Coupling Summarized

élasto-plasticity-damage:

This document presents a method of coupling of an elastoplastic model with a model of damage, which is established in the frame of the KIT_DDI. The method uses an approach of partitioning of the model in three moduli: elastoplasticity, damage and coupler. It is particularly this modularity which represents the principal interest of the method. It makes it possible to use the same modulus “coupler” for couplings between various models of elastoplasticity and D “damage without intervening in L” establishment of the latter. Exactly the same idea is used for the couplings mechanics/creep (see R7.01.19).

Now (version STA9.3 of *the Code_Aster*), the establishment makes it possible to couple only the model GLRC_DM for the damage with the models VMIS_ (ISOT/CINE) _LINE for elastoplasticity and this only for modelizations DKTG. This restricted frame of the use of the method is due exclusively to a lack of definite needs at the present time and will undoubtedly evolve in the next versions.

Contents

1 Introduction

the document is organized in two great parts: the first presents the general method of the couplings élasto-plasticity-damage and the second describes specificities of the various couplings, which currently do not limit to the coupling between GLRC_DM and VMIS_ (ISOT/CINE) _LINE for the "total" models.

In the first part one considers in particular the following aspects: challenges of this kind of couplings, the thermodynamic frame and numerical establishment. In the second part one recalls what are the total models (GLRC_) for the plates and shells out of reinforced concrete and how the general method is declined for this framework.

2 Models of élasto-plasticity-damage in general

2.1 Challenges and difficulties

In the modelization of the behavior of solids it is noted that the nonlinear phenomena most important to take into account during a structure analysis under extreme loading relate to the presence and the evolution of the unelastic strain (plasticity) and the weakening of the stiffness (damage). The relative importance of these two phenomena can drastically vary from one material to another. Thus, generally one considers only plasticity at the time of the modelization of metals while neglecting the damage, while for a concrete material type, the damage is considered to be more important and plasticity is not taken into account. Nevertheless, often this simplification is not justifiable and one is brought to model the two phenomena, plasticity and the damage, at the same time.

Since already a certain time the thermodynamic frame for the modelization of the two phenomena is well drawn up (e.g to see [bib1], [bib2]) and there does not exist any obstacle to extend it to the couplings élasto-plastic-endommageables. As for the numerical resolution of these models in the frame of finite element method (MEF), there exist very robust algorithms in particular for plasticity (e.g to see [bib13]) and also for the damage (e.g to see [bib14]) provided that the phenomenon of localization does not become dominating (see [bib15]). On the other hand, the numerical resolution of a coupled model is much less obvious. The numerical

difficulties concerning the algorithm of resolution of a coupled model intervene on two aspects primarily: satisfactions

- of the conditions (of Kuhn-Tucker) of the thresholds of plasticity and damage, construction
- of the tangent matrix. The first

difficulty is due to the fact that the plastic local variables and endommageables act very differently on the level of the stresses, which makes the system of the equations difficult to solve. The second point is a simple consequence of the increased complexity of the model coupled compared to the models with only one phenomenon. Lately

, one proposed in [bib3] and [bib4] an algorithm of resolution of a model élasto-plastic-endommageable based on strict partitioning in a plastic part and with a endommageable part. It is a question of introducing an additional iterative process in order to satisfy plastic admissibility and admissibility endommageable simultaneously. This iterative process is controlled by what is called the strain of damage, a variable which, contrary to the plastic strain, is not a variable of state, but rather represents a connection between various models. Thus, in the model introduced into [bib3] the choice of the elastoplastic model being free, one was nevertheless to modify the formulation of damage, which in fact envisaged only the modelization of nonlenitive behaviors. In [bib4] one showed the interest to integrate this kind of models in the frame of mixed finite elements. In this

document one generalizes the approach presented in [bib3] being able to practically couple any elastoplastic model with any model of damage. The advantage of such a platform actually goes beyond the coupling élasto-plastic-endommageable. In Code_Aster *one already* has the mechanical

coupling/creep (see [bib16]) . On the other hand a coupling élasto-plasticity-damage-creep was not tried yet (situation of the STA9.3). Tally thermodynamic

2.2 In this

part one points out the usual approach to build models of damage and elastoplasticity, based on the thermodynamics. The combination of the two types of model can be done in a rather natural way, provided that the coupling is made on the level of the computation of the stresses. All the approach presented here is carried out under the assumption that the local variables of damage and plasticity are not coupled directly between them. Plasticity

