Constitutive laws of the joints of the stoppings: JOINT_MECA_RUPT and JOINT_MECA_FROT.

Abstract:

This document describes surface models making it possible to model the fracture and friction between the lips of a crack or a joint. Model JOINT_MECA_RUPT is based on a cohesive formulation of the fracture, model JOINT_MECA_FROT is an elastoplastic version of the Mohr-Coulomb friction law in pure mechanics.

In mechanics those lean on the modelizations of standard joints XXX_JOINT . The models are dedicated to the modelization of the stoppings, more precisely of the joints concrete/rock or the joints between the studs of a stopping. According to the type of loading the use of one or other model can be selected for various parts of the work. To be able to simulate the behavior of the real stoppings in the frame of the same modelization, certain specificities of construction were introduced: in particular the procedure of keying-up (grouting of concrete enters the studs of a stopping: model JOINT_MECA_RUPT , option PRES_CLAVAGE) , or the effect of hydrostatic pressure without coupling due to the presence of fluid (option PRES_FLUIDE for the two models).

These models also admit a hydraulic modelization coupled (XXX_JOINT_HYME) taking into account the propagation of the uplifts with the interface stopping-rock.

Contents

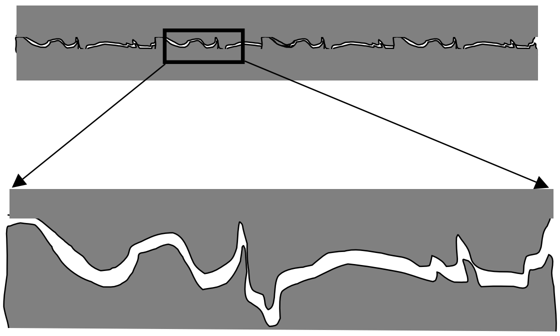
1 Introduction

the incidents which have occurred on concrete dams (Bouzey 1895, Malpasset 1959), as well as the results of sounding, raised that their stability, and consequently their security, depend very largely on the hydraulic behavior of the weakest zones of the valley-stopping group. Located on the level of discontinuities in structure and the rock, these weak points are mainly the faults of the zones of bearing, the concreting resumptions in the stopping, the contact concrete-rock of the foundation and the joined between the studs of the stopping. The structural mechanics behavior of its zones at the risk is strongly nonlinear, but thanks to their surface character, the industrial studies on the important works are complex but possible. Besides these difficulties, the method of construction of the stoppings, the techniques of keying-up/sawing used and its multiple points of drainage make of them works whose modelization by finite elements is all the more complex in a conventional computer code.

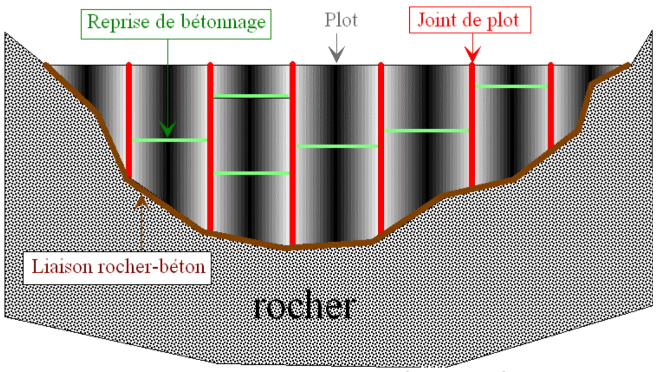
The two constitutive laws described in this document, make it possible to take into account principal non-linearities of the behavior of the works: the phase of crack opening (`JOINT_MECA_RUPT`) and the phase of sliding of its lips (`JOINT_MECA_FROT`).

1.1 Joints – weak links of the stoppings

As mentioned previously, the joints of the stoppings have a varied origin (Figure 1.1-a). Generally, one can represent the joint like a rough discontinuity possibly reinforced by a fill material.



Appear 1.1-a: Physical image of joint



Appears 1.1-b: Various types of joint of a stopping

the friction law of Coulomb, which uses one parameter (coefficient of kinetic friction), collects only one negligible part of the very complex structural mechanics behavior of such a structure. Indeed, except for friction, the joint displays the following important phenomena: the loss of tensile strength, elastic behavior with very weak displacement, the progressive disappearance of the peak of stress shear for a cyclic loading. The relevance of these phenomena strongly depends on several physical parameters in particular: level of roughness, mean size of the asperities, mechanical properties of fill materials and rock matrix (as the Young modulus, the Poisson's ratio, or the coefficient of kinetic friction).

The modelization joint supplements thus requires the introduction of a model depending on many parameters (for example a model with 18 parameters integrated in the Gefdyn *code* by the CIH **[1]** pages 75-77). The implementation of such a model in an implicit computer code not being possible, we propose simpler models, dependant on few parameters but which make it possible nevertheless to collect the essential behavior of the joint in most current conditions of use. These last are baptized: `JOINT_MECA_RUPT` and `JOINT_MECA_FROT`. They make it possible to model the behavior of the joints which one finds between the studs of an arch dam and/or with the interface between the gravity dam and his foundation. We make a brief description Ci of it below, before detailing them in the parts which follow.

1.2 Constitutive laws mechanics

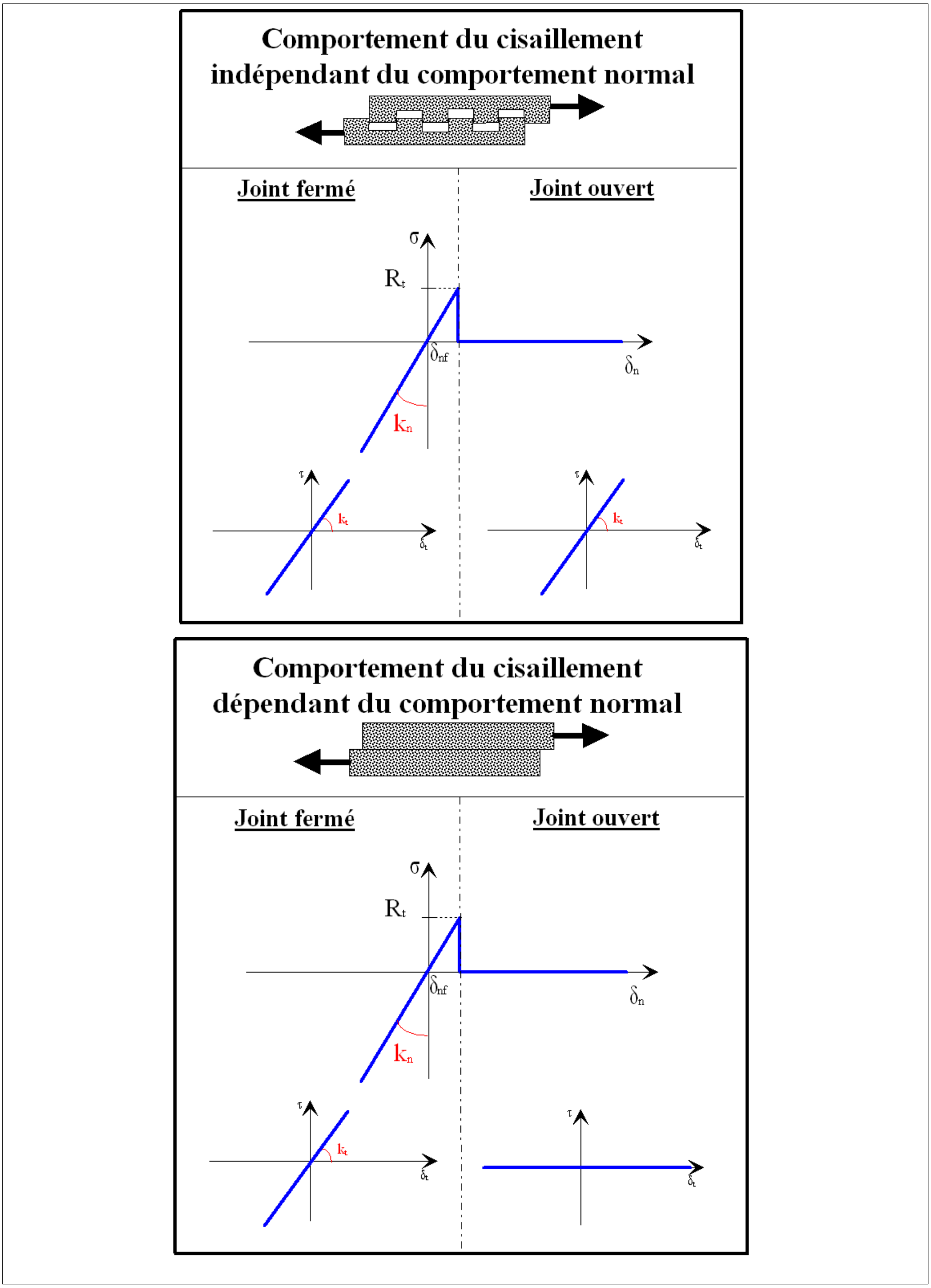
model JOINT_MECA_RUPT is a lenitive elastic model, whose form of normal behavior is based on the cohesive formulation of the fracture. It I opens the possibility of fracture in mode (tension) and takes into account the coupling between the normal opening and the tangencial stiffness. This model collects well the behavior of real joints with weak displacements, as long as the mode of sliding is not reached.

To identify the modes in which it is applicable one can take again the physical image of the joint. They are two rough interfaces possibly containing a fill material between its lips; this can be either of clay or the elements of the rock for the cracks stopping-foundation, or of the coulis of keying-up for the join-studs. A request of crack initially brings into plays the properties of fill material and the geometry of the asperities which define the behavior of structure in weak displacements. As long as the fill material is not damaged or as the asperities are not broken, the behavior of the joint remains elastic as well in opening, that in shears. However the parameters of stiffness norm and tangential, noted K_n and K_t , are not equivalent, because they utilize two distinct physical phenomena. The first depends mainly on the stiffness of fill material, the second depends on advantage of the stiffness in bending of the asperities. The joint has a tensile strength, noted σ_{max} , which can be connected to fill materials, but also with transverse frictions between the asperities of the two lips of crack.

The fracture of the joint occurs in a progressive way. Indeed, the joint is damaged initially by partially decreasing its stiffness before breaking completely. To quantify this phenomenon we introduce an adimensional parameter of penalization in fracture P_{rupt} , which represents a relative opening in softening compared to the elastic opening. For the values P_{rupt} weak fracture is brutal, the great values $P_{rupt} \gg 1$ the transition is more progressive, but this will increase the energy of initial dissipation significantly. The details of this behavior will be presented in the following parts.

In order to model the various types of profiles of the joints of stud of the stoppings (represented on the Figure 1.2-a below), we introduce into the user interface an additional $\alpha \in [0,2]$ parameter. This one binds the normal opening of joint with the fall of its tangencial stiffness. Physically it reflects the depth of the asperities and varies continuously enters $[0,2]$. $\alpha=0$ corresponds to the smooth interface without asperities (the tangencial stiffness falls to zero as of the normal opening of joint, to see Figure 1.2-a on the right). $\alpha=2$ represent another extreme case, where the interface is very rough with the infinite depth of the asperities is the profile of a joint of stud in crenel (Figure 1.2-a on the left). In this case the tangencial stiffness is not affected by the normal opening. The value by default is fixed at 1, which represents an intermediate situation.

