Weibull models, of Bordet and Rice and Summarized

Tracey

One first of all recalls the bases of these models of local approach of the fracture allowing to model, for two of them, brittle fracture (models of Beremin, said WEIBULL, and of Bordet), for the third ductile starting (models of Rice and Tracey).

In the case of the models of brittle fracture (Beremin and Bordet), one describes how is calculated the probability of fracture of a structure from knowledge of the mechanical fields requesting it. While placing oneself in the general case of a nonmonotonous way of thermomechanical loading and by supposing that certain parameters of the models do not depend on the temperature, one draws up the general statement of the cumulated probabilities of fracture. The model of Beremin also allows to include the case of a correction of plastic strain.

Then, one presents the model leading to the model of growth of the cavities of Rice and Tracey as well as the ductile criterion of starting being referred to it. Lastly, indications concerning the implementation of these models in *Code_Aster* are summarized.

Contents

1 Introduction

One is interested here in a metal structure requested thermomécaniquement. One seeks to determine rupture criteria of this structure, representative of the two mechanisms met on certain steels:

- at low temperature, certain metallic materials (as the steel of tank) can behave like brittle materials while breaking brutally by cleavage,
- at higher temperature, appears the ductile tear.

In opposition to the comprehensive approach, models of Beremin, Bordet and Rice - Tracey introduced here lean on the knowledge of the mechanical fields in the zones most requested to obtain a local criterion of fracture representative of the concerned physical mechanisms (instability of the microscopic cracks of cleavage or increase then coalescence in porosity).
Advice of use of the models Beremin and Bordet is given in U2.05.08 documentation.

2 The model of Beremin (or Weibull)

the generally allowed mechanism of cleavage fracture is the following: the plasticization of the material led to the starting of microscopic cracks. Taking into account the metallurgical heterogeneity of the material, these microscopic cracks have a random size and a position. The total fracture is then obtained when the normal stress with one of these microscopic cracks becomes sufficiently large to make it unstable.

The model of Beremin, proposed at the beginning of the years 1980 [bib1] takes up these ideas by leaning on the knowledge of the local mechanical fields requesting structure considered. We present the broad outlines here of them by adopting the frame more general bench in [bib2]. By abuse language, we call this model, in what follows, Weibull model, in reference to the model of probability to which it leads.

For that, one arbitrarily considers a structure subjected to a history of thermomechanical requests from built-in time t=0. One seeks at any moment to determine the probability of cumulated fracture of this structure.

By assumption, this structure is made up (at least partly) of a steel, likely to break by cleavage at low temperature, whose elastoviscoplastic constitutive law uses the criterion of Von Mises, and of which one of the local variables corresponds to the cumulated equivalent plastic strain: a list of these models is provided to paragraph 2.4.

2.1 Bases

2.1.1 general Assumptions

first of all Let us consider a ground volume representative V_{rep} of the material considered. It is supposed that the microstructural heterogeneity of the material led to the existence of sites of damage (microdéfauts) appearing with plasticity. V_0 The volume of each site is noted, so that in a plasticized volume V_{rep}, the number of sites of damage is $\frac{V_{rep}}{V_0}$. For each one of these sites, one notes $g(\sigma)d\sigma$ the probability of having a critical stress of cleavage understood in $[\sigma;\sigma+d\sigma]$. The probability that one of the sites of damage has a stress of cleavage lower than σ_{Ic} is thus:

$$\int_0^{\sigma_{Ic}} g(\sigma)d\sigma$$

In practice, one applies a form of g for the positive stresses: $g(\sigma)=\alpha'\sigma^{m-1}$ ($g(\sigma)=0\,\mathrm{si}\,\sigma<0$).

If, one considers now structure, we suppose that $\sqrt[3]{\delta V}\geq\sqrt[3]{V_0}$, where δV is a ground volume whose characteristic dimension is lower than the macroscopic fluctuations of the mechanical fields.

Lastly, the events of fracture of the sites of damage are supposed to be independent from/to each other, the fracture of one of the sites involving the fracture of the group of the structure (assumption of the weakest link).

2.1.2 Probability of cumulated fracture of structure

One supposes here to know the probability of cumulated fracture of each site, noted $p_r(\mathrm{site})$. One can then successively write the probability of cumulated fracture of a ground volume, then of complete structure. The stress field being homogeneous in δV , the first is worth:

$$1-p_r(\delta V)=\prod_{\mathrm{site}\in\delta V}(1-p_r(\mathrm{site})) ,$$

that is to say:

$$p_r(\delta V)=1-\left(1-p_r(\mathrm{site})\right)^{\frac{\delta V}{V_0}} .$$

The probability so that at the end of the loading, our structure (volume Ω) is not broken raises then with:

$$1-P_r=\prod_{\delta V\in\Omega}(1-p_r(\delta V))=\prod_{\delta V\in\Omega}(1-p_r(\mathrm{site}))^{\frac{\delta V}{V_0}}=\prod_{\delta V\in\Omega}\exp\left(\ln\left(1-p_r(\mathrm{site})\right)\frac{\delta V}{V_0}\right) .$$

