Viscoplastic constitutive law VISC_DRUC_PRAG

Summarized:

This document the model describes viscoplastic constitutive law VISC_DRUC_PRAG based on elastoplastic of Drucker-Prager and taking into account viscosity according to a model power of the Perzyna type. Its scope of application is the mudstone which is the rock host of the concept of storage.

The model proposed comprises only one viscoplastic mechanism. The criterion is hammer-hardened with the viscoplastic strain cumulated via three thresholds: elastic, of peak and ultimate. Flow is nonassociated, the flow potential being a potential of Drucker-Prager being hammer-hardened according to three levels: elastic, of peak and ultimate. Between the thresholds, hardenings are linear.

This model can be used in a pure mechanical modelization as it can be used in a modelization THM. It is available in 3D, plane strains and axisymmetric. It is integrated by the solution of only one scalar equation nonlinear.

Contents

1 Notations

σ indicate the tensor of the effective stresses in small disturbances, noted in the shape of the following vector:

$$\begin{matrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sqrt{2}\,\sigma_{12}\\ \sqrt{2}\,\sigma_{13}\\ \sqrt{2}\,\sigma_{23}\end{matrix}$$

One notes:

D^e	elasticity tensor
$I_1=\mathrm{tr}(\sigma)$	first invariant of the stresses
$s=\sigma-\frac{I_1}{3}I$	tensor of the stresses déviatoires
$s_{II}=\sqrt{S.S}$	second invariant of the tensor of the stresses déviatoires
$\sigma_{\mathrm{eq}}=\sqrt{\frac{3}{2}s_{ij}s_{ij}}$	equivalent stress
I_1^{el}	traces elastic prediction of the stresses
$s^{el}=\sigma^{el}-\frac{I_1^{el}}{3}I$	tensor of the stresses déviatoires of the elastic prediction of the stresses
$\sigma_{\mathrm{eq}}^{el}=\sqrt{\frac{3}{2}s_{ij}^{el}s_{ij}^{el}}$	equivalent stress of the elastic prediction of the stresses
$\tilde{\varepsilon}=\varepsilon-\frac{tr(\varepsilon)}{3}I$	deviator of the strains
$\varepsilon_v=tr(\varepsilon)$	voluminal strain
$\dot{p}=\sqrt{\frac{2}{3}\dot{\tilde{\varepsilon}}_{\mathrm{ij}}^{\mathrm{vp}}\dot{\tilde{\varepsilon}}_{\mathrm{ij}}^{\mathrm{vp}}}$	cumulated viscoplastic deviatoric strains
f	viscoplastic surface of load
G	potential of viscoplastic flow
α_0 , R_0 and β_0	hardening parameters corresponding to the threshold of elasticity ($p=0$)
α_{pic} , R_{pic} and β_{pic}	hardening parameters corresponding to the peak $(p=p_{pic})$
α_{ult} , R_{ult} and β_{ult}	hardening parameters corresponding to the ultimate threshold $(p=p_{ult})$
Φ	amplitude velocity of the unrecoverable deformations
A	parameter of creep
n	power of the creep model
P_{ref}	pressure of reference

2 Introduction

This document describes the integration of viscoplastic constitutive law `VISC_DRUC_PRAG` in Code_Aster. This model comprises only one viscoplastic mechanism. The viscoplastic criterion is hammer-hardened with the deviatoric viscoplastic strain cumulated via three thresholds: elastic for a viscoplastic strain null, a threshold known as of peak for a viscoplastic strain known as of peak (parameter of the model) and an ultimate threshold for a viscoplastic strain known as ultimate (parameter of the model). Between the thresholds, the functions of hardening are linear. In Code_Aster there exists another model based on the model of Drucker_Prager and used in élastoplasticié in a form associated in name `DRUCK_PRAGER` or not associated under name `DRUCK_PRAG_N_A` (see [R7.01.16]).

3 Formulation of viscoplastic model `VISC_DRUC_PRAG`

3.1 Equations of the model

This model is based on a viscoplastic formulation of the Drucker-Prager type, where the surface of load is defined by:

$$f=\sqrt{\frac{3}{2}}s_{II}+\alpha(p)I_1-R(p)$$

$\alpha(p)$ and $R(p)$ are functions of the cumulated viscoplastic strain p ,

One introduces a viscoplastic flow potential G :

$$G=\sqrt{\frac{3}{2}}s_{II}+\beta(p)I_1 \;;$$

For the evolution of the criterion f and potential G we distinguish three thresholds distinct corresponding to three values from the variable from hardening: an elastic threshold, a threshold of peak and an ultimate threshold. Between these thresholds, hardening is linear. Between the elastic threshold and the threshold of peak, hardening is positive, after the peak hardening is negative and becomes constant after the ultimate threshold.