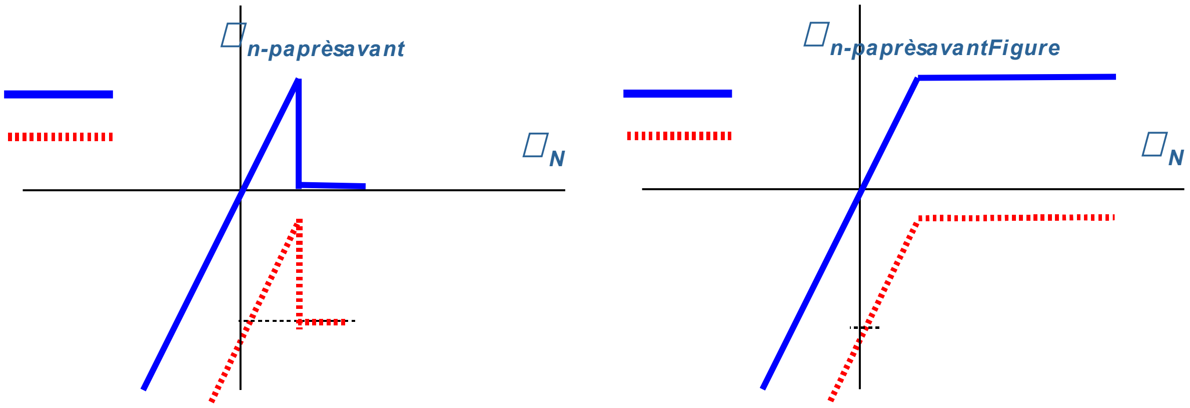
If the joint of advantage in pure shears is requested it will end up slipping with a certain coefficient of kinetic friction. Before passing in this plastic mode, one observes a peak of the frictional force in experiments. This phenomenon is related to the fact that to be able to slip the asperities must leave their inserted position (dilatancy). During this phase the rubbing contacts are not inevitably parallel to the surface of the lips, which increases the effective coefficient of kinetic friction. This procedure of "output" is accompanied besides the increase by normal displacement by the joint. If one repeats this cycle several times, the peak of stress attenuates and disappears completely. One notes that the deeper the asperities are, the more the peak of friction is important. One can even imagine the borderline case where in spite of the normal opening, the joint remains always elastic on the level of the tangential behavior.
Model JOINT_MECA_FROT does not take into account the peak, it represents only the phenomenon of pure sliding, characterized by the coefficient of kinetic friction μ and the adhesion c , which is related to the tensile strength $R_t = c/\mu$. The loss of tensile strength is not taken into account in the current version of friction.



Appear 1.2-a: Behavior normal and tangential according to profile of hydraulic

1.3 joint Coupling

to evaluate the stability of the stoppings it is important to be able to model the propagation of the uplifts between the rock and the foundation of the stopping. Two opportunities are given to user. Initially, a profile of uplift can be imposed, like a parameter among loading. This simple possibility makes it possible to study the stability of the stopping for a conservative hydraulic loading (the least favorable). In addition, the speed of computation has a significant advantage. One can thus test an extreme hydraulic profile without spending more time than in pure mechanical computation. Moreover, this functionality makes it possible to do a hydraulic calculation of sequence. This one consists in starting by a mechanical computation with an initial state of pressure in the joints. According to the damage of the latter, the profile of pressure (of which the form is given *a priori*) is updated. Once the modified pressure, the mechanical state of the joints evolves again, which can generate the fracture of some of them. The fluid is propagated then more easily and the profile of pressure undergoes an evolution again. This process is chained thanks to a fixed loop of point in the command file in order to obtain states converged mechanics and hydraulics.



1.3-a: Taking into account of the hydrostatic pressure. Normal shift of behavior model for the model JOINT_MECA_RUPT **(left) and model** JOINT_MECA_FROT **(right).**

From the theoretical point of view the introduction of the fluid into the joint modifies the normal mechanical stress $\sigma_n \rightarrow \sigma_n - p$. In practice the constitutive law in question is shifted downwards according to the value of pressure p in each point of integration (cf Figure 3.7).

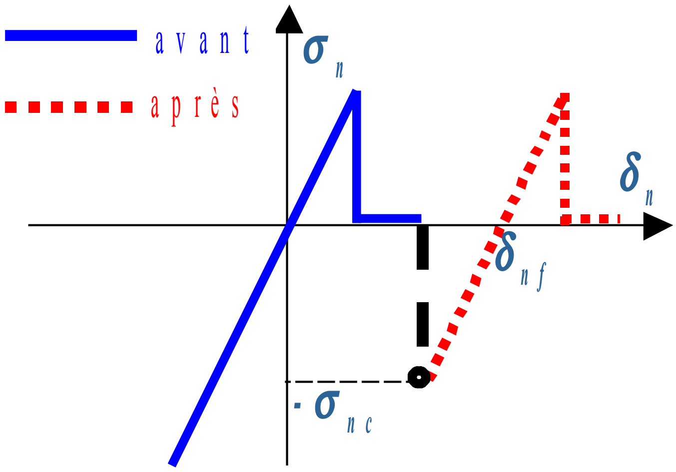
These models accept also a coupled hydraulic modelization, named xxx_JOINT_HYME. The difference enters the model with hydraulic sequence (presented above) door on the more precise taking into account of the action of the mechanics on the hydraulics. Indeed, in the first the profile models is given *a priori*. In the second the opening of the joint amends the flow law of the fluid, the profile of pressure is an unknown of the problem. During the hydraulic coupling, to model flow, the mechanical model is enriched in taking into account cubic flow by One tenth of a poise, which is regularized for very weak crack openings . Thus, the profile of pressure any more is not imposed, but is calculated during simulation. Besides the standard mechanical equation one solves simultaneously the following equation of flow: $\mathrm{div}\,\vec{w}=0\,;\vec{w}=\frac{\rho}{12\bar{\mu}}\delta_n^3\vec{\nabla}p$.

where $\vec{w}$ corresponds to hydraulic flux, δ_n is the normal opening of joint, finally ρ and $\bar{\mu}$ respectively indicate the density and the dynamic viscosity of the fluid.

1.4 Procedure of keying-up

keying-up is a key stage during the construction of an arch dam. It results in a grouting of concrete under pressure between the studs of the stopping. It is thus important to be able to model this process correctly. In practice the casting of the concrete breaks up into several bearings where the concrete is injected at various places and varied pressures. From the mechanical point of view keying-up is interpreted by a setting in compression of the lips of joint clavé until $\sigma_n = -\sigma_{nc}$ (pressure of the concrete injected, key word PRES_CLAVAGE).

option is directly included in the parameters of the constitutive law. Physically keying-up is accompanied by a process of solidification of the coulis injected, the procedure is modelled by the modification of the thickness of the joints concerned. If the joint is in strong compression initially, keying-up does not influence it. So on the other hand the joint is opened or not sufficiently compressed ($\sigma_n > -\sigma_{nc}$), keying-up will result in the change of the parameter of thickness of the noted joint δ_{nf} . And, consequently, of a translation of the normal stress.



Appear 1.4-a: Illustration of the procedure of clavageCette

The normal behavior of the joint after keying-up can be modified. Thus, the tensile strength can be restored (Figure 1.4-a) either partially, or completely according to the damage of the joints before the procedure of keying-up. The implemented procedure consists in not restoring it, this one keeps its current price.

1.5 Limit of application and vocabulary

the constitutive laws presented in this section are simple, robust, depend on few parameters and have the significant advantage to be based on a theoretical formalism tested in the scientific literature. The principal parameters are the following:

Physical parameter	Denomination Aster	Value advised for the concrete dam
K_n normal Stiffness	K_N	$K_n=3\cdot 10^{12}$ Pa/m
K_t Tangencial stiffness	K_T	Default $K_t=K_n$
σ_{max} Threshold of fracture	SIGMA_MAX	$\sigma_{max}=3$ MPa
P_{rupt} Penalization fracture	PENA_RUPT	Default $P_{rupt}=1$
P_{cont} Penalization contacts	PENA_CONTACT	Default $P_{cont}=1$
α relative Roughness	ALPHA	fluid $\alpha=1$
p_{flu} Default Pressure interns	PRES_FLUIDE	Default $p_{flu}=0$ Pa (absence of fluid)
σ_{nc} Pressure of keying-up	PRES_CLAVAGE	Default $\sigma_{nc}=-1$ Pa (not of keying-up)
$\bar{\mu}$ Dynamic viscosity of fluid	VISC_FLUIDE	$\bar{\mu}=10^{-3}$ Pa·s [1]
ρ Density	RHO_FLUIDE	$\rho=1000$ kg/m^3
ϵ_{min} minimal Opening of joint	OUV_MIN	$\epsilon_{min}=10^{-8}$ m
μ Coefficient of kinetic friction	MU	$\mu=1$
c Adhesion	ADHESION	Default $c=0$ Pa
K Hardening	PENA_TANG	Default $K=(K_n+K_t)\cdot 10^{-6}$

These models currently do not make it possible to model the behavior in the phase of transition fracture-friction. For the friction law, which has a tensile strength non-zero, the fracture of joint is not implemented. Moreover the modelization of the hydraulic coupling limits itself to the model of fracture.

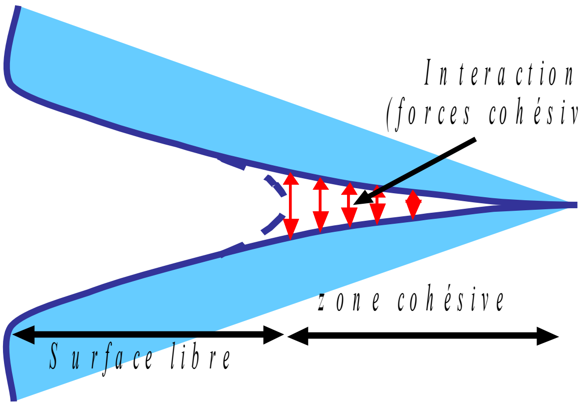
1 value being a hydraulic flux multiplier (see page 7), its unit can be selected in order to have mechanical flux and the forced of the same order of magnitude, which simplifies the analysis of relative errors.

2 Theoretical formulation of JOINT_MECA_RUPT

model JOINT_MECA_RUPT accepts a hydraulic coupled modelization, but these two phenomena can be treated separately. Initially one will describe the mechanical part of the model, which includes: fracture, contact, procedure of keying-up and imposed pressure. It leans on modelizations XXX_JOINT (R3.06.09).
Note: The two next sections present the general theoretical concept. In first reading one can omit them and pass directly to the section 13 , which provides sufficient elements to understand the numerical implementation of the model.