Knowing what $p_r(site)$ remains small, owe the unit, the preceding statement can be simplified to give finally:

$$P_r\approx 1-\prod_{\delta V\in\Omega}\exp(-\ p_r(\mathrm{site})\frac{\delta V}{V_0})=1-\exp\left(-\int_\Omega p_r(\mathrm{site})\frac{\delta V}{V_0}\right) .$$

That is to say:

$$P_r=1-\exp\left(-x\right) \quad \text{with} \quad x=\int_\Omega p_r(\mathrm{site})\frac{\delta V}{V_0}$$

2.2 Statements of the probability of cumulated fracture of the sites.

At any moment, the evolution of the mechanical fields in each element δV is supposed to be radial and not necessarily monotonous. This evolution is characterized in any point by a history of maximum principal stress $\sigma_I(u)_{0\leq u\leq t}$.

2.2.1 Parameters of cleavage independent of the temperature

the loading being radial, the direction of maximum principal stress is supposed to be constant.

When there is plasticization, the sites of damage appear.

We suppose that a requirement of fracture of a site of damage is that plasticity is active. So that this volume did not break at the moment T, it is necessary and it is enough that:

$$\sigma_{Ic}\geq\sigma_I(u),u<t \ \text{such as}\ \dot{p}(u)>0 ,$$

$\dot{p}(u)$ being plastic strain rate cumulated at time u .

Note :

> *Let us stress that this condition of active plasticity is different from the classically adopted condition ($p>0$). It is clear that these two conditions are equivalent in the case of a way of monotonic loading.*
> *For more general ways of loading, this condition of active plasticity leads on the other hand to much better results [bib7].*

One considers only times u for which plasticity is active, since the fracture is not possible that at these times there. One notes $\{u<t, \dot{p}(u)>0\}$ the set of these times for the element dV considered. The preceding condition is thus written:

$$\sigma_{Ic} \geq \max_{\{u<t,\, \dot{p}(u)>0\}} \sigma_I(u) \,.$$

Its probability of fracture being equal, as in the preceding section, with the probability so that it has a critical stress of cleavage lower or equal to the member of right of the preceding inequality, she is thus written:

$$p_r(\text{site}) = \int_0^{\max_{\{u<t,\, \dot{p}(u)>0\}} \sigma_I(u)} g(\sigma)\, d\sigma = \left(\frac{\max_{\{u<t,\, \dot{p}(u)>0\}} \sigma_I(u)}{\sigma_u} \right)^m ,$$

$\sigma_u = \left(\frac{m}{\alpha'} \right)^{\frac{1}{m}}$ indicating the stress of cleavage (forced for which the probability of cumulated fracture of the potential sites of cleavage is worth 1).

The probability of fracture of structure is written then according to [§2.1.2]:

$$P_r = 1 - \exp\left(- \left(\frac{\sigma_W}{\sigma_u} \right)^m \right)$$

where the stress of Weibull at the moment T is given by:

$$\sigma_W(t) = \left[\int_\Omega \tilde{\sigma}_I^m \frac{\delta V}{V_0} \right]^{\frac{1}{m}} \quad \text{with} \quad \tilde{\sigma}_I = \max_{\{u<t,\, \dot{p}(u)>0\}} \sigma_I(u)$$

Let us notice that in the case of a way of monotonic loading, the preceding statement of the stress of Weibull is reduced to:

$$\sigma_W = \left[\int_\Omega \tilde{\sigma}_I^m \frac{\delta V}{V_0} \right]^{\frac{1}{m}} \quad \text{with} \quad \tilde{\sigma}_I = \begin{cases} \sigma_I & \text{si l'élément est plastifié} \\ 0 & \text{sinon} \end{cases} .$$

2.2.2 Forced cleavage depend on the temperature

Is $\theta(u)_{0 \leq u \leq t}$ the evolution of temperature in δV .

For any time (u) , we suppose that in the vicinity of each site of damage, the “microscopic” normal stress checks:

$$\sigma_{I(micro)}(u) = f \,.\, \sigma_I(u) \ ,$$

f being a parameter of localization depending only on the average temperature $\theta(u)$ in δV. So that the site of damage did not break, it is necessary thus that:

$$\sigma_{Ic} \geq \sigma_{I(micro)}(u), \text{u<t}, \text{tel que } \dot{p}(u) > 0 .$$

that is to say:

$$\sigma_{Ic} \geq f \cdot \sigma_{I(micro)}(u), \text{u<t}, \text{tel que } \dot{p}(u) > 0 .$$

so that the probability of cumulated fracture of a site rises with:

$$p_r(\text{site}) = \left[\max_{[u<t,\, \dot{p}(u)>0]} \left\{ \frac{\sigma_I(u) . f(\theta(u))}{\sigma_u} \right\} \right]^m ,$$

or:

$$p_r(\text{site}) = \left[\max_{[u<t,\, \dot{p}(u)>0]} \left\{ \frac{\sigma_I(u)}{\sigma_u(\theta(u))} \right\} \right]^m ,$$

with: $\sigma_u(\theta) = \dfrac{\sigma_u}{f(\theta)}$, a function of the temperature. The introduction of the parameter of localization f thus leads to an apparent dependence of the stress of cleavage.