The functions related to cohesion are written in the following form:

$$\alpha(p)=\left(\frac{\alpha_{pic}-\alpha_0}{p_{pic}}\right)p+\alpha_0 \text{ for } 0<p<p_{pic}$$

$$\alpha(p)=\left(\frac{\alpha_{ult}-\alpha_{pic}}{p_{ult}-p_{pic}}\right)(p-p_{pic})+\alpha_{pic} \quad p_{pic}<p<p_{ult}$$

$$\alpha(p)=\alpha_{ult} \text{ for } p>p_{ult}$$

the functions related to dilatancy are written in the following form:

$$\beta(p)=\left(\frac{\beta_{pic}-\beta_0}{p_{pic}}\right)p+\beta_0 \text{ for } 0<p<p_{pic}$$

$$\beta(p)=\left(\frac{\beta_{ult}-\beta_{pic}}{p_{ult}-p_{pic}}\right)(p-p_{pic})+\beta_{pic} \quad p_{pic}<p<p_{ult}$$

$$\beta(p)=\beta_{ult} \text{ for } p>p_{ult}$$

the functions of hardening are written:

$$R(p)=\left(\frac{R_{pic}-R_0}{p_{pic}}\right)p+R_0 \text{ for } 0<p<p_{pic}$$

$$R(p)=\left(\frac{R_{ult}-R_{pic}}{p_{ult}-p_{pic}}\right)\left(p-p_{pic}\right)+R_{pic} \quad p_{pic}<p<p_{ult}$$

$$R(p)=R_{ult} \text{ for } p>p_{ult}$$

the stresses are connected to the strains by the Hooke's law:

$$\sigma=D^e\left(\varepsilon-\varepsilon^{vp}\right)$$

When the viscoplastic threshold is reached, of the viscoplastic unrecoverable deformations are generated and expressed according to the theory of Perzyna by:

$$d\,\varepsilon_{ij}^{vp}=A\left\langle\frac{f}{P_{ref}}\right\rangle^n\frac{\partial G}{\partial\sigma_{ij}}dt \;;$$

f being the criterion of viscoplasticity; A and n are parameters of the model; P_{ref} a pressure of reference.

$$\frac{\partial G}{\partial\sigma_{ij}}=\sqrt{\frac{3}{2}}\frac{\partial s_{II}}{\partial\sigma_{ij}}+\beta(p)\frac{\partial I_1}{\partial\sigma_{ij}} \text{ and } \dot{p}=\sqrt{\frac{2}{3}\dot{\tilde{\varepsilon}}_{ij}^{vp}\dot{\tilde{\varepsilon}}_{ij}^{vp}}$$

with, $\tilde{\varepsilon}_{ij}^{vp}$ the deviator of the strain tensor,

$$\frac{\partial s_{II}}{\partial\sigma_{ij}}=\frac{\partial s_{II}}{\partial s_{kl}}\frac{\partial s_{kl}}{\partial\sigma_{ij}}=\frac{s_{kl}}{s_{II}}\left(\delta_{ik}\delta_{jl}-\frac{1}{3}\delta_{ij}\delta_{kl}\right)=\frac{s_{ij}}{s_{II}}$$

and

$$\frac{\partial I_1}{\partial\sigma_{ij}}=\frac{\partial tr\left(\sigma_{ij}\right)}{\partial\sigma_{ij}}=\delta_{ij}$$

from where

$$\frac{\partial G}{\partial\sigma_{ij}}=\sqrt{\frac{3}{2}}\frac{s_{ij}}{s_{II}}+\beta\left(p\right)\delta_{ij}$$

Summarized equations:

The criterion:

$$f=\sqrt{\frac{3}{2}}s_{II}+\left[\left(\frac{\alpha_{pic}-\alpha_0}{p_{pic}}\right)p+\alpha_0\right]I_1-\left[\left(\frac{R_{pic}-R_0}{p_{pic}}\right)p+R_0\right] \text{ for } 0<p<p_{pic}$$

$$f=\sqrt{\frac{3}{2}}s_{II}+\left[\left(\frac{\alpha_{ult}-\alpha_{pic}}{p_{ult}-p_{pic}}\right)(p-p_{pic})+\alpha_{pic}\right]I_1-\left[\left(\frac{R_{ult}-R_{pic}}{p_{ult}-p_{pic}}\right)(p-p_{pic})+R_{pic}\right]$$

$p_{pic}<p<p_{ult}$

$$f=\sqrt{\frac{3}{2}}s_{II}+\alpha_{ult}I_1-R_{ult} \text{ for } p\geq p_{ult} \text{ :}$$

Flow potential:

$$G=\sqrt{\frac{3}{2}}s_{II}+\left[\left(\frac{\beta_{pic}-\beta_0}{p_{pic}}\right)p+\beta_0\right]I_1 \text{ for } 0<p<p_{pic}$$

$$G=\sqrt{\frac{3}{2}}s_{II}+\left[\left(\frac{\beta_{ult}-\beta_{pic}}{p_{ult}-p_{pic}}\right)(p-p_{pic})+\beta_{pic}\right]I_1 \quad p_{pic}<p<p_{ult}$$

$$G=\sqrt{\frac{3}{2}}s_{II}+\beta_{ult}I_1 \text{ for } p\geq p_{ult}$$

α_0 , R_0 and β_0 : hardening parameters corresponding to the threshold of elasticity ($p=0$)
α_{pic} , R_{pic} and β_{pic} : hardening parameters corresponding to the parameter p_{pic}
α_{ult} , R_{ult} and β_{ult} : hardening parameters corresponding to the parameter p_{ult}

the Hooke's law:

$\sigma=D^e(\varepsilon-\varepsilon^{vp})$

$f(\sigma,p)\leq 0$ field of elasticity; $\dot{\varepsilon}_{ij}^{vp}=0$

$f(\sigma,p)>0$ viscoplasticity ; $\dot{\varepsilon}_{ij}^{vp}=A\left\langle\frac{f}{P_{ref}}\right\rangle^n\frac{\partial G}{\partial\sigma_{ij}}$; $\dot{p}=\sqrt{\frac{2}{3}\dot{\varepsilon}_{ij}^{vp}\dot{\varepsilon}_{ij}^{vp}}$

4 Integration in Code_Aster

4.1 Decomposition of the strain tensor

the decomposition of the increment of total deflection is written:

$\Delta\varepsilon=\Delta\varepsilon^e+\Delta\varepsilon^{vp}$

where $\Delta\varepsilon^e$ and $\Delta\varepsilon^{vp}$ are the increments of the elastic and viscoplastic tensors.