2.1 Cohesive model in mechanics

the selected theoretical frame to model the fracture of the joints of dams concrete is based on the cohesive models (R7.02.11, [Bar62], [Lav04]), because in brittle fracture, to raise the problem of infinite stresses in crack tip, one can introduce forces of cohesion which impose a criterion of starting in stress. The forces in evanescent matter are exerted then between the particles on both sides of the plane of separation of crack during its opening (see Figure 2.1-a).



Appear 2.1-a: Diagram of a cohesive crack

From the physical point of view one considers that the opening of crack costs an energy proportional to its length in 2D and on its surface in 3D. It is called energy of surface which one expresses using the density of energy $\Psi=\int_{\Gamma}\psi(\vec{\delta})\,d\Gamma$ [2]. The field of displacement to the equilibrium $\mathbf{u}$ is obtained by minimizing the sum of elastic strain energy Φ , the energy of surface, and the work of the external forces $W^{ext.}$. The solution is obtained by means of a variational approach of the fracture. The state which carries out the minimum of total energy corresponds in a state of mechanical equilibrium:

$$\min_{\mathbf{u}}\left(\Phi(u)+\Psi(\vec{\delta}(u))+W^{ext}\right)$$

The surface of discontinuity is discretized in 2D or 3D by of the finite elements of joint (confer to the documentation: R3.06.09). The jump of displacement in the element $\vec{\boldsymbol{\delta}}=(\delta_n,\delta_{t1},\delta_{t2})$ is a linear function of nodal displacements. The force3force[3] of cohesion which is exerted on the lips of crack is noted $\vec{\boldsymbol{\sigma}}$, it is defined by derivative of the density of energy of surface compared to the jump of displacement.
One calls cohesive model a relation enters $\vec{\boldsymbol{\sigma}}$ and the jump of displacement $\vec{\boldsymbol{\delta}}$.
Materials parameters the most relevant, which describes the joint of a stopping are:
- the two stiffness in requests norm K_n and tangential K_t , which characterize surface and the fill materials of crack;

2 Γ represents the contour of crack
3 per unit of area, homogeneous with a stress.

•and the critical stress with the fracture σ_{max}

One introduces, in addition, three adimensional numerical parameters: P_{rupt} , P_{cont} and α . The first controls the regularization of the slope of softening in fracture, the second the penalization of the contact and the third ensures a progressive resumption of tangential stresses according to the normal opening. This last can be associated with the relative size with the asperities with surfaces in contact: $\alpha\in[0,2]$.

In the cohesive models standards (R7.02.11) the energy of surface depends on the vector of déplacement4Le[4] and the forced are defined as derivatives first of energy:

$$\Psi(\vec{\delta})\equiv\int_{\Gamma}\psi(\delta_n,\vec{\delta}_t)\,d\Gamma \qquad \sigma_n=\frac{\partial\psi_n(\delta_n)}{\partial\delta_n}\;;\vec{\sigma}_t=\frac{\partial\psi_t(\vec{\delta}_t)}{\partial\vec{\delta}_t}$$

Unlike these cohesive models standards, in this model, only the normal part of the model is derived from energy from surface, whereas the tangential component of the model is given in an explicit way. [5] In both cases, the irreversibility of cracking is taken into account via a condition of increase in maximum normal opening of joint.

2.2 Energy of surface for the normal behavior

the density of energy of surface ψ , in a given point of crack, depends explicitly on the jump of normal displacement between the lips of crack δ_n . It also varies according to the state of the joint, which is taken into account via a local variable threshold $\kappa\geqslant 0$, which manages the irreversibility of cracking. The latter memorizes the greatest norm of the jump reached during the opening. Its law of evolution between two successive increments of loading - and + is written:

$$\kappa^{+}=\max(\kappa^{-},\delta_n^{+})$$

According to the value of opening of joint one will be able to find oneself in one of the three situations. The compressed joint is in the mode of contact; for a positive normal opening, if the latter exceeds the threshold one speaks about dissipative mode (dissipation of energy during cracking); finally in the intermediate case the joint is in a linear mode (discharge or linear refill without dissipation of energy).

$$\psi_n(\delta_n,\kappa)=\begin{cases}\psi_n^{con}(\delta_n,\kappa) & si \quad \delta_n<0\\ \psi_n^{lin}(\delta_n,\kappa) & si \quad 0\leqslant\delta_n<\kappa\\ \psi_n^{dis}(\delta_n) & si \quad \delta_n\geqslant\kappa\end{cases} \qquad \psi_t(\vec{\delta}_t)=\begin{cases}\psi_t^{fer}(\vec{\delta}_t) & si \quad \delta_n<0\\ \psi_t^{ouv}(\vec{\delta}_t) & si \quad \delta_n\geqslant 0\end{cases}$$

In a synthetic way energy of surface is written in the following way:

$$\psi(\delta_n,\kappa)=H(\delta_n-\kappa)\psi_n^{dis}(\delta_n)\;+\;H(\delta_n)\cdot H(\kappa-\delta_n)\psi_n^{lin}(\delta_n,\kappa)\;+\;+\;H(-\delta_n)\cdot\psi_n^{con}(\delta_n)$$ **éq 2.2-1**

where $H(x)$ is the function of Heaviside ($H(x)=0\;\text{si}\;x<0, H(x)=1\;\text{si}\;x\geqslant 0$).

One of the characteristics of model `JOINT_MECA_RUPT` is that, all the confused modes, the behavior is always linear on the level of the stresses. On the levels of energies we obtain quadratic functions, who are given in following an additive constant close, who it depends on the threshold.

4 vector to two components $\vec{\delta}_t=(\delta_{t1},\delta_{t2})$ indicates the tangential jump.

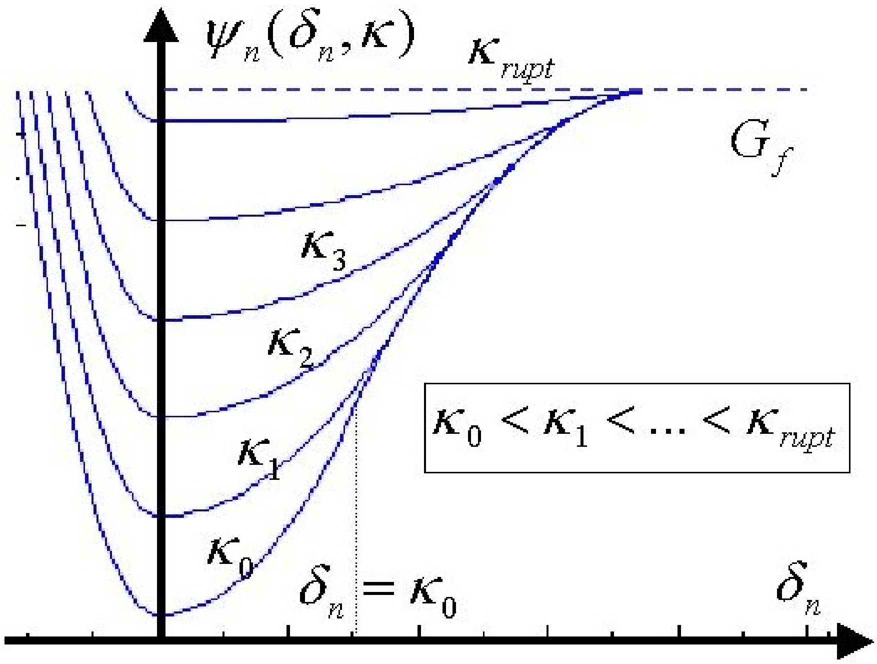
5 formulation thus leaves the energy formalism of the fracture. We can imagine a future improvement of the model by introducing a differentiable function of regularization there.

The normal behavior of joint is separate in three modes: contact, linear and dissipative. The function $\psi_n^{con}(\delta_n)=P_{con} K_n \delta_n^2/2$ ensures the condition of contact (not interpenetration of the lips of crack) it is regularized in order to obtain a better numerical convergence. While varying the parameter P_{con} , one can change the normal stiffness for the closed joint.

In linear mode if an existing crack evolves without dissipating energy, the density of corresponding energy depends only on the parameter of normal stiffness: $\psi_n^{lin}(\delta_n,\kappa)=K_a(\kappa)\delta_n^2/2$, where $K_a(\kappa)=(P_{rupt}^{-1}+1)\sigma_{max}/\kappa-K_n P_{rupt}^{-1}$ is a decreasing function of the threshold of fracture κ .

In the dissipative mode, in order to obtain a linear softening, the density of corresponding energy has a quadratic form according to the opening:

$$\psi_n^{dis}(\delta_n)=\begin{cases}\sigma_{max}(1+P_{rupt}^{-1})\delta_n \text{ - } K_n P_{rupt}^{-1}\delta_n^2/2 & si \quad \delta_n<\sigma_{max}(1+P_{rupt})/K_n \\ \sigma_{max}^2(1+P_{rupt})^2/(2\mathrm{P}_{rupt}K_n) & si \quad \delta_n\geqslant\sigma_{max}(1+P_{rupt})/K_n\end{cases}$$



Appear 2.2-a: Density of energy of surface according to the jump of displacement for various values of the threshold of damage κ

the additive constant makes relocate energies elastic ψ_n^{lin}; ψ_n^{con} defined previously so that it does not matter the state of damage of the joint one always obtains the same rate of refund of the energy (constant of Griffith G_f , to see Figure 2.2-a and eq. 2.2-2). It depends only on the threshold and does not affect the statement of the stresses.
P_{rupt} is introduced so that more it increases more the critical opening to the fracture is important and by consequence plus that made increase the dissipative energy of Griffith G_f .

In short: *the normal behavior of model* `JOINT_MECA_RUPT` *is controlled by the evolution of the density of energy surface, that Ci is appeared as a potential well. It comprises three principal modes: contact, elastic linear tension, damage/softening, whose profiles corresponding are approximated by quadratic functions in opening. The joint starts to be damaged when the normal stress reaches the* *breaking value $\sigma_n=\sigma_{max}$. More the joint is damaged more the energy well is flattened to see (Figure 2.2-a). The parameter of damage (the threshold) evolves to him only in this last mode on the basis of its initial value for the operational joint $\kappa_0=\sigma_{max}/K_n$ up to the ultimate value for the completely damaged joint $\kappa_{rupt}=\sigma_{max}(1+P_{rupt})/K_n$. P_{con} is a constant defines by the user who changes the level of penalization into contact (see Figure 2.3.2-a). P_{rupt} change energy dissipated per unit unit of area: $G_f=\sigma_{max}^2(1+P_{rupt})/(2K_n)$.*

$$\psi_n(\delta_n,\kappa)=\begin{cases}\sigma_{max}(1+P_{rupt}^{-1})(\kappa-\kappa_0)/2 + P_{con}K_n\delta_n^2/2 & si \quad \delta_n<0\\ \sigma_{max}(1+P_{rupt}^{-1})(\kappa-\kappa_0)/2 + \left[\sigma_{max}(1+P_{rupt}^{-1})/\kappa-K_nP_{rupt}^{-1}\right]\delta_n^2/2 & si \quad 0\leqslant\delta_n<\kappa\\ \sigma_{max}(1+P_{rupt}^{-1})(\delta_n-\kappa_0/2) - K_nP_{rupt}^{-1}\delta_n^2/2 & si \quad \kappa\leqslant\delta_n<\kappa_{rupt}\\ G_f & si \quad \delta_n\geqslant\kappa_{rupt}\end{cases}$$

éq 2.2-2:

This function is continuous and differentiable, which ensures the continuity of the stresses.

