Identification of the Summarized

Weibull model

One tackles here the problem of the identification of the parameters of the Weibull model on a sample of tests representative of behavior with fracture of a brittle material (typically, ferritic steel of low temperature). The method of regression linear and the method of the maximum of probability are the two adopted methods. One details of it the principle as well as the associated methods of resolution, being based in both cases on an iterative process. Lastly, one shows their extension if one of the two parameters of this model (the stress of cleavage) depends on the temperature.

Contents

1Introduction

When they call on the Weibull model (cf POST_ELEM [U4.81.22]), the study of modelization of the brittle fracture of steels in general require a preliminary identification of the parameters of this model. In order to avoid a hard identification “with the hand” of these parameters which would require to start again repeatedly operation POST_ELEM with option WEIBULL, an automatic procedure of retiming was established in *Code_Aster*.

In this document, one briefly points out the equations of the Weibull model then one defines the problem of identification posed. One then describes the principle of the two methods of resolution adopted (linear and maximum regression of probability) by including the case where one of the two parameters of the model depends on the temperature.

2Rappels

2.1Le Weibull model

One considers a structure of behavior elastoplastic subjected to a thermomechanical request. It is supposed that the probability of cumulated fracture of this structure follows the model of WEIBULL [bib1] to two parameters following:

$$P_f(\sigma_w)=1-\exp\left[-\left(\frac{\sigma_w}{\sigma_u}\right)^m\right]$$ éq 2.1-1

statement in which the modulus of WEIBULL $m>1$ describes the tail of the statistical distribution of the sizes of the defaults at the origin of cleavage, σ_u is the stress of cleavage and σ_w is the stress of WEIBULL which depends on the principal history of the stress field in the plasticized zone of structure. For example, in the case of a way of monotonic loading, it is written:

$$\sigma_w=\sqrt[m]{\sum_p \left(\sigma_I^p\right)^m \frac{V_p}{V_0}}$$.éq 2.1-2

the summation relates to plasticized volumes of V_p matter, σ_I^p indicating the maximum principal stress in each one of these volumes (V_0 is a volume characteristic of the material).

2.2Identification of the parameters

In a very general way, one considers an experimental base made up of tests of various natures (type 1,2,…, N), each type of test being carried out n_j time so that the nombre total of tests rises with:

$$N=\sum_{j=1}^{j=n} n_j\,.$$

This experimental base could for example be made up of tests on axisymmetric test-tubes notched of radius of notch different led to various temperatures. Taking into account the random nature of the properties with fracture of the material considered, this base constitutes only one sample. The more important the number of these samples will be, the more it will be representative of the behavior of the material considered.

Among the various methods of identification suggested in the literature (see for example [bib2]), we retain two of them: method of regression linear, often used, like that of the maximum of probability recommended by the "Structural European Integrity Society (ESIS)" [bib3].

Note:

A comparative systematic study of the results given by these two methods [bib2] according to the number of sample taken by chance on a theoretical distribution showed that the method of the maximum of probability led to a better estimate of the parameters of the Weibull model. The method of regression linear remaining nevertheless very much used, we integrated it into our developments.

In the two adopted methods of retiming, one carries out the first computation of the stresses of WEIBULL with a clearance of parameter given (typically $m=20$, $s_u=3000\,\mathrm{MPa}$). One classifies these N tests using their stress of WEIBULL reached at the instant of the failure. One thus has an increasing list of stresses of WEIBULL $(\sigma_w^1,\ldots,\sigma_w^i,\ldots,\sigma_w^N)$, such as for each (i) , the number of test-tubes broken with a lower stress of WEIBULL or equalizes with σ_w^i is n_w^i (in general $n_w^i=i$). Among the various possible estimators of the probability of cumulated fracture P_f^i corresponding with σ_w^i [bib2], we choose that generally recommended: $P_f^i=\frac{i}{N+1}$.

Note:

In the cas particulier where the stress of WEIBULL depends on the temperature, the preceding ranking must be made temperature by temperature, each temperature corresponding to a different statistical model. The estimator of the probability of fracture precedent thus becomes: $P_f^i=\frac{i}{N_T+1}$, if the test-tube (i) were broken with the temperature T , for which there were tests N_T .

The two adopted methods of retiming are valid as long as [éq 2.1-1] remains true. If the identification is carried out on test results anisothermals whereas the stress of cleavage is supposed to depend on the temperature, this condition is not checked any more (cf `POST_ELEM` [U4.81.22]). In this case, typical case one will not be able to thus apply the developments which follow.

3Méthode of the linear regression

3.1.1Principe

the variation theory-experiment is measured by the statement:

$$\sum_i \left(LogLog\left(\frac{1}{1-P_f^i}\right) - LogLog\left(\frac{1}{1-P_f(\sigma_W^i)}\right) \right)^2 \qquad \text{éq 3.1.1-1}$$

("Log" indicates the Napierian logarithm). One wants to minimize this variation compared to (m, σ_u).

