# Constitutive law of reinforced concrete GLRC_DAMAGE plates

**Summarized:**

This documentation presents the theoretical formulation and the numerical integration of constitutive law GLRC_DAMAGE [bib1]. She is written in a total way in forces and moments resulting for modelizations in finite elements from plates. This model integrates the elastic behavior and endommageable in bending coming from the concrete and the elastoplastic behavior coming primarily from steel reinforcements starting from the material characteristics of the two materials and the composition of the section of the reinforced concrete plate. It results from it an elastoplastic behavior endommageable cyclic, adapted for dynamic studies of structures out of reinforced concrete. The model GLRC_DAMAGE current does not take into account the damage out of membrane and is thus not very precise when the requests of the plate are dominated by the effects out of membrane. On the other hand, the fracture of a plate depends especially on the steel reaction of, modelled by the elastoplastic part of the model. It is thus estimated that the fracture should be represented correctly even out of membrane. A model similar to that one, GLRC_DM, is able to better represent the damage out of membrane/bending, but does not take into account the phase of plasticization of steels and cannot thus be used to simulate the fracture.

The orthotropic elasticity induced by the orthogonal network of reinforcements is not taken into account that in the frame of a linear analysis; for the nonlinear analysis, one simplifies by building an approximate equivalent isotropic elasticity. At the present time, the general coherent tangent modulus is not available yet for this model.

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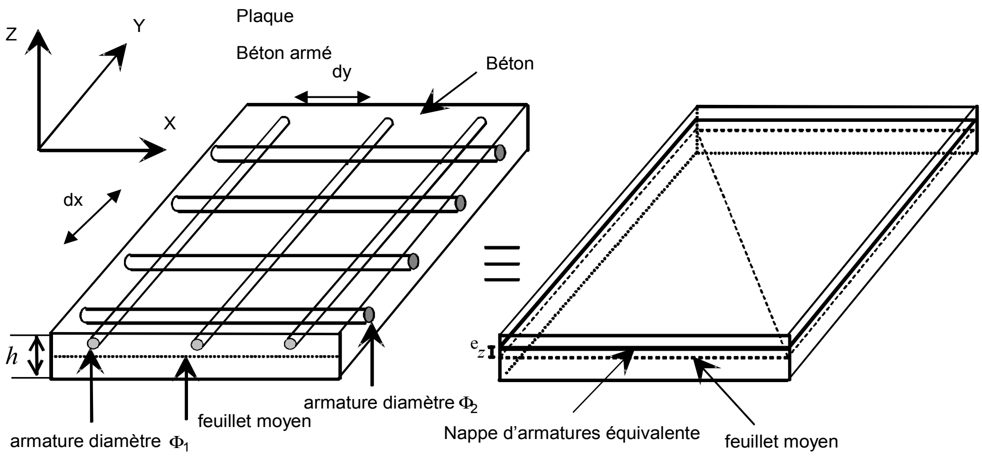
# 1 Introduction

## 1.1 Models of total behaviors

the total models and the represent the evolutions of the material within structure studied – beam and plate – on the basis of a relation between the generalized quantities of strains (extension, curvature, distortion) generalized forces (forces of membrane, bending, shears). These models “are fixed” as a preliminary using a fixed local analysis (for example using the results of limit analysis of the sections), according to the characteristics of the materials concrete and steel constituting the plate and the distribution of those in the section, cf [Appear 1.1-a]. The general diagram is the following:

$$d(\boldsymbol{\epsilon},\boldsymbol{\kappa},\boldsymbol{\gamma}) \underset{\substack{\text{loi globale de comportement}\\ \uparrow \\ \text{analyse locale}}}{\rightarrow} d(\boldsymbol{N},\boldsymbol{M},\boldsymbol{T}) \quad (1.1.1)$$

This local analysis must take account of the various couplings: for example the evolution in bending is dependant on the value of the normal force applied. The nonlinear equilibrium of structure is treated at the total level on the generalized forces, via the kinematics of plate considered.




**Appear 1.1-a: Pave out of reinforced concrete.**

As the local analysis is put in work only in preprocessing (in the frame of an analysis in monotonous load), there is not an immediate layer to return in the course of computation to the local analysis of the stresses starting from the generalized internal forces. Indeed, the dissipative character of the irreversible constitutive laws requires to store during cycles the evolution of the local variables in any time if one wants to calculate the stresses in a particular point. One could plan to launch in parallel the three-dimensional constitutive law according to [éq 1.1-1] and to integrate in the thickness to return to a total behavior, but the cost and the complexity of such a approach seem an obstacle. It should be noted that could be a layer to consider the mistake made by a total constitutive law. On the other hand, this approach is not adopted yet in *Code_Aster*.

This kind of model can be usefully validated by comparison with a direct analysis carried out with a local model.

## 1.2 Purposes of model GLRC_DAMAGE

One finds the formulation initial of the model total of reinforced concrete of plate GLRC_DAMAGE, established by P.Koechlin in 2002, in [bib1], [bib2] and [bib3].

This model was first of all developed for applications of dynamics with failure under impact of works out of reinforced concrete. The elastoplastic response of the model is essential for this kind of applications. Indeed, dissipation of energy by plasticization of steels is important. The taking into account of the damage by cracking of the concrete makes it possible to make more precise the first phases of the behavior nonlinear. In the frame of seismic applications, one can expect a reversed situation: the damage and the response after cracking are essential, while it is rare to go until mobilizing the generalized plasticization of steels. He seems however advantageous to have the same model to treat these two families of applications.

The formulation of the model is established in the frame of the thermodynamics of the irreversible processes. It combines plasticity with hardening, in particular brought by steels, and the damage brought by the concrete fissuring at the time of the bending of the plate. The plastic behavior is built on the basis of limit analysis in bending of a reinforced concrete plate. It is described using the frame of the generalized standard materials. On the basis of results experimental, cf [bib1], a linear kinematic hardening was selected to treat the cyclic behavior. The damage is introduced to represent the elastic loss of stiffness which takes place by cracking of the concrete before the plasticization of steels. The threshold of damage is supposed to be constant. This behavior is supposed to be independent the velocities of requests (dissipations are instantaneous).

# 2 Formulation of the model

One presents hereafter the formulation of model GLRC_DAMAGE, under the formalism of the thermodynamics of the irreversible processes.

One must note that the use of this model is associated with that of a shell element. If one chooses the family of finite elements **DKT** (supported modelization: DKTG), one adopts the theory of **Coils-Kirchhoff**, i.e. one considers no transverse distortion in the thickness of the plate. The model GLRC_DAMAGE could be usable with the finite elements of thick plate Q4G, but this extension was not carried out yet.

The mesh of the finite element is supposed to be placed on the average average of slab (with $z=0$ ).

For being able to use the model behavior GLRC_DAMAGE in two different types of analysis, one chose according to the case:

[1] ***for a linear elastic analysis of reinforced concrete plate:*** to take into account the orthotropy induced by the orthogonal network of steel reinforcements, as well as the coupling bending-membrane in the event of unequal three-dimensions functions of reinforcements, via a steel-concrete homogenized elastic behavior;

[2] ***for an elastoplastic analysis endommageable nonlinear of reinforced concrete plate:*** to neglect the orthotropy and the coupling bending-membrane in the phase of elasticity. This assumption makes it possible to simplify the model, by supposing that in the presence of strongly nonlinear phenomena, orthotropic elasticity becomes negligible, especially at the time of the modelization of the fracture. Moreover, in practice one expects that the walls, veils as well as the other structural elements are reinforced about in the same way between the two orthogonal principal directions. That causes to decrease the effect of the elastic orthotropy. On the other hand, one very often chooses asymmetrical reinforcements in order to optimize them according to the direction of the loading due to the inertia loading. That tends to induce a membrane-flexure coupling in elasticity besides that in plasticity. However, even if however the model asymmetry in elasticity neglects, its influence in elastoplasticity, the dominating behavior during the fracture, can be controlled through functions threshold, which they can be asymmetrical.

The model is defined by the description of the variables of state, which represent the mechanical system in each material point of the mean surface of the plate, the surface density of free energy which includes the form of the behavior models and the type of hardening, the statement of the plasticity criteria and damage, and the laws of irreversible evolution, deduced from the principle of the maximum work of Drücker.

## 2.1 Variables of state

the total variables of state are the following ones. First of all aggregate variables of strain:

[1] a membrane strain tensor: $\boldsymbol{\epsilon}$ defined in the tangent plane in the plate.

