Constitutive law of reinforced concrete GLRC_DM plates

Summarized:

This documentation presents the formulation theoretical and the numerical integration of constitutive law GLRC_DM, usable with modelization DKTG. She belongs to the models known as "total" used for thin structures (beams, plates and shells). The nonlinear phenomena, such as plasticity or the damage, are directly in relation to the generalized strains (extension, curvature, distortion) and the generalized stresses (forces of membrane, of bending and cutting-edges). Thus, this constitutive law applies with a finite element of plate or shell. That makes it possible to save, compared to an approach multi-layer, TEMPS CPU as well as memory. The advantage compared to the multi-layer shells is even more important, when one of the components of the plate behaves in a quasi-brittle way (concrete, for example), since the model total allows to avoid the problems of localization.
Constitutive law GLRC_DM models the damage under membrane request and request of bending of reinforced concrete plates, using "homogenized" parameters. This model of behavior thus represents an evolution compared to the model GLRC_DAMAGE which treats the damage only in membrane request. The structure of the model of damage of GLRC_DM resembles that of GLRC_DAMAGE and are both inspired by ENDO_ISOT_BETON. Contrary to GLRC_DAMAGE, GLRC_DM does not make it possible to model a possible plasticization, which returns it adapted less to simulate extreme loadings. Moreover, behavior GLRC_DM is isotropic before damage: one neglects the orthotropy brought by the three-dimensions functions of steel reinforcements.

One can plan to use behavior GLRC_DM "alone" on a modelization of plates to represent the reinforced concrete, or only to represent the concrete only by then associating it with modelizations of grids of steel reinforcements, which makes it possible to represent the orthotropy and possibly it not symmetry of the three-dimensions functions of reinforcements. This last choice simplifies also the parameter setting of this behavior.

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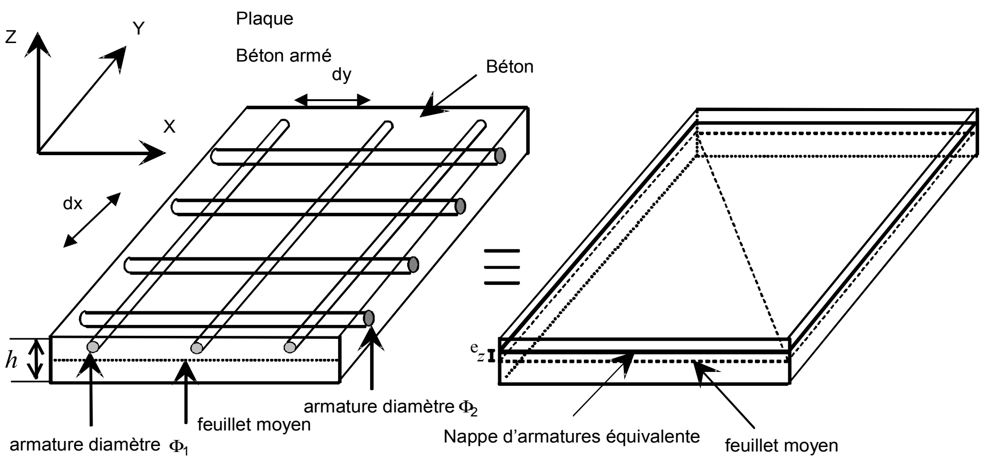
1 Introduction

1.1 total Models

a model of behavior of plate known as total or structural element, in general, means that the constitutive law is written directly in terms of relation between the generalized stresses and the generalized strains. The comprehensive approach of modelization of the behavior of structures applies in particular to composite structures, for example the reinforced concrete (see Figure 1.1-a), and represents an alternative to the approaches known as local or semi-total, which are finer but more expensive modelizations (see [bib5] and [bib6]). In the local approach one uses a fine modelization for each phase (steel, concrete) and their interactions (dependancy) and in the semi-GLOBALE approach one exploits the slenderness of structure to simplify the description of the kinematics, which leads to models PMF (Multifibre Beam) or multi-layer shells.

The advantage of the total model lies in the fact that the finite element corresponding requires only one point of integration in the thickness and especially in obtaining a homogenized behavior. This advantage is even more important in the analysis of the reinforced concrete, since one circumvents the problems of localization encountered at the time of the modelization of the concrete without reinforcements. Obviously, a total model represents the local phenomena in a coarse way and requires more validation before its application to the industrial examples.



Appear 1.1-a : Pave out of reinforced concrete.

1.2 Purposes of model GLRC_DM

The model `GLRC_DM` is able to represent the damage of a reinforced concrete plate, when this one remains restricted, i.e. without reaching the fracture. It was inspired by the model `GLRC_DAMAGE` and is complementary for him. If `GLRC_DAMAGE` can also represent the damage, that is possible only if for the loading in bending without any impact on the behavior out of membrane. On this point `GLRC_DM` is more faithful to physics, but contrary to `GLRC_DAMAGE` it does not make it possible to take into account plasticity. While being simpler, `GLRC_DM` is also more powerful on the level of the cost of computation and the numerical robustness.

One aims at an identical behavior in the directions Ox and Oy ; more three-dimensions functions of higher and lower reinforcement are supposed to be identical.

2 Formulation of the model of behavior

the use of the theory of the plates and thin shells makes it possible to effectively describe the structural mechanics behavior of the reinforced concrete structures, which are generally slender.
In a first stage of the construction of the model, one supposes the existence of a medium homogenized with the same structural mechanics behavior as reinforced concrete structure, in which one is interested. To simplify, the assumption is made that this medium is isotropic and that the studied structural element is symmetric compared to its average average. These assumptions are not essential for the formulation, but were made to simplify the approach. Moreover, it is estimated that their impact on the behavior is less compared to the cracking, which is with area of interest model.

One must note that the use of this model is associated with that of a shell element. It is usable only in the frame of finite elements **DKT** (supported modelization: DKTG), corresponding to the theory of **Coils-Kirchhoff**, where one neglects any transverse distortion in the thickness of the plate.

2.1 Free energy

For an isotropic homogeneous continuum with the linear elastic behavior one can write the voluminal density of the free energy like:

$$\Phi_e(\boldsymbol{\varepsilon})=\frac{\lambda}{2}\mathrm{tr}(\boldsymbol{\varepsilon})^2+\mu\sum_{i=1}^{3}\tilde{\varepsilon}_i^{\,2}$$

where λ,μ are the coefficients of Lamé, $\boldsymbol{\varepsilon}$ the strain tensor and $\tilde{\varepsilon}_i$ its eigenvalues. As for the model ENDO_ISOT_BETON one introduces the damage by a function $\xi(.,d_i)$, d_i being a variable of damage. Therefore, for a endommageable medium, the free energy is written:

$$\Phi_{ed}(\boldsymbol{\varepsilon},d_j)=\frac{\lambda}{2}\mathrm{tr}(\boldsymbol{\varepsilon})^2\xi(\mathrm{tr}(\boldsymbol{\varepsilon}),d_j)+\mu\sum_{i=1}^{3}\tilde{\varepsilon}_i^{\,2}\xi(\tilde{\varepsilon}_i,d_j) \quad (2.1\text{-}1)$$

with the function $(x,d)\in\mathbb{R}^2\rightarrow\xi(x,d)\leq 1$ checking a priori $\frac{\partial\xi}{\partial d}\leq 0$ to represent the loss of stiffness related to the damage, and $\frac{\partial\xi}{\partial x}=0 \quad pour \quad x\in]-\infty,0[\cup]0,+\infty[$, the jump while 0 allowing to represent the discontinuity of behavior between tension and compression.

The equation [eq 2.1-1] is valid for a continuum and one will apply it to a plate of Coils-Kirchhoff $\omega\times\left]\frac{-h}{2};\frac{h}{2}\right[$, of thickness h (one notes $z=x_3$), where one makes kinematical assumptions of Hencky-Mindlin (see [bib8]):

$$\begin{pmatrix}U_1(x_1,x_2,z)\\U_2(x_1,x_2,z)\\U_z(x_1,x_2,z)\end{pmatrix}=\underbrace{\begin{pmatrix}u_1(x_1,x_2)\\u_2(x_1,x_2)\\u_z(x_1,x_2)\end{pmatrix}+z\begin{pmatrix}\theta_2(x_1,x_2)\\-\theta_1(x_1,x_2)\\0\end{pmatrix}}_{\substack{\textit{cinématique de plaque}\\ \mathbf{u}^s\in V_s}}+\underbrace{\begin{pmatrix}u_1^c(x_1,x_2,z)\\u_2^c(x_1,x_2,z)\\u_z^c(x_1,x_2,z)\end{pmatrix}}_{\substack{\textit{déplacement complémentaire}\\ \mathbf{u}^c\in V_c}}$$

where $\mathbf{U}=\left(U_1 \;\; U_2 \;\; U_z\right)^T$ is the field of displacement in 3D, $\mathbf{u}=\left(u_1 \;\; u_2 \;\; u_z\right)^T$ the displacement of the average average and θ_x , θ_y its rotations. Thus, the strain tensor, definite like:

$$\varepsilon_{ij}=\frac{1}{2}\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right),\;\; i,j=1..3$$

is also written like:

$$\begin{aligned}
\varepsilon_{11}&=\underbrace{\epsilon_{11}+z\kappa_{11}}_{\varepsilon^s_{11}}+u^c_{1,1}\\
\varepsilon_{22}&=\underbrace{\epsilon_{22}+z\kappa_{22}}_{\varepsilon^s_{22}}+u^c_{2,2}\\
\varepsilon_{12}&=\underbrace{\epsilon_{12}+z\kappa_{12}}_{\varepsilon^s_{12}}+\frac{1}{2}\left(u^c_{2,1}+u^c_{1,2}\right)\\
\varepsilon_{1z}&=\varepsilon^c_{1z}=\frac{1}{2}u^c_{3,1}\\
\varepsilon_{2z}&=\varepsilon^c_{2z}=\frac{1}{2}u^c_{3,2}\\
\varepsilon_{zz}&=\varepsilon^c_{zz}=u^c_{3,z}
\end{aligned} \qquad (2.1\text{-}2)$$

where $\boldsymbol{\epsilon}$ is the tensor of the membrane extension, defined in the plane:

$$\epsilon_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right),\;\; i,j=1..2$$

and $\boldsymbol{\kappa}$ the tensor of variation of curvature, defined in the plane:

$$\kappa_{11}=\frac{\partial\theta_2}{\partial x_1}\quad \kappa_{22}=\frac{-\partial\theta_1}{\partial x_2},\;\; \kappa_{12}=\frac{1}{2}\left(\frac{\partial\theta_2}{\partial x_2}-\frac{\partial\theta_1}{\partial x_1}\right)$$

relations to which the assumption of plane stresses is added $\sigma_{zz}=0$ $\sigma_{1z}=0$, $\sigma_{2z}=0$ which will determine the complementary field of displacement $\mathbf{u}^c\in V_c$. Into the theory used here, one introduces only two components of rotation θ_x and θ_y , which implies that the tensor of variation of curvature is 2D and has only 3 independent components.

By introducing these kinematical assumptions, cf [éq. 2.1-2], in the statement of the free energy, [éq. 2.1-1], one can determine the eigenvalues of the strain $\tilde{\varepsilon}_i=\widetilde{\left(\epsilon+z\kappa\right)}_i$. These eigenvalues being, in general, of the nonpolynomial functions of the coordinate z , the integral $\int\phi_{ed}\,dz$ is not computable analytically. This formulation would thus not be adapted for the application to the structural elements.

Instead of the formulation [éq. 2.1-1], one will rather use a formulation of the directly definite free energy in generalized strains, $\boldsymbol{\epsilon}$ and $\boldsymbol{\kappa}$:

$$\begin{aligned}\Phi_{ed}\left(\boldsymbol{\epsilon},\boldsymbol{\kappa},\varepsilon_{zz},d_j\right) &= \frac{\lambda}{2}(\mathrm{tr}(\boldsymbol{\epsilon})+\varepsilon_{zz})^2.\xi_m(\mathrm{tr}(\boldsymbol{\epsilon}),d_j)+\mu\left(\sum_{i=1}^{2}\tilde{\epsilon}_i^{\,2}.\xi_m(\tilde{\epsilon}_i,d_j)+\varepsilon_{zz}^2\right)\\ &+ \frac{\lambda}{2}z^2\mathrm{tr}(\boldsymbol{\kappa})^2.\xi_f(\mathrm{tr}(\boldsymbol{\kappa}),d_j)+\mu z^2\sum_{i=1}^{2}\tilde{\kappa}_i^{\,2}.\xi_f(\tilde{\kappa}_i,d_j)+z(.)\end{aligned} \quad (2.1\text{-}3)$$

where $z(.)$ contains all the terms coupling $\boldsymbol{\epsilon}$ and $\boldsymbol{\kappa}$, which disappear after integration on z, if the assumption is made that the plate/beam is symmetric compared to the average average. One thus obtains the surface density of the free energy:

$$\begin{aligned}\Phi_{ed}^{S}\left(\boldsymbol{\epsilon},\boldsymbol{\kappa},\varepsilon_{zz},d_j\right) &= \int_{-\frac{h}{2}}^{\frac{h}{2}}\Phi_{ed}\left(\boldsymbol{\epsilon},\boldsymbol{\kappa},\varepsilon_{zz},d_j\right)dz\\ &= \frac{\lambda_m}{2}\left(\mathrm{tr}(\boldsymbol{\epsilon})+\varepsilon_{zz}\right)^2.\xi_m(\mathrm{tr}(\boldsymbol{\epsilon}),d_j)+\mu_m\left(\sum_{i=1}^{2}\tilde{\epsilon}_i^{\,2}.\xi_m(\tilde{\epsilon}_i,d_j)+\varepsilon_{zz}^2\right)\\ &+ \frac{\lambda_f}{2}\mathrm{tr}(\boldsymbol{\kappa})^2.\xi_f(\mathrm{tr}(\boldsymbol{\kappa}),d_j)+\mu_f\sum_{i=1}^{2}\tilde{\kappa}_i^{\,2}.\xi_f(\tilde{\kappa}_i,d_j)\end{aligned} \quad (2.1\text{-}4)$$

where h is the thickness of the plate and $\lambda_m,\lambda_f,\mu_m,\mu_f$ are defined with the section §3.2. The surface density of the free energy is strictly convex for $d_j=0$ (having correctly chosen the elastic coefficients).