In a general way, the probability of cumulated fracture of structure rises with:

$$p_r = 1 - \exp\left(- \int_{\Omega} \left[\max_{[u<t,\, \dot{p}(u)>0]} \left\{ \frac{\sigma_I(u)}{\sigma_u(\theta(u))} \right\} \right]^m \frac{\delta \mathrm{V}}{V_0} \right)$$

The stress of Weibull has nothing any more but the following conventional meaning then: by noting σ_u^o a value chosen arbitrarily, one can write:

$$p_r = 1 - \exp\left(- \left(\frac{\sigma_w^o}{\sigma_u^o} \right)^m \right),$$

σ_w^o being defined by:

$$\sigma_w^o = \left[\int_{\Omega} \tilde{\sigma}_I^{\circ\, m} \frac{\delta V}{V_0} \right]^{\frac{1}{m}} \text{ with } \tilde{\sigma}_I^{\circ} = \max_{[u<t,\, \dot{p}(u)>0]} \left\{ \frac{\sigma_u^o \sigma_I(u)}{\sigma_u(u)} \right\}$$

2.3 Correction of strain

a large deformation of the sites resulting in de' to decrease harmfulness by it (contraction relative of the microscopic cracks in the transverse plane to the axis of tension), the critical stress of cleavage at one time u increases under the effect of this strain $\underline{\varepsilon}(u)$ according to:

$$\sigma_{Ic}(u)=\sigma_{Ic}(u=0)\exp(\frac{1}{2}\varepsilon_I(u)) \text{ with } \varepsilon_I(u)=\underline{n}_\sigma . \underline{\varepsilon}(u) . \underline{n}_\sigma$$

where $\underline{n}_\sigma(u)$ the direction associated with the maximum principal stress at time indicates u .

The probability of fracture of a site at a given u time is written now:

$$p_r(\text{site})=\max_{\{u<t,\,\dot{p}(u)>0\}}\left\{\frac{\sigma_I(u)}{\sigma_u(\theta(u))}.\exp\left(-\frac{1}{2}\varepsilon_I(u)\right)\right\}^m .$$

For a way of monotonic loading (constant temperature and uniform), the preceding relation leads to the classical statement [bib2]:

$$p_r(\text{site})=\left[\frac{\sigma_I}{\sigma_u}\right]^m \exp\left(-\frac{m}{2}\varepsilon_I\right) .$$

2.4 Establishment in *Code_Aster*

Of the advice of use of this model are given in U2.05.08 documentation.

Let us consider a field Ω_c of the studied structure which can be the group of the studied mesh, a group of mesh or a mesh. Following an elastoplastic thermomechanical computation, one knows the evolution of the stress fields, strain and of plastic strain cumulated in this field and one wishes to determine his probability of cumulated fracture.

For this making, one uses key word `WEIBULL` POST_ELEM `of the command.`

Let us stress that for computation with correction of strain (option `CORR_PLAST: "YES"`), a preliminary computation of the strain field of Green-Lagrange on the zone of studied structure (via command `CALC_CHAMP`) is necessary. In the contrary case, postprocessing stops.

Moreover, the constitutive law of the material must comprise a local variable corresponding to the cumulated equivalent plastic strain p . They is the models in particular (nonexhaustive list): `VMIS_ISOT_*, VMIS_ECMI_*, VMIS_*_CHAB, ROUSS_*, LEMAITRE, MONOCRISTAL`. In the contrary case, postprocessing stops.

Corresponding numerical integration in *Code_Aster* is carried out in two times:

one calculates in each Gauss point $\tilde{\sigma}_I$ if plastic strain rate cumulated in this point is strictly positive,

by squaring on each mesh then simple summation on the field Ω_c concerned, one from of deduced the stress from Weibull as well as the probability of associated fracture. The summation is balanced by a multiplicative coefficient which takes account of possible symmetries and the type of modelization selected (axi,…). One will take care well to define this coefficient (`COEF_MULT)` in accordance with the indications given in [U4.61.04].

The first stage makes it possible to introduce an alternative (key word `SIGM_ELMOY` instead of `SIGM_ELGA`) leading to results appreciably different into the case from a fissured structure (presence of gradient): in each mesh, $\tilde{\sigma}_I$ is given from the average on this mesh of the stress field (and, possibly, of the strain field of Green - Lagrange).

It is non-zero if plastic strain rate cumulated at time considered is strictly positive in a Gauss point at least.