2.3 Vector forced

the vector forced in the element, noted $\vec{\boldsymbol{\sigma}}=(\sigma_n,\vec{\sigma}_t)$ [6], can be separate in several mode.

$$\sigma_n=H(\delta_n-\kappa)\sigma_n^{dis}+H(\kappa-\delta_n)H(\delta_n)\sigma_n^{lin}+H(-\delta_n)\sigma_n^{con}$$
$$\vec{\sigma}_t=H(\delta_n)\vec{\sigma}_t^{ouv}+H(-\delta_n)\vec{\sigma}_t^{fer}$$

éq 2.3-1

For the normal part it is equal to the sum of derivatives of the density of energy of surface and the density of energy of penalization in contact compared to the jump. It is thus enough to derive the statements given in the preceding section (11) to obtain the normal component of the contraintes7Les[7]. The tangential part $\vec{\sigma}_t^{fer}=f(K_t,\vec{\delta}_t)$ is a function of the tangential opening, it brings into plays the tangencial stiffness for the closed joint. According to the profile of the interface the tangential behavior for the open joint can be varied: $\vec{\sigma}_t^{ouv}=\vec{\sigma}_t^{fer}$ for surfaces in crenel (such as for example on the Figure 1.2-a on the right), or $\vec{\sigma}_t^{ouv}\equiv 0$ for very smooth surfaces (Figure 1.2-a on the left).

2.3.1 Normal stresses

Let us look at the evolution of the normal stress in the zone of tension according to the jump (see Figure 2.3.1-a). The deflections represent the possible meaning of evolution of the stress according to whether the process of opening is reversible (linear mode) or not (dissipative mode). With starting, the joint behaves initially in a linear elastic way, then as soon as the normal stress reaches the value criticizes $\sigma_n=\sigma_{max}$, it has a lenitive behavior: it loses its stiffness gradually, which gives the mode linear but not elastic, it is characterized by a slope of softening of the fracture $-K_n/P_{rupt}$. The elastic stiffness for the operational joint defines the initial value of the threshold of damage $\kappa_0=\sigma_{max}/K_n$. The threshold of the fracture is given par. $\kappa_{rupt}=\sigma_{max}(1+P_{rupt})/K_n$ More P_{rupt} is important plus the energy of dissipation increases $G_f=\sigma_{max}^2(1+P_{rupt})/(2K_n)$.

6 our case $\sigma_n=\partial\psi_n(\delta_n)/\partial\delta_n$;

7 statements of the stresses can be applied directly without passing by the energy formulation

$$\sigma_n(\delta_n,\kappa)=\begin{cases} P_{con}K_n\delta_n & si \quad \delta_n<0 \\ \left[\sigma_{max}(1+P_{rupt}^{-1})/\kappa-K_nP_{rupt}^{-1}\right]\delta_n & si \quad 0\leqslant\delta_n<\kappa \\ \sigma_{max}(1+P_{rupt}^{-1})\text{ - }K_nP_{rupt}^{-1}\delta_n & si \quad \kappa\leqslant\delta_n<\kappa_{rupt} \\ 0 & si \quad \delta_n\geqslant\kappa_{rupt}\end{cases}$$ **éq 2.3.1-1**

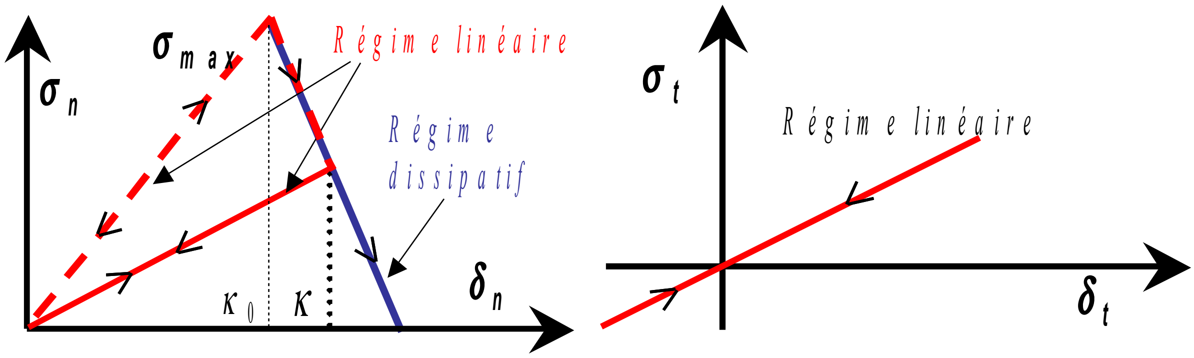


Figure 2.3.1-a: Dependence of the stresses according to the Forced

2.3.2 opening of penalization of the contact

the value of the slope of penalization in contact is given by the relation:

$$\sigma_n(\delta_n)=P_{con}K_n\delta_n,\ \text{si}\ \delta_n<0$$ **éq 2.3.2-1**

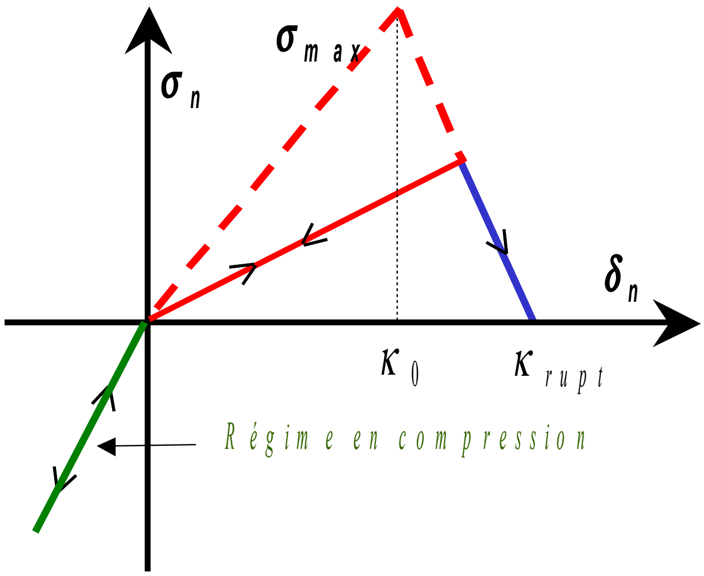


Figure 2.3.2-a: Normal cohesive stress according to the normal jump for the joint partially damaged

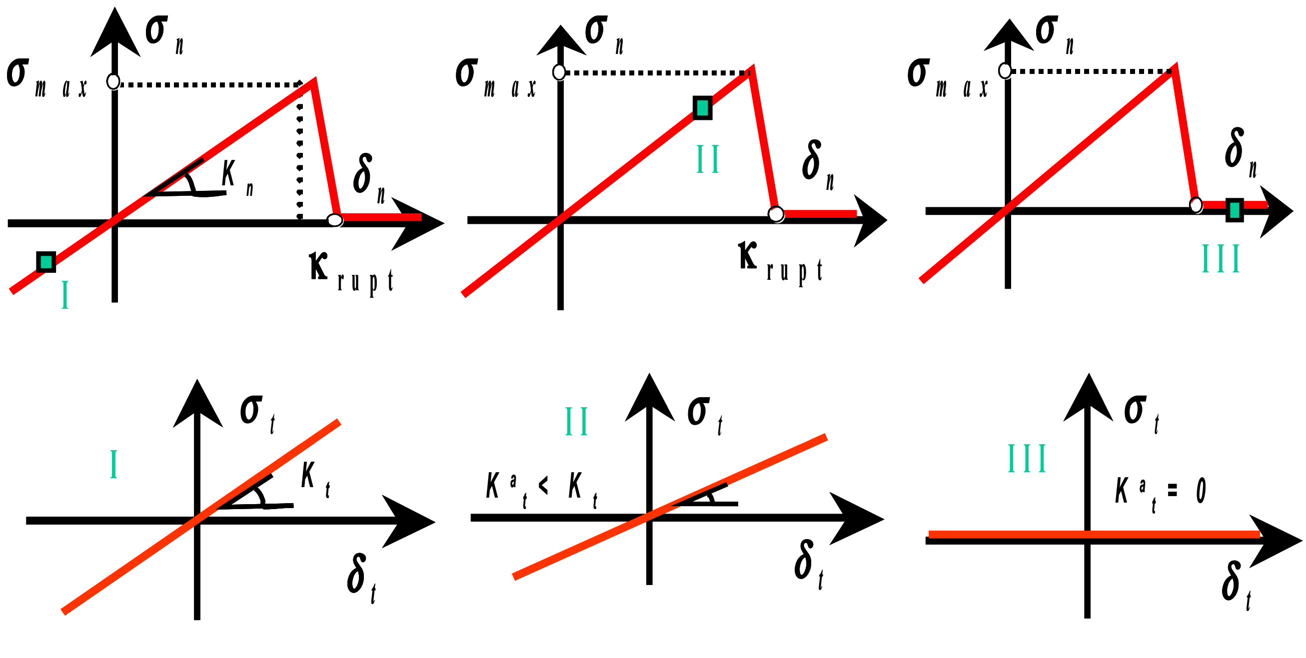
numerical parameter PENA_CONTACT, entered by the user, makes it possible to exploit the slope of the penalization of the contact (see figure 2.3.2-a). This last is worth by default 1, that corresponds if the slope of the contact is identical to that of the stiffness in opening. If one chooses a value higher than 1, one increases the penalization. This makes it possible to model, for example, the resumption of the forces by the concrete partially damaged in tension. For a value lower than 1, one decreases the penalization, which facto makes a modelization of the concrete partially damaged in compression.

2.3.3 Shear stress

For the joints of stopping to weak opening, one observes that independently of the mode of normal loading the shear stress varies always linearly, the tangencial stiffness is function of the normal opening. In the extreme case of a perfectly smooth contact surface the tangencial stiffness falls brutally to zero with the positive normal opening. Consequently the surface energy of the model is not continuous any more, which generates in theory a peak in the normal stress (function delta $\delta(x)$) with the opening. For this reason we give up in the current version model to keep it the complete energy formalism. The tangential model is then applied in an empirical way in incremental form:

$$\Delta\vec{\sigma}_t(\delta_t,\delta_n)=\begin{cases} K_t\Delta\vec{\delta}_t & si \quad \delta_n<0 \\ (1-\delta_n/\kappa_{rupt}^{\tan})K_t\Delta\vec{\delta}_t & si \quad 0\leqslant\delta_n<\kappa_{rupt}^{\tan} \\ 0 & si \quad \delta_n\geqslant\kappa_{rupt}^{\tan} \end{cases} \qquad \text{éq 2.3.3-1}$$

We introduce the threshold of tangential fracture $\kappa_{rupt}^{\tan}=\kappa_{rupt}\tan(\alpha\pi/4)$, whose value can be modified by the user using key word ALPHA $\alpha\in[0,2]$. For a zero value the tangential slope changes brutally with the opening, for the value $\alpha=2$ the tangencial stiffness does not evolve. By preoccupation with a compatibility with the constitutive laws developed in the Gefdyn code, we do not make correction of the normal component of stresses in the phase of transition, that Ci are always given by éq 2.3.1-1, this Ci results by the asymmetric tangent matrix. The evolution of the shear stress separates in three modes: elastic joint in compression; partially open elastic joint with a decreased stiffness; completely broken joint (éq 2.3.3-1, Figure 2.3.3-a).



