Constitutive law (in 2D) for steel-concrete connection: JOINT_BA

Summarized:

Constitutive law JOINT_BA describes the phenomenon of degradation and fracture of connection between the steel bars and the concrete, in reinforced concrete structures. This documentation presents the theoretical writing in the thermodynamic frame and the numerical integration of the model, as well as the parameters which manage the model.

For his use, one will lean on the finite elements of the type joined (see the document [R3.06.09]) already existing in the code.

Contents

1 Introduction

constitutive law `JOINT_BA` describes the phenomenon of degradation and fracture of existing connection between the steel bars (smooth or ribbed) and the concrete surrounding it. Key point for the structural design out of reinforced concrete, the purpose of the modelization of steel-concrete connection is the representation as well as simplification of this complex phenomenon of interaction between the two materials which develops in the interface and which undergoes an increasing degradation when certain thresholds of strength are exceeded, specific for each material. The structural models which do not take into account the linkage effects, are generally unable to predict the localization of cracks as well as the networks created. In addition, the degradation of the stiffness of connection increases the period of vibration, reduced the capacity of dissipation of energy and conduit to a significant redistribution of the internal forces (according to *Bertero*, 1979, cf [bib2]).

Constitutive law `JOINT_BA` is described in the frame of the thermodynamics of the irreversible processes: the writing and L `use of a “classical” model of material coupling cracking and friction make it possible to integrate in a robust way of the fine mechanisms nonlinear concomitant into the particular description of the kinematics of sliding. This last point enables us not to resort to the classical modelizations of type “contact” very often used in this context in spite of the many sources of numerical instabilities. Thus, in monotonic loading the taking into account of the coupling normal force – shears make it possible to treat cases of strong multiaxial pressures; in cyclic, the hysteretic behavior and the corresponding dissipations are expressed thanks to the coupling between the state of damage and kinematic hardening. The use of an implicit scheme makes it possible to obtain a robust implementation.

The paragraph [§2] described in short form the phenomenon of the bond steel concrete. The paragraph [§3] presents the thermodynamic writing of the constitutive law, while the paragraph [§4] specifies the stage of numerical integration of the model. The parameters which manage the model and which could be obtained starting from the properties of the implied materials, are described in the paragraph [§5].

2 Short description of steel-concrete connection

Conceptually, the phenomenon of connection corresponds to the physical interaction of two different materials, which occurs on a zone of interface by allowing the transfer and the continuity of the forces and the stresses between the two bodies in contact. In the case of the reinforced concrete structures, this phenomenon is also known as the “stiffness of tension” which develops around an element of reinforcement, partially or completely drowned in a volume of concrete. The tensile forces which appear inside the reinforcement are transformed into shearing stresses on surface, and are transmitted directly to the concrete in contact which will balance them finally, and vice versa. The response of the group will depend on the capacity of the concrete to become deformed as much as steel, since steel will tend to slip inside the concrete surrounding it. The phenomenon of connection corresponds to this capacity of the concrete to become deformed and to be degraded locally by creating a species of layer, or wraps, around the reinforcement, whose kinematical and material properties differ from those moreover concrete or used reinforcement.

The phenomenon can be broken up into three well defined mechanisms:

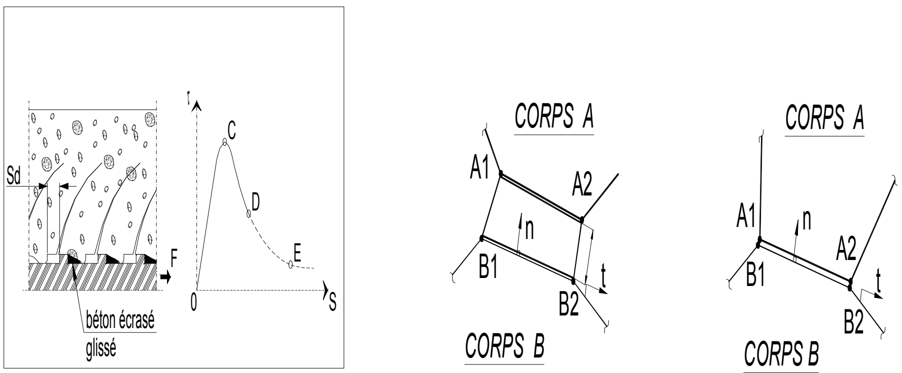
- a chemical dependancy of origin,
- a mechanism of friction between two rough surfaces (steel-concrete or concrete-concrete),
- a mechanical action created by the presence of the veins of the steel bar on the neighbouring concrete.

According to this decomposition, one can deduce clearly that for a bar smoothes, the dominating mechanism is friction between the two materials, while for a bar ribbed (in French usually called “reinforcement ha: High Dependancy”), the mechanism dominating is the mechanical interaction between surfaces. When reinforcement is consisted the strands with steel wire ropes, it is possible to control or combine the various mechanisms since they are function directly of the surface of the cables.

Connection will undergo a different degradation according to the type of loading applied, either monotonous, or cyclic. In addition, among the most important parameters which influence the behavior of connection, one can quote:

1) characteristics of the loading,
2) geometrical characteristics of the steel bar,
3) spacing between active bars,
4) characteristics of the concrete,
5) containment by passive reinforcement,
6) side pressure.

At the time of the study of a cylindrical bar drowned in an infinite medium, one can identify the surface of discontinuity where one will place the linkage effects, which develops in a certain zone of concrete fissured and crushed around the steel bar. A a given moment, this surface will correspond to cylindrical crack created during the coalescence of cracks of shears. By looking at the network of cracks, one can suppose that, in ideal conditions, the plane of cracking is always perpendicular (normal direction) to the surface of the bar and parallel (tangential direction) to its longitudinal axis (see [Figure 2-a]). That enables us to project the components of displacement on the normal and tangential direction of the plane of cracking, and consequently to obtain the corresponding strains and stresses.



Appear 2-a: real description of the phenomenon of connection and simplification finite elements: coordinated in the local coordinate system of the element of interface used as support of theoretical model

3 JOINT_BA Writing

the formulation presented here was developed in the frame of the thermodynamics of the irreversible processes; it gives the constitutive relation between the normal force, the shearing stress and the sliding by considering the influence of the cracking of the concrete, friction and the various couplings in the phenomenon. For that, the constitutive relations which connect the tensor of the stresses and the tensor of the strains must include:

- the cracking of the material of interface by shears
- the inelastic strains because of sliding
- the hysteretic behavior due to friction
- the coupling between the tangential response and the normal stresses

3.1 Presentation of the model

One is placed in the frame of a formulation planes in 2D, in the definite local coordinate system [Figure 2-a]. The tensors of the stresses $\boldsymbol{\sigma}$ and the strains ε are written:

$$\varepsilon=\begin{pmatrix}\varepsilon_N & \varepsilon_\tau\\ \varepsilon_\tau & 0\end{pmatrix} \quad \text{and} \quad \sigma=\begin{pmatrix}\sigma_N & \sigma_\tau\\ \sigma_\tau & 0\end{pmatrix}$$ éq 3.1-1

where σ_N is the normal stress and σ_τ is the shear stress of the element of interface; $\varepsilon_N$corresponds to the normal strain and ε_τ the tangential strain. The normal strain in the tangential direction with the interface is regarded as null. This mode of strain for an element of dependancy is with strain energy null.