4.2 Update of the stresses

the following Notations are adopted: A^-, A and ΔA respectively indicating the quantities at the beginning, time step and its increment lasting the step.
One expresses the stresses brought up to date at time + compared to those calculated at time -:

$$\sigma=\sigma^-+D^e\Delta\varepsilon^e \;;\; s=s^-+2\mu\Delta\tilde{\varepsilon}^e \;;\; I_1=I_1^-+3\mathrm{K}\,\Delta\varepsilon_v^e$$

$$\sigma_{ij}=s_{ij}+\frac{I_1}{3}\delta_{ij} \;;$$

$$\Delta\epsilon_{ij}=\Delta\tilde{\epsilon}+tr\frac{(\Delta\epsilon)}{3}\delta_{ij}=\Delta\tilde{\epsilon}_{ij}+\frac{\Delta\epsilon_v}{3}\delta_{ij}\;;$$

$$I_1=tr(\sigma) \;;\; \varepsilon_v=tr(\Delta\varepsilon)\;;$$

Elastic prediction:

$$\sigma^{el}=\sigma^-+D^e\Delta\varepsilon \;;\; s^{el}=s^-+2\mu\Delta\tilde{\varepsilon} \;;\; I_1^{el}=I_1^-+3\mathrm{K}\,\Delta\varepsilon_v$$

4.2.1 Elastic solution

Computation of the increment of the stresses in elastic mode:

$$\Delta\sigma_{ij}=\Delta s_{ij}+\frac{\Delta I_1}{3}\delta_{ij} \;;\quad \Delta\varepsilon_{ij}=\Delta\tilde{\varepsilon}_{ij}+\frac{\Delta\varepsilon_v}{3}\delta_{ij}$$

$$\Delta\sigma_{ij}=2\mu\Delta\tilde{\epsilon}_{ij}+3\mathrm{K}\frac{\Delta\epsilon_v}{3}\delta_{ij}=2\mu\Delta\tilde{\epsilon}_{ij}+K\Delta\epsilon_v\delta_{ij}=2\mu\left(\Delta\epsilon_{ij}-\frac{tr(\Delta\epsilon)}{3}\delta_{ij}\right)+K\,tr(\Delta\epsilon)\delta_{ij}$$

$$\Delta\sigma_{ij}=2\mu\,\Delta\varepsilon_{ij}+\left(K-\frac{2\mathrm{G}}{3}\right)tr(\Delta\varepsilon)\delta_{ij}$$

$$\begin{Bmatrix}\Delta\sigma_{11}\\ \Delta\sigma_{22}\\ \Delta\sigma_{33}\\ \sqrt{2}\Delta\sigma_{12}\\ \sqrt{2}\Delta\sigma_{13}\\ \sqrt{2}\Delta\sigma_{23}\end{Bmatrix}=\underbrace{\begin{bmatrix}\frac{4\mu}{3}+K & K-\frac{2\mu}{3} & K-\frac{2\mu}{3} & 0 & 0 & 0\\ K-\frac{2\mu}{3} & \frac{4\mu}{3}+K & K-\frac{2\mu}{3} & 0 & 0 & 0\\ K-\frac{2\mu}{3} & K-\frac{2\mu}{3} & \frac{4\mu}{3}+K & 0 & 0 & 0\\ 0 & 0 & 0 & 2\mu & 0 & 0\\ 0 & 0 & 0 & 0 & 2\mu & 0\\ 0 & 0 & 0 & 0 & 0 & 2\mu\end{bmatrix}}_{D^e}\cdot\begin{Bmatrix}\Delta\varepsilon_{11}\\ \Delta\varepsilon_{22}\\ \Delta\varepsilon_{33}\\ \sqrt{2}\Delta\varepsilon_{12}\\ \sqrt{2}\Delta\varepsilon_{13}\\ \sqrt{2}\Delta\varepsilon_{23}\end{Bmatrix}$$

4.2.2 Viscoplastic solution

One expresses the stress field at time +:

$$\sigma_{ij}=\sigma_{ij}^-+D_{ijkl}^e\Delta\varepsilon_{kl}^e=\sigma_{ij}^-+D_{ijkl}^e\left(\Delta\varepsilon_{kl}-\Delta\varepsilon_{kl}^{vp}\right)=\sigma_{ij}^{el}-D_{ijkl}^e\Delta\varepsilon_{kl}^{vp}$$

$s_{ij}=s_{ij}^{el}-2\mu\,\Delta\tilde{\varepsilon}_{ij}^{vp}$ and $I_1=I_1^{el}-3\mathrm{K}\,\Delta\varepsilon_v^{vp}$

$$\sigma_{ij}=s_{ij}+\frac{I_1}{3}\delta_{ij}$$

which is written by replacing the increase in the viscous strains by their statements in the form:
$\sigma_{ij}=\sigma_{ij}^{el}-D_{ijkl}^{el}\langle\Phi\rangle\frac{\partial G}{\partial\sigma_{ij}}(\sigma,p)\Delta t$ with $\Phi=A\left(\frac{f(\sigma,p)}{P_{ref}}\right)^n$ where

Φ and $\frac{\partial G}{\partial\sigma}$ the amplitude and the direction velocity of the unrecoverable deformations characterize.
$f(\sigma,p)$ being the criterion of viscoplasticity, A and n are parameters of the model.