In [éq 2.1-3] and [éq 2.1-4] we also made the assumption that the damage is not affected by the extension in z, which results in the absence of ε_{zz} arguments of the indicator of damage ξ_m. That is justified by the purpose of this model which is to represent cracking perpendicular compared to the average average, which starts either in membrane request or in bending, but never in extension par. ε_{zz} Moreover, from the numerical point of view of resolution approached low, this assumption facilitates the local computation of the variables of damage d_j and the satisfaction of the assumption of plane stress $\sigma_{zz}=0$.

Note:

In [éq. 2.1-4], it is observed that the déformationn' ε_{zz} is introduced explicitly that on the membrane term of energy. The effect of plane stresses on the membrane behavior will be thus affected by the damage. This is not the case in bending. One could imagine that the constraint plane is written by integrating a coupling bending-membrane, itself depend on the damage, but that was not fall here.
One will see section §3.2 how to determine the parameters of the model from his responses on simple cases.

As for the variable of damage, it is described by two components, one of both representative overall the damage on the side of the upper face of the plate and the other representing the damage on the side of the lower face of the plate:

$$d(z)=\begin{cases}d_1 & si \quad z\geq 0\\ d_2 & si \quad z<0\end{cases}$$

It remains to define the functions characteristic of the damage $\xi_m(.,d_j)$ and $\xi_f(.,d_j)$, so that the formulation, [éq2.1-4], is complete:

$$\xi_m(x,d_1,d_2)=\frac{1}{2}\left(\left(\frac{1+\gamma_{mt}d_1}{1+d_1}+\frac{1+\gamma_{mt}d_2}{1+d_2}\right)H(x)+\left(\frac{\alpha_c+\gamma_{mc}d_1}{\alpha_c+d_1}+\frac{\alpha_c+\gamma_{mc}d_2}{\alpha_c+d_2}\right)H(-x)\right)\quad\in\;[0,1]$$

and

$$\xi_f(x,d_1,d_2)=\frac{\alpha+\gamma_f\,d_1}{\alpha+d_1}H(-x)+\frac{\alpha+\gamma_f\,d_2}{\alpha+d_2}H(x)\quad\in\;[0,1]$$

where $H(.)$ is the function of Heaviside.

The functions $(x,d)\in\mathbb{R}^2\to\xi(x,d)\leq 1$, which are expressed same way as that selected in model ENDO_ISOT_BETON, cf [R7.01.04], by operating a change of variable on d_i, offer the advantage of giving a constant slope in the phases of damage. They are decreasing $\frac{\partial\xi}{\partial d}\leq 0$ and convex to represent the loss of stiffness related to the damage, and $\frac{\partial\xi}{\partial x}=0\quad pour\quad x\in]-\infty,0[\cup]0,+\infty[$, the jump while 0 making it possible to represent the behavioral change between tension and compression, however without introducing discontinuity. The variables of damage d_i grow until $+\infty$. The functions characteristic of the damage $\xi_m(.,d_j)$ and $\xi_f(.,d_j)$ vary from 1 with respectively γ_{mt} or γ_{mc}, and γ_f, for $d_j\to+\infty$, to see section §3.2.3. This model thus describes a damage partial but not total of the material.

The parameters of damage γ_{mt} for the tension out of membrane, γ_{mc} compression out of membrane and γ_f bending, can have values in $[0,1]$, so that the model is not lenitive, which would involve difficulties of dependence to the spatial discretization and convergence. One will choose $\gamma_.\approx 0$ when the phenomenon corresponding has more impact on the damage and $\gamma_.\approx 1$ when this one is negligible. Thus, for the reinforced concrete, one expects $\gamma_{mc}\approx 1$ and $\gamma_{mt}\approx 0$. As for the parameter, α it makes it possible to adjust the contribution of bending the threshold of damage (see §3.2). The parameter α_c makes it possible to modulate the evolution of the damage in compression.

2.2 Generalized stresses

According to the usual procedure one defines the generalized stresses (normal force and moments) by derivatives of the density of free energy compared to the generalized strains, $\boldsymbol{\epsilon}$ and $\boldsymbol{\kappa}$:

$$\mathbf{N} = \frac{\partial \Phi_{ed}^S}{\partial \boldsymbol{\epsilon}} \;; \qquad \mathbf{M} = \frac{\partial \Phi_{ed}^S}{\partial \boldsymbol{\kappa}}$$

In our application, the generalized stresses are calculated in the reference of the eigenvectors of the generalized strains and they are written like (see computation in appendix):

$$\begin{aligned}\tilde{N}_i &= \frac{\partial \Phi_{ed}^S}{\partial \tilde{\epsilon}_i} = \lambda_m(\mathrm{tr}(\boldsymbol{\epsilon})+\varepsilon_{zz}).\xi_m\left(\mathrm{tr}(\boldsymbol{\epsilon}), d_j\right)+2\mu_m \tilde{\epsilon}_i.\xi_m(\tilde{\epsilon}_i, d_j), \quad i=1..2\\ \tilde{M}_i &= \frac{\partial \Phi_{ed}^S}{\partial \tilde{\kappa}_i} = \lambda_f \,\mathrm{tr}(\boldsymbol{\kappa}).\xi_f\left(\mathrm{tr}(\boldsymbol{\kappa}), d_j\right)+2\mu_f \tilde{\kappa}_i.\xi_f(\tilde{\kappa}_i, d_j), \quad i=1,2\end{aligned} \tag{2.2-1}$$

It is easy to check (see appendix) that the generalized stresses $\mathbf{N}$ and $\mathbf{M}$, definite starting from the statements [éq 2.2-1], divide the same eigenvectors respectively as the generalized strains $\boldsymbol{\epsilon}$ and $\boldsymbol{\kappa}$. If these eigenvectors are indicated respectively like $\mathbf{Q}_m$ and $\mathbf{Q}_f$, one can write:

$$\mathbf{N}=\mathbf{Q}_m \tilde{\mathbf{N}} \mathbf{Q}_m^T \quad \text{and} \quad \mathbf{M}=\mathbf{Q}_f \tilde{\mathbf{M}} \mathbf{Q}_f^T \tag{2.2-2}$$

where $\tilde{\mathbf{N}}$ and $\tilde{\mathbf{M}}$ are the diagonal matrixes made up of the eigenvalues defined in [éq 2.2-1]. It is important to note that the eigenvectors for the part of membrane $\mathbf{Q}_m$ and bending and $\mathbf{Q}_f$ are completely independent.

In the same way, one defines the pinching stresses as the dual variable of ε_{zz} :

$$\sigma_{zz} = \frac{\partial \Phi_{ed}^S}{\partial \varepsilon_{zz}} = \lambda_m \xi_m(\mathrm{tr}(\boldsymbol{\epsilon}), d_1, d_2).\left(\mathrm{tr}(\boldsymbol{\epsilon})+\varepsilon_{zz}\right)+2\mu_m \varepsilon_{zz} \tag{2.2-3}$$

on which one will impose the constraint plane: $\sigma_{zz} = 0$.

2.3 Thresholds and evolution of the damage

to be able to define a threshold of damage in the frame of the assumption of *a generalized standard material* (see [feeding-bottle 1], [feeding-bottle 7]), one introduces the thermodynamic forces associated with the variables d_1 and d_2 :

$$Y_j = -\frac{\partial \Phi_{ed}^S}{\partial d_j} = Y_j^m + Y_j^f \tag{2.3-1}$$

where

$$Y_j^m\left(\epsilon, d_j, \varepsilon_{zz}\right) = \frac{1}{(1+d_j)^2}\left(\frac{\lambda_m}{4}\left(\mathrm{tr}(\boldsymbol{\epsilon})+\varepsilon_{zz}\right)^2 . G_m(\mathrm{tr}(\boldsymbol{\epsilon}), d_j)+\frac{\mu_m}{2}\left(\sum_{i=1}^{2}\tilde{\epsilon}_i^2 . G_m(\tilde{\epsilon}_i, d_j)\right)\right)$$

by noting

$$G_m(x, d_j)=\left(1-\gamma_{mt}\right)H(x)+\left(\frac{\alpha_c\left(1-\gamma_{mc}\right)\left(1+d_j\right)^2}{\left(\alpha_c+d_j\right)^2}\right)H(-x) \quad \in [0,1]$$

so that: $\dfrac{\partial \xi_m\left(x, d_{1,} d_2\right)}{\partial d_j}=-\dfrac{G_m(x, d_j)}{2(1+d_j)^2}$, (2.3-2)

and

$$Y_j^f\left(\boldsymbol{\kappa}, d_j\right) = \frac{\alpha}{\left(\alpha+d_j\right)^2} Y_j^{f,0}(\boldsymbol{\kappa})$$

with

$$Y_j^{f,0}(\boldsymbol{\kappa})=(1-\gamma_f)\left(\frac{\lambda_f}{2}\mathrm{tr}(\boldsymbol{\kappa})^2 H\left((-1)^j . \mathrm{tr}(\boldsymbol{\kappa})\right)+\mu_f\sum_{i=1}^{2}\tilde{\kappa}_i^2 H\left((-1)^j . \tilde{\kappa}_i\right)\right)$$

the thresholds of damage are defined by:

$$f_{d_j} = Y_j(\boldsymbol{\epsilon}, \boldsymbol{\kappa}, d_j, \varepsilon_{zz}) - k_{0_j} \le 0 \quad (2.3\text{-}3)$$

where k_{0_j} are constants of threshold. These thresholds define the convex field of reversibility within the space of $(\boldsymbol{\epsilon}, \boldsymbol{\kappa})$.

In theory the constants of threshold k_{0_1} and k_{0_2} could be different, but according to the assumption that one makes on the symmetry of the plate compared to the average average, the two values are the same ones: $k_{0_1}=k_{0_2}=k_0$.

One sees in [éq 2.3-2] that the parameter α controls the contribution of bending to the threshold of initial damage, since:

$$f_{d_j}(d_j=0)=Y_j^m(\boldsymbol{\epsilon}, d_j=0)+\frac{1}{\alpha}Y_j^{f,0}(\boldsymbol{\kappa})-k_0$$

The law of evolution of the variables of damage d_1 and d_2 is defined by the normality rule in the thresholds [éq 2.3-3], for which one can define the pseudopotential of dissipation $\boldsymbol{D}(\delta)$:

$$\dot{d}_j = \eta\frac{\partial f_{d_j}}{\partial Y_j} \ , \ \textit{avec} \ \ \eta\ge 0$$
$$\Leftrightarrow \ Y_j \in \partial\boldsymbol{D}(\dot{d}_j) \ \Leftrightarrow \ \boldsymbol{D}(\dot{d}_j)-\boldsymbol{D}(\delta) \ge Y_{j.}(\dot{d}_j-\delta) \ , \ \forall\delta\ge 0 \quad (2.3\text{-}4)$$

the values of damage d_1 and d_2 are determined perfectly by the following conditions:

so $f_{d_j}<0$ then $\dot{d}_j=0$

$$\text{so } f_{d_j}=0 \text{ then } \begin{cases} \dot{d}_j \geq 0, \\ \dot{f}_{d_j}=0, \text{ condition de cohérence} \\ \dot{d}_j f_{d_j}=0, \text{ condition de complémentarité (Kühn-Tücker)} \end{cases} \quad (2.3\text{-}5)$$

the evolution of the variables of damage is thus obtained using the condition of coherence, the functions $\xi(x, d)$ being convex, the hardening moduli $-f_{,d_j}$ are positive (coefficients checking $\gamma_. \in [0,1]$):

$$\dot{d}_j = -\frac{[f_{,Y} . Y_{,\epsilon} . \dot{\epsilon}]_+}{f_{,d_j}} \quad (2.3\text{-}6)$$

It is noted that in damaging uniaxial pure load membrane ($\dot{d}_1=\dot{d}_2 \geqslant 0$), the thermodynamic force is expressed: $Y^m(d)=-E^m_{eq} h \frac{\varepsilon^2 . \xi_{m,d}(d)}{2}$, $E^m_{eq} h$ being uniaxial membrane elastic stiffness. The condition of coherence is written then:

$$\dot{f}_d = E^m_{eq} h \left(-\epsilon \dot{\epsilon} \xi_{m,d} - \frac{\epsilon^2 \dot{d} \xi_{m,dd}}{2} \right) = 0 \quad (2.3\text{-}7)$$

From where: $\dot{d}_m = -\frac{2 \dot{\epsilon} \xi_{m,d}}{\epsilon \xi_{m,dd}}$ and thus the model of damaging uniaxial pure load membrane is:

$$\dot{N} = E^m_{eq} h \left(\xi_{m,d} \dot{d} \epsilon + \xi_m \dot{\epsilon} \right) = E^m_{eq} h \dot{\epsilon} \left(\xi_m - 2 \frac{\xi^2_{m,d}}{\xi_{m,dd}} \right) = \gamma_m E^m_{eq} h \dot{\epsilon} \quad (2.3\text{-}8)$$

what makes it possible to interpret the role of the parameter γ_m : the slope being constant, which is the justification of the algebraic form of the function ξ_m .

2.4 Numerical integration

Contrary in the majority of the nonlinear unelastic constitutive laws, that presented here does not require the discretization of the equations of evolution of the local variables. One adopts a method of direct discretization implicit in time.

The variables d_j can be calculated directly from the condition of coherence, [éq 2.3-4]. The only time where one refers to the “velocity of damage” is to check that it is positive $\dot{d}_j \geq 0$. For an incremental computation this condition results in $d^n_j \geq d^{n-1}_j$ with time step $n>1$, therefore without reference to a particular diagram of temporal integration.

Let us place at a given time t_n of the way of loading. One carries out initially an elastic stage of prediction (elasticity tensor evaluated with the variables of damage d_j^{n-1} solidified at the preceding stage), from where $(\boldsymbol{\epsilon}^n, \boldsymbol{\kappa}^n)$. One calculates then new ε_{zz}^{n0} :

$$\sigma_{zz}^n=0 \Rightarrow \varepsilon_{zz}^{n0}(\boldsymbol{\epsilon}^n, d_1^{n-1}, d_2^{n-1}) = -\frac{\lambda_m \xi_m(\mathrm{tr}(\boldsymbol{\epsilon}^n), d_1^{n-1}, d_2^{n-1})}{2\mu_m+\lambda_m \xi_m(\mathrm{tr}(\boldsymbol{\epsilon}^n), d_1^{n-1}, d_2^{n-1})}\mathrm{tr}(\boldsymbol{\epsilon}^n)$$

to see [éq 2.2-3].