The normal and tangential behaviors being regarded as uncoupled on the level from the state, the thermodynamic potential obtained from the free energy of Helmholtz is expressed in the following way:

$$\rho\cdot\psi=\frac{1}{2}[\langle\varepsilon_N\rangle_- E\langle\varepsilon_N\rangle_- +\langle\varepsilon_N\rangle_+ E\cdot(1-D_N)\langle\varepsilon_N\rangle_+ \\ +\varepsilon_T G(1-D_T)\varepsilon_T+(\varepsilon_T-\varepsilon_T^f)G\cdot D_T(\varepsilon_T-\varepsilon_T^f)+\gamma\alpha^2]+H(z)$$ éq 3.1-2

where ρ is the density, E is the Young modulus, D_N is the local variable of normal damage and D the local variable of tangential damage, both being related to the cracking and ranging between 0 and 1. G is the modulus of stiffness or of shears, ε_T^f is the unrecoverable deformation induced by sliding with friction of cracks, α is the local variable of kinematic hardening, γ is a material parameter and z , the variable of pseudonym "isotropic hardening" by damage, with its function of consolidation $H(z)$. $\langle\rangle_-$ and $\langle\rangle_+$ define respectively the positive and negative parts tensor considered.

One can notice in the equation [éq 3.1-2] that in the normal direction, the damage will be activated during the appearance of the positive strains produced by tensile forces, while if the strains are negative because of effects of compression, the behavior will remain elastic. With regard to the tangential part of the behavior, one can recognize a classical coupling elasticity-damage as well as a new term allowing to associate with the state elasticity-endommageable, a state of sliding with friction. The coupling between sliding and cracking is possible thanks to the presence of the variable of damage like multiplier in the second element of the right part of the equation [éq 3.1-2].

The state models are obtained classically by derivative of the thermodynamic potential, and thus make it possible to define the associated thermodynamic variables. The normal stress is expressed like:

$$\sigma_N=\rho\frac{\partial\psi}{\partial\varepsilon_N}=\begin{cases}E\cdot\varepsilon_N & \text{si} \quad \varepsilon_N\le 0\\ (1-D_N)\cdot E\cdot\varepsilon_N & \text{si} \quad \varepsilon_N>0\end{cases}$$ éq 3.1-3

and the total shear stress like:

$$\sigma_T=\rho\frac{\partial\psi}{\partial\varepsilon_T}=G(1-D_T)\varepsilon_T+G\cdot D_T(\varepsilon_T-\varepsilon_T^f)$$ éq 3.1-4

One can also define the shear stress due to the sliding with friction (strain ε_τ^s):

$$\sigma_T^f=-\rho\frac{\partial\psi}{\partial\varepsilon_T^f}=G\cdot D_T(\varepsilon_T-\varepsilon_T^f)$$ éq 3.1-5

Note::

Such a formulation moves away amply from a classical formulation of coupling plasticity – damage. The assumption bringing to the introduction of the damage into the stress by sliding is based on an experimental observation which is that all the inelastic phenomena in a brittle material come from the growth of cracks.

The rate of energy restored by damage-friction can be written like:

$$-Y=-\rho\frac{\partial\psi}{\partial D_T}=\frac{1}{2}\varepsilon_T\cdot G\cdot\varepsilon_T-\frac{1}{2}\left(\varepsilon_T-\varepsilon_T^f\right)\cdot G\cdot\left(\varepsilon_T-\varepsilon_T^f\right)=-\left(Y_{DT}+Y_{fT}\right) \quad \text{éq 3.1-6}$$

In this last statement, Y_{DT} corresponds to the rate of energy restored by damage and Y_{fT} to the rate of energy restored by friction of cracks.

The state model of kinematic hardening brings to the definition of the stress of recall:

$$X=\rho\frac{\partial\psi}{\partial\alpha}=\gamma\alpha \quad \text{éq 3.1-7}$$

Concerning the model of hardening of the isotropic damage, it is expressed by:

$$Z=\rho\frac{\partial\psi}{\partial z}=\mathrm{H}'(z) \quad \text{éq 3.1-8}$$

It is necessary for us now in the connection to clarify in a more detailed way the evolution of the damage mechanism, in other words to specify the statement of $H(z)$. For a low value of damage, the mechanism which prevails is the interaction of the concrete with the veins of the steel bar, while for a value much larger, it is the friction between the concrete and the steel which prevails. During the evolution of the damage, 2 principal phases could be identified:

1) the first phase corresponds to transverse a stable crack growth related to the presence of veins on steel (positive apparent hardening of the law of evolution),
2) the second does not utilize more but the coalescence of these transverse cracks bringing not to consider but the mechanisms of friction more (negative hardening towards a residual stress of friction).

3.2 Damage in the tangential direction

the law of evolution of the damage analyzes is divided into three stages:

- region of perfect dependancy,
- region of transition of small strains to the great slidings,
- region of maximum strength of connection and degradation until ultimate residual strength.

To identify these regions, two thresholds are established:

1) the threshold of perfect dependancy ε_T^1 ,
2) the threshold of continuity before coalescence of cracks ε_T^2 .

Thus, by taking again the statements related to the damage with knowing that of rate of energy restitution [éq 3.1-6] and that of the local variable associated with isotropic hardening [éq 3.1-8], one can note:

1) a true separation between L “damage and the friction of the cracks (what makes it possible to amend only the law D” evolution of the damage without affecting the part “friction”),
2) the partition in two parts of isotropic hardening since one has two different stages in the damage.

From now on we will write for hardening related to the variable of damage:

$$Z_T=\rho\frac{\partial\psi}{\partial z}=H(z)=\begin{cases}Z_{T1}, & \text{si}\quad \varepsilon_T^1<\varepsilon_T\le\varepsilon_T^2\\ Z_{T1}\cdot Z_{T2}, & \text{si}\quad \varepsilon_T^2<\varepsilon_T\end{cases}$$ éq the 3.2-1

components Z_{T1} and Z_{T2} are expressed in the following way:

$$Z_{T1}=\left[\sqrt{Y_{T1}}+\frac{1}{A_{1_{DT}}}\cdot\sqrt{\frac{G}{2}}\cdot\ln\left((1+z_T)\frac{\varepsilon_T}{\varepsilon_T^1}\right)\right]^2$$ éq 3.2-2

$$Z_{T2}=\left[y_{T2}+\frac{1}{A_{2_{DT}}}\left(\frac{-z_T}{1+z_T}\right)\right]$$ éq 3.2-3

One also defines the function threshold φ_{DT} which depends on Y_{DT} and which S `writes like:

$$\varphi_{DT}=Y_{DT}-(Y_{T1}+Z_T)\le 0$$ éq the 3.2-4

thresholds which manage the law of evolution of the damage are also expressed in terms Y_{DT} (see [Figure 3.2-a]). The first statement corresponds to the threshold of perfect dependancy and S `written:

$$Y_{T1}=Y_{elas}|_T=\frac{1}{2}\varepsilon_T^1\cdot G\cdot\varepsilon_T^1$$ éq 3.2-5

