Reaction of the steel subjected to corrosion

Summarized:

This documentation the model presents steel reaction of subjected to the corrosion which makes it possible to describe the structural mechanics behavior of reinforcement corroded in reinforced concrete structures. This model is developed in 1D and 3D, elastoplastic endommageable with isotropic hardening and leans on the model of Lemaître. This behavior is used for computations of prediction of the bearing reinforced concrete structure capacity reached by corrosion in the case of unidimensional loading.

It is implemented in *Code_Aster* under the name of `CORR_ACIER` in 3D and 1D (for the elements `BARS` and multifibre beams). It also functions with option `DEBORST` for the plane stresses (elements shell); the equations are integrated numerically by an implicit scheme of Mr. Ortiz and J.C. Simo [bib1] with plastic multiplier obtained by linearizing the function threshold compared to the local variables and by imposing the respect of the criterion on convergence.

Contents

1 Introduction

This document present a constitutive law of the steel subjected to corrosion in the works in Civil Engineering. This model was formulated on the basis of the model of Lemaître (plasticity coupled with the damage). Following traction tests carried out on corroded bars, one notes an influence of corrosion on the plastic strain with fracture (the plastic strain of the corroded bars decreases with the increase in the degree of corrosion) [bib2], [bib3]. One connects the plastic strain to fracture of the model of Lemaître to the rate of corrosion (the diameter of the bar corroded compared to that not corroded in 1D or the thickness of flat reinforcement corroded compared to that not corroded in 3D).

2 Description of the model

From a thermodynamic point of view, to translate the behavior of the hammer-hardenable plastic material and endommageable, the model replaces in surface threshold, the stress σ by the effective stress [bib4]:

$$\tilde{\sigma}=\frac{\sigma}{1-D} \quad \text{éq 2-1}$$

where, one a:

- $\tilde{\sigma}=\sigma$ for a virgin material
- $\tilde{\sigma}=0$ at the instant of the failure

One applies a particular form of the free energy of Helmholtz:

$$Y=\frac{1}{\rho}\left[\frac{1}{2}(1-D)\left(\varepsilon-\varepsilon^p\right)E\left(\varepsilon-\varepsilon^p\right)+R(r)\right] \quad \text{éq 2-2}$$

ρ density; Ψ potential of state; E the Young modulus; D the variable of damage; ε total deflection; ε^p plastic strain; $R(r)$ the isotropic function of hardening; r the variable associated with R.

Thus posed the potential is separate in two distinct parts. The first corresponds to the classical coupling damage-elasticity, the second term with hardening.

The state models describing this potential are:

$$\sigma=\rho\frac{\partial\Psi}{\partial\varepsilon} \qquad R=\rho\frac{\partial\Psi}{\partial r} \qquad Y=\rho\frac{\partial\Psi}{\partial D} \quad \text{éq 2-3}$$

By supposing that the material obeys the criterion of Von Mises, the criterion of flow is expressed by:

$$f=\frac{\sigma_{eq}}{1-D}-R-\sigma_y\leq 0 \quad \text{éq 2-4}$$

where $\sigma_{eq}=\left(\frac{3}{2}\boldsymbol{\sigma}':\boldsymbol{\sigma}'\right)^{\frac{1}{2}}$ is the equivalent stress within the meaning of Settings, with $\boldsymbol{\sigma}'=\boldsymbol{\sigma}-\sigma_H 1$ the deviator of the matrix of stresses and $\sigma_H=\frac{1}{3}Tr(\boldsymbol{\sigma})$ the hydrostatic stress.