Appear 2.3.3-a: Illustration of the coupling between the shears and the normal opening of joint: I in the mode in compression; II in the partial mode of opening; III in the mode of complete opening. Tangent $K_t^a\equiv(1-\delta_n/\kappa_{rupt}^{\tan})K_t$

2.4 Operator is noted

As the behavior in each mode is linear, the computation of the tangent matrix is easy:

$$\frac{\partial \sigma_n(\delta_n)}{\partial \delta_n} = \begin{cases} P_{con} K_n & si \quad \delta_n < 0 \\ \sigma_{max}(1+P_{rupt}^{-1})/\kappa - K_n P_{rupt}^{-1} & si \quad 0 \leqslant \delta_n < \kappa \\ -K_n P_{rupt}^{-1} & si \quad \kappa \leqslant \delta_n < \kappa_{rupt} \\ 0 & si \quad \delta_n \geqslant \kappa_{rupt} \end{cases} \qquad \frac{\partial \sigma_n(\delta_n)}{\partial \delta_t} = 0 \qquad \text{éq 2.4-1}$$

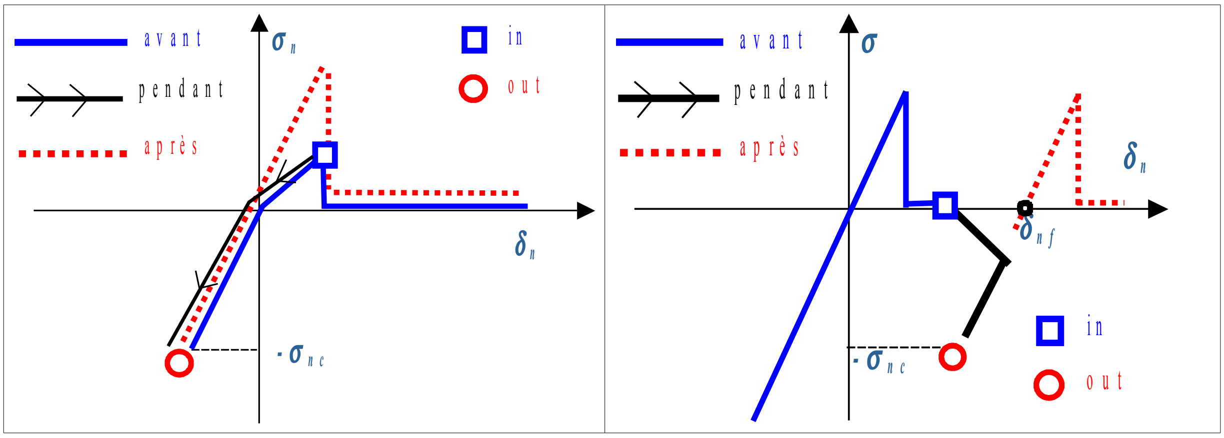
$$\frac{\partial \vec{\sigma}_t(\vec{\delta}_t, \delta_n)}{\partial \vec{\delta}_t} = \begin{cases} K_t \boldsymbol{Id} & si \quad \delta_n < 0 \\ (1-\delta_n / k_{rupt}^{\tan}) K_t \boldsymbol{Id} & si \quad 0 \leqslant \delta_n < \kappa_{rupt}^{\tan} \\ \boldsymbol{0} & si \quad \delta_n \geqslant \kappa_{rupt}^{\tan} \end{cases} \qquad \frac{\partial \vec{\sigma}_t(\vec{\delta}_t, \delta_n)}{\partial \delta_n} = \begin{cases} -K_t \Delta \vec{\delta}_t / k_{rupt}^{\tan} & si \quad 0 \leqslant \delta_n < \kappa_{rupt}^{\tan} \\ 0 & si \quad 0 > \delta_n \geqslant \kappa_{rupt}^{\tan} \end{cases}$$

Let us note that the tangent matrix is not symmetric. This results from nonthe repercussion on the normal stresses of the regularization of the evolution of the shear stress to the opening of the joint (the singular terms are not taken into account) .5[5]

2.5 numerical Realization of keying-up

keying-up corresponds physically to a procedure of grouting of concrete under pressure between the studs of the stopping, it is characterized by a simple parameter the local pressure of coulis injected $\sigma_{nc} \geqslant 0$. To set up keying-up the user must define a function of pressure of keying-up, key key PRES_CLAVAGE, which depends with the faith on space (keying-up at the various places with different pressure) and on time (several successive keying-up). The places where the pressure of keying-up is negative are not clavés.
The procedure is modelled by the modification of the thickness of the joints concerned. If the joint is in strong compression initially, keying-up does not influence it. So on the other hand the joint is opened or not sufficiently compressed ($\sigma_n > -\sigma_{nc}$), keying-up will result in the change of the parameter of total thickness of the noted joint $\delta_{nf}^{+} = \delta_{nf}^{-} + \delta_n^{+} + \sigma_{nc} / (P_{con} K_n)$.
Keying-up then preserves the normal opening of each joint while relocating the constitutive law according to the x-axis. The new equilibrium position room has a normal stress equal to the pressure of the concrete injected $\sigma_n = -\sigma_{nc}$. This procedure is applied locally. Although each joint subjected to keying-up is found in the linear mode in compression, the selection it even of the joints with claver is “nonlinear”. The total procedure becomes, also nonlinear so and it is not enough to just modify the normal stresses of the joints clavés to again obtain the mechanical equilibrium after keying-up. Consequently a mechanical computation of equilibrium is carried out after this “local” keying-up to give the system in its state of equilibrium. If the clearance of the joint with claver is modified, the update of the thickness of the joints is carried out and so on as long as there exist the points where $\sigma_n > -\sigma_{nc}$. The joints concerned are closed and are put gradually in compression while following the curve of normal behavior. Numerically this results in a mixture of implicit scheme and explicit what increases the nombre of iterations of Newton before reaching the convergence criterion. That is not awkward, because this procedure is made only once during the phase of numerical “construction” of stopping. An illustration is given on the Figure 2.5-a above. The points “in” and “out” correspond respectively to the values of the stress before and after keying-up.

The normal behavior of the joint after keying-up can be modified. Thus, the tensile strength can be restored either partially, or completely (it is the case on the Figure 2.5-a) according to the damage of the joints before the procedure of keying-up. *In the procedure such as it was developed we made the choice not to restore the tensile strength, this one keeps its current price.*



Appear 2.5-a: . Evolution of the normal stress during keying-up: joint partially damaged on the left and completely damaged on the right

2.6 Local variables

model JOINT_MECA_RUPT has eighteen local variables. From the point of view of the constitutive law, only the first and it tenth are *stricto sensu* local variables. The others provide indications on the hydraulic state of the joint to a given time.

$V1=\kappa$: threshold in jump (greater norm reached).
$V2$: indicator of dissipation $=0$ if mode linear, $=1$ if dissipative mode.
Mechanical indicators:
$V3$: indicator of healthy normal $=0$ damage, $=1$ damaged, $=2$ broken
$V4\in[0,1]$: percentage of normal damage (in the lenitive zone)
$V5$: indicator of healthy tangential $=0$ damage, $=1$ damaged, $=2$ broken
$V6\in[0,1]$: percentage of tangential damage
Value of the jump in the local coordinate system:
$V7=\delta_n$: normal jump, $V8=\delta_{t1}$ tangential jump, $V9=\delta_{t2}$ tangential jump (no one in 2D)
$V10=\delta_{nf}$: thickness of the clavé joint
$V11=\sigma_n$: normal mechanical stress (without fluid pressure)
Indicating hydraulics:
Components of the gradient of pressure in the total reference (only for xxx_JOINT_HYME):
$V12=\partial_x p$ $V13=\partial_y p$, $V14=\partial_z p$ three components in space
Components of hydraulic flux in the total reference (only for xxx_JOINT_HYME):
$V15=w_x$ $V16=w_y$, $V17=w_z$ three components in space
$V18=p$: fluid pressure imposed by user (PRES_FLUIDE) in the case of the modelizations xxx_JOINT or fluid pressure interpolated from that calculated (degree of freedom of the problem) on the nodes mediums of the elements of joint of the modelizations: xxx_JOINT_HYME.

3 Theoretical formulation of JOINT_MECA_FROT

the simplest possible friction law is the model of Coulomb, which depends only on one parameter of friction $\mu \in [0,\infty]$. It carries out the condition of NON-interpenetration of lips in contact (condition of Signorini) by drawing up a local restrain between the shear stress and norm in the phase of sliding: $\|\vec{\sigma}_t\| = \mu \sigma_n$. Several regularizations of this model are necessary in order to make possible its numerical implementation. Firstly the condition of Signorini must be made differentiable, which is easy if it is supposed that the behavior of surfaces in contact follows an elastic model. In the same way for the slope of change of management of sliding in the tangential behavior. Moreover for the modelization of the concrete dams one observes in experiments a tensile strength considerable between the joints. All these considerations bring back for us towards a model of Mohr-Coulomb of which the representation in the plane of Mohr is given on the Figure 3-a , it describes the phase of sliding of joints between the foundation and the stopping or the studs of a stopping (in a simplified way) all in taking into account the most relevant effects.