One calculates then $f_{d_j}^{n0}(\boldsymbol{\epsilon}^n, \boldsymbol{\kappa}^n, d_j^{n-1}, \varepsilon_{zz}^{n0})$ to check the thresholds of damage. If $f_{d_j}^{n0}\leq 0$, the damage does not evolve: $d_j^n=d_j^{n-1}$ and $\varepsilon_{zz}^n=\varepsilon_{zz}^{n0}$, and the generalized stresses are calculated according to [éq 2.2-1].

When $f_{d_j}^{n0}>0$, the damage can evolve and one must solve the equations:

$$f_{d_j}^n = Y_j(\boldsymbol{\epsilon}^n, \boldsymbol{\kappa}^n, d_j^n, \varepsilon_{zz}^n) - k_0 = 0$$

What corresponds to the resolution of the nonlinear equations, with $\boldsymbol{\epsilon}, \boldsymbol{\kappa}$ given:

$$R_{d_j}(d_j, \boldsymbol{\epsilon}, \boldsymbol{\kappa}, \varepsilon_{zz}) = \frac{1}{(1+d_j)^2}\left(\frac{\lambda_m}{4}\left(\mathrm{tr}(\boldsymbol{\epsilon})+\varepsilon_{zz}\right)^2 . G_m(\mathrm{tr}(\boldsymbol{\epsilon}), d_j)+\frac{\mu_m}{2}\left(\sum_{i=1}^{2}\tilde{\epsilon}_i^2 G_m(\tilde{\epsilon}_i, d_j)\right)\right) - \frac{\alpha(1-\gamma_f)}{(\alpha+d_j)^2}\left(\frac{\lambda_f}{2}\mathrm{tr}(\boldsymbol{\kappa})^2 H\left((-1)^j \mathrm{tr}(\boldsymbol{\kappa})\right)+\mu_f\sum_{i=1}^{2}\tilde{\kappa}_i H\left((-1)^j\tilde{\kappa}_i\right)\right) - k_0 = 0 \quad (2.4\text{-}1)$$

the equation [éq 2.4-1] must be solved in taking into account also the plane constraint, which makes it possible to express $\varepsilon_{zz}(\boldsymbol{\epsilon}, d_j)$, cf [éq 2.2-3]:

$$\sigma_{zz}=0 \Rightarrow \varepsilon_{zz}(\boldsymbol{\epsilon}, d_{1,} d_2) = -\frac{\lambda_m \xi_m(\mathrm{tr}(\boldsymbol{\epsilon}), d_{1,} d_2)}{2\mu_m+\lambda_m \xi_m(\mathrm{tr}(\boldsymbol{\epsilon}), d_{1,} d_2)}\mathrm{tr}(\boldsymbol{\epsilon}) \quad (2.4\text{-}2)$$

One solves the equations [éq 2.4-1] and [éq 2.4-2] by the method of Newton. One starts with a phase of prediction, of explicit Eulerian type:

$$d_j^{n(0)}=d_j^{n-1}+\left.\left(\frac{dR_{d_j}}{d(d_j)}\right)\right|_{d_j^{n-1}}^{-1} . \left(k_0-Y_j(\boldsymbol{\epsilon}^n, \boldsymbol{\kappa}^n, d_j^{n-1}, \varepsilon_{zz}^{n0})\right)$$

Then one treats the phase of correction, with the iteration $(m)>1$:

1. $\Delta d_j^{n(m)}=\left.\left(\frac{dR_{d_j}}{d(d_j)}\right)\right|_{d_j^{n(m-1)}}^{-1} . \left(k_0-Y_j(\boldsymbol{\epsilon}^n, \boldsymbol{\kappa}^n, d_j^{n(m-1)}, \varepsilon_{zz}^{n(m-1)})\right)$
2. $d_j^{n(m)}=d_j^{n(m-1)}+\Delta d_j^{n(m)}$
3. $\varepsilon_{zz}^{n(m)}=\varepsilon_{zz}(\boldsymbol{\epsilon}^n, d_1^{n(m)}, d_2^{n(m)})$

This phase of correction is completed when the convergence criterion expressed in energy term is reached:

$$\Delta d_j^{n(m)} . R_{d_j}^{n(m)} < \eta_{tolerance} . k_0$$

The tangent operator of this nonlinear system is defined like:

$$\begin{aligned}
\frac{dR_{d_j}}{d(d_j)} &= \frac{\partial R_{d_j}}{\partial \varepsilon_{zz}} \frac{\partial \varepsilon_{zz}}{\partial d_j} + \frac{\partial R_{d_j}}{\partial d_j} \\
&= \frac{1}{(1+d_j)^2} \left(\frac{\lambda_m}{2} \left(\mathrm{tr}(\boldsymbol{\epsilon}) + \varepsilon_{zz} \right) G_m(\mathrm{tr}(\boldsymbol{\epsilon}), d_j) \frac{\partial \varepsilon_{zz}}{\partial d_j} \right) \\
&+ \frac{1}{(1+d_j)^2} \left(\frac{\lambda_m}{4} \left(\mathrm{tr}(\boldsymbol{\epsilon}) + \varepsilon_{zz} \right)^2 \frac{\partial G_m(\mathrm{tr}(\boldsymbol{\epsilon}), d_j)}{\partial d_j} \varepsilon_{zz} + \frac{\mu_m}{2} \left(\sum_{i=1}^{2} \tilde{\epsilon}_i^2 \frac{\partial G_m(\tilde{\epsilon}_i, d_j)}{\partial d_j} \right) \right) \\
&- \frac{2}{(1+d_j)^3} \left(\frac{\lambda_m}{4} \left(\mathrm{tr}(\boldsymbol{\epsilon}) + \varepsilon_{zz} \right)^2 G_m(\mathrm{tr}(\boldsymbol{\epsilon}), d_j) + \frac{\mu_m}{2} \left(\sum_{i=1}^{2} \tilde{\epsilon}_i^2 G_m(\tilde{\epsilon}_i, d_j) \right) \right) \\
&- \frac{2\alpha(1-\gamma_f)}{(\alpha+d_j)^3} \left(\frac{\lambda_f}{2} \mathrm{tr}(\boldsymbol{\kappa})^2 H((-1)^j \mathrm{tr}(\boldsymbol{\kappa})) + \mu_f \sum_{i=1}^{2} \tilde{\kappa}_i^2 H((-1)^j \tilde{\kappa}_i) \right)
\end{aligned} \quad (2.4\text{-}3)$$

where:

$$\frac{\partial \varepsilon_{zz}}{\partial d_j} = - \frac{\lambda_m \left(\mathrm{tr}(\boldsymbol{\epsilon}) + \varepsilon_{zz} \right)}{\left(2\mu_m + \lambda_{m.} \xi_m \left(\mathrm{tr}(\boldsymbol{\epsilon}), d_j \right) \right)} \frac{\partial \xi_m(\mathrm{tr}(\boldsymbol{\epsilon}), d_j)}{\partial d_j} \quad (2.4\text{-}4)$$

and:

$$\frac{\partial \xi_m}{\partial d_j}(x, d_j) = - \frac{1}{2} \left(\frac{1-\gamma_{mt}}{(1+d_j)^2} H(x) + \frac{\alpha_c(1-\gamma_{mc})}{(\alpha_c+d_j)^2} H(-x) \right) = - \frac{G_m(x, d_j)}{2(1+d_j)^2} \quad (2.4\text{-}5)$$

It is checked that one has well $\frac{\partial \xi_m}{\partial d_j}(x, d_{1,} d_2) < 0$ as expected.

2.5 Operator of tangent stiffness

As the main aim of a total model is to propose an approach simplified with the modelization of a complex material, such as the reinforced concrete, it is essential that its numerical performance is optimal. Thus, to return the model adapted to computations with implicit schemes in time, either into quasi-static or in transient dynamics, the computation of the coherent tangent stiffness becomes essential to have a quadratic and robust convergence total iterative process of Newton.
The essence of the computation of the model is carried out in the reference of the eigenvectors of the strain tensors generalized (and the generalized stresses, cf [éq 2.2-2]), the tangent stiffness also being thus expressed in the same reference. The transformation necessary to then be able to use it in the assembly of the total stiffness matrix is specified in the following chapter.

In order to simplify the writing one defines a stress vector generalized (membrane force, bending moment) $\boldsymbol{\Sigma}$, and a vector of generalized strains (extension, curvature) $\boldsymbol{E}$, like:

$$\boldsymbol{\Sigma}=\left(\tilde{N}_1 \;\; \tilde{N}_2 \;\; \tilde{M}_1 \;\; \tilde{M}_2\right)^T$$
$$\boldsymbol{E}=\left(\tilde{\epsilon}_1 \;\; \tilde{\epsilon}_2 \;\; \tilde{\kappa}_1 \;\; \tilde{\kappa}_2\right)^T$$

The operator of tangent stiffness $\mathbf{C}$ is defined by the relation in the real evolution:

$$d\boldsymbol{\Sigma} \;=\; \mathbf{C}\cdot d\boldsymbol{E}$$

He can be calculated as the sum of two contributions, that which corresponds to an NON-evolution of the damage and that which is due to the evolution of the damage. These contributions can be named: elastic *contribution* and dissipative *contribution*:

$$\mathbf{C} \;=\; \underbrace{\left.\frac{d\boldsymbol{\Sigma}}{d\boldsymbol{E}}\right|_{\dot{d}_j=0}}_{\mathbf{C}_e} \;+\; \underbrace{\left.\frac{d\boldsymbol{\Sigma}}{d\boldsymbol{E}}\right|_{f_{d_j}=0}}_{\mathbf{C}_d} \qquad (2.5\text{-}1)$$

Moreover, one takes account of structure of the tensor $\mathbf{C}$ made up of the contributions force normal-extension, moment-curvature and their couplings. More particularly the tensors $\mathbf{C}_e$ and $\mathbf{C}_d$ are following form:

$$\mathbf{C}_e \;=\; \begin{pmatrix} \mathbf{C}_e^{mm} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_e^{ff} \end{pmatrix} \;;\; \mathbf{C}_d \;=\; \begin{pmatrix} \mathbf{C}_d^{mm} & \mathbf{C}_d^{mf} \\ (\mathbf{C}_d^{mf})^T & \mathbf{C}_d^{ff} \end{pmatrix} \qquad (2.5\text{-}2)$$

It is seen [éq 2.5-2] that the coupling moments/extension and forces of membrane/curvature are introduced only through the dissipative part. This coupling has a physical justification, since any cracking perpendicular to the average average of the plate affects as well the behavior out of membrane as in bending.

The submatrices $\mathbf{C}_e^{mm}$ $\mathbf{C}_e^{ff}$ $\mathbf{C}_d^{mm}$ $\mathbf{C}_d^{mf}$, $\mathbf{C}_d^{ff}$ are given in the clean reference by the statements which follow:

$$\begin{aligned}(\mathbf{C}_e^{mm})_{ij}&=\left.\frac{\partial\tilde{N}_i}{\partial\tilde{\epsilon}_j}\right|_{d_k=0}=\frac{2\lambda_m\xi_m(\mathrm{tr}(\boldsymbol{\epsilon}),d_k)}{2\mu_m+\lambda_m\xi_m(\mathrm{tr}(\boldsymbol{\epsilon}),d_k)}+2\mu_m\xi_m(\tilde{\epsilon}_j,d_k)\delta_{ij}\\ (\mathbf{C}_e^{ff})_{ij}&=\left.\frac{\partial\tilde{M}_i}{\partial\tilde{\kappa}_j}\right|_{d_k=0}=\lambda_f\xi_f(\mathrm{tr}(\boldsymbol{\kappa}),d_k)+2\mu_f\xi_f(\tilde{\kappa}_j,d_k)\delta_{ij}\end{aligned} \qquad (2.5\text{-}3)$$

$$(\mathbf{C}_d^{mm})_{ij}=\left.\frac{\partial\tilde{N}_i}{\partial\tilde{\epsilon}_j}\right|_{f_{d_k}=0}=\left.\frac{\partial\tilde{N}_i}{\partial d_k}\cdot\frac{d(d_k)}{d\tilde{\epsilon}_j}\right|_{f_{d_k}=0}\text{ (summation on } k\text{)}$$

$$(\mathbf{C}_d^{ff})_{ij}=\left.\frac{\partial\tilde{M}_i}{\partial\tilde{\kappa}_j}\right|_{f_{d_k}=0}=\left.\frac{\partial\tilde{M}_i}{\partial d_k}\cdot\frac{d(d_k)}{\partial\tilde{\kappa}_j}\right|_{f_{d_k}=0}\text{ (summation on } k\text{)}\qquad(2.5\text{-}4)$$

$$(\mathbf{C}_d^{mf})_{ij}=\left.\frac{\partial\tilde{N}_i}{\partial\tilde{\kappa}_j}\right|_{f_{d_k}=0}=\left.\frac{\partial\tilde{N}_i}{\partial d_k}\cdot\frac{d(d_k)}{\partial\tilde{\kappa}_j}\right|_{f_{d_k}=0}\text{ (summation on } k\text{)}$$

where

$$\frac{\partial\tilde{N}_i}{\partial d_k}=\lambda_m(\mathrm{tr}(\boldsymbol{\epsilon})+\varepsilon_{zz})\frac{\partial\xi_m(\mathrm{tr}(\boldsymbol{\epsilon}),d_j)}{\partial d_k}+\lambda_m\xi_m(\mathrm{tr}(\boldsymbol{\epsilon}),d_j)\frac{\partial\varepsilon_{zz}}{\partial d_k}+2\mu_m\tilde{\epsilon}_i\frac{\partial\xi_m(\tilde{\epsilon}_i,d_j)}{\partial d_k}$$

and
$$\frac{\partial\tilde{M}_i}{\partial d_k}=\lambda_f\,\mathrm{tr}(\boldsymbol{\kappa})\frac{\partial\xi_f(\mathrm{tr}(\boldsymbol{\kappa}),d_1,d_2)}{\partial d_k}+2\mu_f\tilde{\kappa}_i\frac{\partial\xi_f(\tilde{\kappa}_i,d_1,d_2)}{\partial d_k}\qquad(2.5\text{-}5)$$

with
$$\frac{\partial\xi_f}{\partial d_k}(x,d_1,d_2)=-\alpha\frac{(1-\gamma_f)}{(\alpha+d_k)^2}H((-1)^k x)$$

Besides the statements [éq 2.5-5], one resorts to the equations [éq 2.4-5] to determine $\frac{\partial\xi_m}{\partial d_k}(x,d_1,d_2)$.