[2] a tensor of curvature: $\boldsymbol{\kappa}$ defined in the tangent plane in the plate.

Then local variables:

[1] two variables of damage associated with the parts higher, $d_1$ and lower, $d_2$ of the plate. They are put a ceiling to each one with a value, $d_1^{max}$ and $d_2^{max}$ .

[2] two tensors of plastic curvature associated with the plasticization of the beds of steel, superior and inferior $\boldsymbol{\kappa}_1^p$, $\boldsymbol{\kappa}_2^p$ .

[3] two strain tensors membrane plastic associated with plasticization with the beds with steel, superior and inferior $\boldsymbol{\epsilon}_1^p$, $\boldsymbol{\epsilon}_2^p$ .

[4] tensors of order 2 of kinematical local variables of hardening $\boldsymbol{\alpha}$ .

## 2.2 Free energy: linear elastic case

the surface density of free energy is an additive statement of the contributions elastic of membrane and bending:

$$\Phi_e^S=\frac{1}{2}\boldsymbol{\epsilon}:\boldsymbol{H}_m:\boldsymbol{\epsilon}+\frac{1}{2}\boldsymbol{\kappa}:\boldsymbol{H}_f:\boldsymbol{\kappa}+\boldsymbol{\epsilon}:\boldsymbol{H}_{mf}:\boldsymbol{\kappa} \quad (2.2.1)$$

the tensors $\boldsymbol{H}_m$ $\boldsymbol{H}_f$, $\boldsymbol{H}_{mf}$ (coupling bending-membrane in the event of unequal three-dimensions functions of reinforcements in the thickness) are described with [§3.1]. In the actual position of the model, one supposes that:

$$\boldsymbol{H}_{mf}=\boldsymbol{0}$$

thus that the plate is symmetric and that there is no elastic membrane-flexure coupling. In the model, membrane-flexure coupling can appear only because of one evolution towards elastoplasticity (cf §2.5).

## 2.3 Free energy: elastoplastic case endommageable

In this case, one neglects the elastic orthotropy induced by reinforcement in the two directions of the plane, as well as the elastic coupling bending-membrane (for dissymmetrical three-dimensions functions of reinforcements). One thus compares steel reinforcements to an isotropic elastic membrane, cf [Appear 1.1-a].

The surface density of free energy is an additive statement of the contributions elastoplastic of membrane, elastoplastic endommageable of bending, and kinematic hardening:

$$\Phi_{epd}^S(\boldsymbol{\epsilon},\boldsymbol{\epsilon}^p,\boldsymbol{\kappa},\boldsymbol{\kappa}^p,d_1,d_2,\boldsymbol{\alpha})=\Phi_{e,m}^S(\boldsymbol{\epsilon}-\boldsymbol{\epsilon}^p)+\Phi_{ed,f}^S(\boldsymbol{\kappa}-\boldsymbol{\kappa}^p,d_1,d_2)+\Phi_p^S(\boldsymbol{\alpha})+H(d_j-d_j^{max}) \quad (2.3.1)$$

with the energy of hardening:

$$\Phi_p^S(\boldsymbol{\alpha})=\frac{1}{2}\boldsymbol{\alpha}:\boldsymbol{C}:\boldsymbol{\alpha} \quad (2.3.2)$$

where $C$ is a tensor of kinematic hardening of Prager. In practice the tensor $C$ is diagonal, with a coefficient $C_m$ out of membrane and another $C_f$ in bending, one thus has:

$$\boldsymbol{C}=\begin{pmatrix} C_m & 0 & 0 & 0 & 0 & 0 \\ 0 & C_m & 0 & 0 & 0 & 0 \\ 0 & 0 & C_m & 0 & 0 & 0 \\ 0 & 0 & 0 & C_f & 0 & 0 \\ 0 & 0 & 0 & 0 & C_f & 0 \\ 0 & 0 & 0 & 0 & 0 & C_f \end{pmatrix}$$

In [éq 2.3.1], $H$ an indicating function of the field of admissibility of the thermodynamic potential indicates. Concretely, it is used to limit the evolution of the damage to the top of $d_j^{max}$. $d_j^{max}$ Are identified by [éq 3.2.11].

The densities of energy of membrane and bending are given by:

[1] Out of membrane:

$$\Phi_{e,m}^S(\boldsymbol{\epsilon}-\boldsymbol{\epsilon}^p)=\frac{1}{2}(\boldsymbol{\epsilon}-\boldsymbol{\epsilon}^p):\boldsymbol{H}_m:(\boldsymbol{\epsilon}-\boldsymbol{\epsilon}^p) \quad (2.3.3)$$

[2] In bending:

$$\Phi_{ed,f}^S(\boldsymbol{\kappa}-\boldsymbol{\kappa}^p,d_1,d_2)=\frac{\lambda_f}{2}tr(\boldsymbol{\kappa}-\boldsymbol{\kappa}^p)^2\xi_f(tr(\boldsymbol{\kappa}-\boldsymbol{\kappa}^p),d_1,d_2)+\mu_f\sum_{i=1}^{2}\widetilde{(\kappa-\kappa^p)}_i^2\xi_f(\widetilde{(\kappa-\kappa^p)}_i,d_1,d_2) \quad (2.3.4)$$

where one introduces the parameters of Lamé in bending $\lambda_f$ and $\mu_f$ :

$$\lambda_f=\frac{h^3}{12}\lambda$$
$$\mu_f=\frac{h^3}{12}\mu$$

$h$ being the thickness of the plate and $\lambda$ , $\mu$ coefficients of Lamé of the homogenized material. $\widetilde{(\kappa-\kappa^p)}_i$ indicate the ième eigenvalue of $\boldsymbol{\kappa}-\boldsymbol{\kappa}^p$. One will note thereafter $\boldsymbol{\kappa}^e$ the tensor of variation of elastic curvature definite according to the assumption of partition of the strains in bending by: $\boldsymbol{\kappa}^e=\boldsymbol{\kappa}-\boldsymbol{\kappa}^p$ .
And finally, one defines the function characteristic of the damage in bending $\xi_f$ :

$$\xi_f(x,d_1,d_2)=\frac{1+\gamma d_1}{1+d_1}H(x)+\frac{1+\gamma d_2}{1+d_2}H(-x) \quad (2.3.5)$$

In this statement $H$ is the Heaviside function and $\gamma$ a parameter of the damage ranging between 0 and 1. This function $\xi_f$ characterizes the weakening of the stiffness by damage. It is decreasing for

$d_1$ , $d_2$ positive. It is convex (thanks to the choice of $\gamma$ , identified by the procedure described in [§3.2.2]) to ensure the stability of the “material” reinforced concrete of slab.

## 2.4 Elastoplastic constitutive law endommageable

elastoplastic constitutive law the endommageable (state model) provides the dual variables: the forces of membrane, the bending moments which are of the tensors of order 2 definite on the tangent level of the plate, irreversible forces of damage and the tensor of hardening. They are written:

[1] Force of membrane:

$$N=\frac{\partial \Phi_{epd}^{S}}{\partial \boldsymbol{\epsilon}}=\boldsymbol{H}_m:(\boldsymbol{\epsilon}-\boldsymbol{\epsilon}^p) \quad (2.4.1)$$

[2] Bending moment:

$$\boldsymbol{M}=\frac{\partial \Phi_{epd}^{S}}{\partial \boldsymbol{\kappa}}=\boldsymbol{H}_f^d(\boldsymbol{\kappa}-\boldsymbol{\kappa}^p, d_1, d_2):(\boldsymbol{\kappa}-\boldsymbol{\kappa}^p) \quad (2.4.2)$$

elasticity tensor membrane $\boldsymbol{H}_m$ is given in [§3.1], while $\boldsymbol{H}_f^d$ is the elasticity tensor endommageable which depends on the variables of damage $d_1$ , $d_2$ and also of the signs of certain components of $\boldsymbol{\kappa}^e=\boldsymbol{\kappa}-\boldsymbol{\kappa}^p$ (of the trace and the eigenvalues, in particular). One recalls that in [éq. 2.4.1] and [éq. 2.4.2] elastic membrane-flexure coupling is neglected (see [§2.2]). Moreover, because of the presence of the eigenvalues of the elastic curvatures in the statement of the free energy (see [éq. 2.3.4]), one calculates the generalized stresses by means of the equations [éq. 2.4.1], [éq. 2.4.2] in the clean reference. The details of the transformation between the references are available in [R7.01.32]. It is also specified that according to the isotropic assumption of elasticity membrane-flexure coupling is due only to the elastoplastic process through $\boldsymbol{\epsilon}^p$ and $\boldsymbol{\kappa}^p$ (cf §2.5).