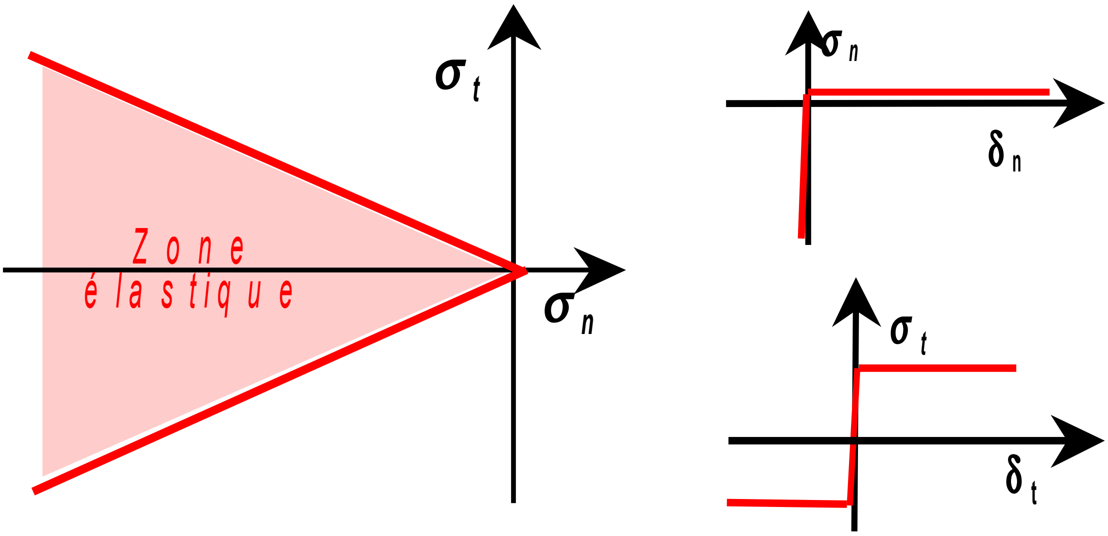
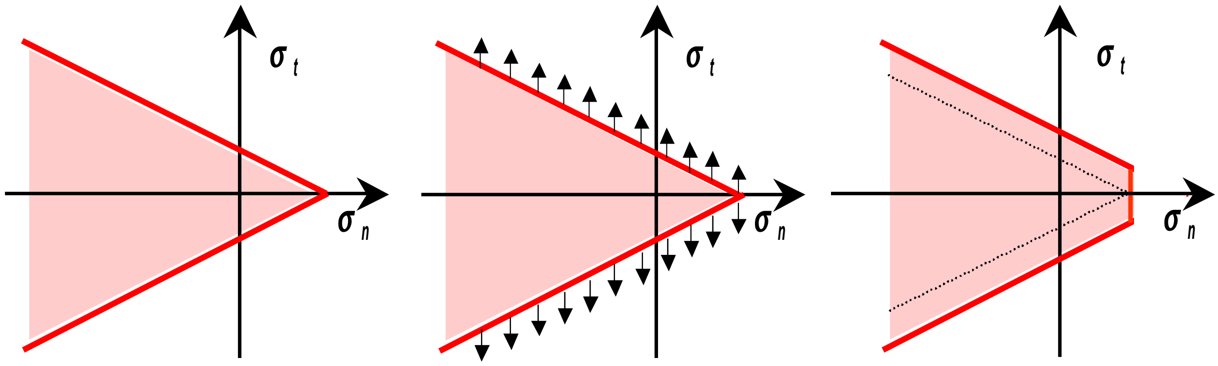


Figure 3-a : 3-a Friction law of Coulomb in 2D

model JOINT_MECA_FROT is an elastoplastic alternative of the model of Mohr-Coulomb, it depends on four parameters: the normal stiffness K_n , the tangencial stiffness K_t , the adhesion c (which are related to the tensile strength maximum $R_t = c/\mu$) and the coefficient of kinetic friction of the joint μ . In addition we introduce an isotropic hardening parameter, which makes it possible to regularize the tangential slope in the phase of sliding, one notes it K . The model elastoplastic introduced relates only to the tangent part of the constitutive law. There is no plastic part of displacement for the normal part: this one is always elastic. The jump of tangential displacement is broken up into an elastic part $\vec{\delta}_t^{el}$ and a plastic part $\vec{\delta}_t^{pl}$, one indicates by λ the jump of cumulated tangential displacement. The flow model is orthogonal to plane of cut of the cone of sliding $\sigma_n = \mathrm{const}$ (circle 2D for a cone 3D). What gives, strictly speaking, a non-aligned model of flow total.
The mechanical formulation of velocity of such a model gives the set of mathematical equations according to:

$$\begin{cases} \vec{\delta}_t = \vec{\delta}_t^{el} + \vec{\delta}_t^{pl} \\ \vec{\sigma}_t = K_t \vec{\delta}_t^{el} \equiv K_t(\vec{\delta}_t - \vec{\delta}_t^{pl}) \\ \sigma_n = min(K_n \delta_n, R_t) \end{cases} \quad \begin{cases} f(\vec{\boldsymbol{\sigma}}, \lambda) = \|\vec{\sigma}_t\| + \mu \sigma_n - c - K\lambda \leqslant 0 \\ f \cdot \dot{\lambda} = 0; \ \dot{\lambda} \geqslant 0 \\ \dot{\vec{\delta}}_t^{pl} = \dot{\lambda} \dfrac{\vec{\sigma}_t}{\|\vec{\sigma}_t\|} \end{cases} \qquad \text{éq 3-1}$$

As long as one is in the elastic zone $f(\vec{\sigma},\lambda)<0$ the relations between the jumps of opening of joint and the forced are linear and the parameter of the plastic tangential jump does not evolve $\vec{\delta}_t^{pl}=const$. As soon as one touches edges of the cone of sliding defined by $f(\vec{\sigma},\lambda)=0$, the evolution of tangential jump plastic is governed by the non-aligned flow model (éq 3-1). The regularization of the function threshold of flow with the isotropic term of $K\lambda>0$ [8] is necessary in order to make the matrix tangent of the model invertible and to avoid thus the problem of multiple solutions in the case of loading in imposed forces. The tensile strength of the joint varies in the interval $(0,R_t)$, it is a function of the shear stress, it is null for a shear stress higher than the parameter of adhesion c and is worth the maximum if the shear stress is null (see fig. 3.1-a). In the current version of the model, the tensile strength maximum is not affected by the phenomenon of sliding, it does not evolve because of term of hardening (Figure 3-b). It is also supposed that once reached the value of the tensile strength maximum, the normal stress does not evolve any more. This last assumption remains valid as long as the number of joints broken by the plastic shears and solicited thereafter in tension is negligible. In order to be able to take into account this phenomenon more rigorously, it is necessary to introduce a regularization of fracture-friction into the phase of sliding in tension, which represents a rather important theoretical difficulty [CR10357] .

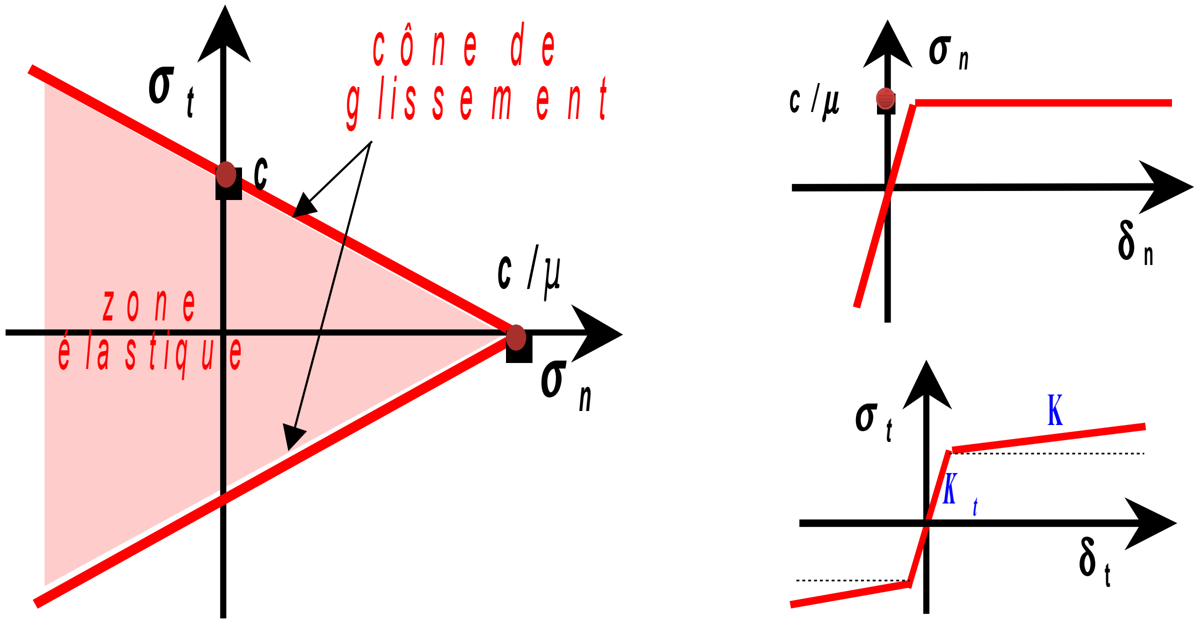


Appear 3-b: Evolution of the cone of sliding due to implicit

3.1 hardening Discretization of the friction law

the elastoplastic version of the friction law is formulated of velocity, which facilitates its numerical discretization. The incremental version of the model remains strictly equivalent to the continuous version on condition that having infinitesimal steps of change. In order to limit the number of steps of loading, we adapt the version continues model with finished increments, in an implicit way, C. - AD. that the conditions of sliding are written in the state of final equilibrium. The algorithm used is that of the radial return with elastic prediction. By convention we note by a sign “-” the variables with the state of preceding equilibrium, the current state is noted by usual variables without additional sign (see [R5.03.02]).

8 8Écrouissage on the tangent level



Appear 3.1-a: Friction law of Coulomb

the continuous equations (éq 3-1) of the model are written in a discretized way:

$$\begin{cases} \lambda=\lambda^{-}+\Delta\lambda \\ \vec{\delta}_t^{pl}=\vec{\delta}_t^{pl-}+\Delta\vec{\delta}_t^{pl} \\ \vec{\sigma}_t=K_t(\vec{\delta}_t-\vec{\delta}_t^{pl}) \\ \sigma_n=min(K_n\delta_n,R_t) \end{cases} \quad \begin{cases} f(\vec{\boldsymbol{\sigma}},\lambda)=\|\vec{\sigma}_t\|+\mu\sigma_n-c-K\lambda\leqslant 0 \\ f(\vec{\boldsymbol{\sigma}},\lambda)\cdot\Delta\lambda=0;\ \Delta\lambda\geqslant 0 \\ \Delta\vec{\delta}_t^{pl}=\Delta\lambda\dfrac{\vec{\sigma}_t}{\|\vec{\sigma}_t\|} \end{cases} \qquad \textbf{éq 3.1-1}$$

In an algorithm of Newton the jumps of displacement $\vec{\boldsymbol{\delta}}=(\delta_n,\vec{\delta}_t)$ as well as the local variables at previous time $\vec{\delta}_t^{pl-},\lambda^{-}$ being known, to solve the model it is enough to obtain the values of stresses $\vec{\boldsymbol{\sigma}}=(\sigma_n,\vec{\sigma}_t)$ and all the local variables at time running (in our case $\vec{\delta}_t^{pl},\lambda$). The model the equation of evolution for the normal component is completely uncoupled from tangential motion, one can thus solve it immediately: $\sigma_n=min(K_n\delta_n,R_t)$. We thus obtain a set of five of equations and an inequality, to obtain five scalar unknowns:

$$\vec{\sigma}_t=K_t(\vec{\delta}_t-\vec{\delta}_t^{pl-}-\Delta\vec{\delta}_t^{pl}) \quad \begin{cases} f(\vec{\boldsymbol{\sigma}},\lambda)\equiv\|\vec{\sigma}_t\|+\mu\sigma_n-c-K\lambda^{-}-K\Delta\lambda\leqslant 0 \\ f(\vec{\boldsymbol{\sigma}},\lambda)\cdot\Delta\lambda=0;\ \Delta\lambda\geqslant 0 \\ \Delta\vec{\delta}_t^{pl}=\Delta\lambda\dfrac{\vec{\sigma}_t}{\|\vec{\sigma}_t\|} \end{cases} \qquad \textbf{éq 3.1-2}$$

to simplify this mathematical problem let us look at more in detail the condition of Karush-Kuhn-Tucker $f(\vec{\boldsymbol{\sigma}},\lambda)\cdot\Delta\lambda=0$. There are two possibilities: either one slips $\Delta\lambda>0$, or one is in the elastic domain $\Delta\lambda=0$. If one is in the elastic domain, then $\Delta\lambda=0 \Rightarrow \Delta\vec{\delta}_t^{pl}=0$ and one obtains the elastic solution if:

$$f_{el}(\sigma_n,\vec{\sigma}_\tau^{-},\lambda^{-})\equiv K_t\|\vec{\delta}_\tau-\vec{\delta}_\tau^{pl-}\|+\mu\sigma_n-c-K\lambda^{-}\leqslant 0 \qquad \textbf{éq 3.1-3}$$