The potential of damage is selected in function power of the variable associated with the damage $-Y$:

$$\varphi_D^*=\left(\frac{S_0}{s_0+1}\right)\left(\frac{1}{1-D}\right)\left(-\frac{Y}{S_0}\right)^{s_0+1} \quad \text{éq 2-5}$$

where s_0 and S_0 is coefficients characteristic of the material.
The generalized normality rule provides the flow model and the evolutions of the local variables:

$$\begin{cases}\dot{\varepsilon}^p=\dot{\lambda}\dfrac{\partial f}{\partial \sigma}\\ \dot{r}=-\dot{\lambda}\dfrac{\partial f}{\partial R}\\ \dot{D}=-\dot{\lambda}\dfrac{\partial \phi_D^*}{\partial Y}\end{cases} \quad \text{éq 2-6}$$

$\dot{\lambda}$ is the plastic multiplier.

In this model `CORR_ACIER`, the local variables introduced into *the Code_Aster* are:

- p : cumulated plastic strain, such as $\dot{p}=\left(\frac{2}{3}\dot{\varepsilon}^p:\dot{\varepsilon}^p\right)^{1/2}$;
- D : scalar variable of isotropic damage.

If $f=\frac{\sigma_{eq}}{1-D}-R-\sigma_y>0$, one is in the plastic range:

$$\begin{cases}\dot{\varepsilon}^p=\dfrac{3}{2}\left(\dfrac{\dot{\lambda}}{1-D}\right)\dfrac{\sigma'}{\sigma_{eq}}\\ \dot{r}=\lambda=\dot{p}(1-D)\\ \dot{D}=\left(-\dfrac{Y}{S_0}\right)^{s_0}\dot{p}\end{cases} \quad \text{éq 2-7}$$

In the unidimensional case, the equivalent stress within the meaning of Settings is $\sigma_{eq}=|\sigma|$ and the cumulated plastic strain is equal to the absolute value of the unidimensional plastic strain: $p=|\varepsilon^p|$

To formulate an isotropic criterion of damage, one applies that the damage mechanism is controlled by total elastic strain energy (energy of distortion + volumetric strain energy). By analogy with the equivalent stress in plasticity, by writing that the energy of a three-dimensional state is equal to that of the unidimensional state are equivalent, definite by an equivalent stress of damage σ_{eq}^* , one finds [bib5]:

$$\sigma_{eq}^*=\sigma_{eq}\left[\frac{2}{3}(1+\nu)+3(1-2\nu)\left(\frac{\sigma_H}{\sigma_{eq}}\right)^2\right]^{1/2} \quad \text{éq 2-8}$$

In addition one shows that the variable associated with the damage is expressed in the isotropic case by $-Y=\frac{\tilde{\sigma}_{eq}^{*2}}{2E}$, therefore:

$$-Y=\frac{\sigma_{eq}^2}{2E(1-D)^2}\left[\frac{2}{3}(1+\nu)+3(1-2\nu)\left(\frac{\sigma_H}{\sigma_{eq}}\right)^2\right] \quad \text{éq 2-9}$$

from where:

$$\dot{D}=\left(\frac{\sigma_{eq}^2}{2ES_0(1-D)^2}\right)^{s_0}\left[\frac{2}{3}(1+\nu)+3(1-2\nu)\left(\frac{\sigma_H}{\sigma_{eq}}\right)^2\right]^{s_0}\dot{p} \quad \text{éq the 2-10}$$

ductile damage intervenes only beyond of a threshold:

$$f_{end}=p-p_D>0 \quad \text{éq 2-11}$$

where p_D is the strain which corresponds to the beginning of damage (see it [Figure 2-a]). One can then consider that hardening is saturated in this field, i.e. the behavior of the material not damaged equivalent would be perfectly plastic. This simplifying assumption allows the analytical integration of the model and to lead to a linear evolution according to the plastic strain. Indeed, one has in this case:

$$\frac{\sigma_{eq}}{1-D}=\tilde{\sigma}_{eq}=\sigma_y=Cte \quad \text{éq the 2-12}$$

If one restricts oneself with the case of the radial loading for which rate of triaxiality σ_H/σ_{eq} is constant, one obtains, while taking as initial condition $D=0$ for $p<p_D$:

$$D=\left(\frac{\sigma_y^2}{2ES_0}\right)^{s_0}\left[\frac{2}{3}(1+\nu)+3(1-2\nu)\left(\frac{\sigma_H}{\sigma_{eq}}\right)^2\right]^{s_0}\langle p-p_D\rangle \quad \text{éq 2-13}$$