**Note:**
*It is noted that the clean reference of the moments is the same one as that of the elastic curvatures. In the same way the clean reference of the forces of membrane is the same one as that of the elastic strain. In the absence of damage* $\xi_f(x, d_1, d_2)=1$ *: one finds well a behavior of elastic plate isotropic.*

[1] Forces of damage, for $j=1,2$ :

$$Y_j=-\frac{\partial \Phi_{epd}}{\partial d_j}=\frac{1-\gamma}{(1+d_j)^2}\left(\frac{\lambda_f}{2} tr(\boldsymbol{\kappa}^e)^2 H((-1)^j tr(\boldsymbol{\kappa}^e))+\mu_f \sum_i (\tilde{\kappa}_i^e)^2 H((-1)^j \tilde{\kappa}_i^e)\right) \quad (2.4.3)$$

that they definite $Y_j$ by [éq are noted. 2.4.3] are positive (it is a surface restitution of energy, of which the unit IF is J/m ²) if $\gamma\in[0,1]$ .

[2] Forces and irreversible moments of plasticity:

$$\boldsymbol{N}^p=\frac{\partial \Phi_{epd}^{S}}{\partial \boldsymbol{\epsilon}^p}=\boldsymbol{H}_m:(\boldsymbol{\epsilon}-\boldsymbol{\epsilon}^p)=\boldsymbol{N} \text{ and } \boldsymbol{M}^p=\frac{\partial \Phi_{epd}^{S}}{\partial \boldsymbol{\kappa}^p}=\boldsymbol{H}_f^d:(\boldsymbol{\kappa}-\boldsymbol{\kappa}^p)=\boldsymbol{M} \quad (2.4.4)$$

[3] Tensor of recall of kinematic hardening:

$$\boldsymbol{X}^m=\frac{\partial \Phi_{epd}^{S}}{\partial \boldsymbol{\alpha}_m}=-\boldsymbol{C}_m:\boldsymbol{\alpha}_m \text{ and } \boldsymbol{X}^f=\frac{\partial \Phi_{epd}^{S}}{\partial \boldsymbol{\alpha}_f}=-\boldsymbol{C}_f:\boldsymbol{\alpha}_f \quad (2.4.5)$$

## 2.5 Criteria – surfaces thresholds

### 2.5.1 Plasticity criterion

**the plasticity criterion** of Johansen with kinematic hardening is duplicated for the plasticity of the upper part (index 1) and the lower part (index 2) of the plate. This criterion couples plasticity out of membrane with that in bending. If $x$ and $y$ the directions of the orthogonal reinforcement of the concrete plate indicate, for $j=1,2$ , the criterion is written:

$$\begin{aligned} f_j^p(\boldsymbol{N}-\boldsymbol{X}^m, \boldsymbol{M}-\boldsymbol{X}^f) &= -(M_{xx}-X_{xx}^f-M_{jx}^p(N_{xx}-X_{xx}^m)) \\ &\times \quad (M_{yy}-X_{yy}^f-M_{jy}^p(N_{yy}-X_{yy}^m))+(M_{xy}-X_{xy}^f)^2 \leq 0 \end{aligned} \quad (2.5.1)$$

define $f_j^p$ Them a convex field (cf [bib4]) of reversibility, parameterized by 4 functions: $M_{jx}^p(N_{xx})$ and $M_{jy}^p(N_{yy})$ . These functions are built using the limit analysis of sections of reinforced concrete beam representative of the section of the studied plate, taken in the meaning of reinforcements of reinforcement, cf [bib1, bib2]. It is noted that only the differences between the state of requests and the tensor of recall in hardening intervene in the statement of the criterion. This is characteristic of the models with kinematic hardening.

The potential of dissipation associated with this criterion is given by:

$$\begin{gathered} \Psi_{tot}=\boldsymbol{N}:\dot{\boldsymbol{\epsilon}}+\boldsymbol{M}:\dot{\boldsymbol{\kappa}}-\dot{\Phi}_{epd}=\Psi_p+\Psi_d \\ \Psi_p=\boldsymbol{N}:\dot{\boldsymbol{\epsilon}}^p+\boldsymbol{M}:\dot{\boldsymbol{\kappa}}^p-\dot{\boldsymbol{\alpha}}_m:\boldsymbol{C}_m:\boldsymbol{\alpha}_m-\dot{\boldsymbol{\alpha}}_f:\boldsymbol{C}_f:\boldsymbol{\alpha}_f \\ \Psi_d=Y_1\dot{d}_1+Y_2\dot{d}_2 \end{gathered} \quad (2.5.2)$$

### 2.5.2 Criterion of damage

**the brittle criterion of** damage without hardening is defined by a scalar. This criterion is duplicated to differentiate positive bendings from negative bendings. He is written:

$$f_j^d(Y_j)=Y_j(\boldsymbol{\kappa}, d_j)-k_j \leq 0 \quad (2.5.3)$$

This criterion represents a convex field (cf [bib2]) of reversibility parameterized by the thresholds $k_1$ and $k_2$ which define the appearance of first cracks in bending of the reinforced concrete plate. Their unit IF is it $J/m^2$ . They correspond to a limitation of the surface density of elastic strain energy. This criterion is associated with the positive potential of dissipation:

$$\Psi_d(d_j, \dot{d}_j)=k_j\dot{d}_j \geq 0 \text{ and } \dot{d}_j \geq 0 \quad (2.5.4)$$

This criterion of damage is basic, but it is its combination with the effect of the damage on the elastic stiffness, cf the function $\xi_f(x, d_{1,} d_2)$ [éq. 2.3.5], which exploits the response of the model.

## 2.6 Models of yielding

the model of yielding is written (according to the normality rule with the criterion [éq. 2.5.1]):

$$\dot{\boldsymbol{\epsilon}}^p=\dot{\boldsymbol{\epsilon}}_1^p+\dot{\boldsymbol{\epsilon}}_2^p=\lambda_1^p\frac{\partial f_1^p}{\partial \boldsymbol{N}}+\lambda_2^p\frac{\partial f_2^p}{\partial \boldsymbol{N}} \quad (2.6.1)$$

$$\dot{\boldsymbol{\kappa}}^p=\dot{\boldsymbol{\kappa}}_1^p+\dot{\boldsymbol{\kappa}}_2^p=\lambda_1^p\frac{\partial f_1^p}{\partial \boldsymbol{M}}+\lambda_2^p\frac{\partial f_2^p}{\partial \boldsymbol{M}} \quad (2.6.2)$$

$$\dot{\boldsymbol{\alpha}}^m=\dot{\boldsymbol{\alpha}}_1^m+\dot{\boldsymbol{\alpha}}_2^m=\lambda_1^p\frac{\partial f_1^p}{\partial \boldsymbol{X}^m}+\lambda_2^p\frac{\partial f_2^p}{\partial \boldsymbol{X}^m}=-(-\dot{\boldsymbol{\epsilon}}_1^p-\dot{\boldsymbol{\epsilon}}_2^p)=\dot{\boldsymbol{\epsilon}}^p \quad (2.6.3)$$

$$\dot{\boldsymbol{\alpha}}^f=\dot{\boldsymbol{\alpha}}_1^f+\dot{\boldsymbol{\alpha}}_2^f=\lambda_1^p\frac{\partial f_1^p}{\partial \boldsymbol{X}^f}+\lambda_2^p\frac{\partial f_2^p}{\partial \boldsymbol{X}^f}=-(-\dot{\boldsymbol{\kappa}}_1^p-\dot{\boldsymbol{\kappa}}_2^p)=\dot{\boldsymbol{\kappa}}^p \quad (2.6.4)$$

where are $\lambda_j^p$ to them the plastic multipliers, positive or null, for positive bendings and the negative bendings. They are divided by flow out of membrane and that in bending. One deduces from [éq. 2.6.3] and [éq. 2.6.4], in a usual way in linear kinematic hardening, that the local variables of hardening out of membrane and bending are equal respectively to the strains and the plastic curvatures. It results from this the following relations on the tensors from recall out of membrane and bending:

$$\boldsymbol{X}^m=-\boldsymbol{C}_m:\boldsymbol{\epsilon}^p \quad (2.6.5)$$

$$\boldsymbol{X}^f=-\boldsymbol{C}_f:\boldsymbol{\kappa}^p \quad (2.6.6)$$

It should be noted that this choice of an identical tensor of $\boldsymbol{C}_m$ Prager at the same time in tension and compression is criticizable. Indeed, in plastic compression, the concrete and steel intervene, while in tension, only steel contributes (concrete being broken).
The plasticity criteria [éq. 2.4.1] can be reached at the same time (for specific schemes of bi--bending), therefore flow can take place since the two criteria reached at the same time. The condition of coherence gives two additional relations:

$$\lambda_j^p \dot{f}_j^p(\boldsymbol{N},\boldsymbol{M})=0 \quad \text{si} \quad \lambda_j^p>0 \quad \text{alors} \quad \dot{f}_j^p(\boldsymbol{N},\boldsymbol{M})=0 \quad (2.6.7)$$