In practice if éq 3.1-3 is satisfied then the elastic prediction is the solution of problem:

$$\text{si } f_{el}(\sigma_n, \vec{\sigma}_t^-, \lambda^-) \equiv K_t \|\vec{\delta}_t - \vec{\delta}_t^{pl-}\| + \mu\sigma_n - c - K\lambda^- \leqslant 0 \Rightarrow \begin{cases} \lambda = \lambda^- \\ \vec{\delta}_t^{pl} = \vec{\delta}_t^{pl-} \\ \vec{\sigma}_t = K_t(\vec{\delta}_t - \vec{\delta}_t^{pl-}) \\ \sigma_n = min(K_n \delta_n, R_t) \end{cases} \quad \text{éq 3.1-4}$$

If the condition éq 3.1-3 is not satisfied, then $\Delta\lambda > 0$ and one is in the phase of sliding and one obtains a system of three nonlinear equations with three unknowns $\Delta\vec{\delta}_t^{pl}, \Delta\lambda$:

$$\begin{cases} f(\vec{\sigma}, \lambda) \equiv \|K_t(\vec{\delta}_t - \vec{\delta}_t^{pl-} - \Delta\vec{\delta}_t^{pl})\| + \mu\sigma_n - c - K\lambda^- - K\Delta\lambda = 0 \\ \Delta\vec{\delta}_t^{pl} = \Delta\lambda \dfrac{(\vec{\delta}_t - \vec{\delta}_t^{pl-} - \Delta\vec{\delta}_t^{pl})}{\|\vec{\delta}_t - \vec{\delta}_t^{pl-} - \Delta\vec{\delta}_t^{pl}\|} \end{cases} \quad \text{éq 3.1-5}$$

This equation perhaps solved by eliminating $\Delta\vec{\delta}_t^{pl}$, following the current procedure for the plastic designs:

$$\Delta\vec{\delta}_t^{pl}\{\|\vec{\delta}_t - \vec{\delta}_t^{pl-} - \Delta\vec{\delta}_t^{pl}\| + \Delta\lambda\} = \Delta\lambda(\vec{\delta}_t - \vec{\delta}_t^{pl-}) \quad \text{éq 3.1-6}$$

By taking the norm of this last equation and by noting that $\|\Delta\vec{\delta}_t^{pl}\| = \Delta\lambda$, one obtains the tangential norm of the stress vector brought up to date, that one inserts in éq 3.1-5 in order to obtain a scalar equation for $\Delta\lambda$:

$$\begin{gathered} \|\vec{\delta}_t - \vec{\delta}_t^{pl-} - \Delta\vec{\delta}_t^{pl}\| = \|\vec{\delta}_t - \vec{\delta}_t^{pl-}\| - \Delta\lambda \\ \|K_t(\vec{\delta}_t - \vec{\delta}_t^{pl-})\| - K_t\Delta\lambda + \mu\sigma_n - c - K\lambda^- - K\Delta\lambda = 0 \end{gathered} \quad \text{éq 3.1-7}$$

Once $\Delta\lambda$ is known it is enough to notice the colinearity of the vectors according to $\Delta\vec{\delta}_t^{pl} \uparrow\uparrow \vec{\delta}_t - \vec{\delta}_t^{pl-} - \Delta\vec{\delta}_t^{pl}$, from where $\Delta\vec{\delta}_t^{pl} \uparrow\uparrow \vec{\delta}_t - \vec{\delta}_t^{pl-}$ (see éq 3.1-6). What makes it possible to rewrite the last equation of 3.1-5 pennies a simplified form, which gives the values of second unknown $\Delta\vec{\delta}_t^{pl}$:

$$\Delta\vec{\delta}_t^{pl} = \Delta\lambda \frac{\vec{\delta}_t - \vec{\delta}_t^{pl-} - \Delta\vec{\delta}_t^{pl}}{\|\vec{\delta}_t - \vec{\delta}_t^{pl-} - \Delta\vec{\delta}_t^{pl}\|} = \Delta\lambda \frac{\vec{\delta}_t - \vec{\delta}_t^{pl-}}{\|\vec{\delta}_t - \vec{\delta}_t^{pl-}\|} \quad \text{éq 3.1-8}$$

This solution corresponds in fact to the slidings in the direction of shear stress in elastic prediction. This implies that the change of the direction of sliding will be done primarily in the elastic zone provided that the steps of loading are small. The final solution in the event of sliding, obtained from éq 3.1-2, is written like:

$$\begin{cases} \sigma_n & = & min(K_n \delta_n, R_t) \\ f_{el}(\sigma_n, \vec{\sigma}_t^-, \lambda^-) & = & K_t \|\vec{\delta}_t - \vec{\delta}_t^{pl-}\| + \mu\sigma_n - c - K\lambda^- \\ \lambda & = & \lambda^- + \dfrac{f_{el}(\sigma_n, \vec{\sigma}_t^-, \lambda^-)}{K_t + K} \\ \vec{\delta}_t^{pl} & = & \vec{\delta}_t^{pl-} + \dfrac{f_{el}(\sigma_n, \vec{\sigma}_t^-, \lambda^-)}{K_t + K} \cdot \dfrac{\vec{\delta}_t - \vec{\delta}_t^{pl-}}{\|\vec{\delta}_t - \vec{\delta}_t^{pl-}\|} \\ \vec{\sigma}_t & = & K_t(\vec{\delta}_t - \vec{\delta}_t^{pl}) \end{cases} \quad \text{éq 3.1-9}$$

In short, from the elastic prediction one then checks initially éq 3.1-3 3.1-3, if it is satisfied, the solution is given by éq 3.1-4 , if not solution is given by éq 3.1-9 .

3.2 Stamp tangent

For model `JOINT_MECA_FROT` the tangent matrix is calculated into implicit, which reinforces the robustness of the calculs9Dans[9]. As it is shown in the ref. [Ngu77] such a numerical diagram is unconditionally stable for the models with positive hardening $K \geqslant 0$. In the case of the elastic mode (the inequality éq 3.1-3 is satisfied), the tangent matrix takes a simple form, it is diagonal:

$$\begin{pmatrix} K_n & 0 & 0 \\ 0 & K_t & 0 \\ 0 & 0 & K_t \end{pmatrix}$$

In the case of sliding (inequality éq 3.1-3 is not satisfied) the tangent matrix is obtained by derivative of éq 3.1-9 . The derivatives compared to the normal opening depend on the state of the joint. For the joint closed that gives:

$$\text{si } \delta_n < \frac{c}{\mu K_n} \Rightarrow \begin{cases} \dfrac{\partial \sigma_n}{\partial \delta_n} &= K_n \\ \dfrac{\partial \vec{\sigma}_t}{\partial \delta_n} &= -\mu \dfrac{\vec{\delta}_t - \vec{\delta}_t^{pl-}}{\|\vec{\delta}_t - \vec{\delta}_t^{pl-}\|} \cdot \dfrac{K_n K_t}{K_t + K} \end{cases} \qquad \textbf{éq 3.2-1}$$

For the open joint all the corresponding derivatives are null:

$$\text{si } \delta_n \geqslant \frac{c}{\mu K_n} \Rightarrow \begin{cases} \dfrac{\partial \sigma_n}{\partial \delta_n} &= 0 \\ \dfrac{\partial \vec{\sigma}_t}{\partial \delta_n} &= 0 \end{cases} \qquad \textbf{éq 3.2-2}$$

derivatives compared to the tangential opening do not depend on the opening of joint:

$$\begin{cases} \dfrac{\partial \sigma_n}{\partial \vec{\delta}_t} &= 0 \\ \dfrac{\partial \vec{\sigma}_t}{\partial \vec{\delta}_t} &= \dfrac{K K_t}{K_t + K} \mathbf{Id} + \dfrac{-\mu \sigma_n + c + K \lambda^-}{K_t + K} \cdot \dfrac{K_t}{\|\vec{\delta}_t - \vec{\delta}_t^{pl-}\|} \cdot \left(\mathbf{Id} - \dfrac{(\vec{\delta}_t - \vec{\delta}_t^{pl-}) \otimes (\vec{\delta}_t - \vec{\delta}_t^{pl-})}{\|\vec{\delta}_t - \vec{\delta}_t^{pl-}\|^2} \right) \end{cases} \qquad \textbf{éq 3.2-3}$$

Note: : The tangent matrix in the plastic phase (in sliding) is NON-symmetric, it is degenerated if hardening is null ($K = 0$). One can null display an eigenvector associated with the eigenvalue:

$$\begin{pmatrix} K_n & 0 \\ \mu K_n \dfrac{\vec{\delta}_t - \vec{\delta}_t^{pl-}}{\|\vec{\delta}_t - \vec{\delta}_t^{pl-}\|} & \dfrac{-\mu \sigma_n + c}{\|\vec{\delta}_t - \vec{\delta}_t^{pl-}\|} \left(\mathbf{Id} - \dfrac{(\vec{\delta}_t - \vec{\delta}_t^{pl-}) \otimes (\vec{\delta}_t - \vec{\delta}_t^{pl-})}{\|\vec{\delta}_t - \vec{\delta}_t^{pl-}\|^2} \right) \end{pmatrix} \cdot \begin{pmatrix} 0 \\ \vec{\delta}_t - \vec{\delta}_t^{pl-} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

For this reason the isotropic hardening parameter is introduced.

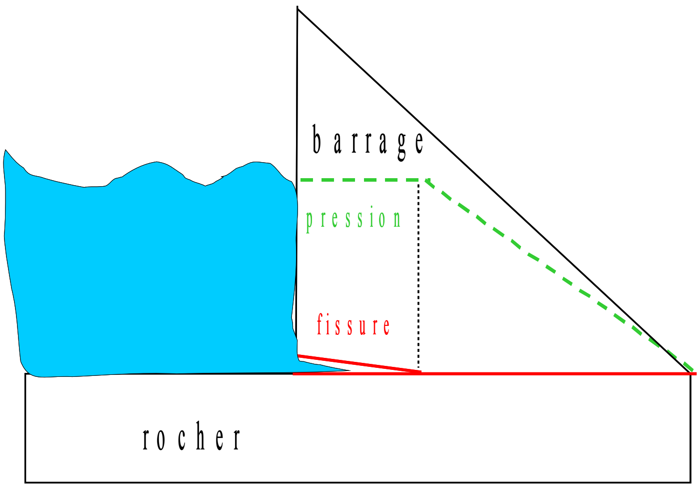
3.3 Local variables

9 the preceding attempts [Kol00],[CR09039] of introduction in explicit version [Div97], it appeared that these modelizations prove generally not very powerful [Div97]

model JOINT_MECA_FROT has eighteen local variables. From the point of view of the constitutive law, only the first, the third and the fourth are *stricto sensu* local variables . The others provide indications on the state of hydromechanics of the joint to a given time.