3.1.2Résolution

the method of retiming usually used leans on successive linear regressions: with the iteration (k) , the values ($m_k, \sigma_{u(k)}$) of the modulus and stress of cleavage are known. It is thus possible, with these values, to calculate the stresses of WEIBULL $\sigma^i_{W(k)}$ at various times of fracture thanks to [éq 2.1-1]. One then classifies these new stresses of WEIBULL by increasing amplitude and one from of deduced the new estimates from the probability of fracture $P^i_{f(k)}$ to the iteration (k) . For these values of stresses of WEIBULL fixed, the minimization of [éq 3.1.1-1] is brought back to a simple linear regression on the group of dots ($Log(\sigma^i_{W(k)})$, $LogLog(\frac{1}{1-P^i_{f(k)}})$) since if one defers $LogLog(\frac{1}{1-P_f})$ according to $Log(\sigma_W)$, one obtains a line of slope m which cuts the x-axis in ($Log(\sigma_u)$). The new values ($m_{k+1}, \sigma_{u(k+1)}$) of these parameters are thus given by (cancellation of derivatives partial of [éq 3.1.1-1] compared to each parameter):

$$m_{k+1}=\frac{\frac{1}{N}\sum_{i,j} X_{i(k)}Y_{j(k)}-\sum_i Y_{i(k)}X_{i(k)}}{\frac{1}{N}\sum_{i,j} X_{i(k)}X_{j(k)}-\sum_i X_{i(k)^2}} \qquad \text{éq 3.1.2-1}$$

$$\sigma_{u(k+1)}=\exp\left(\frac{1}{N}\left(\sum_i X_{i(k)}-\frac{1}{m}\sum_i Y_{i(k)}\right)\right) , \qquad \text{éq 3.1.2-2}$$

with $X_{i(k)}=Log(\sigma^i_{W(k)})$ and $Y_{i(k)}=LogLog(\frac{1}{1-P^i_{f(k)}})$.

These iterations are repeated as long as the difference between the clearances of parameter obtained with the iterations (K) and (k+1) is significant (typically, five iterations). The measurement of this variation is given by: $Max\left[\left|\frac{m_{k+1}-m_k}{m_k}\right|,\left|\frac{\sigma_{u(k+1)}-\sigma_{u(k)}}{\sigma_{u(k)}}\right|\right]$.

Note:

If m is fixed, $\sigma_{u(k+1)}$ is always given by [éq 3.1.2-2]. On the other hand, if σ_u is fixed, m_{k+1} is not given any more by [eq 3.1.2-1] but: $m_{k+1}=\frac{\sum_i X_{i(k)}Y_{i(k)}}{\sum_i X_{i(k)^2}-\log(\sigma_u)\sum_i X_{i(k)}}$.

4Méthode of the maximum of probability

4.1Principe

Let us note $p_f(\sigma_w)$ the density of probability associated with the probability of cumulated fracture $P_f(\sigma_w)$:

$$p_f(\sigma_w)=\frac{m}{\sigma_u}\left(\frac{s_w}{\sigma_u}\right)^{m-1}\exp\left[-\left(\frac{\sigma_w}{\sigma_u}\right)^m\right]$$

The quantity $p_f(\sigma_w)\,d\sigma_W$ is equal to the probability of breaking a test-tube subjected to a request corresponding to a stress of WEIBULL understood in the interval $\left[\sigma_W,\sigma_W+d\sigma_W\right]$. The probability so that all the test-tubes of the base broke thus raises with:

$$p(m,\sigma_u)\,d\sigma_w=\prod_i p_f(\sigma_W^i)\,d\sigma_w\,,$$ éq 4.1-1

p being the function of probability. The method of the maximum of probability then consists in choosing the parameters of the Weibull model so that the function of probability defined by [éq 4.1-1] (in practice rather its Napierian logarithm) that is to say maximum.

4.2Résolution

One uses an iterative process again. There still, with the iteration (k) , ($m_k,\sigma_{u(k)}$) as are $\sigma^i_{W(k)}$ known for them. For these values of stresses of WEIBULL fixed, the maximization of $Log(p)$ conduit to a new couple ($m_{k+1},\sigma_{u(k+1)}$) given by:

$$m_{k+1})=\frac{N}{m_{k+1}}+\sum_{i=1}^{i=N}Log(\sigma^i_{W(k)})-N\frac{\sum_{i=1}^{i=N}(\sigma^i_{W(k)})^{m_{k+1}}Log(\sigma^i_{W(k)})}{\sum_{i=1}^{i=N}(\sigma^i_{W(k)})^{m_{k+1}}}=0$$ éq 4.2-1