2.2.1 the free energy

can be written as the sum of elastic strain energy and energy known as stored, which had with plasticity, eq. 2.2

$$\Psi^{ep}\left(\varepsilon , \varepsilon^{p} \xi^{p}\right)=\Psi^{e}\left(\varepsilon-\varepsilon^{p}\right)+\Xi^{p}\left(\xi^{p}\right) \quad \textbf{.1-1}$$

where is

the total ε deflection, the plastic strain ε^{p} and the vector ξ^{p} of all the other local variables. The essential assumption that one makes in eq. 2.2.1 **- 1 is that** elastic strain energy depends only on the elastic strain, where $\Psi^{e}=\Psi^{e}\left(\varepsilon^{e}\right)$. In $\varepsilon^{e}=\varepsilon-\varepsilon^{p}$

the second phase one defines the rate of plastic dissipation and one secures that it remains always positive, eq. 2.2.1

$$0 \leq D^{p}=\sigma \dot{\varepsilon}-\dot{\Psi}^{ep}=\left(\sigma-\frac{\partial \Psi^{ep}}{\partial \varepsilon^{e}}\right) \dot{\varepsilon}^{e}+\sigma \dot{\varepsilon}^{p}-\frac{\partial \Xi^{p}}{\partial \xi^{p}} \dot{\xi}^{p} \quad \textbf{- 2 First of all}$$

, it is supposed that, when $\dot{D}^{p}=0$ the plastic variables do not evolve, which $\dot{\varepsilon}^{p}=\dot{\xi}^{p}=0$ allows us to define the stress as, that is to say if

$$\sigma=\frac{\partial \Psi^{e}}{\partial \varepsilon^{e}}$$

elasticity $\sigma=C^{e}\left(\varepsilon-\varepsilon^{p}\right)$ is linear, with , where is $\Psi^{e}=\frac{1}{2}\left(\varepsilon-\varepsilon^{p}\right) C^{e}\left(\varepsilon-\varepsilon^{p}\right)$ the elastic C^{e} tensor. Another

essential ingredient of an elastoplastic model is the function of the flow threshold, which is $\Phi^{p}=\Phi^{p}\left(\sigma , q^{p}\right)$ a function of the stress and dual variables with the local variables, therefore of and of. σ Any $q^{p}=-\frac{\partial \Xi^{p}}{\partial \xi^{p}}$ solution

must satisfy the plastic conditions of *admissibility, which are* the following ones, and.

$$\Phi^{p} \leq 0 \quad \text{So that} \quad \dot{\xi}^{p} \geq 0 \quad \Phi^{p} \dot{\xi}^{p}=0$$

our model is established it remains to define the laws of evolution of the local variables. Generally, one resorts to the principle of maximum plastic dissipation, but this is not essential in our approach. One will make use of a more general writing, which becomes

$\dot{\xi}^{p}=\dot{\xi}^{p}(\sigma , q^{p})$, with $\dot{\xi}^{p}=\dot{\gamma}^{p} \frac{\partial \Phi^{p}}{\partial q^{p}}$, the plastic $\dot{\gamma}^{p}$ multiplier, when the principle of maximum dissipation is applied. Damage

2.2.2 For

the models of damage it is more difficult to have a construction as general as for that of plasticity. Consequently one will treat a case simple but rather representative of the desired characteristics of the model. Thus, one defines the free energy like, eq. 2.2

$$\Psi^d = \Psi^e(\varepsilon)\zeta(d) \quad .2\text{-}1$$ where is

the function ζ of damage and D the variable of damage, which is a scalar but which can as well be a tensor. In the same way, the function can in general ζ depend on, in particular ε on the sign of its trace to distinguish from behaviors typically very different in tension and compression. Resorting to eq. 2.2.2-1 one defines the dissipation of damage like. While applying

$$0 \le D^d = \sigma\,\dot{\varepsilon} - \dot{\Psi}^d = \left(\sigma - \frac{\partial \Psi^d}{\partial \varepsilon}\right)\dot{\varepsilon} - \frac{\partial \Psi^d}{\partial d}\dot{d}$$

for, $\dot{D}^d = 0$ one obtains $\dot{d} = 0$ the stress like, where

$$\sigma = \frac{\partial \Psi^d}{\partial \varepsilon} = \frac{\partial \Psi^e}{\partial \varepsilon}\zeta(d) = C(d)\varepsilon$$