The viscoplastic criterion at time + is written:

$$f(\sigma,p)=f\left(\sigma_{ij}^{el}-D_{ijkl}^{e}\langle\Phi\rangle\frac{\partial G}{\partial\sigma_{ij}}(\sigma,p)\Delta t,p\right)$$

The increment of the viscoplastic strain being,

$$\Delta\varepsilon_{ij}^{vp}=\langle\Phi\rangle\frac{\partial G}{\partial\sigma_{ij}}\Delta t=\langle\Phi\rangle\left(\sqrt{\frac{3}{2}}\frac{s_{ij}}{s_{II}}+\beta(p)\delta_{ij}\right)\Delta t$$

the viscoplastic voluminal strain being,

$$\Delta\varepsilon_{v}^{vp}=3\langle\Phi\rangle\beta(p)\Delta t$$

the deviatoric component of the viscoplastic strain is written in the form:

$\Delta\tilde{\varepsilon}_{ij}^{vp}=\langle\Phi\rangle\sqrt{\frac{3}{2}}\frac{s_{ij}}{s_{II}}\Delta t$ or $\Delta\tilde{\varepsilon}_{ij}^{vp}=\langle\Phi\rangle\frac{3}{2}\frac{s_{ij}}{\sigma_{eq}}\Delta t$

like $\sigma_{eq}=\sqrt{\frac{3}{2}}s_{II}$, $s_{II}=\sqrt{s_{ij}s_{ij}}$ and $\sigma_{\mathrm{eq}}^{el}=\sqrt{\frac{3}{2}s_{ij}^{el}s_{ij}^{el}}$

One writes also the following equalities:

$$s_{ij}\frac{\sigma_{eq}^{el}}{\sigma_{eq}}=s_{ij}^{el}$$

$$\Delta p=\sqrt{(\frac{2}{3}\Delta\tilde{\epsilon}_{ij}^{vp}\Delta\tilde{\epsilon}_{ij}^{vp})}$$

$$\frac{\Delta p}{\Delta t}=\langle\Phi\rangle=A\left\langle\frac{f(\sigma,p)}{P_{ref}}\right\rangle^n \quad \textbf{éq 1}$$

from where: $\Delta p=\langle\phi\rangle\Delta t$

By means of these equalities one can find a statement for s_{ij} , σ_{eq} and I_1 according to s_{ij}^{el} σ_{eq}^{el} , I_1^{el} and Δp :

$$s_{ij}=s_{ij}^{el}-2\mu\Delta\tilde{\varepsilon}_{ij}^{vp}=s_{ij}^{el}-3\mu\langle\Phi\rangle\frac{s_{ij}}{\sigma_{eq}}\Delta t=s_{ij}^{el}-3\mu\langle\Phi\rangle\frac{s_{ij}^{el}}{\sigma_{eq}^{el}}\Delta t$$

$$s_{ij}=s_{ij}^{el}\left(1-\frac{3\mu\langle\Phi\rangle}{\sigma_{eq}^{el}}\Delta t\right)=s_{ij}^{el}\left(1-\frac{3\mu}{\sigma_{eq}^{el}}\Delta p\right)$$

$$\sigma_{eq}=\sigma_{eq}^{el}-3\mu\langle\Phi\rangle\Delta t=\sigma_{eq}^{el}-3\mu\Delta p \quad \textbf{éq 2}$$

$$I_1=I_1^{el}-3\mathrm{K}\,\Delta\varepsilon_v^{vp}=I_1^{el}-9\mathrm{K}\,\beta\langle\Phi\rangle\Delta t=I_1^{el}-9\mathrm{K}\,\beta\,\Delta p \quad \textbf{éq 3}$$

4.2.3 Computation of the unknown

the viscoplastic increment of cumulated strain Δp is the only unknown of the problem. To determine it, the viscoplastic flow model is written (éq 1):

$$\frac{\Delta p}{\Delta t}=A\left\langle\frac{\sigma^{eq}+\alpha(p)I_1-R(p)}{P_{ref}}\right\rangle^n$$

$$R(p)=R(p^-+\Delta p)=R^-+R_{const}\Delta p \ ; \ R_{const}=\frac{\partial R}{\partial p}$$

$$\alpha(p)=\alpha(p^-+\Delta p)=\alpha^-+\alpha_{const}\Delta p \ ; \ \alpha_{const}=\frac{\partial\alpha}{\partial p}$$

$$\beta(p)=\beta(p^-+\Delta p)=\beta^-+\beta_{const}\Delta p \ ; \ \beta_{const}=\frac{\partial\beta}{\partial p}$$

By preoccupation with a simplification of the writing of the equation in Δp , one poses:

$$C=\frac{(A\,\Delta t)}{P_{ref}{}^n}$$

Maybe, while replacing σ_{eq} and I_1 by their statements (éq 2 and éq 3), one obtains:

$$F(\Delta p)=C\left\langle\begin{matrix}(\sigma_{eq}^{el}+\alpha I_1^{el}-R^-)-(3\mu+R_{const}-\alpha_{const}I_1^{el}+9\mathrm{k}\,\alpha^-\beta^-)\Delta p\\ -\\ (9\mathrm{k}\,\alpha^-\beta_{const}+9\mathrm{k}\,\alpha_{const}\beta^-)\Delta p^2-(9\mathrm{k}\,\alpha_{const}\beta_{const})\Delta p^3\end{matrix}\right\rangle^n-\Delta p=0$$