## 2.7 Law of evolution of the variables of damage

the law of evolution of the damage in bending is written, for positive bendings and the negative bendings (according to the normality rule with the criterion [éq. 2.5.3]):

$$\dot{d}_j=\lambda_j^d\frac{\partial f_j^d}{\partial Y_j} \quad (2.7.1)$$

are $\lambda_j^d$ to Them the multipliers of damage, positive or null. That they are pointed out $Y_j$ , definite by [éq. 2.4.3], are positive by construction (restitution of energy).

The condition of coherence gives two additional relations:

$$\lambda_j^d \dot{f}_j^d(Y_j)=0 \quad \text{si} \quad \lambda_j^d>0 \quad \text{alors} \quad \dot{f}_j^d(Y_j)=0 \Leftrightarrow \dot{Y}_j=0 \quad (2.7.2)$$

the two variables of damage can evolve simultaneously.

# 3 Parameters of the model

With the total models such as GLRC_DAMAGE one seeks to have a simpler representation of the nonlinear phenomena, by means of the more effective and more robust numerical methods. Consequently, it is difficult to allot a physical meaning to all the parameters of the model, because most between them include several phenomena. Thus, it is strongly recommended that the parameters of the model are validated by a comparative study between approach GLRC_DAMAGE and a finer approach, the such modelizations by multifibre beams, shells multi-layer or 3D, on a part sufficiently representative of structure to be analyzed. Or else, the error of an analysis using the model GLRC_DAMAGE cannot be estimated nor controlled.

In any case, the parameters of the model are given in a way simplified using the analysis of the monotonous behavior of a reinforced concrete section, except for the linear elastic behavior where it is also possible to resort to an approach homogenized out of plate, cf for example [bib4]. It is supposed that the set of parameters describing the elastic behavior is identifiable independently of the parameters of plasticity and damage. Moreover, the methods of homogenization enable us to determine the total elastic behavior with a very good accuracy. One considers in [§3.1] two approaches to homogenize the elastic behavior: in one one makes the assumption of an isotropic equivalent medium and in the other the orthotropy is taken into account. Now, only the isotropic approach is available in *Code_Aster*. Moreover, it is only the isotropic approximation which can be used in combination with plasticity and the damage. In theory, an extension to the orthotropic behaviors into linear and nonlinear could be under consideration in a theoretical frame are equivalent. The limitation with the isotropic cases was selected on the one hand because the phenomenon of the orthotropy was considered to be negligible during the fracture, of which simulation is the principal goal of the model, and on the other hand to reduce the formulation of the model and the identification of the parameters.

The parameters of the nonlinear behaviors are more delicate to determine than the parameters of elasticity. For the damage that is simpler, because their number is smaller. On the other hand, for plasticity, the user must inform a function, determining the limiting flow moment according to the membrane force. Moreover, kinematic hardening is given by four tensors of Drucker, each one having three parameters. In this version their number is artificially tiny room by supposing that these tensors are the same ones for the two thresholds of plasticity. If the behaviors in elasticity and with damage can be identified without having of a modelization (or a test) of reference, for the plastic behavior that is strongly disadvised.

$h$ The height of the section (thickness of the plate) is noted. One and the $\Omega_x^{\sup}=A_x^{\sup}/d_x^{\sup}$ notes $\Omega_y^{\sup}=A_y^{\sup}/d_y^{\sup}$ densities of reinforcement in the two directions, cf [Appear 1.1-a], [Figure 3-a : 3-a]. $A_x^{\sup}$ (*resp*. $A_y^{\sup}$ ) is the area of the section of a steel bar in the direction $x$ (*resp*. $y$ ) of the higher three-dimensions function. One makes in the same way for the lower three-dimensions function: $\Omega_x^{\inf}=A_x^{\inf}/d_x^{\inf}$ $\Omega_y^{\inf}=A_y^{\inf}/d_y^{\inf}$ . In general, all the quantities having $sup$ while exposing correspond to the upper part of the plate, while that with $inf$ correspond at its lower part.

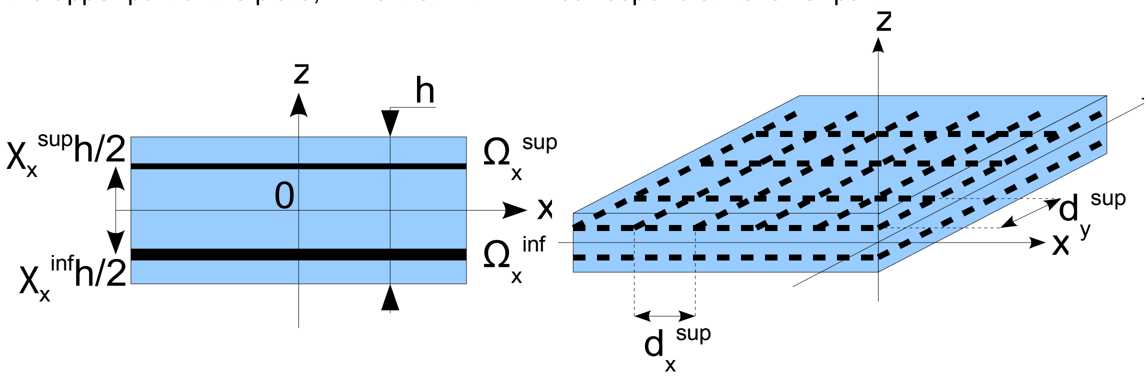



Figure 3-a : 3-a Cut reinforced concrete slab; sight in prospect.

The adimensional positions of the three-dimensions functions of reinforcement in the thickness check:

$$\chi_{x/y}^{\sup} \in ]0,1[ \text{ and } \chi_{x/y}^{\inf} \in ]-1,0[$$

the equivalent density of the reinforced concrete plate is defined by a simple average balanced by the densities $\rho_a$ , $\rho_b$ of the respective proportions of the two materials (model of the mixtures). It is used to establish the kinetic energy of the plate.

$$\rho_{éq} = \rho_b + \frac{\rho_a}{h}\left(\Omega_x^{\sup} + \Omega_x^{\inf} + \Omega_y^{\sup} + \Omega_y^{\inf}\right) \quad (3.1)$$

This equivalent density must be indicated under key word `ELAS` of operator `DEFI_MATERIAU` of definition of concrete material, with Young and the Poisson's ratio modulus of the concrete. This last data is used to draw up an estimation velocity of the waves, used for the control of time step in explicit integration (Flow condition):

| | E | NU | RHO |
|---|---|---|---|
| parameter | $E_b$ | $\nu_b$ | $\rho_{éq}$ |
| Units IF | [Pa] | without | [kg/m3] |

## 3.1 Identification of the parameters of linear elastic behavior

the linear elastic behavior is *a priori* orthotropic and just a membrane-flexure coupling. To carry out an elastic design preliminary to a nonlinear analysis, one wishes to represent the best possible this kind of behavior of reinforced concrete structure.

One proposes to identify the coefficients of linear elastic behavior of two ways:

•by **the orthotropic approach** where one builds the elastic matrix membrane-bending starting from the elastic characteristics of the concrete ( $E_b$ , $\nu_b$ ), of steel ( $E_a$ ) and the geometrical characteristics of the reinforced concrete section, cf (Figure 3-a : 3-a).

•by **the isotropic approach** where one determines the elastic parameters of the medium homogenized equivalent.