$C(d)$ the elasticity tensor evolving according to the damage represents. By making

a linear assumption of elasticity and isotropic damage, by taking the stress $\zeta = 1 - d$ becomes. As

$$\sigma = (1-d)C^e\varepsilon$$ for

plasticity one introduces a threshold of cracking, where Q $\Phi^d = \Phi^d(q^d)$ D is *the dual* variable with D. This time, the threshold is a direct function of the total deflection, and one ε seeks to satisfy the conditions with admissibility of damage, and.

$$\Phi^d \le 0$$ One $\dot{d} \ge 0$ must $\Phi^d \dot{d} = 0$

also formulate the law of evolution of D. *Coupling* $\dot{d} = \dot{d}(q^d)$

2.2.3 Lastly, one

combines the formulations of damage and plasticity to arrive at a coupled model. The free energy of the coupled model is thus written like. In

$$\Psi^{pd}(\varepsilon, \varepsilon^p, \xi^d) = \Psi^e(\varepsilon - \varepsilon^p)\zeta(d) + \Xi^p(\xi^p)$$

the continuation one connects the stages applied to the sub-chapters §2.2.1 and §2.2.2: éfinition **of dissipation. Computation of the stresses:**

$$0 \le \dot{D}^{pd} = \dot{D}^d + \dot{D}^p = \left(\sigma - \frac{\partial \Psi^{pd}}{\partial \varepsilon^e}\right)\dot{\varepsilon}^e + \sigma\,\dot{\varepsilon}^p - \frac{\partial \Xi^p}{\partial \xi^p}\dot{\xi}^p - \frac{\partial \Psi^d}{\partial d}\dot{d}$$

, in general

$\sigma = \dfrac{\partial \Psi^{pd}}{\partial \varepsilon^e} = \dfrac{\partial \Psi^e}{\partial \varepsilon^e}\zeta(d) = C(d)(\varepsilon - \varepsilon^p)$ and, if

$\sigma = (1-d)C^e(\varepsilon - \varepsilon^p)$ elasticity is linear and the isotropic damage. Conditions

of admissibilities of damage and plasticity: , and.

$$\Phi^p \le 0 \ , \ \dot{\xi}^p \ge 0 \text{ and } \Phi^p \dot{\xi}^p = 0 \ .$$

$\Phi^d \leq 0$ Laws $\dot{d} \geq 0$ of $\Phi^d \dot{d} = 0$ evolution

: and. Numerical integration

$$\dot{\xi}^p = \dot{\xi}^p(\sigma, q^p) \quad \dot{d} = \dot{d}(q^d)$$

2.3 Recall of

2.3.1 the classical resolution of the problems nonlinear by EFs In

the method suggested one adds an additional iterative process compared to a classical computation by the MEF, qu" one recalls in this chapter. In a total resolution of a structure problem one seeks the state of equilibrium, defined by: , eq. 2.3

$$f^{\text{int}}(u(t)) = f^{ext}(t) \quad .1\text{-}1$$ where is

the vector f^{int} of the internal forces and the vector f^{ext} of the external forces (the loading). The first is a function of the vector of displacements, the basic variable of the system, and the second a function of the parameter T (pseudonym - time), defining the notion of trajectory as well for the loading as for the evolution of the field of displacements. In the frame of the MEF the internal force is determined by: , eq. 2.3

$$f^{\text{int}}(u) = \int_{\Omega} B^T \sigma(\varepsilon) d\Omega \quad .1\text{-}2$$ where defines

Ω the field considered, the tensor ε of the strain and the tensor σ of the stress. By making the assumption of the small strains, the tensor is defined ε like: , which

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$ becomes

, eq. 2.3

$$\varepsilon_{ij} = B_{ijk} u_k \quad .1\text{-}3$$ after

having discretized the field of displacements by the MEF. In addition, the matrix B in [*eq* . 2.3.1-3] is the same one as in [eq. 2.3.1-2]. When the behavior of the material is nonlinear one generally resorts to the method of Newton to solve [eq. 2.3.1-1], where one is brought to linearize [eq. 2.3.1-1], which gives us: , where

$$f^{\text{int},(k+1)} = f^{\text{int},(k)} + \frac{\partial f^{\text{int},(k)}}{\partial u^{(k)}} \Delta u^{(k)}$$

the index (K) represents the number of the iteration in progress. With convergence one must have and. In

order to $f^{\text{int},(k)} \overset{k\to\infty}{\to} f^{ext}(t)$ $\Delta u^{(k)} \overset{k\to\infty}{\to} 0$

be able to calculate the correction of displacements from one iteration to another, one must $\Delta u^{(k)}$ determine, where is

$$K = \frac{\partial f^{\text{int},(k)}}{\partial u^{(k)}} = \int_{\Omega} B^T C^{epd} B d\Omega$$

the coherent C^{epd} tangent modulus for the constitutive law used: . The method

$$C^{epd} = \frac{\partial \sigma(\varepsilon)}{\partial \varepsilon}$$

detailed below positions on the level of a material point (even at the Gauss point) and intervenes on the level of computation of and from σ C^{epd} the strain, given ε for each call of the total process of the iterations of Newton. With