Where Y_{T1} is the initial threshold of damage defined according to the limiting strain of perfect dependancy ε_T^1, which will correspond to the limiting strain of shears – or tension – concrete before the initialization of the damage. In addition, Y_{T2} is the threshold of initiation of coalescence of microphone - cracks which is defined according to the initial tangential strain of the great slidings ε_T^2 :

$$Y_{T2}=\frac{1}{2}\varepsilon_T^2\cdot G\cdot\varepsilon_T^2$$ éq 3.2-6

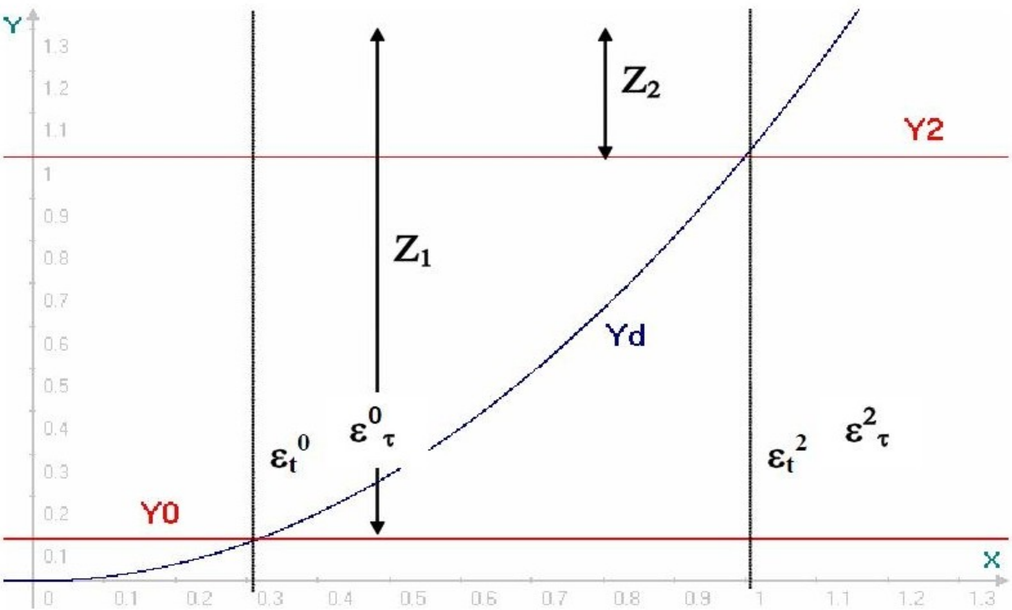


Figure 3.2-a: construction of the functions thresholds in terms of energy

the laws of evolution of the local variables in the frame of the standard associated models make it possible to obtain derivative of the multiplier of damage λ_D :

$$\dot{D}=\dot{\lambda}_D\cdot\frac{\partial\varphi_D}{\partial Y_D}=\dot{\lambda}_D \quad \text{et} \quad \dot{z}=\dot{\lambda}_D\cdot\frac{\partial\varphi_D}{\partial Z}=-\dot{\lambda}_D \qquad \text{éq 3.2-7}$$

By means of in addition the condition of consistency, one obtains the statement of the damage:

$$D_T=1-\sqrt{\frac{Y_{T1}}{Y_{DT}}}\cdot\exp\left\{A_{1_{DT}}\cdot\left[\sqrt{\frac{2}{G}}\cdot\left(\sqrt{Y_{DT}}-\sqrt{Y_{T1}}\right)\right]^{B_{1DT}}\right\}* \\ *\left\{\frac{1}{1+A_{2_{DT}}\cdot\langle Y_{DT}-Y_{T1}\rangle_+^{B_{2DT}}}\right\} \qquad \text{éq 3.2-8}$$

In this statement, one can identify the part which corresponds to the region of the transition of $A_{1_{DT}}$ the small strains to the great slidings with two parameters: and $B_{1_{DT}}$, as well as the final part of damage in mode 2, with the parameters $A_{2_{DT}}$ and $B_{2_{DT}}$. It should be noted that the relation $\langle Y_{DT}-Y_{T1}\rangle$ is managed by a function of *Macaulay*, i.e. this difference in energy must be always positive or null.

The functions which manage isotropic hardening in the tangential direction are expressed like:

$$Z_{T1}=Y_{DT}-Y_{T1}\ ; \qquad \text{éq 3.2-9}$$

$$Z_{T2}=\begin{cases}0, & \text{si } Y_{T1}<Y_{DT}\le Y_{T2}\\ Y_{DT}-Y_{T2}, & \text{si } Y_{T2}<Y_{DT}\end{cases} \qquad \text{éq 3.2-10}$$

According to these statements, one can notice that Z_{T2} is not taken into account in the region of transition from the small strains to great slidings.

3.3 Damage in the normal direction analyzes

the two most important mechanisms which can appear on the normal direction are the detachment between the concrete and the steel bars, and the penetration of reinforcement in the body of the concrete. These two conditions can be interpreted respectively like an opening or a closing of crack, and can be described by a particular constitutive law in the normal direction uncoupled from the tangential behavior.

In order to simplify the resolution for compression between surfaces, one decided to allow a small penetration between those, which implies that $\varepsilon_N\le 0$, and by adopting an elastic constitutive law, one will have:

$$\sigma_N=E\cdot\langle\varepsilon_N\rangle^- \quad \text{si} \quad \varepsilon_N\le 0 \qquad \text{éq the 3.3-1}$$

cases of the decoherence of the interface can be described by a behavior endommageable in the normal direction, that is to say:

$$\sigma_N = (1-D_N)\cdot E\cdot\langle\varepsilon_N\rangle^+ \quad \text{si} \quad \varepsilon_N > 0$$ éq 3.3-2

with D_N scalar variable of the damage in the normal direction, calculated with the following statement:

$$D_N = \begin{cases} 0 & \text{si} \quad \varepsilon_N \le \varepsilon_N^1 \\ \dfrac{1}{1+A_{DN}\cdot\langle Y_{DN}-Y_{NI}\rangle_+^{B_{DN}}} & \text{si} \quad \varepsilon_N^1 < \varepsilon_N \end{cases}$$ éq 3.3-3

In this statement, two material parameters, A_{DN} and B_{DN}, control decoherence by the damage in tension of the concrete. In addition, Y_{NI} is the threshold of damage defined in term of energy, equivalent to the elastic threshold in the normal direction $Y_{elas}|_N$ and which is expressed like:

$$Y_{NI} = Y_{elas}|_N = \frac{1}{2}\varepsilon_N^1\cdot E\cdot\varepsilon_N^1$$ éq 3.3-4

ε_N^1 being limiting strain of perfect dependancy, which corresponds to the limiting strain of the concrete in tension before the initialization of the damage. It should be mentioned that when the detachment – or the crack opening – reached the maximum value of strength to tension, no shear force must be transmitted between the two materials: it east is the single condition in which the scalar variable of damage in the tangential direction becomes 1 because of the damage in the normal direction

3.4 Analyzes contribution of the crack friction by sliding

With regard to the “sliding” part of the formulation, one supposes that it has a behavior pseudo-plastic, with nonlinear kinematic hardening. Initially introduced by *Armstrong & Frederick*, 1966, cf [bib1], nonlinear kinematic hardening makes it possible the formulation to overcome the principal disadvantage of the model of kinematic hardening of *Prager*, namely, the linearity of the state model which connects the forces associated with kinematic hardening. Here, the nonlinear terms are added in the potential of dissipation. The criterion of sliding takes the classical shape of the function threshold of *Drucker-Prager* which takes into account the effect of radial containment on the sliding:

$$\varphi_f = |\sigma_T^f - X| + c\cdot I_1 \le 0$$ éq 3.4-1

Here X is the stress of recall, c is a parameter related to the material, translating the influence of containment, while I_1 corresponds to the first invariant of the tensor of the stresses, which for our case is expressed like:

$$I_1 = \frac{1}{3}\mathbf{Tr}[\sigma] = \frac{1}{3}\sigma_N$$ éq 3.4-2

In addition, the initial threshold for the sliding is 0. Moreover, by considering the principle of maximum plastic dissipation, the laws of evolution can be derived from the statement of the plastic potential which is:

$$\varphi_f^p = |\sigma_T^f - X| + c \cdot I_1 + \frac{3}{4} \cdot a \cdot X^2 \qquad \text{éq 3.4-3}$$

Where a is a material parameter. It should be mentioned that the quadratic term X makes it possible some to introduce the non-linearity of kinematic hardening. The laws of evolution for the strain of sliding as for kinematic hardening take the following shapes:

$$\dot{\varepsilon}_T^f = \dot{\lambda}_f \cdot \frac{\partial \varphi_f^p}{\partial \sigma_T^f} \text{ et } \dot{\alpha} = \dot{\lambda}_f \cdot \frac{\partial \varphi_f^p}{\partial X} \qquad \text{éq 3.4-4}$$

the multiplier of sliding $\dot{\lambda}_f$ is calculated numerically by imposition of the condition of consistency.