The total elastic model of reinforced concrete slab with coupling membrane and bending is written with the tensors $\boldsymbol{H}_m$ $\boldsymbol{H}_f$, $\boldsymbol{H}_{mf}$ and is given in the orthogonal local coordinate system related to reinforcement by:

$$\begin{pmatrix} N_{xx} \\ N_{yy} \\ 2\mathrm{N}_{xy} \\ M_{xx} \\ M_{yy} \\ 2\mathrm{M}_{xy} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} H_{1111}^m & H_{1122}^m & 0 \\ H_{1122}^m & H_{2222}^m & 0 \\ 0 & 0 & H_{1212}^m \end{pmatrix} & \begin{pmatrix} H_{1111}^{mf} & H_{1122}^{mf} & 0 \\ H_{1122}^{mf} & H_{2222}^{mf} & 0 \\ 0 & 0 & H_{1212}^{mf} \end{pmatrix} \\ \begin{pmatrix} H_{1111}^{mf} & H_{1122}^{mf} & 0 \\ H_{1122}^{mf} & H_{2222}^{mf} & 0 \\ 0 & 0 & H_{1212}^{mf} \end{pmatrix} & \begin{pmatrix} H_{1111}^{f} & H_{1122}^{f} & 0 \\ H_{1122}^{f} & H_{2222}^{f} & 0 \\ 0 & 0 & H_{1212}^{f} \end{pmatrix} \end{pmatrix} \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ 2\epsilon_{xy} \\ \kappa_{xx} \\ \kappa_{yy} \\ 2\kappa_{xy} \end{pmatrix} \quad (3.1.1)$$

the orthogonal local coordinate system related to reinforcement is defined with `AFFE_CARA_ELEM` (factor key word `COQUE`, key word `ANGL_REP`).

In this statement, $H_{ijkl}^m$ are the stiffness of membrane, are $H_{ijkl}^f$ to them the flexural stiffness and the $H_{ijkl}^{mf}$ are the stiffness of membrane-flexure coupling. The orthotropy imposes in this reference which the terms $H_{ij12}$ are null. In the case of two grids of symmetric reinforcements, there is decoupling membrane-bending: $H^{mf}_{ijkl}=0$ .

It is checked that necessarily: $H^{m}_{1111}H^{f}_{1111}-\left(H^{mf}_{1111}\right)^2>0$ , in the same way in the other direction. Same way one must have: $H^{m}_{1111}H^{m}_{2222}-\left(H^{m}_{1212}\right)^2>0 \quad H^{f}_{1111}H^{f}_{2222}-\left(H^{f}_{1212}\right)^2>0$ , always because of definite-positive character of the elasticity tensor.

•**Orthotropic approach (**unavailable in `the Code_Aster` now)

One directly builds the coefficients by the approximate following relations:

$$\begin{cases} H^{m}_{1111}=\dfrac{E_b h}{1-\nu_b^2}+E_a\langle\Omega\rangle_x \\ H^{m}_{2222}=\dfrac{E_b h}{1-\nu_b^2}+E_a\langle\Omega\rangle_y \end{cases} \begin{cases} H^{m}_{1122}=\dfrac{\nu_b E_b h}{1-\nu_b^2} \\ H^{m}_{1212}=\dfrac{E_b h}{1+\nu_b} \end{cases}$$

$$\begin{cases} H^{f}_{1111}=\dfrac{E_b h^3}{12(1-\nu_b^2)}+\dfrac{E_a h^2}{4}\langle\chi^2\Omega\rangle_x \\ H^{f}_{2222}=\dfrac{E_b h^3}{12(1-\nu_b^2)}+\dfrac{E_a h^2}{4}\langle\chi^2\Omega\rangle_y \end{cases} \begin{cases} H^{f}_{1122}=\dfrac{\nu_b E_b h^3}{12(1-\nu_b^2)} \\ H^{f}_{1212}=\dfrac{E_b h^3}{12(1+\nu_b)} \end{cases}$$

$$\begin{cases} H^{mf}_{1111}=\dfrac{E_a h}{2}\langle\chi\Omega\rangle_x \\ H^{mf}_{2222}=\dfrac{E_a h}{2}\langle\chi\Omega\rangle_y \end{cases} , \begin{cases} H^{mf}_{1122}=0 \\ H^{mf}_{1212}=0 \end{cases} \quad (3.1.2)$$

where one posed, to simplify, statements:

$$\langle\Omega\rangle_x=\Omega_x^{\sup}+\Omega_x^{\inf} \quad \langle\Omega\rangle_y=\Omega_y^{\sup}+\Omega_y^{\inf}$$

$$\langle\chi\Omega\rangle_y=\chi_y^{\sup}\Omega_y^{\sup}+\chi_y^{\inf}\Omega_y^{\inf} \quad \langle\chi\Omega\rangle_y=\chi_y^{\sup}\Omega_y^{\sup}+\chi_y^{\inf}\Omega_y^{\inf}$$

$$\langle\chi^2\Omega\rangle_x=\chi_x^{\sup^2}\Omega_x^{\sup}+\chi_x^{\inf^2}\Omega_x^{\inf} , \langle\chi^2\Omega\rangle_x=\chi_x^{\sup^2}\Omega_x^{\sup}+\chi_x^{\inf^2}\Omega_x^{\inf}$$

One supposes thus that steels do not bring stiffness in membrane distortion of the plate, nor in torsion.

**Note:**

The orthotropic approach is not available in the current version. It is planned to introduce it into the next evolutions of the model.

**•Isotropic approach**

One builds the total elastic matrix of éq. 3.1.1 while supposing:

$$\begin{pmatrix} H^m_{1111} & H^m_{1122} & 0 \\ H^m_{1122} & H^m_{2222} & 0 \\ 0 & 0 & H^m_{1212} \end{pmatrix} = \frac{E^m_{éq}\, h}{1-(\nu^m_{éq})^2} \begin{pmatrix} 1 & \nu^m_{éq} & 0 \\ \nu^m_{éq} & 1 & 0 \\ 0 & 0 & 1-\nu^m_{éq} \end{pmatrix} \quad (3.1.3)$$

for the membrane part and

$$\begin{pmatrix} H^f_{1111} & H^f_{1122} & 0 \\ H^f_{1122} & H^f_{2222} & 0 \\ 0 & 0 & H^f_{1212} \end{pmatrix} = \frac{E^f_{éq}\, h^3}{12(1-(\nu^f_{éq})^2)} \begin{pmatrix} 1 & \nu^f_{éq} & 0 \\ \nu^f_{éq} & 1 & 0 \\ 0 & 0 & 1-\nu^f_{éq} \end{pmatrix} \quad (3.1.4)$$

for the bending part. Membrane-flexure coupling is also neglected:

$$\begin{pmatrix} H^{mf}_{1111} & H^{mf}_{1122} & 0 \\ H^{mf}_{1122} & H^{mf}_{2222} & 0 \\ 0 & 0 & H^{mf}_{1212} \end{pmatrix} = \mathbf{0} \quad (3.1.5)$$

By comparison with [éq. 3.1.3] and [éq. 3.1.4], one chooses the following relations, by privileging the average behavior in the plane and while realising on the directions $X$ and *there*, which give the four elastic coefficients necessary, starting from the notations [éq 3.1.2]:

$$\nu^m_{éq} = \nu_b \frac{2 E_b h}{2 E_b h + E_a (1-\nu_b^2)(\langle \Omega \rangle_x + \langle \Omega \rangle_y)} \quad (3.1.6)$$

$$\nu^f_{éq} = \nu_b \frac{2 E_b h}{2 E_b h + 3 E_a (1-\nu_b^2)(\langle \chi^2 \Omega \rangle_x + \langle \chi^2 \Omega \rangle_y)} \quad (3.1.7)$$

$$E^m_{éq} = E_b \frac{\nu_b (1-(\nu^m_{éq})^2)}{\nu^m_{éq}(1-\nu_b^2)} \quad E^f_{éq} = E_b \frac{\nu_b (1-(\nu^f_{éq})^2)}{\nu^f_{éq}(1-\nu_b^2)},\ D_{éq} = \frac{E^f_{éq} h^3}{12(1-(\nu^f_b)^2)} \quad (3.1.8)$$

Among the two elastic approaches only the second (*II)* is currently available.