The derivatives $\left.\frac{d(d_k)}{d\tilde{\epsilon}_i}\right|_{f_{d_k}=0}$ and $\left.\frac{d(d_k)}{d\tilde{\kappa}_i}\right|_{f_{d_k}=0}$ are calculated by differentiating the equation $R_{d_k}=0$ respectively compared to d_k, $\tilde{\epsilon}_i$ and $\tilde{\kappa}_i$, (see [éq. 2.4-1]). When the two damage mechanisms are activated, one is brought to solve the systems which follow:

$$\begin{pmatrix}A_{11}&A_{12}\\A_{21}&A_{22}\end{pmatrix}\begin{pmatrix}\frac{d(d_1)}{d\tilde{\epsilon}_i}\\\frac{d(d_2)}{d\tilde{\epsilon}_i}\end{pmatrix}=\begin{pmatrix}B_1^m\\B_2^m\end{pmatrix}\text{ and }\begin{pmatrix}A_{11}&A_{12}\\A_{21}&A_{22}\end{pmatrix}\begin{pmatrix}\frac{d(d_1)}{d\tilde{\kappa}_i}\\\frac{d(d_2)}{d\tilde{\kappa}_i}\end{pmatrix}=\begin{pmatrix}B_1^f\\B_2^f\end{pmatrix}\qquad(2.5\text{-}6)$$

with $A_{jk}=\frac{\partial\boldsymbol{Y}_j}{\partial d_k}$ $B_k^m=\frac{-\partial\boldsymbol{Y}_k}{\partial\boldsymbol{\epsilon}}=\frac{\partial\tilde{N}}{\partial d_k}$ $B_k^f=\frac{-\partial\boldsymbol{Y}_k}{\partial\boldsymbol{\kappa}}=\frac{\partial\tilde{M}}{\partial d_k}$;

thus:

$$\frac{d(d_1)}{d\tilde{\epsilon}_i}=\frac{A_{22}B_1^m-A_{12}B_2^m}{A_{22}A_{11}-A_{12}A_{21}}\text{ and }\frac{d(d_2)}{d\tilde{\epsilon}_i}=\frac{A_{11}B_2^m-A_{21}B_1^m}{A_{11}A_{22}-A_{21}A_{12}}$$

$$\frac{d(d_1)}{d\tilde{\kappa}_i}=\frac{A_{22}B_1^f-A_{12}B_2^f}{A_{22}A_{11}-A_{12}A_{21}}\qquad\frac{d(d_2)}{d\tilde{\kappa}_i}=\frac{A_{11}B_2^f-A_{21}B_1^f}{A_{11}A_{22}-A_{21}A_{12}}$$

where:

$$\begin{aligned}
A_{jk} &= \frac{1}{(1+d_j)^2}\left(\frac{\lambda_m}{2}(\mathrm{tr}(\boldsymbol{\epsilon})+\varepsilon_{zz})\frac{\partial\varepsilon_{zz}}{\partial d_k}G_m(\mathrm{tr}(\boldsymbol{\epsilon}),d_j)\right)\\
&+\left(\frac{1}{(1+d_j)^2}\left(\frac{\lambda_m}{4}(\mathrm{tr}(\boldsymbol{\epsilon})+\varepsilon_{zz})^2\frac{\partial G_m(\mathrm{tr}(\boldsymbol{\epsilon}),d_j)}{\partial d_k}+\frac{\mu_m}{2}\sum_{i=1}^{2}\tilde{\epsilon}_i^2\frac{\partial G_m(\tilde{\epsilon}_i,d_j)}{\partial d_k}\right)\right.\\
&-\frac{2}{(1+d_j)^3}\left(\frac{\lambda_m}{4}(\mathrm{tr}(\boldsymbol{\epsilon})+\varepsilon_{zz})^2G_m(\mathrm{tr}(\boldsymbol{\epsilon}),d_j)+\frac{\mu_m}{2}\sum_{i=1}^{2}\tilde{\epsilon}_i^2G_m(\tilde{\epsilon}_i,d_j)\right)\\
&\left.-\frac{2\alpha(1-\gamma_f)}{(\alpha+d_j)^3}\left(\frac{\lambda_f}{2}\mathrm{tr}(\boldsymbol{\kappa})^2H((-1)^j\mathrm{tr}(\boldsymbol{\kappa}))+\mu_f\sum_{i=1}^{2}\tilde{\kappa}_i^2H((-1)^j\tilde{\kappa}_i)\right)\right)\delta_{jk}
\end{aligned} \quad (2.5\text{-}7)$$

with:

$$\frac{\partial G_m(x,d_j)}{\partial d_k}=\left(\frac{2\alpha_c(1+\gamma_{mc})(1+d_j)(\alpha_c-1)}{(\alpha_c+d_j)^3}\right)H(-x) \quad (2.5\text{-}8)$$

and $\frac{\partial\varepsilon_{zz}}{\partial d_k}$ given by [éq. 2.4-1]. The matrix (A_{jk}) is quite invertible, cf section §2.4.

2.6 Change of reference

the approach of change of reference is identical to that developed for the model ENDO_ISOT_BETON (see section [2.4.4.1] [R7.01.04]) with the only difference which it applies to the generalized stresses and strains. We obtain the components thus of:

$$\mathbf{C}=\begin{pmatrix}\mathbf{C}^{mm} & \mathbf{C}^{mf}\\ (\mathbf{C}^{mf})^T & \mathbf{C}^{ff}\end{pmatrix}$$

as being:

$$\begin{aligned}
(\mathbf{C}^{mm})_{ijkl} &= \frac{\partial N_{ij}}{\partial\epsilon_{kl}} = \sum_{m,n}Q_{im}^mQ_{jm}^mQ_{kn}^mQ_{ln}^m\cdot\frac{\partial\tilde{N}_m}{\partial\tilde{\epsilon}_n}\\
&+\frac{1}{2}\sum_{\substack{m,n\\m\neq n}}\left(\frac{(Q_{km}^mQ_{ln}^m+Q_{lm}^mQ_{kn}^m)(Q_{in}^mQ_{jm}^m+Q_{jn}^mQ_{im}^m)}{\tilde{\epsilon}_n-\tilde{\epsilon}_m}\right)\tilde{N}_m\\
(\mathbf{C}^{ff})_{ijkl} &= \frac{\partial M_{ij}}{\partial\kappa_{kl}} = \sum_{m,n}Q_{im}^fQ_{jm}^fQ_{kn}^fQ_{ln}^f\cdot\frac{\partial\tilde{M}_m}{\partial\tilde{\kappa}_n}\\
&+\frac{1}{2}\sum_{\substack{m,n\\m\neq n}}\left(\frac{(Q_{km}^fQ_{ln}^f+Q_{lm}^fQ_{kn}^f)(Q_{in}^fQ_{jm}^f+Q_{jn}^fQ_{im}^f)}{\tilde{\kappa}_n-\tilde{\kappa}_m}\right)\tilde{M}_m
\end{aligned}$$

$$(\mathbf{C}^{mf})_{ijkl} = \frac{\partial N_{ij}}{\partial \kappa_{kl}} = \sum_{m,n} Q^m_{im} Q^m_{jm} Q^f_{kn} Q^f_{ln} \cdot \frac{\partial \tilde{N}_m}{\partial \tilde{\kappa}_n}$$

The generalized stresses as for them are written like, cf [éq 2.2-2]:

$$\mathbf{N} = \mathbf{Q}_m \tilde{\mathbf{N}} \mathbf{Q}_m^T$$
$$\mathbf{M} = \mathbf{Q}_f \tilde{\mathbf{M}} \mathbf{Q}_f^T$$

2.7 Computation of dissipation

By definition, the density of power of dissipation at the time of the damage is worth:

$$\dot{D} = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}} - \dot{\Phi}^S_{ed} = -\sum_{j=1,2} \frac{\partial \Phi^S_{ed}}{\partial d_j} \dot{d}_j = \sum_{j=1,2} Y_j \dot{d}_j$$

In this statement, one used the definition of Y_j [éq. 2.3-2]. In the damaging phase, the functions thresholds always satisfy $f_{d_j} = Y_j - k_0 \equiv 0$. Consequently, one can calculate the dissipation cumulated like:

$$D = \int \dot{D} \; dt = k_0 (d_1 + d_2) \qquad (2.7\text{-}1)$$

It was shown that the cumulated dissipation of the process of damage is directly related to the local variables. It is enough to make the sum of the two contributions and to multiply it by the constant of threshold k_0.
The computation of dissipation is realized by options `DISS_ELGA` and `DISS_ELNO` of `CALC_CHAMP`. These fields have only one named component `ENDO`.

2.8 Local variables of model

The model requires two local variables, d_1 and d_2 (corresponding to the variables $V1$ and $V2$ of *the Code_Aster*), which represents the damage on the side of the upper face and the side of the lower face, respectively. The distinction between the sides higher and lower is carried out through the directional sense of the local coordinate system of each Gauss point. Thus, the lower face is damaged with $\dot{d}_2 > 0$ for positive curvatures, and the upper face with $\dot{d}_1 > 0$ for negative curvatures, which results directly from the definition from ξ_f, (cf section 2.1).

In any case the choice of the directional sense of the reference in a Gauss point and the does not affect the final result with regard to displacements rotations. There can be an impact on the interpretation of the damage and the generalized stresses if the local directional senses are not coherent in a structure. It is strongly advised to ensure this coherence by informing key word `ANGL_REP` with the nautical angles of the local coordinate system, (see [U4.42.01]):

`AFFE_CARA_ELEM (COQUE = _F (ANGL_REP =` (a, b) `))`

Besides the variables $V1$ and $V2$, one also introduces $V3$ and $V4$, of binary value (0 or 1), which indicate the instantaneous evolution of $V1$ and $V2$. More precisely, $V3$ is worth 1 when $V1$ evolves and 0 if not. In the same way, $V4$ 1 is worth when $V2$ evolves and 0 if not.

Lastly, one also introduces $V5$, $V6$ and $V7$, whose role is to measure the relative weakening of stiffness of reinforced concrete slab in a rational way, for example by visualization in each material point:

$$V5=1-\frac{1}{2}\left(\frac{1+\gamma_{mt}d_1}{1+d_1}+\frac{1+\gamma_{mt}d_2}{1+d_2}\right) \quad V6=1-\frac{1}{2}\left(\frac{\alpha_c+\gamma_{mc}d_1}{\alpha_c+d_1}+\frac{\alpha_c+\gamma_{mc}d_2}{\alpha_c+d_2}\right) \text{ and}$$

$$V7=1-Max\left(\frac{1+\gamma_f d_1}{1+d_1},\ \frac{1+\gamma_f d_2}{1+d_2}\right)$$

respectively in tension, compression and bending. These variables always will lie between 0 and the $1-\gamma_.$ respective, and will be always increasing, being null in absence of damage. These variables are more “speaking” that the variables $V1$ and $V2$.

3 Parameters of the reinforced concrete

slab model The model endommageable `GLRC_DM` thus needs parameters characteristic of elasticity, supplemented of 5 parameters to describe the behavior of damage: k_0 , to define the yield stress, α to determine the participation of bending (see §2.2), γ_{mt} and γ_{mc} , γ_f to describe the nonlinear response. All these parameters can be identified from monotonous uniaxial pure traction tests and of bending. Some of them are replaced in the data file *Code_Aster* by "speaking" parameters more, to see hereafter with the §3.2.5.

It is possible to proceed is from simple analytical estimates (which give the orders of magnitude) that is to say from a retiming on a response curve provided by another model of behavior, possibly by integrating compromises.

One describes in the paragraphs below the approach and one draws up the balance sheet with the § 3.2.53.2.5.

In this section, it is considered that $\alpha_c=1$. Indeed, the development of the methodology of identification with $\alpha_c\neq 1$ was not carried out yet.

3.1 Identification of the parameters of linear elastic behavior

In this model one supposes that the reinforced concrete medium is homogenized and one leaves to the user the care to choose (to calculate or measure) the parameters: E_{eq}^m (effective Young's modulus out of membrane), E_{eq}^f (effective Young's modulus in bending), ν_m (effective Poisson's ratio out of membrane) and ν_f (effective Poisson's ratio in bending). One applies the following relations to determine the coefficients of Lamé λ_m , μ_m and λ_f μ_f :

$$\lambda_m=\frac{\nu_m E_{eq}^m h}{(1+\nu_m)(1-2\nu_m)} \quad , \quad \mu_m=\frac{E_{eq}^m h}{2(1+\nu_m)}$$
$$\lambda_f=\frac{\nu_f E_{eq}^f h^3}{12(1-\nu_f^2)} \quad , \quad \mu_f=\frac{E_{eq}^f h^3}{24(1+\nu_f)} \qquad (3.1\text{-}1)$$

the relations above are not interchangeable by *F↔M* for the parameters membrane and bending, since out of membrane the relations correspond to the general case (elasticity 3D) and the constraint plane is treated within the formulation of the model, while for bending one placed oneself from the start in elasticity 2D with plane stresses. In the elastic domain one has as follows:

$$N_{\alpha\beta} = \frac{E_{eq}^m h}{1-\nu_m^2}\left(\nu_m . \mathrm{tr}\,\boldsymbol{\epsilon} . \delta_{\alpha\beta}+(1-\nu_m)\epsilon_{\alpha\beta}\right)$$
$$M_{\alpha\beta} = \frac{E_{eq}^f h^3}{12(1-\nu_f^2)}\left(\nu_f . \mathrm{tr}\,\boldsymbol{\kappa} . \delta_{\alpha\beta}+(1-\nu_f)\kappa_{\alpha\beta}\right)$$

α and β being indices going from 1 to 2.