3.5 Summarized equations

We show here, abstract of the equations which constitute the constitutive law of the bond steel - concrete:

Free energy of Helmholtz	$\rho \ . \ \psi = \frac{1}{2}[\langle \varepsilon_N \rangle^- E \langle \varepsilon_N \rangle^- + \langle \varepsilon_N \rangle^+ E \ . \ (1-D_N) \langle \varepsilon_N \rangle^+$ $+ \varepsilon_T G(1-D_T)\varepsilon_T + (\varepsilon_T - \varepsilon_T^f) G \ . \ D_T(\varepsilon_T - \varepsilon_T^f) + \gamma \alpha^2] + H(z)$
Function threshold	$\varphi_{DT} = Y_{DT} - (Y_{TI} + Z_T) \leq 0$; $\varphi_f = \lvert \sigma_T^f - X \rvert + c \ . \ I_1 \leq 0$
State models	$\sigma_N = \begin{cases} E \ . \ \varepsilon_N & \text{si } \varepsilon_N \leq 0 \\ (1-D_N) \ . \ E \ . \ \varepsilon_N & \text{si } \varepsilon_N > 0 \end{cases}$; $\sigma_T = G(1-D_T)\varepsilon_T + G \ . \ D_T(\varepsilon_T - \varepsilon_T^f)$; $\sigma_T^f = G \ . \ D_T(\varepsilon_T - \varepsilon_T^f)$
Dissipation	$-Y = -\rho \frac{\partial \psi}{\partial D} = -(Y_D + Y_f)$; $X = \rho \frac{\partial \psi}{\partial \alpha} = \gamma \alpha$; $Z = \rho \frac{\partial \psi}{\partial z} = H'(z)$
Laws of evolution	$\dot{D} = \dot{\lambda}_D \ . \ \frac{\partial \varphi_D}{\partial Y_D} = \dot{\lambda}_D$; $\dot{z} = \dot{\lambda}_D \ . \ \frac{\partial \varphi_D}{\partial Z} = -\dot{\lambda}_D$; $\dot{\varepsilon}_T^f = \dot{\lambda}_f \ . \ \frac{\partial \varphi_f^p}{\partial \sigma_T^f}$; $\dot{\alpha} = \dot{\lambda}_f \ . \ \frac{\partial \varphi_f^p}{\partial X}$

3.6 Form of the tangent matrix

In order to ensure the robustness and the effectiveness of the model in the numerical establishment and for the total analysis of massive structures, it is necessary to calculate the tangent matrix, which can be given from the following statement:

$$\dot{\sigma}_T = H \cdot \dot{\varepsilon}_T$$ éq 3.6-1

After some analytical computations, one can by means of deduce the statement from the tangent modulus the condition of consistency and the respective laws of evolution:

$$H = \frac{G \cdot \left(1 - \left(\partial g(\varepsilon_T) / \partial \varepsilon_T\right) \cdot \varepsilon_T^f\right)}{1 + G \cdot D_T \left(\frac{\left(\partial \varphi_f / \partial \sigma_T\right) \cdot \left(\partial \varphi_f^p / \partial \sigma_T\right)}{\left(\partial \varphi_f / \partial X\right)\left(\partial^2 \rho \psi / \partial \alpha^2\right)\left(\partial \varphi_f^p / \partial X\right)}\right)}$$ éq 3.6-2

With

$$\frac{\partial g(\varepsilon_T)}{\partial \varepsilon_T} = \frac{\partial D_T}{\partial Y_{DT}} \cdot \frac{\partial Y_{DT}}{\partial \varepsilon_T} = \left[\frac{f \cdot g \cdot h' - f' \cdot g \cdot h - f \cdot g' \cdot h}{h^2}\right] \cdot G \cdot \varepsilon_T$$ éq 3.6-3

Where f, g and h are the following functions, obtained thanks to [éq 3.2-8]:

$$f = \sqrt{\frac{Y_{T1}}{Y_{DT}}}$$ éq 3.6-4

$$g = \exp\left\{ A_{1_{DT}} \cdot \left[\sqrt{\frac{2}{G}} \cdot \left(\sqrt{Y_{DT}} - \sqrt{Y_{T1}} \right) \right]^{B_{1DT}} \right\}$$ éq 3.6-5

$$h = 1 + A_{2_{DT}} \cdot \langle Y_{DT} - Y_{T1} \rangle_+^{B_{2DT}}$$ éq 3.6-6

Note::

In practice in Aster, the tangent matrix was not established, only the secant matrix is used either $H = \begin{pmatrix} E(1-D_N) & 0 \\ 0 & G(1-D_T) \end{pmatrix}$.

4 Numerical integration

the separation in two parts in the formulation: damage – sliding, enables us to treat each one of it separately. Thus, the integration of the damage part is carried out explicitly by the definition of two surfaces threshold. On the other hand, the "sliding" part is solved in an implicit way by a classical method with knowing the algorithm of the type "return-mapping" suggested by *Ortiz & Simo*, cf [bib4], which will ensure convergence in an effective way.

4.1 Computation of the part "friction of cracks" with an implicit integration method

the effects on connection associated with the phenomenon of friction with cracks can be calculated in the frame of a behavior pseudo-plastic with a nonlinear kinematic hardening. For the establishment with the integration method suggested, we will carry out a linearization of the function threshold around the current values of the associated local variables. With the iteration $(i+1)$, surface threshold is written:

$$\varphi_f=\varphi_f^{(i)}+\frac{\partial\varphi_f^{(i)}}{\partial\sigma_T^f}:\left(\sigma_T^{f^{(i+1)}}-\sigma_T^{f^{(i)}}\right)+\frac{\partial\varphi_f^{(i)}}{\partial X}:\left(X^{(i+1)}-X^{(i)}\right)\approx 0 \quad \text{éq 4.1-1}$$

According to the equations [éq 3.1-7], [éq 3.1-8], and [éq 3.6-5], one a:

$$\dot{X}=\gamma\cdot\dot{\alpha}=-\gamma\cdot\dot{\lambda}_f\cdot\frac{\partial\varphi_f^p}{\partial X} \quad \text{éq 4.1-2}$$

$$\dot{\sigma}_T^f=-G\cdot D_T\cdot\dot{\varepsilon}_T^f=-G\cdot D_T\cdot\dot{\lambda}_f\cdot\frac{\partial\varphi_f^p}{\partial\sigma_T^f} \quad \text{éq 4.1-3}$$

Which one can discretize in the following way:

$$\Delta X=X^{(i+1)}-X^{(i)}=\gamma\cdot\Delta\alpha=-\gamma\cdot\Delta\lambda_f\cdot\frac{\partial\varphi_f^p}{\partial X} \quad \text{éq 4.1-4}$$

$$\Delta\sigma_T^f=\sigma_T^{f^{(i+1)}}-\sigma_T^{f^{(i)}}=-G\cdot D_T\cdot\Delta\varepsilon_T^f=-G\cdot D_T\cdot\Delta\lambda_f\cdot\frac{\partial\varphi_f^p}{\partial\sigma_T^f} \quad \text{éq 4.1-5}$$

By combining these statements with the statement of surface threshold and by writing that is equal φ_f to zero, one can deduce the increment from multiplier $\Delta\lambda_f$ to each iteration i :

$$\Delta\lambda_f=\frac{\varphi_{f^{(i)}}}{\dfrac{\partial\varphi_f}{\partial\sigma_T^f}^{(i)}\cdot G\cdot D_T\cdot\dfrac{\partial\varphi_f^{p\,(i)}}{\partial\sigma_T^f}+\dfrac{\partial\varphi_f}{\partial X}^{(i)}\cdot\gamma\cdot\dfrac{\partial\varphi_f^{p\,(i)}}{\partial X}} \quad \text{éq 4.1-6}$$

After obtaining the value of $\Delta\lambda_f$, one can substitute it in the equations [éq 4.1-4] and [éq 4.1-5] in order to bring up to date the thermodynamic forces σ_T^f and X . The iterations will have to continue until the moment when the condition of consistency is checked.