3 The model of Bordet

One presents here quickly the model of Bordet. For more details, one will be able to refer to [8], [9] or [10].
The model of Bordet bases itself on the same bases as that of Beremin. It defines a probability of cleavage fracture room.
However, in the model of Beremin, one supposes the creation of microscopic cracks at the time of the attack of the threshold of plasticity, and these microscopic cracks remain potentially active throughout the loading which is followed from there. However, in steels, the total fracture is mainly related to microscopic cracks lately created. It is thus advisable to take into account the level of plastic strain reaches at every moment. This is already possible in the model of Beremin *via* the option of plastic correction defined in Doc. [U4.81.22].
In the model of Bordet, this is taken into account by considering that the probability of cleavage fracture is the product of the probability of nucleation and propagation at the same time.

3.1 Probability of local fracture per cleavage of the model of bordet

the local criterion is here defined like the statistical event simultaneously to meet the conditions of nucleation and propagation of the microscopic cracks. In the model, the nucleation and the propagation of the microscopic cracks are regarded as independent events; nucleation indicates the fracture of a carbide leading to the training of a microscopic crack, whereas the propagation is defined like the local instability of cleavage only guided by the local stresses.
The probability of local fracture per cleavage is written then:

$$P_{cliv}=P_{nucl}P_{propa}$$

Attention, subsequently, one understands by plastic strain ε_p the equivalent plastic strain defined by:

$$\varepsilon_p=\sqrt{\frac{2}{3}\boldsymbol{\varepsilon}_p:\boldsymbol{\varepsilon}_p}$$

This definition is valid only for the elastoplastic constitutive laws with criterion of von Mises. In *Code_Aster*, only the constitutive laws of this type are thus adapted to this model.

3.1.1 Local probability of nucleation

With the definition given higher, the probability of nucleation of a microscopic crack during an increment of plastic strain $d\,\varepsilon_p$ is proportional to the yield stress $\sigma_{ys}(T,\dot{\varepsilon}_p)$ with the temperature T and the plastic strainrate $\dot{\varepsilon}_p$.

$$P_{nucl}\propto N_{unc}(\varepsilon_p)\sigma_{ys}(T,\dot{\varepsilon}_p)d\,\varepsilon_p$$

This remains true as long as the number of microcracked carbides remains weak in front of the number of healthy carbides $N_{unc}(\varepsilon_p)$. The rate of microfissuring of carbides is constant for a given plastic strain; the number of healthy sites varies exponentially with
ε_p . By calling $\sigma_{ys,0}$ the yield stress at a temperature and a plastic strainrate of reference, and $\varepsilon_{p,0}$ a plastic strain of reference, the probability P_{nucl} can be expressed as follows:

$$P_{nucl} \propto \frac{\sigma_{ys}(T,\dot{\varepsilon}_p)}{\sigma_{ys,0}} \exp\left(\frac{-\sigma_{ys}(T,\dot{\varepsilon}_p)}{\sigma_{ys,0}} \cdot \frac{\varepsilon_p}{\varepsilon_{p,0}}\right)$$

If the plastic strain is small, therefore if the carbide cracking is limited enough, i.e., $\frac{\sigma_{ys,0}\varepsilon_{p,0}}{\sigma_{ys}(T,\dot{\varepsilon}_p)} \gg \varepsilon_p$

then the probability of nucleation are reduced to. $P_{nucl} \propto \frac{\sigma_{ys}(T,\dot{\varepsilon}_p)}{\sigma_{ys,0}}$ Local

3.1.2 probability of propagation As

in the model of Beremin presented in Section 2, the density of size of the carbide microscopic cracks is supposed to be distributed according to an opposite model power (with and α of β the material parameters independent of the temperature and the strainrates and). l

$$f(l)=\frac{\alpha}{l^\beta}$$

The cracking of a carbide will not be propagated in ferrite (more deeply only because of inertia alone of propagation, by dynamic effect) that with the condition of the presence of a sufficiently high local stress. A higher limit of size of nucleate ferritic microscopic crack $l_{max}=l(\sigma_{th})$ is thus introduced; it makes it possible to define the minimum of local stress necessary to the propagation of the largest possible ferritic microscopic crack. with

$$P_{propa}(\sigma_1)=\int_{l_c(\sigma_1)}^{l_{max}} f(l)\,dl$$

σ_1 the maximum principal stress, σ_{th} the threshold in lower part of which the propagation cannot take place and l_c the ferritic critical size of microscopic crack, obeying the relation of Griffith for an elliptic microscopic crack: with

$$l_c(\sigma)=\frac{\pi E \gamma_p}{2(1-\nu^2)\sigma^2}$$

E, ν et γ_p the Young modulus, the Poisson's ratio and surface density of energy. One obtains finally: With

$$\begin{cases} P_{propa}(\sigma_1) = 0 & \sigma_1 < \sigma_{th} \\ P_{propa}(\sigma_1) = \left(\left(\frac{\sigma_1}{\sigma_u}\right)^m - \left(\frac{\sigma_{th}}{\sigma_u}\right)^m\right) & \sigma_1 \geqslant \sigma_{th} \end{cases}$$

parameter m independent of the temperature, following the example of and α , β and which σ_u can depend on it (if the Young modulus depends on it). As
for the model of Beremin, the effects of directional sense of the microscopic cracks compared to the direction of the maximum principal stress are taken into account only via the parameter. σ_u Local