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$$\sigma_{(k+1)}=\sqrt[m_{k+1}]{\frac{1}{N}\sum_{i=1}^{i=N}(\sigma^i_{W(k)})^{m_{k+1}}}$$.éq 4.2-2

A each steps, the resolution of [éq 4.2-1] can be realized using the method of Newton, the gradient of $f(m)$ being given by:

$$\frac{df}{dm}(m)=-N\left(\frac{1}{m^2}+\frac{\left(\sum_{i=1}^{i=N}(\sigma^i_W)^m Log^2(\sigma^i_W)\right)\left(\sum_{i=1}^{i=N}(\sigma^i_W)^m\right)-\left(\sum_{i=1}^{i=N}(^i_W)^m Log(\sigma^i_W)\right)^2}{\left(\sum_{i=1}^{i=N}(\sigma^i_W)^m\right)^2}\right).$$

Note:

If m is fixed, $\sigma_{u(k+1)}$ is given by [4.2-2]. On the other hand, if σ_u is fixed, m_{k+1} is not any more solution of [4.2-1] but of:

$$f(m_{k+1})=\frac{N}{m_{k+1}}+\sum_{i=1}^{i=N} Log\left(\frac{\sigma^i_{W(k)}}{\sigma_u}\right)\left(1-\left(\frac{\sigma^i_{W(k)}}{\sigma_u}\right)^{m_{k+1}}\right)=0 \ .$$

This equation can be again solved using the method of Newton, the gradient being now given by:

$$\frac{df}{dm}(m)=-\frac{N}{m^2}-\sum_{i=1}^{i=N}\left(\frac{\sigma^i_W}{\sigma_u}\right)^m Log^2\left(\frac{\sigma^i_W}{\sigma_u}\right) \ .$$

5Dépendance of the parameters with the temperature

If one wishes to fix independently the two parameters temperature by temperature, it is enough to break up the base of tests into as much of under - bases by temperature and to apply to each one of these subbases the preceding methods. If, on the other hand, one only wishes to vary the stress of cleavage σ_u with the temperature, one proceeds in the following way.

linear 5.1Régression

the estimate of the probabilities of fracture being now carried out temperature by temperature (cf notices [§2.2]), it is enough to fix the stress of cleavage on each group of dots associated with the various temperatures (T). The equation [éq 3.1.2-2] thus becomes:

$$\sigma_{u(k+1)}=\exp\left(\frac{1}{N_T}\left(\sum_{i\in T} X_{i(k)}-\frac{1}{m}\sum_{i\in T} Y_{i(k)}\right)\right)$$

(N_T indicating the number of tests for the subbase corresponding to the temperature (T)), the modulus of WEIBULL being given by:

$$m_{k+1}=\frac{\sum_T\left(\frac{1}{N_T}\sum_{i\in T,\, j\in T} X_{i(k)}Y_{j(k)}\right)-\sum_i Y_{i(k)}X_{i(k)}}{\sum_T\left(\frac{1}{N_T}\sum_{i\in T,\, j\in T} X_{i(k)}X_{j(k)}\right)-\sum_i X_{i(k)^2}} \ .$$

5.2Maximum of probability

the stress of cleavage is given for each temperature (T) considered by:

$$\sigma_{u(k+1)}(T)=\sqrt[m_{k+1}]{\frac{1}{N_T}\sum_{i\in T}(\sigma^i_{W(k)}(T))^{m_{k+1}}},$$

m_{k+1} being solution of:

$$f(m_{k+1})=\frac{N}{m_{k+1}}+\sum_{i=1}^{i=N}Log(\sigma^i_{W(k)})-\sum_T N_T\frac{\sum_{i\in T}(\sigma^i_{W(k)})^{m_{k+1}}Log(\sigma^i_{W(k)})}{\sum_{i\in T}(\sigma^i_{W(k)})^{m_{k+1}}}=0\ .$$

6Conclusion

command `RECA_WEIBULL` of *the Code_Aster* makes it possible to carry out the chock of the parameters of the Weibull model [U4.82.06].

The user gives as starter this command the concepts results associated with various nonlinear computations carried out. The possible dependence of the stress of cleavage with the temperature is implicitly specified when different temperatures are associated with each one of these concepts results (if all these temperatures are identical or if they are not specified, it does not have there dependence with the temperature of this parameter).

The user can carry out this retiming by the method of the maximum of probability (`METHODE: "MAXI_VRAI"`) or that of the linear regression (`METHODE: "REGR_LIN"`).

Quantities determined by the command `RECA_WEIBULL` are deferred in an array in which one finds the value of the identified parameters, probabilities of fracture estimated starting from the experimental results as well as the probabilities of theoretical fracture calculated with the identified parameters.

7Bibliography

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Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
05/01/00	*R. MASSON, W. LEFEVRE (EDF/RNE/MTC)*	initial Text