4.2 The algorithm of resolution

In a general way, one seeks to check the equilibrium of structure at every moment, in an incremental form. As clarified previously, for the damage a simple scalar equation makes it possible to obtain the corresponding value, which makes it possible to avoid a recourse to the iterative methods. On the other hand, an iterative method is applied for integration of the friction part of cracks. Then, the algorithm is the following:

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1) Geometrical reactualization:

$$(\boldsymbol{\varepsilon}_T)_{n+1} = (\boldsymbol{\varepsilon}_T)_n + \nabla^s \mathbf{u}_T$$

2) Elastic prediction:

$$(\boldsymbol{\varepsilon}_T^f)_{n+1}^{(0)} = (\boldsymbol{\varepsilon}_T^f)_n \;;$$

$$(\boldsymbol{\varepsilon}_T^e)_{n+1}^{(0)} = (\boldsymbol{\varepsilon}_T)_{n+1} - (\boldsymbol{\varepsilon}_T^f)_{n+1} \;;$$

$$\alpha_{n+1}^{(0)} = \alpha_n$$

3) Evaluating of the threshold:

$$(\phi_{\mathbf{f}})_{n+1}^{(0)} \le 0 ?$$

if SO, end of the cycle; so NON, beginning of the iterations

YES: $(\boldsymbol{\varepsilon}_T^f)_{n+1} = (\boldsymbol{\varepsilon}_T^f)_{n+1}^{(0)}$; $(\boldsymbol{\varepsilon}_T^e)_{n+1} = (\boldsymbol{\varepsilon}_T^e)_{n+1}^{(0)}$; $\alpha_{n+1} = \alpha_{n+1}^{(0)}$; $\boldsymbol{\sigma}_{n+1} = \boldsymbol{\sigma}_{n+1}^{(0)}$

NON: $i = 0$

$$\Delta\lambda_f = \frac{(\varphi_f)_{n+1}^{(i)}}{(\partial\varphi_f / \partial\sigma_T^f)_{n+1}^{(i)} G \;.\; D_T (\partial\varphi_f^p / \partial\sigma_T^f)_{n+1}^{(i)} + (\partial\varphi_f / \partial X)_{n+1}^{(i)} \;.\; \gamma \;.\; (\partial\varphi_f^p / \partial X)_{n+1}^{(i)}}$$

4) Plastic correction:

$$\sigma_{n+1}^{(i+1)} = \sigma_{n+1}^{(i)} - G \;.\; D_T \;.\; \Delta\lambda_f \;.\; (\partial\varphi_f^p / \partial\sigma_T^f)^{(i)} - \gamma \;.\; \Delta\lambda_f \;.\; (\partial\varphi_f^p / \partial X)^{(i)}$$

$$\alpha_{n+1}^{(i+1)} = \alpha_{n+1}^{(i)} + \Delta\lambda_f . (\partial\phi_f^p / \partial X)^{(i)}$$

5) Checking of convergence:

$$(\phi_f)_{n+1}^{(i+1)} \le TOL \left| (\phi_f)_{n+1}^{(0)} \right| ?$$

if SO, end of the cycle; so NON, to continue the iterations in (iv)

YES:

$$\boldsymbol{\sigma}_{n+1} = \boldsymbol{\sigma}_{n+1}^{(i+1)} \;;$$

$$\alpha_{n+1} = \alpha_{n+1}^{(i+1)} \;;$$

$$(\boldsymbol{\varepsilon}_T^e)_{n+1} = \boldsymbol{\varepsilon}_T^e(\boldsymbol{\sigma}_{n+1}, \alpha_{n+1}) \;;$$

$$(\boldsymbol{\varepsilon}_T^f)_{n+1} = (\boldsymbol{\varepsilon}_T)_{n+1} - (\boldsymbol{\varepsilon}_T^e)_{n+1}$$

NON: $i = i + 1$

4.3 Local variables of the model

We show here the local variables stored in each Gauss point in the implementation of the model:

Number of physical	local variable Meaning
1	D_N : Scalar variable of the damage in the normal direction
2	D_T : Scalar variable of the damage in the tangential direction
3	z_{T1} : Scalar variable of isotropic hardening for the damage in mode 1
4	z_2 : Scalar variable of isotropic hardening for the damage in mode 2
5	ε_T^f : Strain of sliding cumulated by friction of cracks
6	α : Value of kinematic hardening by friction of the cracks

5 Parameters of the model

the constitutive law presented here is controlled by 14 parameters, of which 3 manage and the response in the normal direction others affect the response in the tangential direction. In addition, the Young modulus is recovered starting from the elastic data provided by the operator `ELAS`, which must always appear in the command file.

These parameters, or the analytical statements which make it possible to obtain them, were obtained or determined from the computational simulation of the experimental tests carried out by *Eligehausen and al.*, 1983, cf [bib3]. The realization of multiple simulations made it possible question to determine a relation between the geometrical and material characteristics of the materials in (steel and concrete) and the parameters which manage interface the model.

5.1 The initial parameters

5.1.1 the parameter "hpen"

the joined element functioning on the notion of jump of displacement, it is necessary to introduce a dimension characteristic of the zone of degraded interface making it possible to define the notion of strain in the interface. With this intention it was introduced the principle of penetration between surfaces: the parameter " $hpen$ " makes it possible to define this zone surrounding the bar of steel. This parameter corresponds to the possible maximum penetration which depends on the thickness of the compressed concrete - crushed. At the same time, " $hpen$ " dissipation of energy in the element as well as the kinematics of the sliding manages.

In order to give a reference to the user for the choice of this parameter, one proposes to calculate it starting from the diameter of the bar d_b and the relative area of the veins α_{sR} defined by:

$$\alpha_{sR}=\frac{k\cdot F_R\cdot\sin\beta}{\pi\cdot d_b\cdot c}$$ éq 5.1.1-1

where k is the number of veins on the perimeter; F_R the transverse area of a vein; β is the angle between the vein and the axis longitudinal of the steel bar; and c between veins center in center is the measured distance. Finally, " $hpen$ " will be calculated with the statement:

$$h_{pen}=d_b\cdot\alpha_{sR}$$ éq 5.1.1-2

According to *Eligehausen and al.*, the reinforcements usually used in the United States have values of α_{sR} enters 0.05 and 0.08. For the smooth bars, since one needs a small value of "*hpen*", one proposes values of α_{sR} enters 0.005 and 0.02.
The following table gives the values of "$hpen$" according to the diameter of the bar:

Diameter (mm)	relative Area	$Hpen$ (mm)	Description
8	0.01	(0.08) □ 0.1	Bar commercial smoothes
8	0.08	0.64	ribbed commercial Bar
20	0.08	1.50	Bars commercial ribbed
25	0.08	2.00	ribbed commercial Bar
32	0.08	2.54	ribbed commercial Bar

the unit of "hpen" must of course correspond to the unit used for the mesh.