3.1.3 probability of cleavage As

previously specified, the local probability of cleavage is supposed of being the product of the probability of nucleation and the probability of propagation; for that one considers that during an infinitesimal increment of plastic strain, the active stress is constant. From where, if: $\sigma_1 \geqslant \sigma_{th}$ This

$$P_{cliv}(\sigma_{1,} d\varepsilon_p) \quad \propto \quad P_{nucl}(d\varepsilon_p) . P_{propa}(\sigma_1)$$
$$\propto \quad \frac{\sigma_{ys}(T,\dot{\varepsilon}_p)}{\sigma_{ys,0}} \exp\left(\frac{-\sigma_{ys}(T,\dot{\varepsilon}_p)}{\sigma_{ys,0}} . \frac{\varepsilon_p}{\varepsilon_{p,0}}\right)\left(\left(\frac{\sigma_1(\varepsilon_p)}{\sigma_u}\right)^m - \left(\frac{\sigma_{th}}{\sigma_u}\right)^m\right) d\varepsilon_p$$

equation does not state that the microscopic cracks nucleate remain active during the increment of plastic strain, but although the condition of propagation for each one of these microscopic cracks is determined by the value of the local stress field at the time of creation.
The probability that a ferritic microscopic crack is created and is propagated on an interval of plastic strain is $[0,\epsilon_{p,u}]$ then: This

$$P_{cliv}(\sigma_{1,} \varepsilon_{p,u}) \propto \int_0^{\epsilon_{p,u}} \frac{\sigma_{ys}(T,\dot{\varepsilon}_p)}{\sigma_{ys,0}} \exp\left(\frac{-\sigma_{ys}(T,\dot{\varepsilon}_p)}{\sigma_{ys,0}} . \frac{\varepsilon_p}{\varepsilon_{p,0}}\right)\left(\left(\frac{\sigma_1(\varepsilon_p)}{\sigma_u}\right)^m - \left(\frac{\sigma_{th}}{\sigma_u}\right)^m\right) d\varepsilon_p$$

probability is reduced well to 0 if the stress remains lower than during σ_{th} all the way of loading. Total

3.2 probability of cleavage fracture of Bordet

the preceding paragraph made it possible to determine the local probability of cleavage fracture. While following the principle of weak link, the total probability of cleavage fracture on potential n_c sites of initiation is written: This

$$P_{Bordet} = 1 - \exp\left(-\sum_{i=1}^{n_c} P_{cliv}(\sigma_{1,i}, \varepsilon_{p,u,i})\right)$$

equation can be expressed according to the volume of the process zones by introducing an infinitesimal volume on dV which the strains and stresses are constant (in the numerical case, the Gauss point). In order to simply compare the probabilities of Beremin and Bordet on an example given, one can define a stress of Bordet of the same type as that of Weibull: and

$$\sigma_{Bordet} = \left\{\int_{V_p}\left(\int_0^{\varepsilon_{p,u}} \frac{\sigma_{ys}(T,\dot{\varepsilon}_p)}{\sigma_{ys,0}}\left((\sigma_1(\varepsilon_p, dV))^m - (\sigma_{th})^m\right) \exp\left(\frac{-\sigma_{ys}(T,\dot{\varepsilon}_p)}{\sigma_{ys,0}} . \frac{\varepsilon_p}{\varepsilon_{p,0}}\right) d\varepsilon_p\right) \frac{dv}{V_0}\right\}^{\frac{1}{m}}$$

the probability of total fracture per cleavage is written with a distribution of Weibull: If

$$P_{Bordet} = 1 - \exp\left(-\left(\frac{\sigma_{Bordet}}{\sigma_u}\right)^m\right)$$

the number of microcracked carbides is weak in front of the number of healthy carbides, the term into exponential is close to 1 and the stress of Bordet is some simplified. Discussion

3.3 Bordet

3.3.1 or Beremin? The model

of Bordet is slightly more complex and end that the model of Beremin. One of its advantages is to consider the maximum principal stress at every moment, and not the maximum principal stress during the loading; consequently nothing prevents the probability of Bordet of decreasing, contrary to that of Beremin. Moreover

, the model of Bordet account returns owing to the fact that an area with a lower stress but a more important plastic strain can be more critical than a zone in which the stresses are higher but the lower level of plastic strain. However

, the model of Bordet the knowledge of additional material parameters requires, as one will describe it in the paragraph hereafter. Material parameters

3.3.2 The model

of usual Beremin requires the knowledge of three materials parameters: two parameters of form of the model of Weibull, and m , σ_u as well as the ground volume of the plastic zone; V_0 only the parameter depends σ_u on the temperature. A these three parameters, it is possible to add the plastic strain threshold making it possible to define the plastic zone on which integration is carried out.

The first three parameters are formally preserved by the model of Bordet; CAUTION:
formally only; they can be different and require a calibration of the same type as that done for Beremin, and presented in [R7.02.09].

Other parameters are added to it.