2.3.2 the problem coupled In this document

one approaches does not detail numerical integration in time for each model. It is supposed quite simply that one can do it and this with an implicit scheme of Eulerian (back). Let us say, that in an incremental resolution by finite element method (MEF), for each iteration of Newton of the total iterative process, one wants to calculate, for each Gauss point, the stresses and the local variables σ_{n+1} , and from ε_{n+1}^p ξ_{n+1}^p d_{n+1} the values of the total deflection, while ε_{n+1} supposing that one converged at previous time and that one knows, and σ_n . ε_n^p ξ_n^p Indices d_n N or n+ *1* indicate that the value of the variable corresponds at the moment T N or *T* $_{n+}$ 1, *respectively* . Thus one

supposes that the two models can be integrated in decoupled mode and that one is thus able to write, in

$\sigma_{n+1}^p = \sigma^p(\tilde{\varepsilon}_{n+1})$ the modulus of plasticity and

$\sigma_{n+1}^d = \sigma^d(\tilde{\varepsilon}_{n+1})$ the modulus of damage. The exhibitors

D or p *in and* mean σ^d σ^p that the stress is calculated with the model damage and the model of plasticity, respectively. is $\tilde{\varepsilon}$ a variable of the standard strain. Into the algorithm

presented in the continuation one introduces an additional iterative process so that the stresses and converge σ^d σ^p towards only one value. He similar to that is proposed in [bib3] except that here the construction of the model of damage is more general, which require to consider another variable that the strain of damage (used in [bib3]) on which one reiterates. Construction

2.3.3 of the nonlinear system Compared to

a classical resolution of the constitutive law, one must make converge here two values of stresses by introducing another unknown of the standard strain. This system partitionné is built more easily while introducing decomposition following of deflection total, where one

$$\varepsilon = \varepsilon^e + \varepsilon^p + \varepsilon^d$$

introduced the strain of damage, which enables us to write the relation of elasticity like, eq. 2.3

$$\sigma = C^e\left(\varepsilon - \varepsilon^p - \varepsilon^d\right) \quad .3 \qquad \text{- 1 or}$$

. eq. 2.3

$$\varepsilon^d\left(\varepsilon,\sigma,\varepsilon^p\right) = \varepsilon - \varepsilon^p - C_e^{-1}\sigma \quad .3 \qquad \text{- 2 One can}$$

summarize the equations to be solved in three groups: Modulate plasticity

the notation

$$\sigma_{n+1}^p = \sigma^p\left(\varepsilon_{n+1} - \varepsilon_{n+1}^d\right)_e$$

meaning $;C_e$ that the elastoplastic model is calculated with the operator of elasticity eq. 2.3.3

C_e

$$\varepsilon_{n+1}^p = \varepsilon^p\left(\varepsilon_{n+1} - \varepsilon_{n+1}^d\right) \qquad \text{- 3 Modulus damage}$$

eq. 2.3.3

$$\sigma_{n+1}^d = \sigma^d\left(\varepsilon_{n+1} - \varepsilon_{n+1}^p\right) \qquad \text{- 4 Modulus coupling}$$

eq. 2.3.3

$$\varepsilon_{n+1}^d = \varepsilon_{n+1} - \varepsilon_{n+1}^p - C_e^{-1}\sigma_{n+1}^d$$

$$\sigma_{n+1}^{p}=\sigma_{n+1}^{d}$$ - 5 In [feeding-bottle

3] one chose like ε_{n+1}^{d} the additional unknown, which is not possible in our case, since is not σ^{d} a function otherwise of ε^{d} , the variable $\bar{\varepsilon}^{ed}=\varepsilon-\varepsilon^{p}$ which is finally used to control the iterative processes described below. In addition
it is worth too. Algorithm $\bar{\varepsilon}^{ed}=\varepsilon^{e}+\varepsilon^{d}=\varepsilon-\varepsilon^{p}$