5.1.2 The parameter G or Generally modulates stiffness of

connection, because of difficulty in measuring the strains by shears, the modulus of stiffness of a material is calculated starting from the Poisson's ratio and Young modulus, current parameters obtained in experiments. However, for our case, the interface is a pseudo-material whose characteristics must depend on the properties corresponding to the materials in contact, steel and concrete. Being given that the material which one expects to damage is the concrete, one proposes to initially use for connection the same value of G for the studied concrete but it can be higher up to a value similar to the value of the Young modulus E, when one increases the value of "hpen". In the case of reinforcements with stiffness higher than those of the current commercial bars (because of a provision or special geometry of the veins), one can make a correction of the value chosen, by multiplying the modulus of stiffness by a coefficient of correction calculated starting from the relative areas of the commercial bars, with the statement:

$$C_{arm}=\frac{(\alpha_{SR})barre}{(\alpha_{SR})barre_{comm}} \quad \text{éq 5.1.2-1}$$

Then, the modulus of stiffness of connection G will be:

$$G_{liai}=C_{arm}\cdot G_{beton} \quad \text{éq 5.1.2-2}$$

In the last statements, G_{lia} is the modulus of stiffness of connection; G_{beton} is the modulus of stiffness of the concrete; C_{arm} is the coefficient of correction per reinforcement; $(\alpha_{SR})barre$, relative area of the veins of the bar concerned; and $(\alpha_{SR})barre_{com}$, relative area of the veins of the commercial bar of the same diameter (preferably, 0.08).

5.2 The parameters of damage

5.2.1 the limit of elastic strain ¹ T or threshold of perfect dependancy

to define the threshold of perfect dependancy, one considers that the damage by shears must during initiate the going beyond a certain threshold of strain. So one proposes to adopt the limiting strains of the concrete in tension, i.e., enters 1×10^{-4} and 0.5×10^{-3}, which correspond to the shearing stresses enters 0.5 and $4\ MPa$ in perfect dependancy.

5.2.2 The parameter of damage A1DT for the transition of the small strains to the great slidings

In this region, the law of evolution of the damage is expressed in term of strains and its construction depends on the definite elastic slope for the linear behavior (shearing stress versus strain) in the region of perfect dependancy: this parameter controls the value of the stress compared to the sliding in the transition of small strains to the great slidings.

The determination of the value of this parameter is a key and delicate point model, since the evolution of the damage must be carried out with certain conditions noticed by several researchers; for example:

- the strength of connection is directly proportional to the compressive strength of the concrete. However, as one increases the strength of the concrete, the behavior becomes more rigid, bringing to the brittle fracture of connection,
- the particular stiffness of the reinforcement, which is related on the diameter and the quantity of the veins on surface, must increase the strength of connection,
- the relation between the elasticity moduli of the two materials concerned must manage the kinematics of connection directly.

From computational simulations which one carried out, one observed that this value is located between a minimum of 1 and a maximum of 5, and that it will have to be adjusted according to the selected test of reference. Optionally, one proposes a statement which makes it possible to adopt an initial value and which depends on the particular characteristics of the materials:

$$A_{1_{DT}}=\frac{1}{\left(1+\alpha_{SR}\right)}\cdot\sqrt{\frac{f'c}{30}}\cdot\sqrt{\frac{E_a}{E_b}} \qquad \text{éq 5.2.2-1}$$

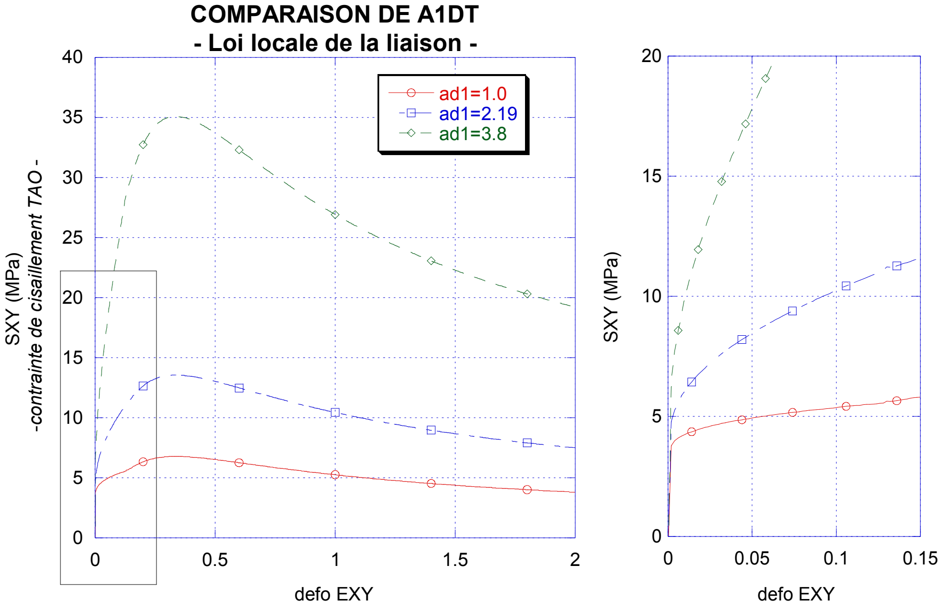
In the last statement, E_b will be calculated with the statement provided in the section A.2.1, 2 of BAEL' 91:

$$E_b=11000\times\left(f'c\right)^{1/3} \qquad \text{éq 5.2.2-2}$$

In the two last statements, one a:

1) $f'c$, compressive strength of the concrete in MPa ;
2) E_a, elasticity modulus of steel, in MPa ;
3) E_b, elasticity modulus of the concrete, in MPa ;
4) α_{sR}, relative area of the veins of the bar concerned.

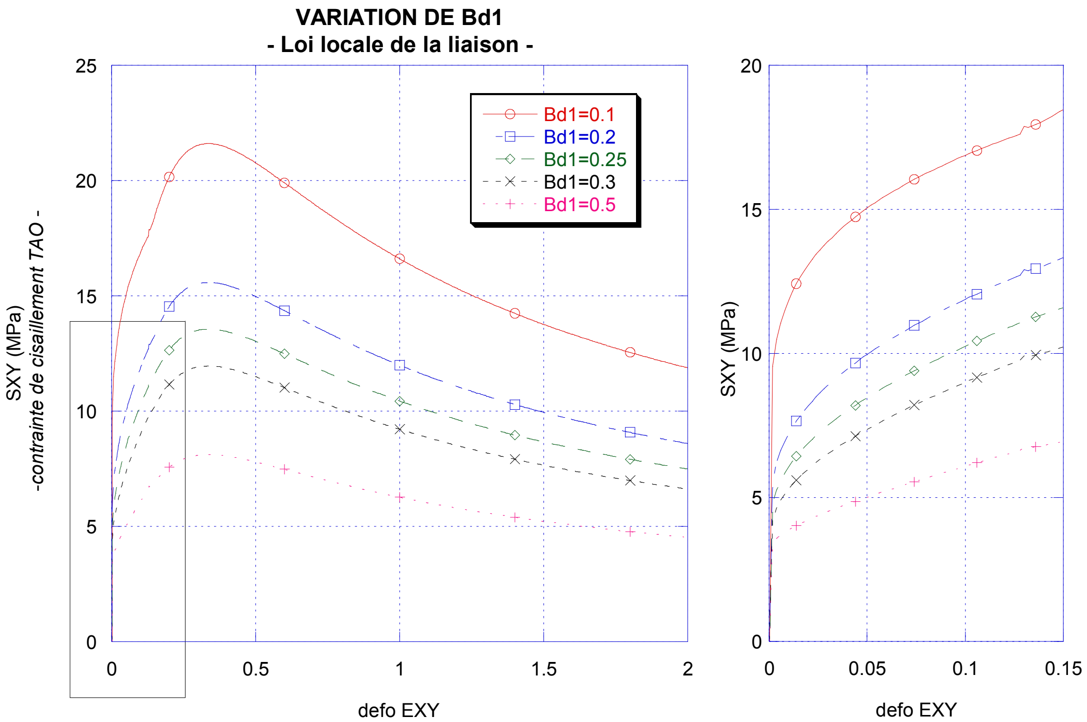
[Figure 5.2.2-a] gives some a graphic comparison.