The threshold of plasticity function has *minimum* temperature and potentially of the plastic strainrate and $\sigma_{ys}(T, \dot{\varepsilon}_p)$ its value of reference. $\sigma_{ys,0}$
The critical stress below which the propagation of the ferritic microscopic cracks cannot take place, σ_{th} independent of the conditions of loading. In
the full version only (with the exponential term taken into account), a plastic strain of reference of which $\varepsilon_{p,0}$ the method of identification is not given and who seems rather delicate. It will be noted that the author him even (cf [8], [9]) seems to use for certain studies and validation the version of the model in which this parameter does not intervene (for the user of Code_Aster, it is enough to specify PROBA _NUCL=' NON') Implementation

3.4 in Code_Aster *The computation*

of the probability and of the stress of Bordet by the operator POST _BORDET is carried out. It requires to have carried out an elastoplastic thermomechanical computation via operator STAT_NON_LINE .
Advice of use of this model is given in U2.05.08 documentation. In

the currently established version, the temperature of the medium on which computation is carried out must be uniform (but can evolve in the course of time); this limitation does not seem crippling insofar as the plastic zone at a peak of crack is in general rather small. It
is possible to carry out computation with or without the term into exponential; if this term is required, the material parameter must $\varepsilon_{p,0}$ be indicated. In
all the cases, one at every moment calculates in Code_Aster the quantities such as the maximum principal stress and the equivalent plastic strain. One determines the value of the parameters and $\sigma_u(T)$ according to $\sigma_{ys}(T, \dot{\varepsilon}_p)$ the material characteristics provided by the user, then one calculates the total stress of Bordet by summation on the points of gauss of the elements of the mesh group indicated by the user, then finally the total probability of fracture of Bordet on all the urgent until that requested. The computation
carried out is written with final as follows: with

$$\sigma_{Bordet}=\left(\sum_{o=1}^{n_{inst}}\sum_{el=1}^{n_{elem}}\sum_{pg=1}^{n_{pg}}\left(\begin{array}{c}\frac{\sigma_{ys}\left(T(o),\dot{\bar{\varepsilon}}_{p,el,pg}(o)\right)}{\sigma_{ys,0}}\\ Max\left(\bar{\sigma}^m_{1,el,pg}(o)-\sigma^m_{th},0\right)\\ \exp\left(\frac{-\sigma_{ys}\left(T(o),\dot{\bar{\varepsilon}}_{p,el,pg}(o)\right)}{\sigma_{ys,0}}\frac{\bar{\varepsilon}_{p,el,pg}(o)}{\varepsilon_{p,0}}\right)\\ \Delta\varepsilon_{p,el,pg}(o)\frac{V_{el,pg}}{V_0}\end{array}\right)\right)^{1/m}$$

n_{inst} the number of times over which is calculated, n_{elem} the number of elements contained in the mesh group requested by the user and n_{pg} the number of points of gauss of each one of these elements; moreover, for any scalar a , $\bar{a}(o)=\frac{a(o)+a(o-1)}{2}$ and $\dot{a}(o)=\frac{a(o)-a(o-1)}{t(o)-t(o-1)}$. $\Delta a(o)=a(o)-a(o-1)$

Result is an array containing the global values of the stress of Bordet and the probability of fracture of Bordet. So that

computation is correct, it is necessary that the user informs key word COEF_MULT as recommended in the user's documentation of POST _BORDET. The model

4 of Rice and Tracey One

is interested now in the case of ductile starting. By considering an initially healthy volume element, ductile this element tears it results from the following elementary mechanisms: nucleation

of cavities caused by the decoherence of inclusions present in the material, growth then coalescence of these cavities. Cavity

4.1 isolated in an infinite plastic rigid matrix In

a approach from comprehension analytical, Rice and Tracey [bib3] studied the behavior of a cavity, initially spherical (surface), S_v isolated in an infinite isotropic medium (volume V), of behavior of plastic rigid Von Mises (elastic limit), σ_0 incompressible, ad infinitum subjected at an unspecified strainrate $\dot{\varepsilon}^{\infty}$ (forced noted with σ^{∞} L `infinite). They

show that the velocity field of displacement, solution of the posed mechanical problem, minimizes the functional calculus: Approximate

$$Q(\dot{u})=\int_V\left[s_{ij}(\dot{\varepsilon})-s_{ij^{\infty}}\right]\dot{\varepsilon}_{ij}dV-\sigma^{\infty}_{ij}\int_{S_v}n_i\dot{u}_j dS$$

4.2 model of the growth of the cavities While

managing to minimize this functional calculus in various situations, Rice and Tracey then showed the dominating influence of the rate of triaxiality ($\frac{\sigma^{\infty}_m}{\sigma^{\infty}_{eq}}$ with: $\sigma^{\infty}_m,\sigma^{\infty}_{eq}$ trace and equivalent of von