2.3.4 of resolution to solve

the system nonlinear, defined in the equations éq. 2.3.3-3, éq. 2.3 .3-4 and éq. 2.3.3-5 by allotting them to the various moduli (plasticity, damage and coupling), one applies the classical method of Newton. In order to linearize the system, one must define following derivatives: , eq. 2.3

$$C^{ep}(\tilde{\varepsilon})=\frac{\partial \sigma_{n+1}^{p}}{\partial \tilde{\varepsilon}_{n+1}}$$

$$C^{d}(\tilde{\varepsilon})=\frac{\partial \sigma_{n+1}^{d}}{\partial \tilde{\varepsilon}_{n+1}}$$

$$\frac{\partial \varepsilon_{n+1}^{d}}{\partial \bar{\varepsilon}_{n+1}^{ed}}=I-C_{e}^{-1}C^{d}(\bar{\varepsilon}_{n+1}^{ed})$$ .4 - 1. The nonlinear

system (of éq. 2.3.3-3 with éq. 2.3.3-5) is first of all brought back to the last of the equations, i.e. with, since $\sigma_{n+1}^{p}=\sigma_{n+1}^{d}$ one can express all the unknowns compared to the value of. One can $\bar{\varepsilon}_{n+1}^{ed}$ thus define the residue like, eq. 2.3

$$R_{\sigma}^{(k)}=\sigma^{p}\left(\varepsilon_{n+1}-\varepsilon_{n+1}^{d,(k)}\right)-\sigma^{d}\left(\bar{\varepsilon}_{n+1}^{ed,(k)}\right)$$ .4 - 2 where the exhibitor

(K) indicates the current iteration and where. By applying

$$\varepsilon_{n+1}^{d,(k)}=\varepsilon_{n+1}-\varepsilon_{n+1}^{p,(k)}-C_{e}^{-1}\sigma^{d}(\bar{\varepsilon}_{n+1}^{ed,(k)})$$

the method of Newton, one calculates the new approximation of like, $\bar{\varepsilon}_{n+1}^{ed,(k+1)}$ where.

$$\bar{\varepsilon}_{n+1}^{ed,(k+1)}=\bar{\varepsilon}_{n+1}^{ed,(k)}+\Delta\bar{\varepsilon}_{n+1}^{ed,(k)}$$

The tangent

$$\Delta\bar{\varepsilon}_{n+1}^{ed,(k)}=-\left(\frac{\partial R_{\sigma}^{(k)}}{\partial \bar{\varepsilon}_{n+1}^{ed}}\right)^{-1}R_{\sigma}^{(k)}$$

modulus corresponding can be entirely expressed by the uncoupled moduli, eq. 2.3.4-1. It is pointed out

$$\frac{\partial R_{\sigma}^{(k)}}{\partial \bar{\varepsilon}_{n+1}^{ed}}=-C^{ep,(k)}\frac{\partial \varepsilon_{n+1}^{d,(k)}}{\partial \bar{\varepsilon}_{n+1}^{ed,(k)}}-C^{d,(k)}=-\left(C^{d,(k)}+C^{ep,(k)}\left(I-C_{e}^{-1}C^{d,(k)}\right)\right)$$

here that the total deflection, resulting ε_{n+1} *from* the total process of the iterations of Newton, is constant for the local iterative process determining. L" algorithm $\bar{\varepsilon}_{n+1}^{ed}$ of the computation of the variables to the iteration (k+1) is carried out in the following way: debut of

$$\bar{\varepsilon}_{n+1}^{ed,(k=0)}=\varepsilon_{n+1}-\varepsilon_{n}^{p}$$

1) the loop, where where $k=0\ldots N_{max}$

$\bar{\varepsilon}_{n+1}^{ed,(k+1)}=\bar{\varepsilon}_{n+1}^{ed,(k)}+\Delta\{\bar{\varepsilon}¿_{n+1}^{ed,(k)}$ If $\Delta\bar{\varepsilon}_{n+1}^{ed,(k)}=\left(C^{d,(k)}+C^{ep,(k)}-C^{ep,(k)}C_e^{-1}C^{d,(k)}\right)^{-1}R_\sigma^{(k)}$

2) $\sigma_{n+1}^{d,(k+1)}=\sigma^d\left(\bar{\varepsilon}_{n+1}^{ed,(k+1)}\right)$ $;C_e$

3) $\varepsilon_{n+1}^{d,(k+1)}=\bar{\varepsilon}_{n+1}^{ed,(k+1)}-\varepsilon_{n+1}^{el,(k+1)}$ $\varepsilon_{n+1}^{el,(k+1)}=C_e^{-1}\sigma_{n+1}^{d,(k+1)}$

4) $\sigma_{n+1}^{p,(k+1)}=\sigma^p\left(\varepsilon_{n+1}-\varepsilon_{n+1}^{d,(k+1)}\right)$ $;C_e$