By default $E_{eq}^m=E_{eq}^f=E$ and $\nu_m=\nu_f=\nu$, where E and ν are the elastic coefficients well informed in the command file under key word `ELAS`. On the other hand, as the reinforced concrete is

not a homogeneous material, the actual value of E_{eq}^{f} can be different from E_{eq}^{m}. Consequently, one leaves to the user the possibility of introducing values E_{eq}^{f} and ν_f (`EF` and `NUF` under factor key word `the GLRC_DM`) different from E and ν, which in this case are only used for to describe elasticity out of membrane.
The condition of the plane stresses for the membrane $\sigma_{zz}=0$ is satisfied with the way described in the §2.3.

Note:

In [eq. 3.1-1], one obtains a different relation enters λ_f, ν_f and E_{eq}^{f} on the one hand, and enters λ_m, ν_m and E_{eq}^{m} on the other hand. This difference is directly related to the taking into account different out of membrane and bending from the constraint plane. More particularly, one defines E_{eq}^{f} and ν_f through a test of pure bending, where $\kappa_{yy}=-\nu_f\kappa_{xx}$, and $M_{ij}=0$, except $M_{xx}\neq 0$. One makes use then of the following equations to find the relation enters λ_f, μ_f and E_{eq}^{f} ν_f :

$$M_{yy}=(\lambda_f(1-\nu_f)-2\mu_f\nu_f)\kappa_{xx}=0$$

and

$$M_{xx}=(\lambda_f(1-\nu_f)-2\mu_f)\kappa_{xx} = \frac{E_{eq}^{f}h^3}{12}\kappa_{xx}$$

from where one obtains:

$$E_{eq}^{f}=\frac{12}{h^3}\left(\lambda_f(1-\nu_f)+2\mu_f\right) \text{ and } \lambda_f(1-\nu_f)-2\mu_f\nu_f=0 \qquad (3.1\text{-}2)$$

By solving [éq. 3.1-2], there are the relations expressed in [éq. 3.1-1].

The identification of the elastic parameters E_{eq}^{m} ν_m, E_{eq}^{f} and ν_f the model starting from the characteristics of the concrete and steels rests on two cases of loading: pure tension and pure bending.

Let us consider the following characteristics for the concrete: Young's modulus E_b, Poisson's ratio ν_b, thickness of slab h, and for steels: Young's modulus E_a, Poisson's ratio ν_a, total section per linear meter (for the two three-dimensions functions, presumedly symmetric in the thickness and identical in the two directions) S_a, relative position of a three-dimensions function in the thickness $\chi_a\in]0,1[$.

One obtains thus by the uniaxial test in **pure elastic tension** :

$$\begin{aligned} N_{xx}&=E_{eq}^{m}h\epsilon_{xx}=E_aS_a\epsilon_{xx}+\frac{E_bh}{1-\nu_b^2}(\epsilon_{xx}+\nu_b\epsilon_{yy}) \\ N_{yy}&=0=E_aS_a\epsilon_{yy}+\frac{E_bh}{1-\nu_b^2}(\epsilon_{yy}+\nu_b\epsilon_{xx}) \end{aligned} \quad ; \ \epsilon_{yy}=-\nu_m\epsilon_{xx} \qquad (3.1\text{-}3)$$

From where (key words `E` and `NU`):

$$E_{eq}^m = E_a \frac{S_a}{h} + E_b \cdot \frac{E_b h + E_a S_a}{E_b h + E_a S_a (1-\nu_b^2)} \;;\; \nu_m = \nu_b \cdot \frac{E_b h}{E_b h + E_a S_a (1-\nu_b^2)} \quad (3.1\text{-}4)$$

One observes that this identification produced an error on the stiffness in plane elastic shears of slab, case for which steels do not contribute (they are grids of welded rods), which makes the behavior homogenized orthotropic and not isotropic. Indeed, one obtains with the values [éq.3.1-43.1-4]:

$$G_{eq}^m = \frac{E_{eq}^m}{2(1+\nu_m)} = \frac{E_b}{2(1+\nu_b)} \cdot \frac{E_b^2 h^2 + 2 E_a E_b h S_a + E_a^2 S_a^2 (1-\nu_b^2)}{E_b^2 h^2 + E_a E_b h S_a (1-\nu_b)} \neq \frac{E_b}{2(1+\nu_b)} \quad (3.1\text{-}5)$$

If one prefers to firstly ensure the identification on the case of **plane elastic shears** of slab, and on the case of the response according to the direction of pure tension (thus by accepting the error on the effect orthogonal Fish), one obtains:

$$E_{eq}^m = E_b + \frac{E_a S_a (1-\nu_b)}{h} \;;\; \nu_m = \nu_b + \frac{E_a S_a (1-\nu_b^2)}{E_b h} \;;\; G_{eq}^m = \frac{E_b}{2(1+\nu_b)} \quad (3.1\text{-}6)$$

One will take care so that this coarse identification (nonacceptable thermodynamically compared to the traction test pure) does not give whimsical values of ν_m.

Then, one obtains by the uniaxial test in **pure elastic bending** :

$$\begin{aligned} M_{xx} &= \frac{E_{eq}^f h^3}{12} \kappa_{xx} = \frac{1}{4} E_a S_a h^2 \chi_a^2 \kappa_{xx} + \frac{E_b h^3}{12(1-\nu_b^2)} (\kappa_{xx} + \nu_b \kappa_{yy}) \\ M_{yy} &= 0 = \frac{1}{4} E_a S_a h^2 \chi_a^2 \kappa_{yy} + \frac{E_b h^3}{12(1-\nu_b^2)} (\kappa_{yy} + \nu_b \kappa_{xx}) \end{aligned} \;;\; \kappa_{yy} = -\nu_f \kappa_{xx} \quad (3.1\text{-}7)$$

From where (key words `EF` and `NUF`):

$$\begin{aligned} E_{eq}^f &= \frac{3}{h} E_a S_a \chi_a^2 + E_b \cdot \frac{E_b h + 3 E_a S_a \chi_a^2}{E_b h + 3 E_a S_a \chi_a^2 (1-\nu_b^2)} \;; \\ \nu_f &= \nu_b \cdot \frac{E_b h}{E_b h + 3 E_a S_a \chi_a^2 (1-\nu_b^2)} \end{aligned} \quad (3.1\text{-}8)$$

One also observes that this identification produced an error on the stiffness in anticlastic elastic bending M_{xy} of the slab (coefficient $G_{eq}^f = \frac{E_{eq}^f h^3}{24(1+\nu_f)}$ instead of $G_b^f = \frac{E_b h^3}{24(1+\nu_b)}$), case for which steels do not contribute.

3.2 Identification of the parameters of elastic behavior endommageable

the way in which one obtains the parameters of linear elasticity being presented to the §3.1, one proposes to calculate the parameters of damage of the model from two tests: a traction test pure and a uniaxial monotonous test of pure bending.

From this way one obtains the values of the threshold k_0, of the two parameters relating to the effects of membrane (γ_{mt} and γ_{mc}) independently of the two parameters relating to the effects of bending (α , γ_f).

To determine the parameters of damage one thus considers primarily two monotonous uniaxial tests:

•**Pure tension**, where:

$$\boldsymbol{\Sigma}=(N_{xx} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)^T$$

•**Pure bending**, where:

$$\boldsymbol{\Sigma}=(0 \quad 0 \quad 0 \quad M_{xx} \quad 0 \quad 0)^T$$

3.2.1 Parameters defining the thresholds (key words NYT, NYC and MYF)

the parameters of the model defining the thresholds (k_0 , α) are expressed respectively starting from the values of the force of membrane in pure tension and of the bending moment threshold in pure bending corresponding to the appearance of the damage : N_D (key word NYT) and M_D (key word MYF).

3.2.1.1 Case pure uniaxial tension

In particular, for **the pure** uniaxial **elastic tension** with the appearance of the damage one can write the value of the threshold, cf [éq. 2.3-2]:

$$f_{d_j} = Y_j^m - k_0 = \epsilon_D^2 \left(\frac{\lambda_m}{4}(1-2\nu_m)^2(1-\gamma_{mt}) + \frac{\mu_m}{2}\left(1-\gamma_{mt}+\nu_m^2(1-\gamma_{mc})\right)\right) - k_0 = 0$$

ϵ_D being elastic strain with the appearance of the damage, having then $\epsilon_{yy}=-\nu_m \epsilon_D = \varepsilon_{zz}$ and $\xi_m(x,0,0)=1$, from where:

$$k_0 = \frac{\lambda_m(1-2\nu_m)^2(1-\gamma_{mt})+2\mu_m\left(1-\gamma_{mt}+\nu_m^2(1-\gamma_{mc})\right)}{4(\lambda_m(1-2\nu_m)+2\mu_m)^2} N_D^2 \text{, that is to say:}$$

$$k_0 = \frac{N_D^2}{4E_{éq}^m h(1+\nu_m)} \cdot \left((1-\nu_m)(1+2\nu_m)(1-\gamma_{mt})+\nu_m^2(1-\gamma_{mc})\right) \qquad (3.2\text{-}1)$$

having

$$N_D = (\lambda_m(1-2\nu_m)+2\mu_m)\epsilon_D = E_{eq}^m h \epsilon_D$$

Note::

It is pointed out that $\gamma_{mt} \le 1$ (cf éq. 3.2-2) and that $\gamma_{mc} \le 1$ so that the damage results well in a weakening of the stiffness. One also observes on [éq. 3.2-1] that one cannot have at the same time $\gamma_{mt}=1$ and $\gamma_{mc}=1$, because then $k_0=0$ (the model does not have elastic domain), or then it would be necessary to give $N_D=\infty$ (key word NYT).

By continuing the analysis made to3.1, just with the appearance of the damage, the longitudinal stress in the concrete is worth:

$$E_b \epsilon_D \frac{1-\nu_b \nu_m}{1-\nu_b^2}$$

so that one can express the threshold N_D (key word NYT) with the limit of cracking of the concrete σ_b^t in tension, while supposing validates the local criterion $\sigma_{xx} \le \sigma_b^t$:

$$N_D = \sigma_b^t \frac{E_{eq}^m h}{E_b} \cdot \frac{1-\nu_b^2}{1-\nu_b \nu_m} \quad (3.2\text{-}2)$$

3.2.1.2 Case pure uniaxial pressing

If one considers a compression test **uniaxial pure**, the value of the threshold of first damage, cf [éq. 2.3-2], is written while indicating by N_C the normal force corresponding:

$$k_0 = \frac{N_C^2}{4 E_{éq}^m h (1+\nu_m)} \cdot \left| (1-\nu_m)(1+2\nu_m)(1-\gamma_{mc}) + \nu_m^2 (1-\gamma_{mt}) \right|$$

One must thus necessarily have the relation:

$$\frac{N_C^2}{N_D^2} = \frac{(1-\nu_m)(1+2\nu_m)(1-\gamma_{mt}) + \nu_m^2 (1-\gamma_{mc})}{(1-\nu_m)(1+2\nu_m)(1-\gamma_{mc}) + \nu_m^2 (1-\gamma_{mt})}$$

Thus, with the value of $|N_C|$ (provided by the key word NYC), the coefficient γ_{mc} results from γ_{mt} and from N_D :

$$\gamma_{mc} = 1 - (1-\gamma_{mt}) \cdot \frac{N_D^2 (1-\nu_m)(1+2\nu_m) - N_C^2 \nu_m^2}{N_C^2 (1-\nu_m)(1+2\nu_m) - N_D^2 \nu_m^2} \quad (3.2\text{-}3)$$

Note::

It is necessary that $\gamma_{mc} \le 1$, just like $\gamma_{mt} \le 1$, cf [éq. 2.3-2]. One also recalls, cf [éq. 3.2-1], that one cannot have at the same time $\gamma_{mt} = 1$ and $\gamma_{mc} = 1$. [Éq. 3.2-3], the requirement is obtained:

$$|N_C| \le N_D \frac{\sqrt{(1-\nu_m)(1+2\nu_m)}}{\nu_m}$$

Equality in the relation above conduit with $\gamma_{mc} = 1$.
For reinforced concrete, having $\nu_m \approx 0{,}2$, this condition is written: $|N_C| < 5{,}2\, N_D$.

3.2.1.3 Case pure uniaxial distortion

Let us check the effect of these parameters on the appearance of the damage following a loading of **pure distortion** elastic $\epsilon_{xy} = \tilde{\epsilon}_1 = -\tilde{\epsilon}_2$, with $\epsilon_{xx} = \epsilon_{yy} = 0$. As follows: $N_{xy} = \tilde{N}_1 = -\tilde{N}_2 = 2 \mu_m \epsilon_{xy}$. The threshold of appearance of the damage [éq. 2.3-2] with the model GLRC_DM is reached for the shearing force:

$$N_{xy}^{D} = 2\frac{\sqrt{2\mu_m k_0}}{\sqrt{2-\gamma_{mc}-\gamma_{mt}}} = \frac{N_D}{1+\nu_m}\cdot\sqrt{\frac{(1-\nu_m)(1+2\nu_m)(1-\gamma_{mt})+\nu_m^2(1-\gamma_{mc})}{2-\gamma_{mc}-\gamma_{mt}}}$$

It can be useful to confront this prediction with that which one the model obtains with concrete `ENDO_ISOT_BETON` [R7.01.04], [2.3.2.3], with the limit of cracking of the concrete σ_b^t in tension:

$$N_{xyEIB}^{D} = 2\sigma_b^t h\sqrt{\frac{(1-\nu_b)(1+2\nu_b)}{(1+\nu_b)^2}} = 2N_D\frac{E_b(1-\nu_b\nu_m)}{E_{éq}^m(1-\nu_b^2)}\cdot\sqrt{\frac{(1-\nu_b)(1+2\nu_b)}{(1+\nu_b)^2}}$$

Note:

In any situation combined (compression+cisaillement, etc) into membrane pure, the statement of the threshold of first damage $Y_j^m=k_0$ of the model `GLRC_DM`, cf [éq. 2.3-2], according to the membrane forces $N_{xx}, N_{xy}\dots$, is made up by the same students' rag processions as the “usual” criterion in plane stresses of concrete material considered not to resist beyond σ_b^t. This comes from the choice of the formulation of model `GLRC_DM` out of membrane, in direct filiation of model `ENDO_ISOT_BETON`.