Mises of the stress imposed on the volume element considered) on the growth rate of the cavities. They

display even a model of growth of the cavities, certainly approached, but very near to the results of the preceding model. Thus, in each principal direction associated (K) at the strainrate, $\dot{e}^{\infty}$ the rate of elongation of a cavity rises with: (

$$\dot{R}_K = \left\{ c\,\dot{\varepsilon}_K^{\infty} + \dot{\varepsilon}_{eq}^{\infty} D \right\} R_K$$

: R_K radius of the cavity in the direction (K) $\dot{\varepsilon}_K^{\infty}$: $\dot{\varepsilon}_{eq}^{\infty}$ principal value in the direction (K) and equivalent of von Mises of the strainrate imposed ad infinitum), relation in which the coefficients and χ depend D on the situation considered: for

$\chi = \frac{5}{3}$ a linear matrix of hardening or a perfectly plastic matrix withlow level of triaxiality or

in the case of $\chi = 2$ a perfectly plastic matrix withstrong rate of triaxiality, for

$D = \alpha \exp\left(\frac{3\sigma_m^{\infty}}{2\sigma_0}\right)$ a perfectly plastic matrix or $D = \frac{\sigma_m^{\infty}}{4\sigma_{eq}^{\infty}}$ a matrix of linear hardening. is

$\alpha = 0,283$ the value given by Rice and Tracey whereas more precise computations (cf [bib4]) showed that this coefficient is higher (). $\alpha = 1,28$ Mudry

[bib6] then proposed to apply these theoretical results to the case of the steel of tank, i.e. *: intermediate*

behavior enters the extreme cases of behavior studied by Rice and Tracey with a non-zero but reasonable hardening, fissured
structures (high rate of triaxiality). He

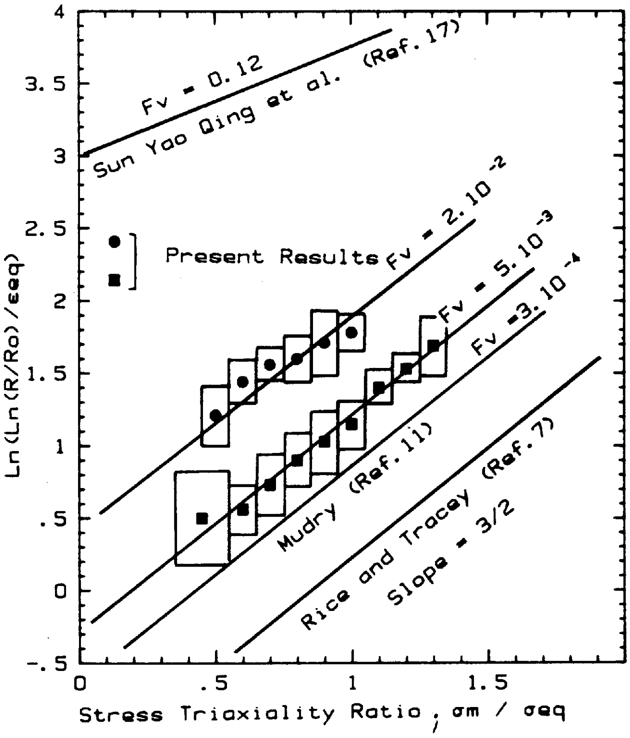
deduced from it the approximate model following, valid for sufficiently high rates of triaxiality (superiors with 0,5): , statement

$$\dot{R} = \alpha\,\dot{\varepsilon}_{eq}^{p^{\infty}} \exp\left(\frac{3\sigma_m^{\infty}}{2\sigma_{eq}^{\infty}}\right) R$$

in which:

$\dot{\varepsilon}_{eq}^{\infty}$ was substituted by (equivalent $\dot{\varepsilon}_{eq}^{p^{\infty}}$ (von Mises) of the plastic part the strainrate) in order to extending the model of Rice and Tracey to the elastoplastic case, the elastic limit σ_0 was substituted by in order to σ_{eq}^{∞} taking account of the hardening of the matrix around the cavity. Experimental

measurements of growth of porosity for various rates of triaxiality made it possible to validate this statement (cf Appears following). These results show that, when the initial proportion of air voids weak rest, the exponential character of the relation between the radius of the cavities and the rate of triaxiality is well confirmed. On the other hand, the coefficient depends α on the material considered thus that initial fraction of porosity. Experimental



results of measurement of the growth of cavities in various metallic materials (figure extracted the ref. 6, indicating εeq the equivalent of the plastic strain noted in ε_{eq}^{p} the body text) according to the rate of triaxiality ductile $\left(\frac{\sigma_m}{\sigma_{eq}}\right)$

4.3 Criterion of starting and indicating

R_0 $R(t)$ the initial radius of the cavities and at time considered t , the ductile criterion of starting adopted here is: , statement

$$\frac{R(t)}{R_0}=\left(\frac{R}{R_0}\right)_c$$

in which the first member results from the integration of the model of growth, in accordance with the indications of the preceding paragraph. One can

object several arguments of principle against the direct use of this model of growth of the cavities of Rice and Tracey like ductile criterion of starting. As follows: inclusions