5) $R_\sigma^{(k+1)}=\sigma_{n+1}^{p,(k+1)}-\sigma_{n+1}^{d,(k+1)}$

6) $|\Delta\bar{\varepsilon}_{n+1}^{ed,(k+1)}\cdot R_\sigma^{(k+1)}|>tol.$ the loop begins again with 1), if not FIN Notices

1: the only *places where physical specificities of the coupled models are taken into account are the points 2) and 4), where one calls on the moduli of plasticity and damage, respectively. Notice*

2: the value *of is considered* ε_{n+1} *constant in the process of balancing of the stresses and during the variation of one varies* $\bar{\varepsilon}_{n+1}^{ed}$ *actually the value of. This* ε_{n+1}^{p} *plastic strain is in theory given by the modulus of elastoplasticity like a function (see [eq* ε_{n+1} *. 2.3.3-3]). Actually, is regarded* $\bar{\varepsilon}_{n+1}^{ed}$ *here as a variable and its restrain with* ε_{n+1}^{p} *is restored only with the convergence of the iterative process. As follows: but. Construction*

$\bar{\varepsilon}_{n+1}^{ed,(k)}\neq\varepsilon_{n+1}-\varepsilon^p\left(\varepsilon_{n+1}-\varepsilon_{n+1}^{d,(k)}\right)$ $\bar{\varepsilon}_{n+1}^{ed,(k)}\overset{k\to\infty}{\to}\varepsilon_{n+1}-\varepsilon^p\left(\varepsilon_{n+1}-\varepsilon_{n+1}^{d,(k)}\right)$

2.3.5 of the coherent tangent modulus Lastly, in order to

obtain an optimal performance in the total iterative process one must calculate the coherent tangent modulus, once the process describes above converged, i.e.. Initially $\sigma_{n+1}=\sigma_{n+1}^p=\sigma_{n+1}^d$

$$C_{n+1}^{epd}=\frac{\partial\sigma_{n+1}}{\partial\varepsilon_{n+1}}$$ it

is observed that if one derives [eq. 2.3.3-3] compared to one can write σ_{n+1}^p , and that

$$I=C_{n+1}^{ep}\left(C_{n+1}^{epd^{-1}}-\frac{\partial\varepsilon_{n+1}^d}{\partial\sigma_{n+1}^p}\right)$$

same way, by deriving [eq. 2.3.3-4] compared to one obtains, σ_{n+1}^d with, in the end

$$I=C_{n+1}^{d}\left(C_{n+1}^{epd^{-1}}-\frac{\partial\varepsilon_{n+1}^p}{\partial\sigma_{n+1}^d}\right)$$

of the iterative process, one obtains $\sigma_{n+1}^p=\sigma_{n+1}^d$ finally: and By means of

$$\frac{\partial\varepsilon_{n+1}^d}{\partial\sigma_{n+1}}=C_{n+1}^{epd^{-1}}-C_{n+1}^{ep^{-1}}$$ $$\frac{\partial\varepsilon_{n+1}^p}{\partial\sigma_{n+1}}=C_{n+1}^{epd^{-1}}-C_{n+1}^{d^{-1}}$$

then and thus, $\varepsilon_{n+1}=\varepsilon^{e}_{n+1}+\varepsilon^{p}_{n+1}+\varepsilon^{d}_{n+1}$ one obtains $\frac{\partial \varepsilon_{n+1}}{\partial \sigma_{n+1}}=\frac{\partial \varepsilon^{e}_{n+1}}{\partial \sigma_{n+1}}+\frac{\partial \varepsilon^{p}_{n+1}}{\partial \sigma_{n+1}}+\frac{\partial \varepsilon^{d}_{n+1}}{\partial \sigma_{n+1}}$

the coherent tangent modulus directly like. Limitations

$$C^{epd}_{n+1}=\left(C^{ep^{-1}}_{n+1}+C^{d^{-1}}_{n+1}-C^{e^{-1}}_{n+1}\right)^{-1}$$

2.4 of the coupler There exist some

conditions to satisfy for the use of this method of coupling concerning at the same time physical and numerical aspects: Distinct