The elastic coefficients of the concrete ( $E_b$ , $\nu_b$ ) as those of steels $E_a$ are indicated under key word `ELAS` of `DEFI_MATERIAU`. The characteristics of the provision of steels in the concrete plate ( $\Omega_x^{\inf}$ $\Omega_x^{\sup}$ $\Omega_y^{\inf}$ $\Omega_y^{\sup}$ $\chi_x^{\inf}$ $\chi_x^{\sup}$ $\chi_y^{\inf}$, $\chi_y^{\sup}$ ) are indicated under the key word `THREE-DIMENSIONS FUNCTION` of `DEFI_GLRC`.

The height $h$ of the section is also provided by the operator `DEFI_GLRC`, key word `BETON`, operand `EPAIS`. The directions of the local coordinate system of orthotropy are defined by the operator `AFFE_CARA_ELEM`, key word `COQUE` with operand `ANGL_REP`. The linear elastic behavior is usable in nonlinear analysis under key word `COMP_INCR` with the operand `RELATION` = "`GLRC_DAMAGE`".

## 3.2 Identification of the parameters of elastoplastic behavior endommageable

The model of damage, [§2.3], is formulated under the assumption of isotropy (see [§3.1, II]), which is a reasonable approximation in most case. Moreover, it is admitted that the coupling bending-membrane (terms $H^{mf}_{ijkl}$ ) in the elastic phase of the behavior is negligible.

According to [§3.1, II] one identifies the elasticity tensor bending-membrane (cf the tensors $\boldsymbol{H}_m$ and $\boldsymbol{H}_f$ defined by [the éq 2.3.3] and [éq. 2.3.4]):

$$\begin{pmatrix} N_{xx} \\ N_{yy} \\ 2\mathrm{N}_{xy} \\ M_{xx} \\ M_{yy} \\ 2\mathrm{M}_{xy} \end{pmatrix} = \begin{pmatrix} \dfrac{E^m_{éq} h}{1-(\nu^m_{éq})^2}\begin{pmatrix} 1 & \nu^m_{éq} & 0 \\ \nu^m_{éq} & 1 & 0 \\ 0 & 0 & 1-\nu^m_{éq} \end{pmatrix} & (\mathbf{0}) \\ (\mathbf{0}) & \dfrac{E^f_{éq} h^3}{12(1-(\nu^f_{éq})^2)}\begin{pmatrix} 1 & \nu^f_{éq} & 0 \\ \nu^f_{éq} & 1 & 0 \\ 0 & 0 & 1-\nu^f_{éq} \end{pmatrix} \end{pmatrix} \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ 2\epsilon_{xy} \\ \kappa_{xx} \\ \kappa_{yy} \\ 2\kappa_{xy} \end{pmatrix} \quad (3.2.1)$$

### 3.2.1 Identification of the thresholds of damage

One must identify the thresholds of damage defined by [éq. 2.4.3] starting from the limits of cracking in monoaxial pure tension and monoaxial pure bending (in the directions positive $M^d_1$ and negative $M^d_2$ ) of slab out of reinforced concrete, themselves definite starting from the threshold of strength in tension of the concrete $\sigma_{ft} \geq 0$ (cf [bib2]). One with this intention uses the analytical resolution of the case of a concrete beam reinforced in the same way as slab. One preserves the approximation consisting in considering three-dimensions functions of reinforcements realised according to the directions $x$ and $y$. It will thus be admitted that one has $H^m_{1111} \equiv H^m_{2222}$ , $H^f_{1111} \equiv H^f_{2222}$ and $H^{mf}_{1111} \equiv H^{mf}_{2222}$ . Not to take into account this approximation leads to computations heavy and not very necessary.

In positive monoaxial pure bending $M_{xx}$ , with $N_{xx}=0$ $\epsilon_{\alpha y}=0$ , $\kappa_{xy}=0$ and $\kappa_{yy}=-\nu^f_{éq}\kappa_{xx}$ $\alpha=\{x,y\}$ , the damage of the concrete is reached initially in lower skin of reinforced concrete slab: $\sigma_{ft}=\dfrac{E_b}{1-\nu^2_b}(\epsilon_{xx}+\kappa_{xx} h/2)$ . There are thus the following relations, cf [éq. 3.1.2], [éq. 3.1.3] and [éq. 3.1.4]:

$$\epsilon_{xx}=-\frac{H^{mf}_{1111}}{H^m_{1111}}\kappa_{xx}$$

then

$$\begin{aligned} M_{xx} &= \left( H^f_{1111} - H^f_{1122}\nu^f_{éq} - \frac{(H^{mf}_{1111})}{H^m_{1111}} \right)\kappa_{xx} \\ &= \left( H^f_{1111}(1-(\nu^f_{éq})^2) - \frac{(H^{mf}_{1111})}{H^m_{1111}} \right)\kappa_{xx} \end{aligned} \tag{3.2.2}$$

$$\sigma_{ft} = \frac{E_b \kappa_{xx}}{1-\nu_b^2}\left(\frac{h}{2} - \frac{H^{mf}_{1111}}{H^m_{1111}}\right) = \frac{E_b M_{xx}}{1-\nu_b^2}\frac{H^m_{1111}}{(1-(\nu^f_{éq})^2)H^m_{1111}H^f_{1111} - (H^{mf}_{1111})^2}\left(\frac{h}{2} - \frac{H^{mf}_{1111}}{H^m_{1111}}\right) \tag{3.2.3}$$

from where:

$$M_1^d = (1-\nu_b^2)\frac{\sigma_{ft}}{E_b}\frac{(1-(\nu^f_{éq})^2)H^m_{1111}H^f_{1111} - (H^{mf}_{1111})^2}{H^m_{1111}}\left(\frac{h}{2} - \frac{H^{mf}_{1111}}{H^m_{1111}}\right)^{-1} \tag{3.2.4}$$

If the coupling bending-membrane is neglected $H^{mf}_{1111}=0$ , one has simply:

$$M_1^d = \frac{\sigma_{ft} h^2}{6}\frac{\nu_b}{\nu^f_{éq}}(1-(\nu^f_{éq})^2) \tag{3.2.5}$$

In the same way, in negative monoaxial pure bending $M_{xx}$ , with $N_{xx}=0$ $\epsilon_{\alpha y}=0$ , $\kappa_{xy}=0$ and $\kappa_{yy}=-\nu^f_{éq}\kappa_{xx}$ $\alpha=\{x, y\}$ , the damage of the concrete is reached initially in higher skin of reinforced concrete slab: $\sigma_{ft} = \frac{E_b}{1-\nu_b^2}(\epsilon_{xx} - \kappa_{xx} h/2)$ . There are thus the following relations:

$$\sigma_{ft} = -\frac{E_b \kappa_{xx}}{1-\nu_b^2}\left(\frac{h}{2} - \frac{H^{mf}_{1111}}{H^m_{1111}}\right) = -\frac{E_b M_{xx}}{1-\nu_b^2}\frac{H^m_{1111}}{(1-(\nu^f_{éq})^2)H^m_{1111}H^f_{1111} - (H^{mf}_{1111})^2}\left(\frac{h}{2} + \frac{H^{mf}_{1111}}{H^m_{1111}}\right) \tag{3.2.6}$$

from where:

$$M_2^d = -(1-\nu_b^2)\frac{\sigma_{ft}}{E_b}\frac{(1-(\nu^f_{éq})^2)H^m_{1111}H^f_{1111} - (H^{mf}_{1111})^2}{H^m_{1111}}\left(\frac{h}{2} + \frac{H^{mf}_{1111}}{H^m_{1111}}\right)^{-1} \tag{3.2.7}$$

If the coupling bending-membrane is neglected, one has simply:

$$M_1^d = -\frac{\sigma_{ft} h^2}{6}\frac{\nu_b}{\nu^f_{éq}}(1-(\nu^f_{éq})^2) \tag{3.2.8}$$

**Note::**

It is checked that: $M_1^d \geq 0$ and $M_2^d \leq 0$ .