One can simplify this statement by introducing the condition of fracture $p=p_R$ which is plastic strain rate cumulated with fracture => $D=D_c$ *(* damage criticizes) [bib4]

$$D_c=\left(\frac{\sigma_y^2}{2ES_0}\right)^{s_0}\left[\frac{2}{3}(1+\nu)+3(1-2\nu)\left(\frac{\sigma_H}{\sigma_{eq}}\right)^2\right]^{s_0}\langle p_R-p_D\rangle \quad \text{éq 2-14}$$

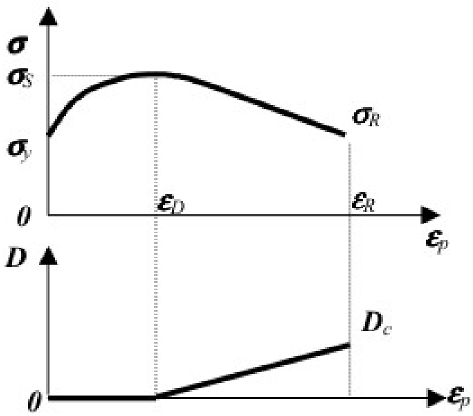
While paying in the statement of *D*, one obtains:

$$D=\frac{D_c}{p_R-p_D}(p-p_D) \quad \text{éq the 2-15}$$

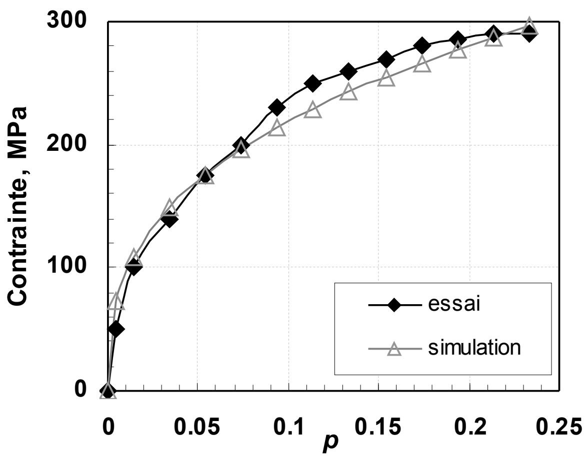
model of hardening of steel integrated in the model of steel is the following one (except damage):

$$\sigma_{eq}-\sigma_y=Kp^{1/m} \quad \text{éq 2 - 16}$$

K m , σ_y are the parameters of the material, provided in `DEFI_MATERIAU/CORR_ACIER` by key keys `ECRO_K, ECRO_M` .



Appear 2-a: Evolution of the damage



Appears 2-b: Model of hardening of steel not corroded

3 Computation of the material parameters

the results of the traction tests of A.A. Almusallam [bib2] were used to identify the constitutive law of steel not corroded and the dependence of the plastic strain with fracture according to the rate of corrosion.

3.1 Constitutive law of steel

nonallied steel is the type of principal steel used in the civil engineer. The model of unidimensional behavior of nonallied steel in one-way monotonic loading must be given starting from the results of a traction test on a bar or a not corroded flat test-tube. An example of computational simulation necessary to determine this constitutive law is presented on [Figure 3.2-a].

3.2 Taken into account of corrosion

the presence of corrosion has two effects on reinforcement in reinforced concrete structures:

- a reduction of the section;
- a reduction of ε_R according to T_c :

The reduction of section results by a reduction in the diameter for the bars or in a reduction in thickness for sheets:

$$T_c=100\left(\frac{d^2_{corrodé}}{d^2_{noncorrodé}}\right) \quad \text{or} \quad T_c=100\left(\frac{e_{corrodé}}{e_{noncorrodé}}\right) \qquad \text{éq 3.2 - 1}$$

Note:: The reduction of section is not treated with the level of the model of behavior, it must be taken into account on the level of the command file in AFFE_CARA_ELEM for example.