Local variables :
$V1=\lambda$: parameter growing indicating cumulated plastic tangential displacement (without directional sense).
$V2$: slip meter $=0$ if linear mode, $=1$ if mode is plastic
$V3, V4=\vec{\delta}^{pl}$: plastic tangential displacement vector compared to the starting point (indicates the current equilibrium position). $V4$ is put at zero in 2D
Indicating mechanics:
$V5$: indicator of complete opening $=0$ closed $(\sigma_n<c/\mu)$, $=1$ opened $(\sigma_n=c/\mu)$
$V6=\|\vec{\sigma}_\tau\|$: tangent stress Value
of the jump in the local coordinate system normalizes:
$V7=\delta_n$: normal jump, $V8=\delta_{t1}$ tangential jump, $V9=\delta_{t2}$ tangential jump (no one in 2D)
Value of the jump in the local coordinate system:
$V10$: unused variable
$V11=\sigma_n$: normal mechanical stress (without fluid pressure)
Indicating hydraulics:
Components of the gradient of pressure in the total reference (only for xxx_JOINT_HYME):
$V12=\partial_x p$, $V13=\partial_y p$, $V14=\partial_z p$ three components in space
Components of hydraulic flux in the total reference (only for xxx_JOINT_HYME):
$V15=w_x$, $V16=w_y$, $V17=w_z$ three components in space
$V18=p$: fluid pressure imposed by user (PRES_FLUIDE) in the case of the modelizations xxx_JOINT or fluid pressure interpolated from that calculated (degree of freedom of the problem) on the nodes mediums of the elements of joint of the modelizations: xxx_JOINT_HYME.

4 Taking into account of the hydrostatic pressure without coupling



Appears 4-a: Illustration of a possible computation of stability of a stopping with the profile of pressure imposed

Although modelization XXX_JOINT does not couple the mechanics and the hydraulics, one can however explicitly introduce the influence of a fluid on the mechanics via an imposed pressure. The presence of the fluid in the joint modifies the normal mechanical stress $\sigma_n \to \sigma_n - p$. By putting an important pressure one is able to make break the joint by a simple hydraulic effect. To take into account the hydrostatic effects the mechanical model is shifted to the bottom (Figure 1.3-a) according to the value of pressure p in each point of integration.
At the numerical level of implementation its R ealisation is easy in the event of complete writing of the mechanical models in form clarifies nonincremental according to displacements and of the local variables (it is necessary to exclude the dependence from the stresses at time more according to stress S in previous time). In this case the only modification of normal curve is sufficient to introduce the coupling: $\sigma_n = \sigma_n^{meca}(\delta_n, \delta_t) - p$

While being limited to this kind of physical phenomenon, it is possible to make studies where profile of pressure which is imposed by user, for example a study of stability of stopping under conservative assumption (Figure 4-a), c-a-d in the presence of uplift, whose form is very penalizing. In order to make to a computation with an imposed pressure the user must define a function, key key PRES_FLUIDE, which depends with the faith on space (profile of NON-homogeneous pressure) and on time (evolution of the profile of pressure).

5 Theoretical formulation of the hydraulic coupling

the introduced models can lean on a coupled hydraulic modelization, noted XXX_JOINT_HYME. In this part one will speak about the hydraulic part of the model, as well as coupling him even; all the details on the mechanical part of the model were described previously.

5.1 Hydraulic modelization

the fluid runs out of the zones of high pressure towards those basic pressure. A theoretical way to take into account the steady flow is to associate at the hydraulic state given a énergie10Nous[10] $H(p(x))$ depending on the distribution of pressure. The first assumption consists in supposing that energy depends explicitly on the variation of pressure and not on the pressure it even $H = H(\nabla p(x))$. By taking the convex shape simplest possible of this dependence in gradient, one obtains energy thus: $H = C(\vec{\nabla} p)^2 / 2$ where C is a parameter of the model, which does not depend on the pressure.

By calculating the forces generalized corresponding to the field of gradient of pressure one obtains the first model of Fick. The hydraulic flux is proportional to the gradient of pressure:

$$\vec{w} = \frac{\partial H}{\partial \vec{\nabla} p} = C \vec{\nabla} p \qquad \textbf{éq 5.1-1}$$

In this energy formalism one seeks the field of pressure to the equilibrium by minimization of the hydraulic power $\min_{p(\vec{x})} \int_\Omega H(\vec{\nabla} p(\vec{x})) \, d\Omega$. What gives a balance equation resembling that of the mechanics: $\operatorname{div} \vec{w} = 0$. In the frame of this model the solution of balance equation hydraulic is equivalent to a resolution of mechanical problem into quasi-static, where the hydraulic flux is equivalent to the stress $\vec{w} \Leftrightarrow \boldsymbol{\sigma}$, the field of pressure corresponds to the field of displacement $p(\vec{x}) \Leftrightarrow u(\vec{x})$ and finally the gradient of pressure is connected at the strain field $\vec{\nabla} p \Leftrightarrow \varepsilon$.

5.2 Influence hydraulics on the mechanics: hydro => méca

the presence of the fluid in the joint adds a hydrostatic stress and this fact modifies the normal mechanical stress $\sigma_n \to \sigma_n - p$. By putting an important pressure one is able to make break the joint

10 use a simplified notation, the exact term would be: rate of density of energy.

by a simple hydraulic effect. One can downwards shift the mechanical model according to the value of pressure p in each point to take into account the effects of the pressure, to see 1.3-a.

5.3 Influence mechanics on the hydraulics: méca => hydro

In the case of flow of fluid through a crack the hydraulic flux must increase with the opening (δ_n) of the latter ($\vec{w}\sim O(\delta_n)\vec{\nabla}p$). In the model of One tenth of a poise, which was found empirically for the laminar flow of a viscous and incompressible fluid, the flux dependence in opening is cubic (the model is often called the cubic model). The hydraulic part of the model uses this kind of coupling. The equations to be solved are written in the following way $\operatorname{div}\vec{w}=0\,;\vec{w}=\frac{\rho}{12\bar{\mu}}\delta_n^3\vec{\nabla}p$. In the case of fluid flow through junctions of a stopping, one notes significant flux even for the closed joints. It is necessary then to define one thickness minimal ϵ_{min} , key key OUV_MIN, below which the flux reaches its minimal value. We regularize the flow equations in the following way:

$$\vec{w}=\frac{\rho}{12\bar{\mu}}\max(\epsilon_{min},\epsilon_{min}+\delta_n)^3\vec{\nabla}p \qquad \textbf{éq 5.3-1}$$

A gradient of pressure NON-no one the flux never reaches the value zero $\min\vec{w}\sim\epsilon_{min}^3\vec{\nabla}p$, which corresponds to flow through permeable walls of the closed joint.

5.4 Hydraulic coupling

the hydraulic coupling utilizes the two mechanisms described previously: of with dimensions fluid acts by pressure on the lips of joint, other with dimensions plus crack is open plus the fluid flow is easy. In absence of the external forces hydraulic computation arises schematically in this form.

$$\begin{cases}\vec{w}=\vec{w}(\vec{\boldsymbol{\delta}}(u),\vec{\nabla}p); & \operatorname{div}\vec{w}=0\\ \vec{\boldsymbol{\sigma}}=\vec{\boldsymbol{\sigma}}(\vec{\boldsymbol{\delta}}(u),p); & \operatorname{div}\vec{\boldsymbol{\sigma}}=0\end{cases}\Rightarrow\quad\vec{Y}=\vec{Y}(\vec{X});\operatorname{div}\vec{Y}=0$$

The hydraulic resolution of the balance equations is equivalent to a resolution of mechanical problem into quasi-static, where the generalized stresses are introduced $\vec{Y}=(\vec{w},\vec{\boldsymbol{\sigma}})$, and the vector field of the unknowns $\vec{X}=(p,u)$.

5.5 To compute: stamp

tangent Considering the generalized forces depend u on only through $\vec{\boldsymbol{\delta}}(u)$, the tangent matrix of hydraulic coupling, it is necessary to know only the four following terms:

$\frac{\partial\vec{\boldsymbol{\sigma}}}{\partial\vec{\boldsymbol{\delta}}}\;\frac{\partial\vec{\boldsymbol{\sigma}}}{\partial p}$, $\frac{\partial\vec{\mathbf{w}}}{\partial\vec{\nabla}p}$ and $\frac{\partial\vec{\mathbf{w}}}{\partial\vec{\boldsymbol{\delta}}}$.

The first term is the same one as in pure mechanics, it is given inéq 2.4-1. The second term is commonplace, because it only component NON-no one is equal to $\partial\sigma_n/\partial p=-1$. The diagonal hydraulic term takes a simple form because the hydraulic flux depends only on the gradient of pressure:

$$\frac{\partial\vec{\mathbf{w}}}{\partial\vec{\nabla}p}=\frac{\rho}{12\bar{\mu}}\max(\epsilon_{min},\epsilon_{min}+\delta_n)^3 .$$

In the last term only the derivative compared to the normal opening is not null:

$$\frac{\partial\vec{\mathbf{w}}}{\partial\delta_n}=\frac{\rho}{4\bar{\mu}}(\epsilon_{min}+\delta_n)^2\vec{\nabla}p ,$$

it is equal to zero for a closed crack $\delta_n<0$.
The tangent matrix thus formulated is not symmetric.

6 Functionalities and validation

Two constitutive laws JOINT_MECA_RUPT and JOINT_MECA_FROT are introduced. They are validated on the elementary cases tests **ssnp162** and the pseudonym gravity dam **ssnp142**. The procedure of keying-up is validated on the simulation of injection of the coulis between two rectangular blocks embedded on the ground **ssnp143**.

Validation in pure mechanics, modelizations type XXX_JOINT	
Model: JOINT_MECA_RUPT	Model: JOINT_MECA_FROT
Tests: ssnp162a/b/c; ssnp142a/b; Keying-up: ssnp143a/b	Tests: ssnp162d/e/f; ssnp142c/d

coupled hydraulic Validation, modelizations type XXX_JOINT_HYME	
Model: JOINT_MECA_RUPT	Model: JOINT_MECA_FROT
Tests: ssnp162g/h/i; ssnp142e/f	Tests: ssnp162j/k/l; ssnp142g/h

7 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
04/29/11	K.KAZYMYRENKO, J.LAVERNE EDF-R&D/AMA	initial Text
12/18/12	K.KAZYMYRENKO, J.LAVERNE EDF-R&D/AMA	Coupling HM for the friction law
1/11/13	K.KAZYMYRENKO, J.LAVERNE EDF-R&D/AMA	Transition with tangential model JOINT_MECA_RUPT incremental

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