One seeks $\Delta p/(\Delta p)=0$

$F(\Delta p)=0$ is a nonlinear scalar equation. The lower limit being $x_{\inf}=0$ and the higher limit can be built-in with $x_{\sup}=A\left\langle\frac{\sigma_{eq}^{el}+\alpha I_1^{el}-R^-}{P_{ref}}\right\rangle^n\Delta t$

One uses the method of the ropes with a control of the interval of search while taking as a starting point the document [R5.03.04].

$$\Delta p\in[x_{\inf},x_{\sup}] \ ;$$
$$x=\Delta p$$

So $|F(x_{\inf})|<\eta$ then $\Delta p=x_{\inf}$

So $|F(x_{\sup})|<\eta$ then $\Delta p=x_{\sup}$

So $F(x_{\inf})>0$ then $x_2=x_{\inf}$ and $y_2=F(x_{\inf})$

So $F(x_{\sup})<0$ then one makes a loop by cutting out $x_{\sup}$ by 10 until obtaining a value of $x_{\sup}$ for which $F(x_{\sup})>0$ in this case one multiplies the last solution by 10 and one fixes $x_1=x_{\sup}$ and $y_1=F(x_{\sup})$

So $F(x_{\sup})>0$ then one makes a loop by multiplying $x_{\sup}$ by 10 until obtaining a value of $x_{\sup}$ for which $F(x_{\sup})<0$ and one fixes $x_1=x_{\sup}$ and $y_1=F(x_{\sup})$

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So $F(x_{\sup})<0$ then one makes a loop while multiplying $x_{\sup}$ by 10 until obtaining a value of $x_{\sup}$ for which $F(x_{\sup})>0$ and one fixes $x_2=x_{\sup}$ and $y_2=F(x_{\sup})$

Of the checks are made on the values which the limits can take and in particular if they are weaker than a tolerance built-in with 1.E-12, they will be considered equal to 0. and thus the solution Δp also. If the limits are equal, one makes a recutting of time step.

The values x_1 x_2 , y_1 and y_2 will be the values to be given as starter to the routine zeroco which is based on the method of the ropes. The solution is calculated by the following formula:

$$x^{n+1}=x^{n-1}-F(x^{n-1})\frac{x^n-x^{n-1}}{F(x^n)-F(x^{n-1})}$$

With the following values, one represents the scalar function to solve.

σ_{eq}^{el}	$6{,}315\ MPa$	α^-	$6{,}86\,10^{-2}$
I_1^{el}	$-21{,}061\ MPa$	β^-	$-0{,}147$
N	$4{,}5$	R^-	$1{,}394\ MPa$
Δt	$10\ s$	α_{const}	$13.$
A	$1{,}5\,10^{-12}$	β_{const}	$10.$
P_{ref}	$0{,}1\ MPa$	R_{const}	$329{,}732\ MPa$

The unknown x for which $F(x)$ cancels itself locates between 6.10^{-5} and 7.10^{-5} who is well between the lower limit x_{inf} and the higher limit $x_{\sup}$ which is worth in this precise case $1{,}2913\,10^{-4}$.

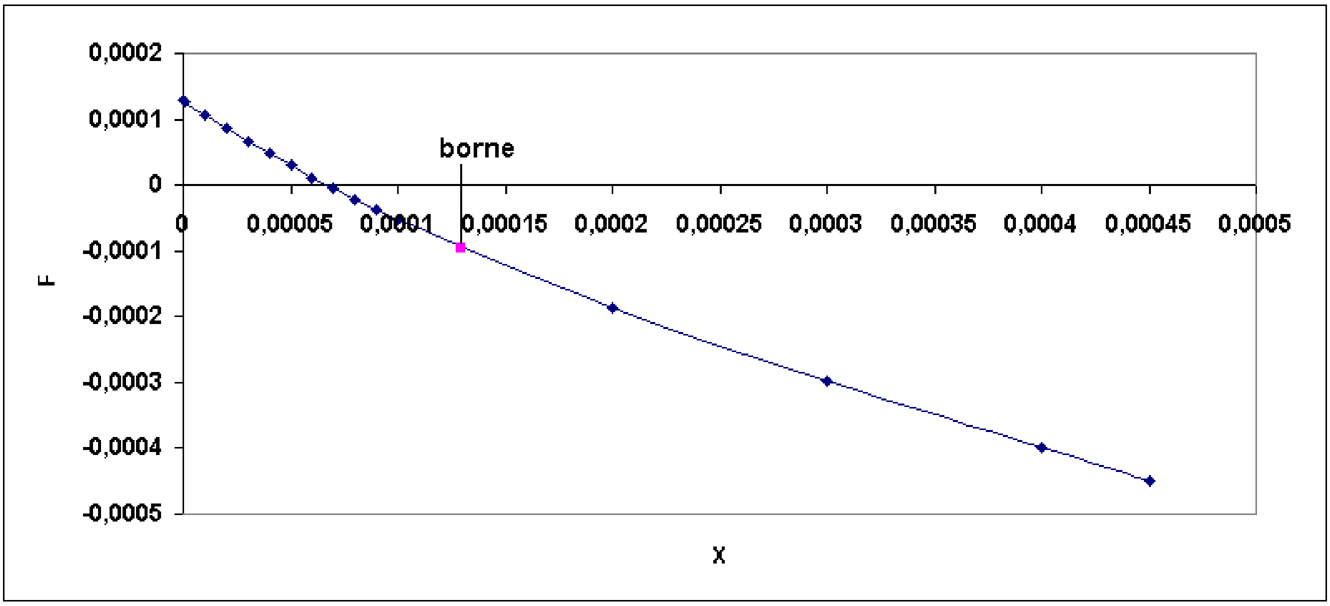

Figure 4-1: Pace of the scalar function

4.3 coherent tangent Operator

One seeks to calculate: $\frac{\partial\sigma}{\partial\varepsilon}=\frac{\partial s}{\partial\varepsilon}+\frac{I}{3}\otimes\frac{\partial I_1}{\partial\varepsilon}$

