plastic correction applied is the addition of $\Delta K_{\max}$ obtained on all the preceding phases of load:

- no plasticization in phase of discharge,
- the correction corresponds to the plasticized residue of the preceding phases of load.

Algorithmic
One initializes $K_{max}=0$
One initializes $K_{\mathrm{I\_last}}$ with a high arbitrary value

- at the first time one will be in phase of discharge per comparison with $K_{\mathrm{I\_last}}$
- not of plasticization at the first time

Buckles over times of the history of the Sialors
loading $K_I(t_n)\leq K_{\mathrm{I\_last}}$ (phase of discharge)
$K_{\mathrm{CP}}(t_n) = K_I(t_n) = \Delta K_{\max}$
If not (phase of load)

So $\beta(t_n)\times K_I(t_n)>K_I(t_n)+\Delta K_{\max}$ then

$$K_{\text{CP}}(t_n)=\beta(t_n)\times K_I(t_n)$$

$$\Delta K_{\max}=K_{\text{CP}}(t_n)-K_I(t_n)$$

If not

$$K_{\text{CP}}(t_n)=K_I(t_n)+\Delta K_{\max}$$

End So

Fine So

$K_{\text{I\_last}}=K_I(t_n)$

Fine Buckles

the same algorithmic one described above is put in work for the plastic corrections of the FIC at the two points A and B crack as the history of the loading.

2 Bibliographical references

1 "CUVE1D Version 2 - Note of validation" H-T26-2007-00833-FR

2 "CUVE1D Version 2 - Note of reference" H-T26-2007-00803-FR.

3 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
6	*A. DAHL, S. BUGAT, R. FERNANDES (EDF-R&D/MMC,* R & D /AMA)	initial Text
7.4	*A. DAHL, S. BUGAT, R. FERNANDES (EDF-R&D/MMC, R & D /AMA)*	
11.2	*C. DURAND (R & D /AMA)*	Addition of the paragraph of bibliographical references.