- **local variables. As specified**
  in the introduction of the thermodynamic frame, the two coupled models should not share the same local variables. The method is conceived to be able to combine models representing of the physical phenomena uncoupled, which is judged to be the case in most target applications. Existence of
- **the coherent tangent moduli. So that**
  the approach functions, it is necessary absolutely that one has the coherent tangent moduli for the two moduli, elastoplastic and endommageable. Under “coherent tangent modulus” one implies that it is the coherent derivative with the temporal discretization employed of the stress tensor at the time given compared to the strain tensor at the same time. NON-singularity
- **of the tangent moduli. One is obliged**
  several time to reverse the tangent matrixes of individual moduli. If the behavior of a modulus has a nonlinear slope punt (perfect elastoplasticity for example) it is necessary to give him a value non-nulle. This value non-nulle must be selected such that it is sufficiently large so that the algorithm functions and sufficiently small so that it coherent with physical is the model studied. Total Model

3 specific models

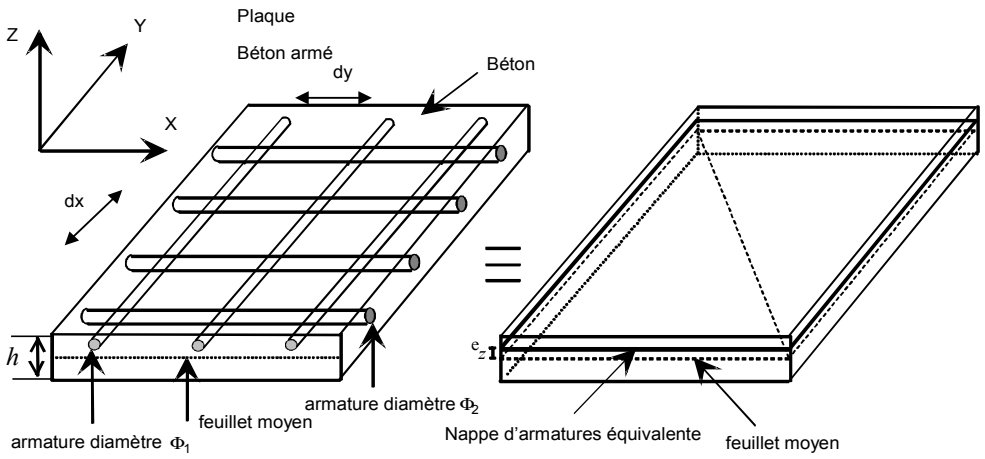
3.1 Recall of

3.1.1 the total models In this context

, a model of behavior of plate known as total or structural element in general, means that the constitutive law is written directly in terms of relation between the generalized stresses and the generalized strains. The comprehensive approach of modelization of the behavior of structures applies in particular to composite structures, for example the reinforced concrete (see Figure 1), and represents an alternative to the approaches known as local or semi-total, which are finer but more expensive modelizations (see [bib6] and [bib7]). In the local approach one uses a fine modelization for each phase (steel, concrete) and their interactions (dependancy) and in the semi-GLOBALE approach one exploits the slenderness of structure to simplify the description of the kinematics, which leads to models PMF (Multifibre Beam) or multi-layer shells. The advantage

of the total model lies in the fact that the finite element corresponding requires only one point of integration in the thickness and especially in obtaining a homogenized behavior. This advantage is even more important in the analysis of the reinforced concrete, since one circumvents the problems of localization encountered at the time of the modelization of the concrete without reinforcements. Obviously, a total model represents the local phenomena in a coarse way and requires more validation before its application to the industrial examples. Now

(version STA9.3) Code_Aster *has* two models of this type: GRLC_DAMAGE (see [bib10]) and GRLC_DM (see [bib11]). Figure 3.1



- has. Pave out of reinforced concrete. Establishment

3.1.2 of coupling GLRC_DM/VMIS _the

most important assumption which is made in this particular coupling is that elastoplasticity can be activated only by requesting the diaphragm disc. In other words, in pure bending the structure will never plasticize. On the one hand, this simplification is justified by the target applications, where the requests are more important out of membrane than in bending and where one thus does not expect dominant influences of bending. In addition, the assumption is essential for technical reasons, because currently one does not have a “total” model elastoplastic suitable for the coupling with a “total” model of damage. The total

models are formulated in term of stresses and strains generalized, which imposes to us, in the coupler (see §2), to replace the stresses, and strains σ , by: and ε , where

$$\sigma \to \Sigma = \begin{pmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ M_{xx} \\ M_{yy} \\ M_{xy} \end{pmatrix} \qquad \text{the generalized} \qquad \varepsilon \to E = \begin{pmatrix} e_{xx} \\ e_{yy} \\ e_{xy} \\ \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{pmatrix}$$