It any more but does not remain to connect these moments of cracking to the thresholds $k_1$ , $k_2$ defined in [éq. 2.5.3]. Since the loading is exerted from a virgin state $d_1=d_2=0$ . From where forces of damage (restitution of energy), cf [éq. 2.4.3]:

$$Y_j=\frac{1-\gamma}{(1+d_j)^2}\left(\frac{\lambda_f}{2}tr(\boldsymbol{\kappa}^e)^2 H((-1)^j tr(\boldsymbol{\kappa}^e))+\mu_f\sum_i(\tilde{\kappa}_i^e)^2 H((-1)^j\tilde{\kappa}_i^e)\right)$$

where one applies $d_j=0$ $\boldsymbol{\kappa}^p=0$ $\boldsymbol{\kappa}^e=\boldsymbol{\kappa}$ , $\kappa_{xy}=0$ and $\kappa_{yy}=-\nu_{éq}^f\kappa_{xx}$ in order to obtain:

$$Y_j=\frac{1}{2}(1-\gamma)\left(\lambda_f(1-\nu_{éq}^f)^2+\mu_f\right)\kappa_{xx}^2$$

By means of [éq. 3.1.1] of the document [R7.01.32]:

$$\lambda_f=\frac{h^3\nu_f E_{éq}^f}{12(1-\nu_f^2)}\ ,\ \mu_f=\frac{h^3 E_{éq}^f}{24(1+\nu_f)}$$

one obtains:

$$\begin{aligned}
Y_j &= \frac{h^3}{24}(1-\gamma)E_{éq}^f\frac{1+\nu_{éq}^f(1-\nu_{éq}^f)}{1+\nu_{éq}^f}\kappa_{xx}^2\\
&= \frac{h^3}{24}(1-\gamma)E_{éq}^f\frac{1+\nu_{éq}^f(1-\nu_{éq}^f)}{1+\nu_{éq}^f}\left(\frac{M_{xx}}{H_{1111}^f(1-(\nu_{éq}^f)^2)}\right)^2\\
&= \frac{h^3}{24}(1-\gamma)E_{éq}^f\frac{1+\nu_{éq}^f(1-\nu_{éq}^f)}{1+\nu_{éq}^f}\left(\frac{12}{E_{éq}^f h^3}M_{xx}\right)^2\\
&= \frac{6}{h^3}\frac{(1-\gamma)}{E_{éq}^f}\frac{1+\nu_{éq}^f(1-\nu_{éq}^f)}{1+\nu_{éq}^f}M_{xx}^2
\end{aligned}$$

then:

$$k_1=\frac{6}{h^3}\frac{(1-\gamma)}{E_{éq}^f}\frac{1+\nu_{éq}^f(1-\nu_{éq}^f)}{1+\nu_{éq}^f}(M_1^d)^2\ ,\ k_2=\frac{6}{h^3}\frac{(1-\gamma)}{E_{éq}^f}\frac{1+\nu_{éq}^f(1-\nu_{éq}^f)}{1+\nu_{éq}^f}(M_2^d)^2 \quad (3.2.9)$$

These thresholds have as units IF the Joule.

The linear elastic behavior endommageable is usable in nonlinear analysis under key word COMP_INCR with the operand RELATION = "GLRC_DAMAGE". The parameters of operator DEFI_MATERIAU MF1 and MF2 correspond to $M_1^d$ and $M_2^d$ .

### 3.2.2 Identification of the slope of damage in bending

According to the relations developed in [§3.2] of [R7.01.32], the damaging slope is proportional to the parameter $\gamma$ :

$$p_f=\gamma\, p_{élas} \quad (3.2.10)$$

the parameter $\gamma$ corresponds to parameter GAMMA informed in operator DEFI_GLRC.

### 3.2.3 Identification of the maximum level of damage in bending

In [éq. 2.3.1], one envisaged to limit the level of damage in bending of the reinforced concrete plate, using the values of $d_j^{max}$ . One associates these values with the slopes moment-curvature, for the two directions of loading $j=1,2$ , and compared to the elastic slope, to also see [Appear 3.2.4-a]:

$$\frac{p_{2,j}}{p_{élas}}=\frac{1+\gamma d_j^{max}}{1+d_j^{max}}$$

from where, for $j=1,2$ :

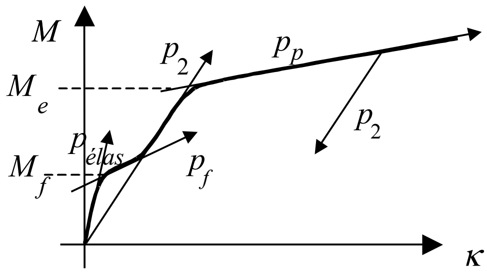
$$d_j^{max}=\frac{p_{élas}-p_{2,j}}{p_{2,j}-p_f} \quad (3.2.11)$$

In operator `DEFI_GLRC` one informs `QP1` and `QP2` for $p_{2,1}$ and $p_{2,2}$ .

### 3.2.4 Identification of the parameters of plastic behavior

For the behaviors in elasticity and with damage, it is possible analytically to obtain the values of the parameters from materials properties and geometrical of the reinforced concrete. To characterize the parameters of the plastic behavior, it is imperative to refer to a finer modelization (beam multifibre, shell multi-layer or 3D). A software of type MOCO (see [bib10]) is recommended for the automatic identification of models `GLRC`. In the long term, it is expected that such tools are integrated in *Code_Aster*. In the current version, the identification of the elastoplastic part is completely left with the care of the users.

For the identification, it is more reasonable to identify the parameters of nonlinear behavior from tests, numerical or experimental, with a monotonic loading. For example, one can use a test with the curvatures and the homogeneous times bending. Such a test is pseudo-unidimensional and can be entirely represented with only one graph, on which one can identify the thresholds of damage and plasticity just as the slopes corresponding to the various phases of loading, to see Appear 3.2.4-a. To measure the effect of the membrane force the monotonous test of bending must be combined with a loading out of membrane.

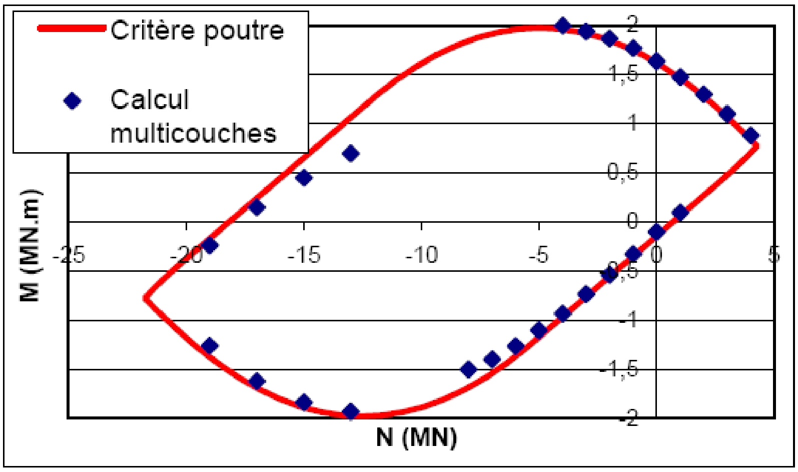



**Appear 3.2.4-a: Monotonous uniaxial bending.**

On Appear 3.2.4-a, one distinguishes five phases:

i) elastic phase characterized by the slope $p_{élas}$
ii) phase corresponding to the damage of the concrete (slope $p_f$ ),
iii) taken again stiffness due to steels after the attack of the maximum damage (slope $p_2$ )
iv) plasticization of steels (slope $p_p$ ).
v) discharge elastic: the value of slopes of discharge is in the interval $[p_{élas}, p_f]$ for phases I) to III) and applies $p_2$ to phase iv).

To describe the plastic behavior it is necessary to inform the functions $M^p_{jx}(N_{xx}-X^m_{xx})$ and $M^p_{jy}(N_{yy}-X^m_{yy})$ $j=1,2$, as functions "aster", FMEX1, FMEX2, FMEY1, FMEY2 in DEFI_GLRC. It is recommended by the command to define functions symbolic systems FORMULA, which must then be transformed into functions discretized by the command CALC_FONC_INTERP. Besides these functions, one must also inform their derivatives first and seconds, which, on the other hand, can be calculated by the operator CALC_FONCTION. Typically, they are functions close to parabolic functions. For a given direction, let us say $x$, the two functions $M^p_{1x}$ and $M^p_{2x}$ define the elastic domain, as in Appear 3.2.4-b.




**Appear 3.2.4-b: The elastic domain is between the two curves membrane bending moment/force. The graph the model presents a comparison between the thresholds used by GLRC_DAMAGE and those obtained by a computation multi-layer on the case of a beam.**

# 4 Numerical integration of the constitutive law

model GLRC_DAMAGE was initially conceived for analyses of fast dynamics having recourse to the explicit diagrams of temporal integration. The version of the model is almost identical to the initial version and is thus not optimized for computations in static or implicit dynamics. Consequently, the model risk to be not very robust and not very powerful for an analysis with the large ones time step. It is expected that the numerical integration of the model is improved.