, and thus the cavities, are not actually insulated. Worse, they are often gathered in cluster, the coalescence
of the cavities undoubtedly results from interactions which are also not described in the model not established, in
a fissured structure, the presence of gradients in crack tip makes less directly applicable the preceding analysis which relates to an infinite medium subjected to homogeneous boundary conditions. Nevertheless

, by means of the preceding criterion, one hopes that this model remains realistic, on average, even in clusters or zones of strong gradients (average on an element of size comparable to that of the model of Beremin). In addition, one makes the assumption that the critical size retained, in general readjusted on geometries given (test-tube CT, for example), represented coalescence , which amounts supposing that coalescence does not depend too much on the nature of the mechanical requests imposed on the volume element (triaxiality, shears,…). Let us notice

to finish that the model of Rice and Tracey is only one approached model, valid for important rates of triaxiality (i.e higher than 0,5). Establishment

4.4 in Code_Aster *Let us consider*

a field of the studied Ω_c structure which can be the group of the studied mesh, a group of mesh or a mesh. Following an elastoplastic thermomechanical computation, one knows the evolution of the stress fields, of strain and plastic strain in this field and one wishes to determine the spatial and temporal variations growth rate of the cavities in this field. For this

making, one uses key word RICE_TRACEY `POST_ELEM` of the command . `In each`

Gauss point of the field, one compares Ω_c at every moment calculated stresses and strainrates to the quantities applied to the infinite medium considered previously. The model of growth of Rice and Tracey is thus integrated step by step using the following approximate formula: One thus

$$Log\left(\frac{R(t_n)}{R_0}\right)=Log\left(\frac{R(t_{n-1})}{R_0}\right)+0,283\,signe\left(\frac{\sigma_m(t_n)}{\sigma_{eq}(t_n)}\right)Exp\left(1,5.|\frac{\sigma_m(t_n)}{\sigma_{eq}(t_n)}|\right)\left(\varepsilon_{eq}^p(t_n)-\varepsilon_{eq}^p(t_{n-1})\right)$$

at every moment obtains the values of the ratio in each $\frac{R}{R_0}$ Gauss point of the field, the sign Ω_c of the rate of triaxiality allowing the taking into account of evolutions as well in tension as in compression. Two features are then offered in Code_Aster *: Search*

4.4.1 of the maximum value of growth rate At every moment

, one seeks on the group of the field the Gauss point Ω_c (and the volume of the associated under-mesh) maximizing. Computation $\frac{R}{R_0}$ of

4.4.2 the mean value of growth rate By squaring

on each mesh then moyennation on the field concerned, one Ω_c at every moment deduces the mean value from on Like $\frac{R}{R_0}$ in the case of the model Ω_c

of Weibull, an alternative is introduced: preceding temporal integration is then carried out from the stress and of the average plastic strain by mesh. Bibliography

5 F. BEREMIN

1) : "A local criterion for cleavage fracture of has nuclear presses vessel steel", Metall. Trans. 14 A, pp 2277-2287, 1981. R. MASSON

2) and al.: "Definition of the local approach (into brittle) in the case of an unspecified way of loading", EDF HT-26/00/021 Notes. J.R. RICE

3) and D.M. TRACEY: "One the ductile enlargement of voids in triaxial stress fields", J. Mech. Phys. Solids, vol. 17, pp. 202-217, 1969. Y. HUANG

4) : "Accurate Thermal expansion Disastrous for Spherical Voids in Triaxial Stress Fields", Transactions of the ASME J. a. Mech., vol. 58, n°4, pp 1084-86, 1991. F. MUDRY

5) : "Study of the Ductility fracture and the Cleavage fracture of Low alloy steels", Thesis of State, University of Technology of Compiegne, 1982. B. MARINI

6) , F. MUDRY, A. PINEAU: "Experimental study of cavity growth in ductile fracture", Engineering Fractures Mechanics Flight 22, No 6, pp. 989-996, 1985. W. LEFEVRE

7) , R. MASSON, G. BARBER: "Computational simulations using the model improved of Beremin of tests of preloading hot carried out with steel 18MND5". Note EDF HT-26/00/028. S.R BORDET

8) , A.D. KARSTENSEN. D.M. KNOWLES, C.S. WIESNER: "A new statistical local criterion for cleavage fracture in steel. Leaves I: model presentation", Eng. Tailcoat. Mech., vol. 72, pp.435-452, 2005. S.R BORDET

9) , A.D. KARSTENSEN. D.M. KNOWLES, C.S. WIESNER: "A new statistical local criterion for cleavage fracture in steel. Leaves II: application to structural offshore oil rig year steel", Eng. Tailcoat. Mech., vol. 72, pp.453-474, 2005. B. TANGUY

10) , C. BOUCHET, S.R. BORDET, J. BESSON, A. PINEAU: "Toward has better understanding of the cleavage in RPV steels: Room mechanical conditions and of evaluating has nucleation enriched Weibull model and of the Beremin model over has broad temperature arranges", 9 HT European Mechanics of Materials Conference, Room approach to fracture, pp 129-134, Moret on Far, 2006. Description

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