With,

$$\frac{\partial s}{\partial\varepsilon}=\frac{\partial s^{el}}{\partial\varepsilon}\left(1-\frac{3\mu}{\sigma_{eq}^{el}}.\Delta p\right)+\frac{3\mu}{\left(\sigma_{eq}^{el}\right)^2}.\Delta p\left(s^{el}\otimes\frac{\partial\sigma_{eq}^{el}}{\partial\varepsilon}\right)-\frac{3\mu}{\sigma_{eq}^{el}}.\left(s^{el}\otimes\frac{\partial\Delta p}{\partial\varepsilon}\right)$$

$$\frac{\partial I_1}{\partial\varepsilon}=\frac{\partial I_1^{el}}{\partial\varepsilon}-9\mathrm{K}\beta(p)\frac{\partial\Delta p}{\partial\varepsilon}$$

Computation of $\frac{\partial s^{el}}{\partial\epsilon}$:

$$\frac{\partial s^{el}}{\partial\varepsilon}=2\mu\left(I_d-\frac{1}{3}1\otimes1\right)$$

$$\frac{\partial s_{ij}}{\partial\varepsilon_{pq}}=2\mu\left(\delta_{ip}\delta_{jq}-\frac{1}{3}\delta_{ij}\delta_{pq}\right)$$

Computation of $\frac{\partial I_1^{el}}{\partial\varepsilon}$:

$$\frac{\partial I_1^{el}}{\partial\varepsilon}=3\mathrm{K}1$$

$$\frac{\partial I_1^{el}}{\partial \varepsilon_{pq}}=3\mathrm{K}\,\delta_{pq}$$

Computation of $\frac{\partial \sigma_{eq}^{el}}{\partial \varepsilon}$:

$$\frac{\partial \sigma_{eq}^{el}}{\partial \varepsilon}=\frac{\partial \sigma_{eq}^{el}}{\partial \sigma^{el}}\frac{\partial \sigma^{el}}{\partial \varepsilon}=\frac{\partial \sigma_{eq}^{el}}{\partial s^{el}}\frac{\partial s^{el}}{\partial \sigma^{el}}\frac{\partial \sigma^{el}}{\partial \varepsilon}=\frac{3}{2}\frac{s^{el}}{\sigma_{eq}^{el}}\left(I_d-\frac{1}{3}1\otimes 1\right)D^e=\frac{3}{2}\frac{s^{el}}{\sigma_{eq}^{el}}D^e$$

Computation of $\frac{\partial \Delta p}{\partial \varepsilon}$:

$$\frac{\Delta p}{\Delta t}=A\left\langle\frac{f(\sigma,p)}{P_{ref}}\right\rangle^n$$

that is to say $F(\Delta p)=\frac{A\Delta t}{P_{ref}^n}\langle f(\sigma,p)\rangle^n-\Delta p$

$$\frac{\partial \Delta p}{\partial \varepsilon}=\frac{\partial \Delta p}{\partial \sigma^{el}}\frac{\partial \sigma^{el}}{\partial \varepsilon}$$

to compute: $\frac{\partial \Delta p}{\partial \sigma^{el}}$, one uses $F\left(\sigma^{el},p\right)=0$

$$\frac{\partial F\left(\sigma^{el},p\right)}{\partial \sigma^{el}}\delta\sigma^{el}+\frac{\partial F\left(\sigma^{el},p\right)}{\partial \Delta p}\delta\Delta p=0$$

$$\frac{\delta \Delta p}{\delta \sigma^{el}}=-\frac{\dfrac{\partial F\left(\sigma^{el},p\right)}{\partial \sigma^{el}}}{\dfrac{\partial F\left(\sigma^{el},p\right)}{\partial \Delta p}}$$

$$F(\Delta p)=C\left\langle\begin{array}{c}(\sigma_{eq}^{el}+\alpha I_1^{el}-R^-)-(3\mu+R_{const}-\alpha_{const}I_1^{el}+9\mathrm{k}\,\alpha^-\beta^-)\Delta p\\ -\\ (9\mathrm{k}\,\alpha^-\beta_{const}+9\mathrm{k}\,\alpha_{const}\beta^-)\Delta p^2-(9\mathrm{k}\,\alpha_{const}\beta_{const})\Delta p^3\end{array}\right\rangle^n-\Delta p=0$$

$$\frac{\partial F\left(\sigma^{el},p\right)}{\partial \sigma^{el}}=C.n.\langle f(\sigma^{el},p)\rangle^{n-1}\frac{\partial f\left(\sigma^{el},p\right)}{\partial \sigma^{el}}$$

where $\frac{\partial f\left(\sigma^{el},p\right)}{\partial \sigma^{el}}=\left(\frac{\partial \sigma_{eq}^{el}}{\partial \sigma^{el}}+\alpha\frac{\partial I_1^{el}}{\partial \sigma^{el}}\right)+\alpha_{const}\left(\frac{\partial I_1^{el}}{\partial \sigma^{e}}\right)\Delta p$

$$\frac{\partial F\left(\sigma^{el},p\right)}{\partial \Delta p}=C.n.\langle f(\sigma^{el},p)\rangle^{n-1}\frac{\partial f\left(\sigma^{el},p\right)}{\partial \Delta p}-1$$

$$\frac{\partial f(\sigma^{el},p)}{\partial \Delta p}=-\left(3\mu+R_{const}-\alpha_{const}I_1^{el}+9\mathrm{k}\alpha^-\beta^-\right)$$
$$-2\Delta p\,9\mathrm{k}\left(\alpha^-\beta_{const}+\alpha_{const}\beta^-\right)-3\Delta p^2\,9\mathrm{k}\left(\alpha_{const}\beta_{const}\right)$$