## 4.1 Evaluating of the damage

The model of damage used in GLRC_DAMAGE was extended to the membrane-flexure coupling and put in the model GLRC_DM (see [R7.01.32]). One thus returns the reader to [R7.01.32] for the details concerning the numerical integration of the damage part. The computation in GLRC_DAMAGE is simpler, because one neglects the influence of membrane energy on the evolution of the damage.

## 4.2 Evaluating of yielding

the integration of the elastoplastic part is the most delicate part of the model. For time, one does not have method having a satisfactory robustness for great increments of (pseudonym) - time. One summarizes below the characteristics of the model presented in [§2], that one can solve with difficulty by the classical approaches, based on the method of Newton.

- **Double cones:** The elastic domain of each function threshold defined in [§2.5.1] is not convex and can be represented by a double cone within the space of generalized stresses (see Figure 4.2-a : 4.2-a). Obviously, it is only the cone close to the origin which represents the true elastic domain. Thus, the resolution of plastic admissibility must be carried out by adding two inequations:

$$g_1^p(\boldsymbol{N}-\boldsymbol{X}^m, \boldsymbol{M}-\boldsymbol{X}^f)=(M_{xx}-X_{xx}^f)-M_{1x}^p(N_{xx}-X_{xx}^m)+(M_{yy}-X_{yy}^f)-M_{1y}^p(N_{yy}-X_{yy}^m)\leq 0 \quad (4.2.1)$$

$$g_2^p(\boldsymbol{N}-\boldsymbol{X}^m, \boldsymbol{M}-\boldsymbol{X}^f)=(M_{xx}-X_{xx}^f)-M_{2x}^p(N_{xx}-X_{xx}^m)+(M_{yy}-X_{yy}^f)-M_{2y}^p(N_{yy}-X_{yy}^m)\leq 0 \quad (4.2.2)$$

With the inequations [éq. 4.2.1] and [éq. 4.2.2], one eliminates the solutions NON-physics from the plastic equations of admissibilities $f_j^p(\boldsymbol{N}-\boldsymbol{X}^m, \boldsymbol{M}-\boldsymbol{X}^f)\leq 0 \quad \lambda_j^p \geq 0$ . The functions $g_j^p$ define the two planes $g_j^p(\boldsymbol{N}-\boldsymbol{X}^m, \boldsymbol{M}-\boldsymbol{X}^f)=0$ , separating the cones NON-physics from the cones determining the elastic domain. On the other hand, the introduction of the inequations into the system prevents us from using the algorithms of the Newton type without important modifications.

- **Summits of the cones** : the other disadvantage, also related to the form of the elastic domain within the space of generalized stresses, comes from the two tops of the cones of the elastic domain (see Figure 4.2-a : 4.2-a). This property can also return the convergence of the difficult iterative algorithm.

Because of the two disadvantages mentioned above one cannot apply the algorithm of the radial return, generally used for the resolution of the problems of plasticity. A its core one implemented an algorithm of “cutting planes”, combined with dichotomy. The details of the algorithm are available in [bib1].

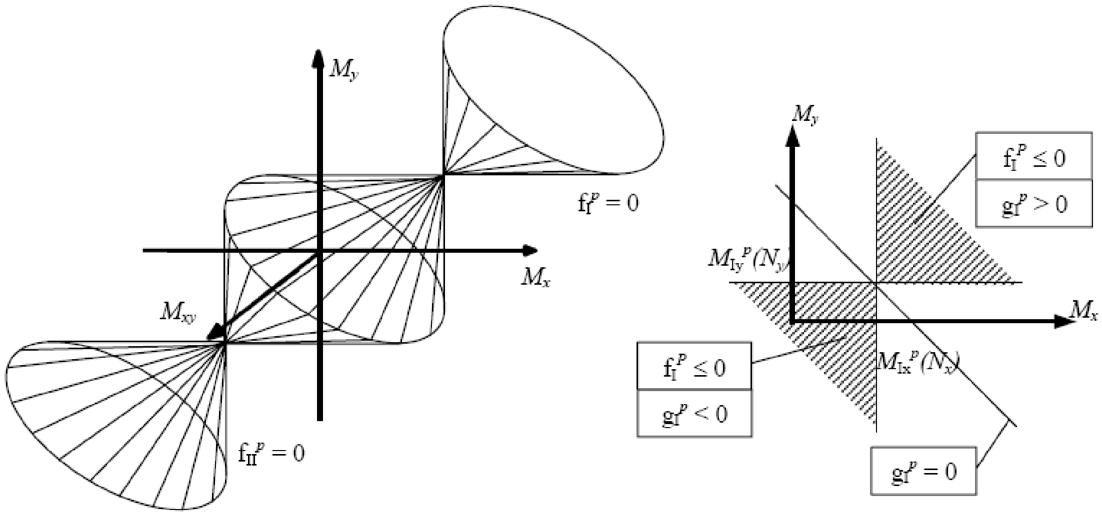



**Figure 4.2-a : 4.2-a Plastic threshold within the space of generalized stresses in form of two double cones.**

## 4.3 Evaluating of the tangent operator

Currently, the tangent operator of model GLRC_DAMAGE is not coherent and does not guarantee the quadratic convergence of the total process of Newton. Its establishment is based on an older version of the model. This part of the model is still in building site.

## 4.4 Local variables of the model

Us listels here local variables stored in each Gauss point in the model installation of.

| Number of local variable | physical meaning |
|---|---|
| membrane | V1 EXXP extension plastic |
| membrane | V2 EYYP extension plastic |
| membrane | V3 EXYP extension plastic |
| plastic | V4 KXXP cumulated curvature |
| V5 | KYYP cumulated plastic curvature |
| plastic | V6 KXYP cumulated curvature |
| V7 | cumulated Plastic dissipation |
| V8 | D1 variable of endom. upper face |
| variable | V9 D2 of endom. lower face |
| V10 | Dissipation of damage |
| V11 | angle of orthotropy |
| V12 | angle of orthotropy |
| V13 | angle of orthotropy |
| V14 | NXX force of kinematical membrane of recall |
| V15 | NYY force of kinematical membrane of recall |
| V16 | NXY force of kinematical membrane of recall |
| V17 | MXX moment of kinematical recall |
| V18 | MYY kinematical moment of recall |
| V19 | MXY kinematical moment of recall |

# 5 Checking

constitutive law `GLRC_DAMAGE` is checked by the cases following tests:

| | | | |
|---|---|---|---|
| linear static | SSLS126 | Bending of a reinforced concrete slab (GLRC_DAMAGE `models`) leaned on two with dimensions: mode of elastic beam | [V3.03.126] |
| static linear | SSLS127 | Bending of a reinforced concrete slab (GLRC_DAMAGE `models`) leaned on 4 with dimensions: mode of elastic plate | [V3.03.127] |
| dynamic clarifies nonlinear | SDNS106 | Transient response of a reinforced concrete slab: model `GLRC_DAMAGE` | [V5.06.106] |

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- [V3.03.126] SSLS126 – Bending of a reinforced concrete slab (glrc MODELS) leaned on 2 sides.
- [V3.03.127] SSLS127 – Bending of a reinforced concrete slab (glrc MODELS) leaned on 4 sides.
- [V5.06.106] SDNS106 – Transient response of a reinforced concrete slab (GLRC_DAMAGE models).
- F.VOLDOIRE. Homogenization of heterogeneous structures. Note EDF/DER/MMN HI-74/93/055, 10/27/1993.
- P. KOECHLIN, MOCO 2007: Documentation of reference and of use, Note EDF/R & D HT-T62-2007-00635-FR, 4/12/2007.


# 7 Description of the versions of the document

| Version Aster | Author (S) Organization (S) | Description of the modifications |
|---|---|---|
| 8.4 | D.Markovic<br>F.Voldoire<br>EDF-R&D/AMA | initial Text |
| 9.5 | S.Fayolle<br>EDF-R&D/AMA | Introduction of DEFI_GLRC, rewriting of the equations, reformulations of certain sentences,… |
| 10.2 | S.Fayolle<br>EDF-R&D/AMA | Cleaning and consistency with GLRC_DM |