3.2.1.4 Case pure uniaxial bending

In addition, in **pure** uniaxial elastic bending only one damage mechanism is activated, according to its meaning, positive or negative. Here the positive bending is chosen, for which one always has $f_{d_1}<f_{d_2}$. The maximum value of elastic curvature κ_{xx} to the appearance of the damage is noted κ_D ($\kappa_{yy}=-\nu_f\kappa_{xx}$), such as only the threshold $f_{d_2}=0$ can be reached, while $f_{d_1}<0$ for each point of this trajectory of the loading, cf [éq. 2.3-2]:

$$f_{d_2} = Y_2^f-k_0 = \kappa_D^2\frac{(1-\gamma_f)}{\alpha}\left(\frac{\lambda_f}{2}(1-\nu_f)^2+\mu_f\right) - k_0$$

$\gamma_f\leq 1$ being obtained with the §3.2.2, from where:

$$\alpha = (1-\gamma_f)\frac{\lambda_f(1-\nu_f)^2+2\mu_f}{2(\lambda_f(1-\nu_f)+2\mu_f)^2}\frac{M_D^2}{k_0} \quad (3.2\text{-}4)$$

having

$$M_D=(\lambda_f(1-\nu_f)+2\mu_f).\kappa_D = \frac{E_{eq}^f h^3}{12}\kappa_D$$

As the reinforced concrete plate is supposed to be symmetric compared to the average average, one needs to make the identification only for positive bending (negative bending giving the same value).

By continuing the analysis made with the §3.1, just with the appearance of the damage, the longitudinal stress in the concrete is worth out of wall of the plate (it is known that then the damage progresses immediately in a good portion of the thickness of the section):

$$E_b \kappa_D h \frac{1-\nu_b \nu_f}{2(1-\nu_b^2)}$$

so that one can express the threshold M_D with the limit of cracking of the concrete σ_t^b (key word MYF):

$$M_D = \sigma_t^b \frac{E_{eq}^f h^2}{6 E_b} \cdot \frac{1-\nu_b^2}{1-\nu_b \nu_f} \quad (3.2\text{-}5)$$

3.2.1.5 uniaxial Cases tension-bending

Let us check the effect of these parameters on the appearance of the damage following a loading mixing **uniaxial** monotonous tension and **uniaxial** monotonous bending concomitant. The thresholds are written, for each variable of damage:

$$k_0 \geq Y_1 =$$
$$\epsilon_{xx}^2 \left(\frac{\lambda_m}{4}(1-2\nu_m)^2(1-\gamma_{mt}) + \frac{\mu_m}{2}\left(1-\gamma_{mt}+\nu_m^2(1-\gamma_{mc})\right)\right) + \kappa_{xx}^2 \nu_f^2 \mu_f \frac{(1-\gamma_f)}{\alpha}$$
$$k_0 \geq Y_2 =$$
$$\epsilon_{xx}^2 \left(\frac{\lambda_m}{4}(1-2\nu_m)^2(1-\gamma_{mt}) + \frac{\mu_m}{2}\left(1-\gamma_{mt}+\nu_m^2(1-\gamma_{mc})\right)\right) + \kappa_{xx}^2 \frac{(1-\gamma_f)}{\alpha}\left(\frac{\lambda_f}{2}(1-\nu_f)^2+\mu_f\right)$$

It is checked easily that $Y_1 < Y_2$: the first damage appears on the variable d_2 as expected. Let us exploit the results [éq. 3.2-1] and [éq. 3.2-3], the threshold $Y_2 = k_0$ is thus written:

$$\frac{N_{xx}^2}{N_D^2} + \frac{M_{xx}^2}{M_D^2} = 1 \quad (3.2\text{-}6)$$

It thus defines the elastic domain predicted by the model GLRC_DM in the quadrant (N_{xx}, M_{xx}) positive in an elliptic form.

By continuing the analysis made with the §3.1, just with the appearance of the damage, the longitudinal stress in the concrete is worth:

$$\frac{E_b}{1-\nu_b^2}\left(\epsilon_{xx}(1-\nu_b \nu_m) + \kappa_{xx}\frac{h}{2}(1-\nu_b \nu_f)\right)$$

While confronting this result with the limit of cracking of the concrete σ_t^b , one notes that one obtains an elastic domain in the quadrant (N_{xx}, M_{xx}) positive of polygonal form:

$$\frac{N_{xx}}{N_D} + \frac{M_{xx}}{M_D} = 1$$

It is known that this incipient damage is followed immediately in a model 3D of the reinforced concrete plate by the appearance of a zone damaged on a good share of the thickness.

This difference in prediction with the model GLRC_DM is inherent in the choice operated in [éq. 2.1-4] of an energy of elasto-damageable plate in membrane-bending, compound with the threshold [éq.2.3-12.3-1].

As this difference can vary between 0% and 30%, one suggests lowering the values numerical of N_D and M_D defined by [éq. 3.2-2] and [éq. 3.2-4] of 10%.

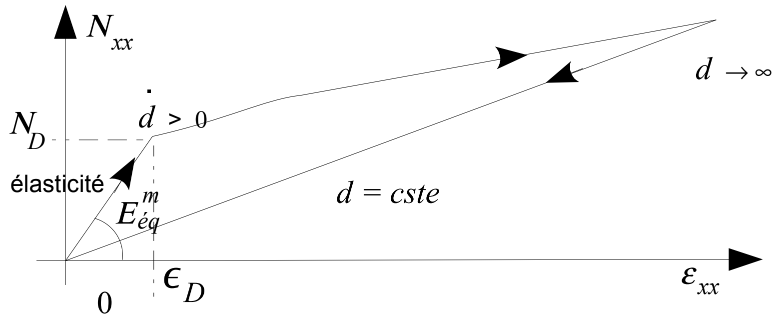
3.2.2 Parameters defining the slopes of damage (key words GAMMA_T and GAMMA_F): ENDO_NAISS

the coefficients $\gamma_{mt}\leq 1$ and $\gamma_f\leq 1$ (key word GAMMA_T and key word GAMMA_F) are respectively related to the slopes of monotonous pure uniaxial curve of tension and pure uniaxial bending, by considering the infinitesimal evolution just after appearance of the first damage ($d_j=0$, $\dot{d}_j>0$). This method of identification was established in DEFI_GLRC under key word simple METHODE_ENDO=ENDO_NAISS (cf [U4.42.06]).

3.2.2.1 Case pure uniaxial tension

Let us start with the case of **uniaxial pure** tension in $\epsilon_{xx}=\epsilon_D$, to the appearance of the first damage, with $\dot{\epsilon}_{xx}>0$ and $\dot{d}_1=\dot{d}_2$, to see Appear 3.2.2.1-a. The equilibrium conditions, the constitutive law and the evolution of the threshold give the following system of equations in $\dot{\epsilon}_{xx},\dot{\epsilon}_{yy},\dot{d}_1=\dot{d}_2$:

$$\begin{cases} \dot{N}_{xx}=\dfrac{E_{eq}^m h}{1-\nu_m^2}\left(\dot{\epsilon}_{xx}+\nu_m\dot{\epsilon}_{yy}-\epsilon_D\dot{d}_1(1-\gamma_{mt})(1-2\nu_m^2)\right) \\ \dot{N}_{yy}=\dfrac{E_{eq}^m h}{1-\nu_m^2}\left(\nu_m\dot{\epsilon}_{xx}+\dot{\epsilon}_{yy}-\nu_m\epsilon_D\dot{d}_1\left((1-\gamma_{mt})(1-2\nu_m)-(1-\gamma_{mc})(1-\nu_m)\right)\right)=0 \\ \dot{Y}_j^m=\mu_m\epsilon_D\left(\dot{\epsilon}_{xx}(1-\gamma_{mt})\dfrac{1-2\nu_m^2}{1-\nu_m}\;+\;\nu_m\dot{\epsilon}_{yy}\left((1-\gamma_{mt})\dfrac{1-2\nu_m}{1-\nu_m}\;-\;(1-\gamma_{mc})\right)\;+\right. \\ \qquad\left.\epsilon_D\dot{d}_1\left(\nu_m^2(1-\gamma_{mt})^2\dfrac{1-2\nu_m}{1-\nu_m}\;-\;(1-\gamma_{mt})(1-\nu_m)(1+2\nu_m)-\nu_m^2(1-\gamma_{mc})\right)\right)=0 \end{cases}$$



Appear 3.2.2.1-a **: Way in uniaxial tension: charge – discharge. Typical response of model GLRC_DM.**

One from of deduced the transverse strainrate in particular:

$$\dot{\epsilon}_{yy}=-\nu_m\left(\dot{\epsilon}_{xx}-\epsilon_D\dot{d}_1\left((1-\gamma_{mt})(1-2\nu_m)-(1-\gamma_{mc})(1-\nu_m)\right)\right)$$

Then:

$$\begin{cases}\dot{N}_{xx}=E_{eq}^m h\left(\dot{\epsilon}_{xx}\;-\;\dfrac{\epsilon_D\dot{d}_1}{1+\nu_m}\left((1-\nu_m)(1+2\nu_m)(1-\gamma_{mt})+\nu_m^2(1-\gamma_{mc})\right)\right)\\ 0\;=\;\dot{\epsilon}_{xx}\left((1-\gamma_{mt})(1+2\nu_m)(1-\nu_m)+\nu_m^2(1-\gamma_{mc})\right)-\\ \quad\epsilon_D\dot{d}_1\left(\nu_m^2(1-\gamma_{mc})+(1-\nu_m)(1+2\nu_m)(1-\gamma_{mt})-\nu_m^2(1-\nu_m)(1-\gamma_{mc})^2-2\nu_m^2(1-2\nu_m)(1-\gamma_{mt})(\gamma_{mc}-\gamma_{mt})\right)\end{cases}$$

From where:

$$\begin{cases}\dot{N}_{xx}=\dfrac{E_{éq}^m h.\dot{\epsilon}_{xx}}{1+\nu_m}\cdot\dfrac{A}{\nu_m^2(1-\gamma_{mc})+(1-\nu_m)(1+2\nu_m)(1-\gamma_{mt})-\nu_m^2(1-\nu_m)(1-\gamma_{mc})^2-2\nu_m^2(1-2\nu_m)(1-\gamma_{mt})(\gamma_{mc}-\gamma_{mt})}\\ avec\;:\;A=\\ (1-\nu_m^2)(1+2\nu_m)(1-\gamma_{mt})+\nu_m^2(1+\nu_m)(1-\gamma_{mc})-(1+2\nu_m-\nu_m^2-6\nu_m^3)(1-\gamma_{mt})^2-\nu_m^2(1-\gamma_{mc})^2-4\nu_m^3(1-\gamma_{mt})(1-\gamma_{mc})\\ \dot{d}_1=\dot{d}_2\;=\;\dfrac{\dot{\epsilon}_{xx}}{N_D}\cdot\dfrac{E_{éq}^m h.\left((1-\gamma_{mt})(1+2\nu_m)(1-\nu_m)+\nu_m^2(1-\gamma_{mc})\right)}{\nu_m^2(1-\gamma_{mc})+(1-\nu_m)(1+2\nu_m)(1-\gamma_{mt})+\nu_m^2(1-\nu_m)(1-\gamma_{mc})^2-2\nu_m^2(1-2\nu_m)(1-\gamma_{mt})(\gamma_{mc}-\gamma_{mt})}\end{cases} \quad (3.2\text{-}7)$$

One also recalls, cf [éq. 3.2-1], that one cannot have at the same time $\gamma_{mt}=1$ and $\gamma_{mc}=1$.

One thus obtains the slope of the curved normal force N_{xx} – extension ϵ_{xx} as of the appearance of the damage, according to the elastic characteristics and of γ_{mt}. It is observed in practice that the curved normal force – extension obtained with the model GLRC_DM after damage is quasi rectilinear. One can thus assimilate the slope obtained with [éq. 3.2-7] with that of the wished response, obtained by comparison with a curve resulting from another model of reinforced concrete plate, in the range of strain which one wishes to represent, possibly by neglecting the phase of negative hardening corresponding to the brutal phase of damage of the concrete in the section.

Let us note by $\chi_m=\dfrac{C_m^{\tan}}{E_{eq}^m h}$ this slope $C_m^{\tan}$ standardized by the elastic slope; by supposing the concrete completely degraded and the still elastic steels, one can express this slope: $\chi_m=\dfrac{2E_a S_a}{E_b h}$.

One must finally solve them [éq numerically. 3.2-7] and [éq. 3.2-3] to compute: γ_{mt} (key word GAMMA_T). One can also operate using a numerical method of identification per reference to one result obtained with another modelization.