Appear 5.2.2-a: Comparison of A_{IDT} : the purpose of growth of the strength of connection

5.2.3 the parameter of damage B_{1DT}

This parameter is softening the shape of the curve of behavior, like facilitating the transition from the elastic slope towards the nonlinear region. It can have a value understood enters 0.1 and 0.5 (never higher than 0.5 since it is the equivalent of the square root of the formula). One can advise to adopt the value of 0.3 for ordinary computations. (See [Figure 5.2.3-a]).



Appear 5.2.3-a: Comparison of B_{1DT} : Modification of the curvature

5.2.4 the limit of strain ε_T^2 or threshold of the great slidings

According to several authors, the great slidings are overall higher than $1\,mm$ displacement, but that is an indicator which depends on the form and dimensions of the specimens tested; therefore, it is proposed that this strain never exceeds 1.00 (adimensional value). In a more precise way, one proposes to apply the following statement:

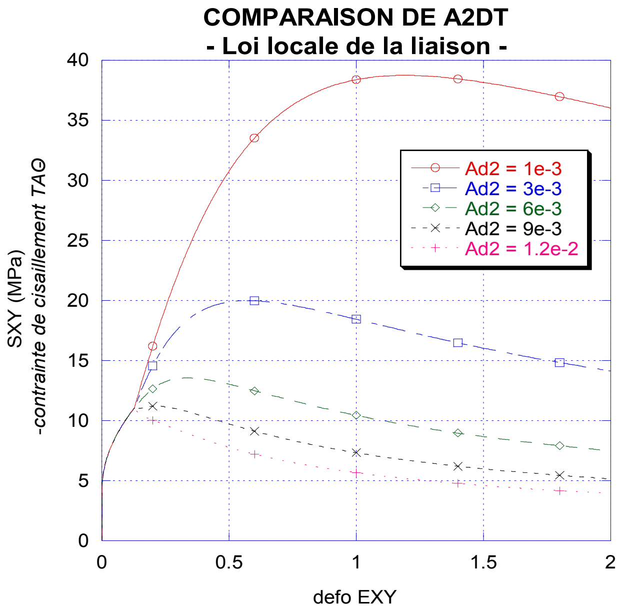
$$\varepsilon_T^2=\frac{1}{(h_{pen})^2}\cdot\left(1-\frac{(A_{1DT})^n}{C+(A_{1DT})^n}\right)=\frac{1}{(h_{pen})^2}\cdot\left(1-\frac{(A_{1DT})^4}{9+(A_{1DT})^4}\right)\leq 1.0 \quad \text{éq 5.2.4-1}$$

In this statement, one applied a sigmoid function whose coefficients C and n make it possible to adjust the kinematical effect of A_{1DT} on the sliding, i.e., when connection becomes more resistant because of an increase in stiffness, the sliding is reduced gradually. One adopted the values 9.0 and 4.0 respectively, but they are always optional.

The choice of the value of the limit of strain ε_T^2 is very important because it introduces a more or less great brittleness of the response by translation of the threshold of transition of the small strains to the great slidings. This brittleness is related to the stiffness of the concrete via the parameter A_{1DT} . It should be noted that the following parameters which manage the damage must be also adjusted at the local level to ensure the correct continuity of behavior in shears of connection and to thus be able to obtain the desired or expected response of a real system steel – connection – concrete.

5.2.5 The parameter of damage A_{2DT}

the damage, such as it was conceived in the model, obeys two laws of evolution which are expressed using one only variable classical scalar which will ensure the coherence of the damage. The parameters of each of the 2 models are independent and numerically stable, but they are likely to generate serious errors in the continuity of the behavior if one does not pay attention to the shape of the local curve stress-strain: to see the case of the curve shown in graphics of [Figure 5.2.5-a], with a value $A_{2DT}=1\times10^{-3}\,MPa^{-1}$. We are not able to propose an analytical relation for the choice of this parameter, but the gained experience enables us to affirm that the value of this parameter must be understood enters 1×10^{-3} and $9\times10^{-2}\,MPa^{-1}$ roughly.

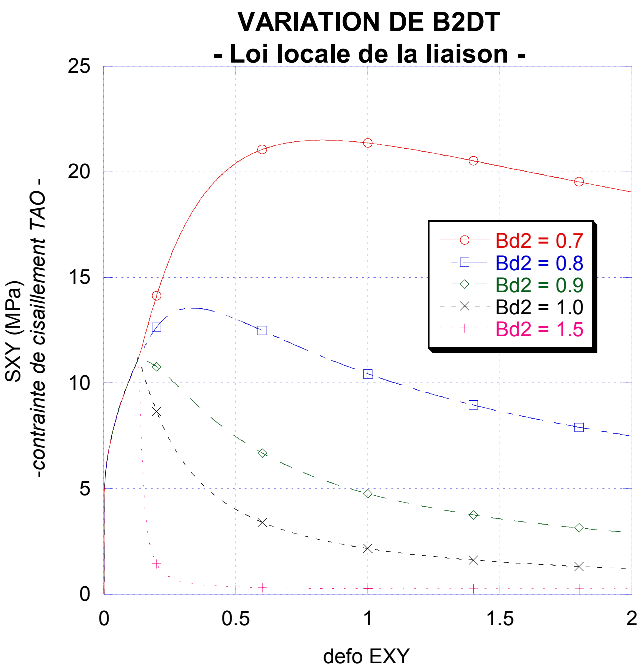


Appear 5.2.5-a: Comparison of A_{2DT} : damage and fracture of connection

5.2.6 the parameter of damage B_{2DT}

This parameter, which supplements the law of evolution of damage in great slidings, controls not only the growth of the strength of connection or the shape of the curve of behavior with the peak and in the region post-peak, but also the kinematics of the response, which implies the determination of the sliding for the maximum shearing stress as well as the amplitude of the curve to the peak of the behavior. Then, although the values of the parameters of damage A_{2DT} and B_{2DT} will have to be adjusted at the same time when one builds the curve of behavior of connection in order to respect the continuity of the pace, one can say that the value of B_{2DT} is inversely proportional to the amplitude of sliding at the top, i.e., a value of 0.8 allows great broader slidings in the top than a value of 1.2, for example.

For practical cases, one recommends to use a value understood enters 0.8 and 1.1 to reproduce a coherent curve of behavior (See [Figure 5.2.6-a]).



Appear 5.2.6-a: Comparison A_{2DT} of : damage and fracture of connection

5.3 the parameters of damage on the normal direction

5.3.1 the limit of strain □ ¹ ɴ or threshold of large displacements

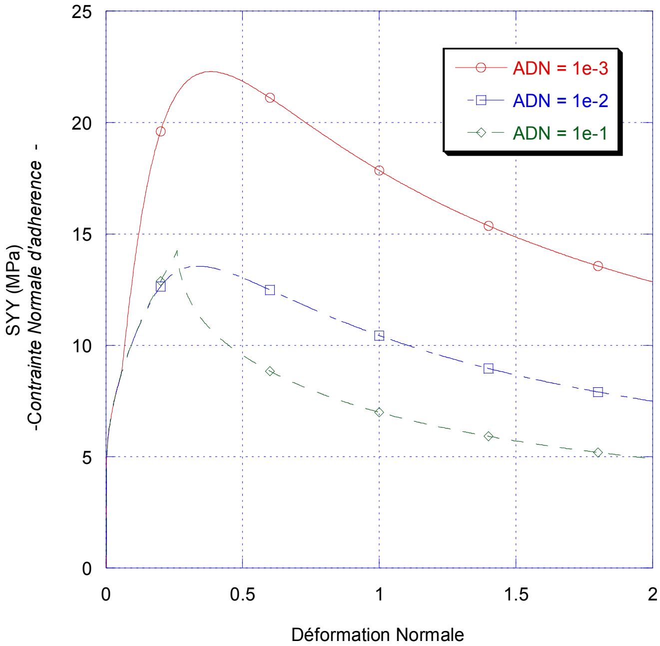
In a way similar to the elastic behavior in the tangential direction, one considers that decoherence must during initiate the going beyond a certain threshold of strain. We propose to adopt a value enters 10^{-4} and 10^{-3}.