Computation of $\frac{\partial\sigma_{eq}^{el}}{\partial\sigma^{el}}$:

$$\frac{\partial\sigma_{eq}^{el}}{\partial\sigma^{el}}=\frac{\partial\sigma_{eq}^{el}}{\partial s^{el}}\frac{\partial s^{el}}{\partial\sigma^{el}}=\frac{3}{2}\frac{s^{el}}{\sigma_{eq}^{el}}.\left(I_d-\frac{1}{3}1\otimes 1\right)=\frac{3}{2}\frac{s^{el}}{\sigma_{eq}^{el}}$$

Computation of $\frac{\partial I_1^{el}}{\partial\sigma^{el}}$:

$$\frac{\partial I_1^{el}}{\partial\sigma^{el}}=\frac{\partial tr\left(\sigma^{el}\right)}{\partial\sigma^{el}}=1$$

4.1 Data materials

the 16 parameters of the model are:

under ELAS

E : Young modulus (Pa or MPa)
ν : Poisson's ratio

under VISC_DRUC_PRAG

P_{ref} : pressure of reference (Pa or MPa)
A : viscoplastic parameter (in s^{-1})
n : power of the model creep
p_{pic} : rate of hardening on the level of the threshold of peak
p_{ult} : rate of hardening on the level of the ultimate threshold
α_0 , α_{pic} and α_{ult} : parameters of the function of cohesion $\alpha(p)$
R_0 , R_{pic} and R_{ult} : parameters of the function of hardening $R(p)$
β_0 , β_{pic} and β_{ult} : parameters of the function of dilatancy $\beta(p)$

4.2 local variables

$v_1=p$;

$v_2=(0 \text{ ou } 1)$; indicator of plasticity;

$v_3=pos$; position of the point of load compared to the thresholds;

($\mathrm{pos}=1 \text{ si } 0<p<p_{\mathrm{pic}}$; $\mathrm{pos}=2 \text{ si } \mathrm{p_{pic}}<p<p_{\mathrm{ult}}$; $\mathrm{pos}=3 \text{ si } \mathrm{p}>p_{\mathrm{ult}}$)

v_4 ; nombre of iterations local;

4.3 Summarized algorithm of resolution

the algorithm of resolution such as it is implemented in Code_Aster:

$$\sigma^{el}=\sigma^{-}+D^{e}\Delta\varepsilon$$

The criterion: $f\left(\sigma^{el},p^{-}\right)=\sigma_{eq}^{el}+\alpha\left(p^{-}\right)I_1^{el}-R\left(p^{-}\right)$

Elasticity : $f\left(\sigma^{el},p^{-}\right)\le 0 \quad \Delta p=0$;

Viscoplasticity: $f\left(\sigma^{el},p^{-}\right)=0 \quad \Delta p\ge 0$ with Δp solution of the equation $F\left(\Delta p\right)=0$

where,

$$\frac{\Delta p}{\Delta t}=A\left\langle\frac{\sigma^{eq}+\alpha\left(p\right)I_1-R\left(p\right)}{P_{ref}}\right\rangle^{n}=\frac{A}{P_{ref}^{n}}\left\langle f\left(\sigma,p\right)\right\rangle^{n}$$

and

$$F=\frac{A\,\Delta t}{P_{ref}^{n}}\left\langle f(\sigma,p)\right\rangle^{n}-\Delta p$$

Put up to date of the stresses:

$$\sigma=\sigma^{el}-D^{e}\Delta\epsilon^{vp}$$

$$s=s^{el}\left(1-3\frac{\mu}{\sigma_{eq}^{el}}\Delta p\right)$$

$$\sigma_{eq}=\sigma_{eq}^{el}-3\mathrm{G}\,\Delta p$$

$$I_1=I_1^{el}-9\mathrm{K}\,\beta\,\Delta p$$

$$\sigma=s+\frac{I}{3}\otimes I_1$$

Once Δp is calculated, the stresses and the up to date put local variables, one checks the position from p ratio with p^{-} and signs it $f\left(\sigma,p\right)$:

If $0<p^{-}<p_{pic}$; to test 1) if not 2) if not 3)
If $p_{pic}<p^{-}<p_{ult}$; to test 2) if not 3)
If $p^{-}>p_{ult}$; to test 3)

If $p^{-}+\Delta p<p_{pic}$;

one checks $f\left(\sigma,p\right)>0$ with R , α and β corresponding to $0<p<p_{pic}$,
so $f\left(\sigma,p\right)>0$ then one updates the stress fields
and of local variables,
if not, one considers that Δp is not valid and one redécoupe time step

If $p_{pic} < p^- + \Delta p < p_{ult}$;

one checks $f(\sigma, p) > 0$ with R, α and β corresponding to $p_{pic} < p < p_{ult}$
so $f(\sigma, p) > 0$ then one updates the stress fields
and of local variables,
if not, one considers that Δp is not valid and one redécoupe time step

If $p^- + \Delta p \geq p_{ult}$;

one checks $f(\sigma, p) > 0$ with R, α and β corresponding to $p \geq p_{ult}$
so $f(\sigma, p) > 0$ then one updates the stress fields
and of local variables,
if not, one considers that Δp is not valid and one redécoupe time step

5 the Results of a triaxial compression test

It acts to simulate a triaxial compression test (see the case test ssnv211) while imposing like a stress of containment of $5\,MPa$. A uniaxial strain is imposed in compression and which evolves in time. The velocity of the loading is fixed at $10^{-5}\,m/s$. The deviator of the stresses and the voluminal strain according to the imposed axial strain are represented Ci against.