strains, and the E break up into membrane extensions (,) curvatures e_{xx} e_{yy} e_{xy} (,), while κ_{xx} κ_{yy} κ_{xy} the generalized stresses are made up of the membrane forces (,) and of the bending moments N_{xx} N_{yy} N_{xy} (,). It is M_{xx} M_{yy} important M_{xy}

to note that in this frame one makes the assumption of thin plates where the transverse distortion and the shears are negligible. Below one and the points out to the kinematics of Hencky-Mindlin (see [bib9] for the details) for the shells plates as well as the definition of the generalized stresses: where is

$$\begin{pmatrix} U_1(x,y,z) \\ U_2(x,y,z) \\ U_3(x,y,z) \end{pmatrix} = \begin{pmatrix} u_1(x,y) \\ u_2(x,y) \\ u_3(x,y) \end{pmatrix} + z \begin{pmatrix} \theta_y(x,y) \\ -\theta_x(x,y) \\ 0 \end{pmatrix}$$

the field $U=(U_1 U_2 U_3)^T$ of displacement in 3D, the displacement $u=(u_1 u_2 u_3)^T$ of the average average and, its rotations θ_x θ_y . Thus, the strain tensor, definite like, is also written

$$\varepsilon_{ij}=\frac{1}{2}\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right) \quad i,j=1..3$$

like, éq 3.1.2

$$\begin{aligned}\varepsilon_{xx}&=e_{xx}+z\kappa_{xx}\\ \varepsilon_{yy}&=e_{yy}+z\kappa_{yy}\\ \varepsilon_{xy}&=e_{xy}+z\kappa_{xy}\\ \varepsilon_{xz}&=\varepsilon_{yz}=0\end{aligned} \quad -1$$

where is the tensor e of the membrane extension, and the tensor

$$e_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right) \quad i,j=1..3$$

κ of curvature, to which

$$\kappa_{xx}=\frac{\partial \theta_y}{\partial x} \qquad \kappa_{yy}=-\frac{\partial \theta_x}{\partial y} \qquad \kappa_{xy}=\frac{1}{2}\left(\frac{\partial \theta_y}{\partial y}-\frac{\partial \theta_x}{\partial x}\right)$$

the assumption of plane stresses is added. Into the theory $\sigma_{zz}=0$ used here, one introduces only two components of rotation and, which θ_x implies θ_y that the tensor of curvature is in 2D and has only 3 independent parameters. Among the two

coupled models, GLRC_DM is a model total and thus formulated directly in term of, but $\Sigma=\Sigma(E)$ models VMIS_ISOT_LINE and VMIS_CINE_LINE are the constitutive laws 3D classical. Considering one wishes especially that the elastoplastic part be represented out of membrane, one applies constitutive law VMIS_ only to the relation, then it is supposed $N=N(e)$ that the behavior in bending remains elastic linear. One thus has: Damage

: Elastoplasticity

$$\Sigma^d(e,\kappa)=\Sigma_{GLRC_{DM}}(e,\kappa)$$

: , where represents

$$\Sigma^p(e,\kappa)=\begin{pmatrix}N_{VMIS}(e)\\ H_{ELAS}\cdot\kappa\end{pmatrix}$$

$\Sigma_{GLRC_{DM}}(e,\kappa)$ total constitutive law GLRC_DM, the model of Von Mises $N_{VMIS}(e)$ in plane stresses and the elastic H_{ELAS} tensor acting on the curvature (see [bib9] for its exact form). These two moduli, of damage and elastoplasticity, are then used in the coupler (see §2) while replacing and. Local variables $\sigma^p\to\Sigma^p$ $\sigma^d\to\Sigma^d$

3.1.3 the local variables

of the coupled model are stored in series: the first correspond to the model of damage, which are followed of that of the elastoplastic model. The coupler needs six (6) local variables to store the tensor of the standard strain, which controls $\bar{\varepsilon}^{ed}$ the internal loop (see §2.3). In the case of coupling GLRC_DM/VMIS _one adds of them four (4) because of the algorithm making it possible to satisfy the condition with the plane stresses. These four local variables have the same meaning as those used for method DEBORST (see [bib12]). For GLRC_

DM/VMIS _there are thus 21 local variables: V1-V4: GLRC_DM ; V5 - V11: VMIS_ (one makes use only of V5 and V6 for VMIS_ISOT_LINE); V12-V17: strain tensor ; V18-V21: $\bar{\varepsilon}^{ed}$ method DEBORST. Validation

3.1.4 This model

is validated by tests SSNS106F, G (see [bib8]). Bibliography

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16 of the coupling creep/plasticity for the concrete. History of the versions

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