3.2.2.2 Case pure uniaxial distortion

Let us study the case of **uniaxial pure** distortion in $\epsilon_{xy}=\tilde{\epsilon}_I=-\tilde{\epsilon}_{II}=\epsilon_{xy}^D$, to the appearance of the first damage, with $\dot{\epsilon}_{xy}>0$ and $\dot{d}_1=\dot{d}_2$. The threshold provides (one recalls, cf [éq. 3.2-1], that one cannot have at the same time $\gamma_{mt}=1$ and $\gamma_{mc}=1$) the statement of the distortion:

$$(\epsilon_{xy}^D)^2 \;=\; \frac{2\,k_0(1+\nu_m)}{E_{éq}^m h(2-\gamma_{mc}-\gamma_{mt})}$$

The constitutive law provides, having $\varepsilon_{zz}=0$, the normal force in the principal directions with 45°:

$$\dot N_I \;=\; \frac{E_{éq}^m h}{(1+\nu_m)}\left(\dot\epsilon_{xy}-\dot d_1\,\epsilon_{xy}^D(1-\gamma_{mt})\right)\;;\qquad \dot N_{II}\;=\;-\frac{E_{éq}^m h}{(1+\nu_m)}\left(\dot\epsilon_{xy}-\dot d_1\,\epsilon_{xy}^D(1-\gamma_{mc})\right)$$

from where in the initial reference:

$$\dot N_{xx}=\dot N_{yy}=-\frac{E_{éq}^m h(2-\gamma_{mt}-\gamma_{mc})\epsilon_{xy}^D\dot d_1}{2(1+\nu_m)}\;;\quad \dot N_{xy}=\frac{E_{éq}^m h}{(1+\nu_m)}\left(\dot\epsilon_{xy}-\epsilon_{xy}^D\dot d_1\frac{\gamma_{mc}-\gamma_{mt}}{2}\right)$$

The maintenance on the threshold of damage $\dot Y_j^m=0$ (loading case) gives:

$$0 \;=\; \mu_m\epsilon_{xy}^D(2-\gamma_{mc}-\gamma_{mt})\left(\dot\epsilon_{xy}-\dot d_1\,\epsilon_{xy}^D\right)$$

One obtains as follows:

$$\begin{cases}\dot N_{xx} \;=\; \dot N_{yy} \;=\; -\dfrac{E_{éq}^m h(2-\gamma_{mc}-\gamma_{mt}).\dot\epsilon_{xy}}{2(1+\nu_m)}\\ \dot N_{xy} \;=\; \dfrac{E_{éq}^m h(2-\gamma_{mc}+\gamma_{mt}).\dot\epsilon_{xy}}{2(1+\nu_m)}\\ \dot d_1=\dot d_2 \;=\; \dfrac{\dot\epsilon_{xy}}{\epsilon_{xy}^D}\end{cases} \qquad (3.2\text{-}8)$$

One sees by there that the damage makes lose the isotropy of the initial elastic behavior.

3.2.2.3 Case pure uniaxial bending

now Let us treat the case of **uniaxial** pure bending in $\kappa_{xx}=\kappa_D$, to the appearance of the first damage, with $\dot\kappa_{xx}>0$ and $\dot d_1=0,\ \dot d_2>0$. The equilibrium conditions, the constitutive law and the evolution of the threshold give the following system of equations in $\dot\kappa_{xx},\dot\kappa_{yy},\dot d_2$:

$$\begin{cases}\dot M_{xx}=\dfrac{E_{eq}^f h^3}{12(1-\nu_f^2)}\left(\dot\kappa_{xx}+\nu_f\dot\kappa_{yy}-\kappa_D\dot d_2\dfrac{1-\gamma_f}{\alpha}(1-\nu_f^2)\right)\\ \dot M_{yy}=\dfrac{E_{eq}^f h^3}{12(1-\nu_f^2)}\left(\nu_f\dot\kappa_{xx}+\dot\kappa_{yy}-\nu_f\kappa_D\dot d_2\dfrac{1-\gamma_f}{\alpha}(1-\nu_f)\right)=0\\ \dot Y_f=\dfrac{\kappa_D(1-\gamma_f)E_{eq}^f h^3}{12\alpha^2}\left(\alpha\left(\dot\kappa_{xx}+\dot\kappa_{yy}\dfrac{\nu_f}{1+\nu_f}\right)-\dot d_2\kappa_D\dfrac{1+\nu_f-\nu_f^2}{1+\nu_f}\right)=0\end{cases}$$

One from of in particular deduced the velocity of variation of transverse curl:

$$\dot{\kappa}_{yy}=-\nu_f\left(\dot{\kappa}_{xx}-\kappa_D\dot{d}_2\frac{1-\gamma_f}{\alpha}(1-\nu_f)\right)$$

Then:

$$\begin{cases}\dot{M}_{xx} &= \dfrac{E^f_{eq}h^3}{12}\left(\dot{\kappa}_{xx}-\kappa_D\dot{d}_2\dfrac{1-\gamma_f}{\alpha(1+\nu_f)}(1+\nu_f-\nu_f^2)\right)\\ \dot{d}_2 &= \dfrac{\alpha\dot{\kappa}_{xx}}{\kappa_D}\cdot\dfrac{1+\nu_f-\nu_f^2}{1+\nu_f-\nu_f^2-\nu_f^2(1-\nu_f)(1-\gamma_f)}\end{cases}$$

From where:

$$\begin{cases}\dot{M}_{xx} &= \dfrac{E^f_{eq}h^3\dot{\kappa}_{xx}}{12}\cdot\dfrac{\nu_f^3+\gamma_f(1+2\nu_f-2\nu_f^3)}{(1+\nu_f)(1+\nu_f-\nu_f^2-\nu_f^2(1-\nu_f)(1-\gamma_f))}\\ \dot{d}_2 &= \dfrac{12\alpha\dot{M}_{xx}}{E^f_{eq}h^3\kappa_D}\cdot\dfrac{(1+\nu_f)(1+\nu_f-\nu_f^2)}{\nu_f^3+\gamma_f(1+2\nu_f-2\nu_f^3)}\end{cases} \quad (3.2\text{-}9)$$

One thus obtains the slope of the curved moment M_{xx}– variation of curvature κ_{xx} as of appearance of the damage, according to the elastic characteristics and of γ_f. It is observed in practice that the curved moment – variation of curvature obtained with the model GLRC_DM after damage is quasi rectilinear. One can thus assimilate the slope obtained with [éq. 3.2-9] with that of the wished response, obtained by comparison with a curve resulting from another model of reinforced concrete plate, in the range of strain which one wishes to represent, possibly by neglecting the phase of negative hardening corresponding to the brutal phase of damage of the concrete in the section, to see Appear 3.2.2.1-a.

One must note that a fine model of reinforced concrete plate in bending describes as of the appearance of the damage of the concrete a phase of negative hardening in stress, and even in strain (if a suitable control is used, cf [R5.03.80]), which is followed by a phase where steels, still in their field of elasticity, take again the exerted loading, before entering the ultimate phase where the section perishes at the same time by the concrete and steels which plasticize.

Let us note by $\chi_f=\dfrac{12C_f^{\tan}}{E^f_{eq}h^3}$ this slope $C_f^{\tan}$ standardized by the elastic slope. One obtains then γ_f (key word GAMMA_F):

$$\gamma_f = \frac{\chi_f(1+\nu_f)(1+\nu_f-2\nu_f^2+\nu_f^3)-\nu_f^3}{1+2\nu_f-2\nu_f^3-\chi_f\nu_f^2(1-\nu_f^2)} \quad (3.2\text{-}10)$$

If one wishes to identify the slope $C_f^{\tan}$ starting from the characteristics of the materials and the section, one can adopt a simple model of section in pure bending where the concrete is damaged on the interval $]-h/2;\ \zeta h[$ and the elastic steels. The equilibrium of the section (not of normal force) gives the relation: $E_b h+\zeta(16E_aS_a-4E_bh)+4E_bh\zeta^2=0$, of which the resolution provides ζ. The relation moment – curvature is written then:

$$M_{xx} \;=\; \kappa_{xx}\left(2\,E_a\,S_a\,\rho_a^2\,h^2+\frac{E_b\,h^3}{24}(1-3\,\zeta+4\,\zeta^3)\right)$$

This slope describes also the discharge. Thus, like the model GLRC_DM the phase of brutal damage does not describe after the attack of the threshold of cracking, it is preferable to adopt like compromise the slope corresponding to the attack of the threshold of plasticization of steels (σ_y^{acier}) since the threshold of cracking of the concrete, [éq. 3.2-4]. One thus obtains:

$$C_f^{\tan} \;=\; \frac{E_b\,\sigma_y^{acier}\,(12\,E_a\,S_a\,\rho_a^2\,h^2+E_b\,h^3(1-3\,z^{ta}+4\,\zeta^3)/2)-E_{éq}^f\,h^3\,E_a(\zeta+\rho_a)}{6\,E_b\,\sigma_y^{acier}-12\,E_a\,\sigma_t^b\,(\zeta+\rho_a)}$$

3.2.3 Another method to define parameters GAMMA_T and GAMMA_F: ENDO_LIM

If this method of identification of the parameters γ_{mt}, γ_f appears complicated or not very relevant compared to with the problem which one wishes to deal, one can as well use the observation quoted with the §2.1 owing to the fact that the functions characteristic of the damage $\xi_m(.,d_j)$ and $\xi_f(.,d_j)$ vary from 1 with respectively γ_{mt} or γ_{mc}, and γ_f, for important damages $d_j \to +\infty$, to see Appear 3.2.2.1-a, which corresponds of course to important generalized strains. It is checked easily that if one is interested, for $d_j \to +\infty$ with the two cases: that of the load and that of the discharge, the two slopes are identical. This method of identification was established in DEFI_GLRC under key word simple METHODE_ENDO=ENDO_LIM (cf [U4.42.06]).

3.2.3.1 Case pure uniaxial tension

Let us start with the case of **uniaxial pure** tension, in load with $\dot{\epsilon}_{xx}>0$ and $d_1=d_2\to\infty$. One obtains then, like $N_{yy}=0$:

$$\varepsilon_{zz}=\frac{-\nu_m\gamma_{mt}}{1-2\,\nu_m+\nu_m\gamma_{mt}}\,\mathrm{tr}\,\varepsilon\;;\qquad \varepsilon_{yy}=\frac{-\nu_m\,\gamma_{mt}\,\varepsilon_{xx}}{\nu_m\gamma_{mt}+\gamma_{mc}(1-2\,\nu_m+\nu_m\,\gamma_{mt})}$$

from where:

$$N_{xx} \;=\; \frac{E_{éq}^m\,h\,\gamma_{mt}\,\varepsilon_{xx}}{(1+\nu_m)}\cdot\frac{(1-\nu_m+\nu_m\,\gamma_{mt})\left(\nu_m+\gamma_{mc}(1-2\,\nu_m+\nu_m\,\gamma_{mt})\right)-\nu_m^2}{(1-2\,\nu_m+\nu_m\,\gamma_{mt})\left(\nu_m\,\gamma_{mt}+\gamma_{mc}(1-2\,\nu_m+\nu_m\,\gamma_{mt})\right)}$$

what defines an asymptotic secant slope on the curve $N_{xx}\leftarrow\varepsilon_{xx}$ in the positive quadrant. For γ_{mt} taken in $[0,1]$, this slope remains positive. Numerically, it is noted that the variation of this slope with γ_{mt} is quasilinear.

3.2.3.2 Case pure uniaxial pressing

In the case of **uniaxial pure** compression, in load with $\dot{\epsilon}_{xx}<0$ and $\dot{d}_1=\dot{d}_2$, one deduces in a similar way:

$$\varepsilon_{zz}=\frac{-\nu_m\gamma_{mc}}{1-2\,\nu_m+\nu_m\gamma_{mc}}\,\mathrm{tr}\,\varepsilon\;;\qquad \varepsilon_{yy}=\frac{-\nu_m\,\gamma_{mc}\,\varepsilon_{xx}}{\nu_m\gamma_{mc}+\gamma_{mt}(1-2\,\nu_m+\nu_m\,\gamma_{mc})}$$

from where:

$$N_{xx} = \frac{E^m_{éq} h \gamma_{mc} \varepsilon_{xx}}{(1+\nu_m)} \cdot \frac{(1-\nu_m+\nu_m\gamma_{mc})(\nu_m+\gamma_{mt}(1-2\nu_m+\nu_m\gamma_{mc}))-\nu_m^2}{(1-2\nu_m+\nu_m\gamma_{mc})(\nu_m\gamma_{mc}+\gamma_{mt}(1-2\nu_m+\nu_m\gamma_{mc}))}$$

what defines an asymptotic secant slope on the curve $N_{xx} \leftarrow \varepsilon_{xx}$ in the negative quadrant.

3.2.3.3 Case pure uniaxial bending

In the case of **uniaxial** pure bending, in load with $\dot{\kappa}_{xx}>0$ and $d_1=0,\ d_2\rightarrow\infty$, like $M_{yy}=0$, one deduces in a similar way:

$$\kappa_{yy} = \frac{-\nu_f \gamma_f \kappa_{xx}}{1-\nu_f+\nu_f\gamma_f}$$

from where:

$$M_{xx} = \frac{E^f_{éq} h^3 \gamma_f \kappa_{xx}}{12(1+\nu_f)} \cdot \frac{1+\nu_f\gamma_f}{1-\nu_f+\nu_f\gamma_f}\text{, for } d_1=0,\ d_2\rightarrow\infty$$

what defines an asymptotic secant slope on the curve $M_{xx} \leftarrow \kappa_{xx}$ in the positive quadrant. For γ_f taken in $[0,1]$, this slope remains positive. Numerically, it is noted that the variation of this slope with γ_f is quasilinear.

3.2.3.4 Case pure uniaxial distortion

Finally let us consider the case of **uniaxial pure** distortion, in load with $\dot{\epsilon}_{xy}>0$ $\epsilon_I=-\epsilon_{II}=\epsilon_{xy}$, $\epsilon_{xx}=\epsilon_{yy}=0$ and $d_1=d_2\rightarrow\infty$. One obtains then, like $\varepsilon_{zz}=0$ the normal force on the principal directions with 45°:

$$N_I = \frac{E^m_{éq} h \gamma_{mt} \epsilon_I}{(1+\nu_m)} \quad ; \quad N_{II} = -\frac{E^m_{éq} h \gamma_{mc} \epsilon_I}{(1+\nu_m)}$$

from where in the initial reference:

$$N_{xx} = N_{yy} = \frac{E^m_{éq} h (\gamma_{mt}-\gamma_{mc}) \epsilon_{xy}}{2(1+\nu_m)} \quad ; \qquad N_{xy} = \frac{E^m_{éq} h (\gamma_{mt}+\gamma_{mc}) \epsilon_{xy}}{2(1+\nu_m)}$$

In general, one will have chosen $\gamma_{mt}<\gamma_{mc}$: the normal force will be of compression, supplemented of course shears N_{xy}, whose slope function of ε_{xy} is given by the second equation. This slope is weaker than that given in [éq. 3.2-8] for the incipient damage, as expected.

3.2.4 Another method to define parameters `GAMMA_T` and `GAMMA_F`: `ENDO_INTER`

a last method consists in considering that the parameters γ_{mt}, γ_{mc} and γ_f are equal to the relationship between the slope corresponding to the stiffness of the phase damaged, provided by the key word `PENTE` in `DEFI_GLRC` and the slope corresponding to the elastic stiffness.

This method of identification was established in DEFI_GLRC under key word simple METHODE_ENDO=ENDO_INTER (cf [U4.42.06]).