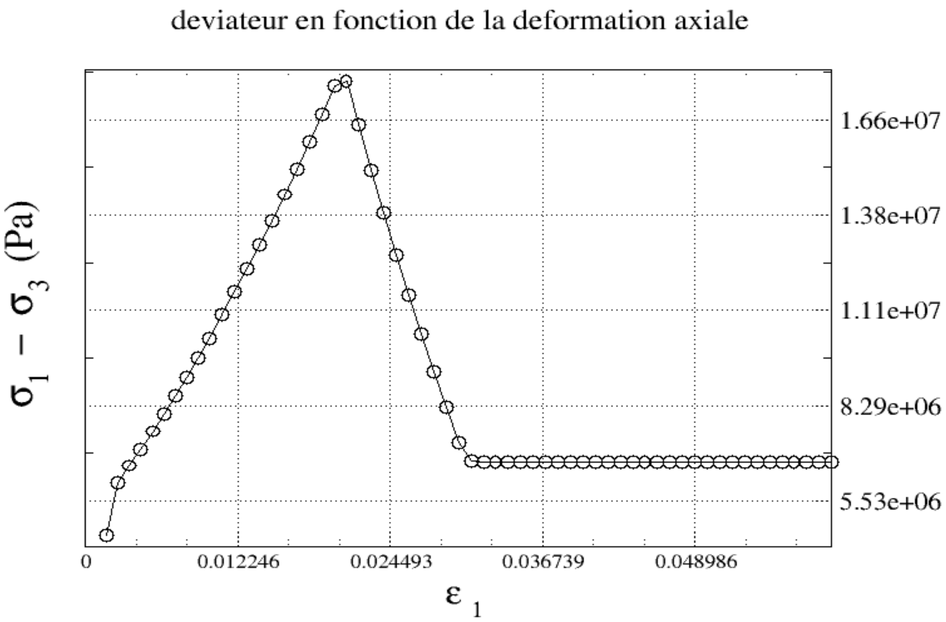

Figure 5-1: Deviator of the stresses according to the uniaxial strain

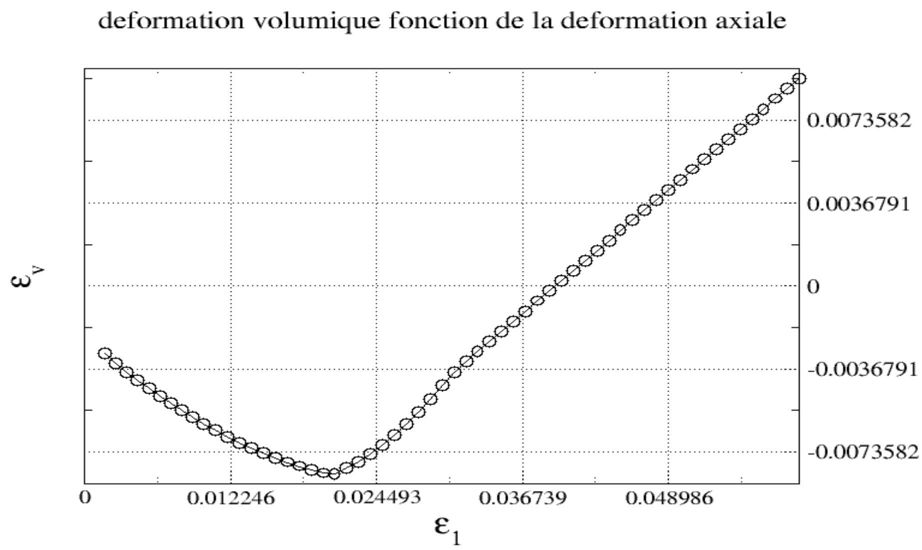

Figure 5-2: Voluminal strain according to the uniaxial strain

6 Functionalities and checking

the constitutive law can be defined by the key word `VISC_DRUC_PRAG` (command `STAT_NON_LINE`, key word factor `COMP_INCR`). It is associated with material `VISC_DRUC_PRAG` (command `DEFI_MATERIAU`).

Model `VISC_DRUC_PRAG` is checked by the cases following tests:

SSNV211	[V6.04.211]	triaxial Compression test drained with the model `VISC_DRUC_PRAG`
WTNV137	[V7.31.137]	triaxial Compression test drained with the model `VISC_DRUC_PRAG`
WTNV138	[V7.31.138]	triaxial Compression test not drained with the model `VISC_DRUC_PRAG`

7 References

[1] J. EL GHARIB and C. CHAVANT, “Chock on triaxial compression tests of a viscoplastic constitutive law for the mudstone based on the model Drucker_Prager”, H-T64-2008-04194-FR,

[2] J. EL GHARIB and C. CHAVANT, “Implemented in Code_Aster of a simplified viscoplastic model”, H-T64-2007-01800-FR,

8 Description of the versions of the document

Version Aster	**Author (S) Organization (S)**	**Description of the modifications**
10.0	J. EL GHARIB, C.CHAVANT EDF R &D/AMA	initial Text