5.3.2 The parameter of damage A_{DN}

This parameter compared to the controls primarily the slope of degradation of the normal stress strain due to the opening of the interface. We propose to use a minimal value of $1\times10^{-1} MPa^{-1}$, which corresponds to a degradation similar to that of the concrete. Nevertheless, if one wishes to have a behavior of connection even more brittle, it is enough to increase this value.

5.3.3 The parameter of damage B_{DN}

In combination with the preceding parameter, this parameter controls the damage of connection, in particular the shape of the curve of behavior in phase post-peak.
We propose to use a value equal to $1,$ or $1{,}2$ for more marked curves.



Appear 5.3.3-a: behavior of connection on the normal direction at the time of the opening of the interface (normal tension on connection).

5.4 The parameters of friction

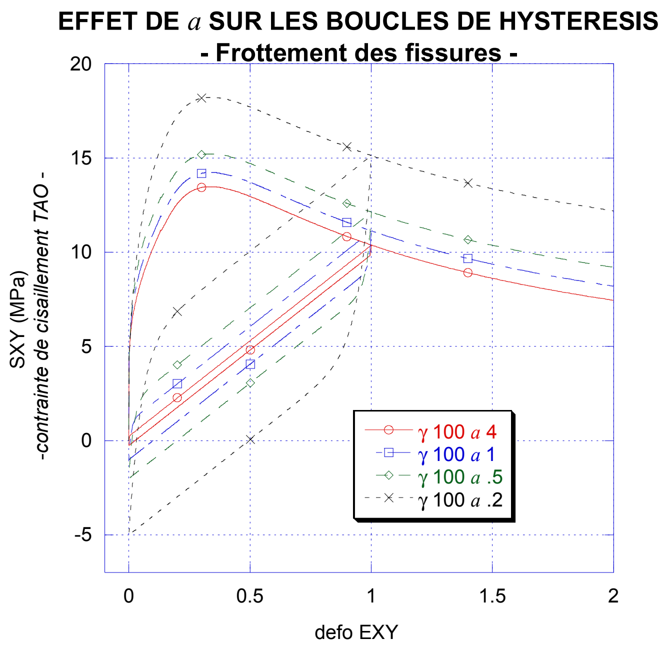
5.4.1 the material parameter γ of friction of cracks

One of the assets of the model suggested here is that it is able to take into account the effects of friction of the cracks, which, in the case of monotonic loading, appears by a positive contribution to the shear strength of connection; in addition, in the cyclic cases of loadings, it is obvious that the pace of the loops of hysteresis depends directly on the choice of the value of this material parameter. However, the corresponding values were not gauged, since we did not simulate tests with cyclic loadings yet to validate them. Temporarily, one proposes to use values lower than $10\,MPa$, with a maximum value of α equal to $1.0\,MPa^{-1}$.

5.4.2 The material parameter α of kinematic hardening

On [Figure 5.4.2-a], one can appreciate that the reduction in the value of α increases hysteretic dissipation, but also the strength of shears and the strain residual pseudo-plastic. That is very important for the cyclic modelization of connection since in reality, when one exceeds the peak of maximum strength, one notices that at the time of the discharge there is no more elastic contribution of the sliding, i.e. the strain residual pseudo-plastic corresponds exactly to the total sliding reached. In other keys, once connected all the cracks in the potential of fracture, longitudinal and tangential layer with the steel bar, the single strength which will prevent the displacement of reinforcement is the friction resistance of connection, produced by the contact and the tangle of the asperities between surfaces concrete – concrete.

As previously, our experiment is restricted: one proposes to use a maximum value of $0.1\,MPa^{-1}$ which gives correct results for applications in monotonic loading, and which seems suitable for cyclic loadings.



Appear 5.4.2-a: Comparison of a : effects on the loops of hysteresis in cyclic

5.4.3 the parameter of influence of containment c

In our model, the influence of containment was taken into account thanks to the application of this parameter which controls these effects on the connection, and which appears by an increase in the maximum shearing stress as by the increase in maximum displacement to the peak when containment increases.

For the calibration, we carried out simulations with containments of $0\quad 5$, 10 and $15\,MPa$, by means of always a value of 1.0 for this parameter. It was noticed that if one wants to produce a kinematical translation of the sliding caused by containment, it is enough D `to adopt a value of 1.2 or 1.5 (adimensional). Optionally, it is advised to maintain the value of 1.0 for ordinary computations.

5.5 Summarized parameters

to facilitate the use of the model, the following table presents a synthesis of all the parameters of the model of behavior.

It is pointed out that the values or the statements suggested have only one indicative value, and that the arbitrary combination can give inaccurate and unexpected results compared to the hoped behavior of connection; in other words, a bad choice of the parameters can produce a strong stiffness or a weak response of the steel-concrete interface.

Parameter	Unit	Value suggested	Analytical statement	Variables concerned	
h_{pen}	mm	-	$h_{pen}=d_b.\alpha_{sR}$	d_b	Diameter of the bar
				α_{sR}	relative Area of the veins
G_{liai}	MPa	-	$G_{liai}=C_{arm}\cdot G_{beton}$	C_{arm}	Coefficient of correction by reinforcements
				G_{beton}	Modulates stiffness of the concrete
ε_T^1	-	min 1.0x10-4 max 1.5x10-3			
A_{1DT}	-	min 1.0 max 5.0	$A_{1DT}=\frac{1}{(1+\alpha_{SR})}\cdot\sqrt{\frac{f'c}{30}}\cdot\sqrt{\frac{E_a}{E_b}}$	$f'c$	Compressive strength of the concrete (MPa)
				E_{has}	Elasticity modulus of steel
				E_B	Elasticity modulus of the concrete
B_{1DT}	-	min 0.1 max 0.5			
ε_T^2	-	-	$\varepsilon_T^2=\frac{1}{(h_{pen})^2}\cdot\left(1-\frac{(A_{1DT})^4}{9+(A_{1DT})^4}\right)\leq 1.0$		
A_{2DT}	MPa-1	min 1.0x10-4 max 9.x10-2			
B_{2DT}	-	min 0.8 max 1.5			
γ	MPa	max 10.0			
a	MPa $^{-1}$	min 0.01 max 1.0			
c	-	1.0	(value recommended)		
α	-	min 10-4 max 0.9 10-3			
A_{DN}	MPa $^{-1}$	min 1.0x10 $^{-1}$	(value recommended, not gauged)		
B_{DN}	-	1.	(value recommended, not gauged)		

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7 constitutive law

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112 Test	of axisymmetric wrenching (Borderie & Pijaudier- Pooch) for the study of steel-concrete connection: model JOINT_BA [V6.01	.112] SSNP
126 Validation	of constitutive law JOINT_BA (steel-concrete connection) in 2D plane [V6.03	.126] Description

8 of the versions of the document Version

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7.4	- *PONNELLE, N. DOMINGUEZ EDF* R & D /AMA initial	Text