3.2.4.1 Case pure uniaxial tension

Let us start with the case of uniaxial pure tension $N_{xx} - \varepsilon_{xx}$, the evolution of the normal force when $d>0$ provides γ_{mt} from the slope :

$$E_{éq}^{m} h \gamma_{mt}$$

3.2.4.2 Case pure uniaxial pressing

In the case of **uniaxial pure** compression $N_{xx} - \varepsilon_{xx}$, evolution of the normal force when $d>0$ provides γ_{mc} from the slope :

$$E_{éq}^{m} h \gamma_{mc}$$

3.2.4.3 Case pure uniaxial bending

In the case of **uniaxial** pure bending $M_{xx} - \kappa_{xx}$, evolution of the bending moment when $d>0$ provides γ_f from the slope :

$$\frac{E_{éq}^{f} h^3 \gamma_f}{12}$$

3.2.5 Assessment of the identification of the parameters of model GLRC_DM

One draws up the balance sheet of the simplified analytical statements suggested in the paragraphs above, exploiting geometrical data and material of the reinforced concrete (see their definitions with sections 3.2.1 to 3.2.2) being used to establish the values to be given to the key words of model `GLRC_DM` in *Code_Aster*. It is useful to confront these estimates with the response given by another model of behavior – as the model `ENDO_ISOT_BETON` – on a simple case, to even operate a retiming on response curves, in the interval $\left[0,|\varepsilon_{xx}^{max}|\right]$ estimated in the study in sight.

Parameter	key word	Identification	Analytical statement suggested
E_{eq}^{m}	`E`	effective Young's modulus out of membrane (units: force/surface)	$E_{eq}^{m} = E_a\frac{S_a}{h}+E_b\cdot\frac{E_b h+E_a S_a}{E_b h+E_a S_a(1-\nu_b^2)}$
ν_m	`NU`	effective Poisson's ratio out of membrane	$\nu_m=\nu_b\cdot\frac{E_b h}{E_b h+E_a S_a(1-\nu_b^2)}$
E_{eq}^{f}	`EF`	effective Young's modulus in bending (units: force/surface)	$E_{eq}^{f}=\frac{3}{h}E_a S_a\chi_a^2+E_b\cdot\frac{E_b h+3E_a S_a\chi_a^2}{E_b h+3E_a S_a\chi_a^2(1-\nu_b^2)}$
ν_f	`NUF`	effective Poisson's ratio in bending	$\nu_f=\nu_b\cdot\frac{E_b h}{E_b h+3E_a S_a\chi_a^2(1-\nu_b^2)}$
Note: these values can be amended to privilege the loadings of shears, to see [éq. 3.1-5 and 8].			
N_D	`NYT`	threshold in pure tension with the appearance of the damage (units: force/length)	$N_D = \sigma_b^t\frac{E_{eq}^m h}{E_b}\cdot\frac{1-\nu_b^2}{1-\nu_b\nu_m}$
$\|N_C\|$	`NYC`	threshold in pure compression with the appearance of the damage (units: force/length)	$\|N_C\| \le N_D\frac{\sqrt{(1-\nu_m)(1+2\nu_m)}}{\nu_m}$, from where γ_{mc}
$\gamma_{mc}\le 1$	`GAMMA_C`	parameter of degradation of stiffness in compression	, from where $\|N_C\|$ to see §3.2.1.2.
M_D	`MYF`	threshold in pure bending with the appearance of the damage (units: force)	$M_D = \sigma_t^b\frac{E_{eq}^f h^2}{6E_b}\cdot\frac{1-\nu_b^2}{1-\nu_b\nu_f}$
Note:: these values of N_D and M_D can be reduced to limit the variation on the border of the elastic domain for the mixed loadings in tension-bending, to see §3.2.1. One can also check the evaluating of N_D on a situation of pure shears, cf §3.2.1.3.			
$\gamma_{mt}\le 1$	`GAMMA_T`	parameter of degradation of stiffness in tension	Solution of a nonlinear equation established with the local analytical slope of curve $N_{xx}\leftarrow\epsilon_{xx}$
$\gamma_f\le 1$	`GAMMA_F`	parameter of degradation of stiffness in bending	Solution of a nonlinear equation established with the local analytical slope of the curve $M_{xx}\leftarrow\kappa_{xx}$
Note: these values of γ_{mt} and γ_f can be identified either on the situation of incipient damage, see §3.2.2, or on the situation of maximum damage (asymptotic slope), see §3.2.3, or an average estimate, see §31. One can in this process privilege the loadings of shears.			

4 Checking

This model is checked by tests SSNS106A, B, C, D, E, F (see [bib8]), by comparison with a multi-layer modelization exploiting behavior ENDO_ISOT_BETON and of the elastic steel three-dimensions functions. The studied cases are:

ssns106 has	2 ways of loading traction and compression then compression-tension
ssns106 B	2 ways of loading after (bending - + then bending +), double cycle
ssns106 C	way of loading combined with cycling in tension 2 times faster than in bending
sns106 D	cycles pure shears
ssns106 E	cycles combined shears and bending
ssns106 F and G	cycles of traction and compression and pure shears with *the kit_ddi* GLRC_DM + VMIS_ISOT_LINE

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9. [R3.07.03] – Shell elements DKT, DST, DKQ, DSQ and Q4g.

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6 Appendix: Eigenvalues of the tensor of the strains

One considers an orthonormal base $\left(\mathbf{e}_i\right)_{i=1,2,3}$ of Euclidean space three-dimensional, and a tensor $\boldsymbol{\varepsilon}$ of order 2, symmetric, therefore diagonalisable. One notes ε^i_j the mixed components of the tensor $\boldsymbol{\varepsilon}=\varepsilon^i_j.\mathbf{e}_i\otimes\mathbf{e}^{*j}$ in the base $\left(\mathbf{e}_i\right)_{i=1,2,3}$. One uses the convention of Einstein on the repeated mixed indices.

One notes: $\mathbf{Q}_k, \eta_k$ the triplet of the normalized eigenvectors and eigenvalues associated with problem:

$$\boldsymbol{\varepsilon}.\mathbf{Q}_k=\eta_k\mathbf{Q}_k \Leftrightarrow \varepsilon^i_j.\left(\mathbf{Q}_k\right)^j\mathbf{e}_i=\eta_k\left(\mathbf{Q}_k\right)^i\mathbf{e}_i \quad pour\ k=1,2,3 \tag{6-1}$$

Note::

It is noted that: $(\boldsymbol{\varepsilon}+\xi\,\mathbf{Id}).\mathbf{Q}_k=(\lambda_k+\xi).\mathbf{Q}_k$, $\forall\xi\in\mathbb{R}$ *, therefore to add to* $\boldsymbol{\varepsilon}$ *any tensor diagonal does not modify the clean directions of* $\boldsymbol{\varepsilon}$ *.*

It is known that the eigenvectors $(\boldsymbol{Q_k})^j\boldsymbol{e}_j$ form an orthonormal base (principal reference):

$$\left(\mathbf{Q}_k\right)_i\mathbf{e}^{*i}.\left(\mathbf{Q}_l\right)^j\mathbf{e}_j=\delta_{kl}.\delta^{ij} \Rightarrow \left(\mathbf{Q}_k\right)_j\left(\mathbf{Q}_\ell\right)^j=\delta_{kl} \tag{6-2}$$

Let us differentiate these two relations:

$$(d\,\boldsymbol{\varepsilon})^i_j.\left(\mathbf{Q}_k\right)^j+\varepsilon^i_j.\left(d\,\mathbf{Q}_k\right)^j=d\,\eta_k\left(\mathbf{Q}_k\right)^i+\eta_k\left(d\,\mathbf{Q}_k\right)^i \quad pour\ k=1,2,3 \tag{6-3}$$

$$\left(d\,\mathbf{Q}_k\right)_j.\left(\mathbf{Q}_l\right)^j+\left(\mathbf{Q}_k\right)_j.\left(d\,\mathbf{Q}_l\right)^j=0 \tag{6-4}$$

Let us project the equation (6-3) on the eigenvector $\left(\mathbf{Q}_l\right)^i\mathbf{e}_i$ and use the equation (6-4):

$$\begin{aligned}
&(d\,\boldsymbol{\varepsilon})^i_j.\left(\mathbf{Q}_k\right)^j\left(\mathbf{Q}_l\right)_i+\varepsilon^i_j.\left(d\,\mathbf{Q}_k\right)^j\left(\mathbf{Q}_l\right)_i=d\,\eta_k\left(\mathbf{Q}_k\right)^i\left(\mathbf{Q}_l\right)_i+\eta_k\left(d\,\mathbf{Q}_k\right)^i\left(\mathbf{Q}_l\right)_i \quad pour\ k,l=1,2,3\\
\Leftrightarrow&(d\,\boldsymbol{\varepsilon})^i_j.\left(\mathbf{Q}_k\right)^j\left(\mathbf{Q}_l\right)_i+\eta_l.\left(d\,\mathbf{Q}_k\right)^j\left(\mathbf{Q}_l\right)_j=d\,\eta_k.\delta_{kl}+\eta_k\left(d\,\mathbf{Q}_k\right)^i\left(\mathbf{Q}_l\right)_i \quad pour\ k,l=1,2,3\\
\Leftrightarrow&(d\,\boldsymbol{\varepsilon})^i_j.\left(\mathbf{Q}_k\right)^j\left(\mathbf{Q}_l\right)_i=d\,\eta_k.\delta_{kl}+\left(\eta_k-\eta_l\right).\left(d\,\mathbf{Q}_k\right)^i\left(\mathbf{Q}_l\right)_i \quad pour\ k,l=1,2,3
\end{aligned} \tag{6-5}$$

From where:

$$\begin{cases}
d\,\eta_k=(d\,\boldsymbol{\varepsilon})^i_j.\left(\mathbf{Q}_k\right)^j\left(\mathbf{Q}_k\right)_i \quad pour\ k=1,2,3\\
\left(\eta_k-\eta_l\right).\left(d\,\mathbf{Q}_k\right)^i\left(\mathbf{Q}_l\right)_i=(d\,\boldsymbol{\varepsilon})^i_j.\left(\mathbf{Q}_k\right)^j\left(\mathbf{Q}_l\right)_i \quad pour\ k\neq l=1,2,3
\end{cases} \tag{6-6}$$

Let us note $\tilde{\varepsilon}^i_j$ the mixed components of a tensor in the base $\left(\mathbf{Q}_k\right)_{k=1,2,3}$. Then:

$$\begin{cases}
d\,\eta_k=(d\,\tilde{\varepsilon})^k_k \quad pour\ k=1,2,3\ (\ pas\ de\ sommation\ sur\ k\ )\\
\left(\eta_k-\eta_l\right).\left(d\,\mathbf{Q}_k\right)^i\left(\mathbf{Q}_l\right)_i=(d\,\tilde{\varepsilon})^k_l \quad pour\ k\neq l=1,2,3
\end{cases} \tag{6-7}$$

One checks obviously on the trail of $\mathrm{tr}\,\boldsymbol{\varepsilon}=\mathbf{Id}\underline{\underline{\otimes}}\boldsymbol{\varepsilon}$ tensor of the strains (which is independent of the selected reference):

$$\sum_{k=1,2,3} d\,\eta_k = \mathrm{tr}\left(d\,\tilde{\boldsymbol{\varepsilon}}\right) = \mathrm{tr}\left(d\,\boldsymbol{\varepsilon}\right) = d\left(\mathrm{tr}\,\boldsymbol{\varepsilon}\right) \tag{6-8}$$

Let us consider the density of free energy of isotropic elasticity:

$$\phi(\boldsymbol{\varepsilon})=\frac{1}{2}\lambda\,(\operatorname{tr}\boldsymbol{\varepsilon})^2+\mu\sum_{k=1,2,3}(\eta_k)^2 \quad (6\text{-}9)$$

then the state model gives the tensor of the stresses:

$$\boldsymbol{\sigma}=\phi_{,\varepsilon}(\boldsymbol{\varepsilon})=\lambda\,(\operatorname{tr}\boldsymbol{\varepsilon})\frac{d\operatorname{tr}\boldsymbol{\varepsilon}}{d\boldsymbol{\varepsilon}}+\mu\sum_{k=1,2,3}\eta_k\frac{d\eta_k}{d\boldsymbol{\varepsilon}}=\lambda\,(\operatorname{tr}\boldsymbol{\varepsilon})\,\mathbf{Id}+2\mu\sum_{k=1,2,3}\eta_k\cdot(\mathbf{Q}_k)^j(\mathbf{Q}_k)_i\cdot\mathbf{e}_j\otimes\mathbf{e}^{*i} \quad (6\text{-}10)$$

By applying the remark passed higher, the clean reference of the tensor of the stresses $\boldsymbol{\sigma}$ is thus identical to that of the strains $\boldsymbol{\varepsilon}$.

The principal stresses are thus naturally in the principal reference $(\boldsymbol{Q_k})^j\,\boldsymbol{e}_j$:

$$s_k=\tilde{\sigma}_k^k=\lambda\,tr(\boldsymbol{\varepsilon})+2\mu\,\eta_k \quad (6\text{-}11)$$

7 Description of the versions of the document

Version *Code_Aster*	Author (S) Organization (S)	Description of the modifications
8.4	D.Markovic EDF-R&D/AMA	initial Text
9.5	S.Fayolle EDF-R&D/AMA	Rewriting of the equations and reformulations of certain sentences
9.6	F.Voldoire, S.Fayolle EDF-R&D/AMA	Corrections of equations and reformulation partial of the model; Re-drafting of the § 3 (identification of the parameters). Drafting of the appendix: demonstration of derivative of the eigenvalues.
10.1	F.Voldoire, S.Fayolle EDF-R&D/AMA	Some small corrections and complements.
10.2	F.Voldoire EDF-R&D/AMA	Modifications of the definition of the position of reinforcements; addition of 3 local variables, §2.8.
11.1	F.Voldoire, S.Fayolle EDF-R&D/AMA	Addition of the methods of identification for `DEFI_GLRC` and the coefficient α_c .
11.3	F.Voldoire EDF-R&D/AMA	Addition of a transition p. 11 explaining the slope in uniaxial load.