In the uniaxial case, the plastic strain with fracture ε_R depends on the rate of corrosion. This evolution is presented on [Figure 3.2-b].

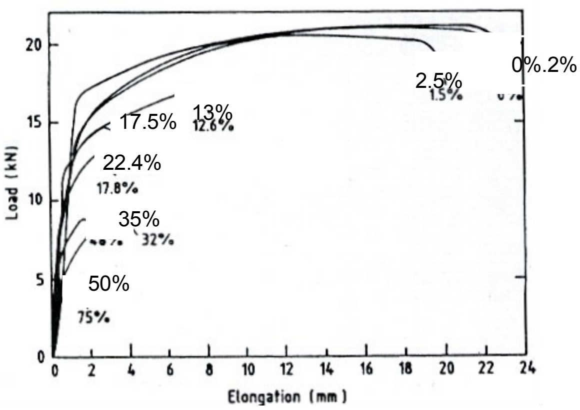
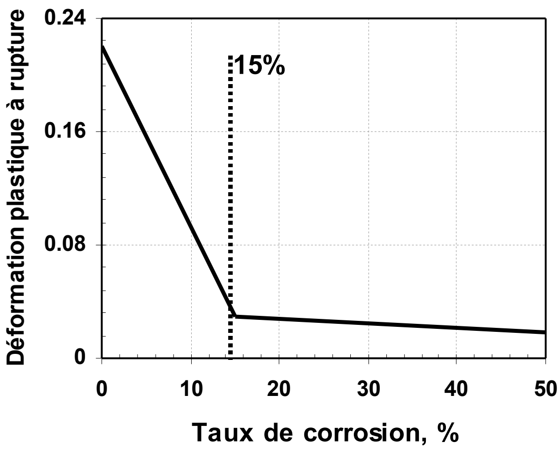


Figure 3.2-a: Influence corrosion on the reaction of steel according to the rate of corrosion



Appears 3.2-b: The evolution of the plastic strain with fracture according to the rate of corrosion

One deduces from these experimental data (traction tests) the variation of the plastic strain with fracture according to rate of corrosion:

$$T_c<15\% \Rightarrow \varepsilon_R=-0.0111\,Tc+0.2345 \qquad \text{éq 3.2 - 2}$$

$$T_c>15\% \Rightarrow \varepsilon_R=-0.0006\,Tc+0.051 \qquad \text{éq 3.2 - 3}$$

By analysis of various traction tests, one notes that the behavior of corroded reinforcement is quasi-brittle and $\varepsilon_D=0.8\,\varepsilon_R$ (ε_D with the peak) **.**

In order to integrate the model in 3D, the critical cumulated plastic strain is calculated by means of $p_R=\left(\frac{2}{3}\boldsymbol{\varepsilon_R}:\boldsymbol{\varepsilon_R}\right)^{1/2}$ in taking into account that the uniaxial state is defined by a unidimensional in stress but three-dimensional state in strain [bib5]:

$$\varepsilon_R=\begin{bmatrix}\varepsilon_R & 0 & 0\\ 0 & -\nu^*\varepsilon_R & 0\\ 0 & 0 & -\nu^*\varepsilon_R\end{bmatrix} \qquad \text{éq 3.2-4}$$

where ν^* is the coefficient of contraction, equal to the Poisson's ratio ν in elasticity:

$$\nu^* = \nu \frac{\varepsilon^e}{\varepsilon} + \frac{1}{2}\frac{\varepsilon^p}{\varepsilon} = \frac{1}{2} - \frac{\varepsilon^e}{\varepsilon}\left(\frac{1}{2} - \nu\right)$$ éq 3.2-5

here $\varepsilon = \varepsilon_R$ and one approximates ε^e by: $\varepsilon_y = \frac{\sigma_y}{E}$

$$\nu^* = \frac{1}{2} - \frac{\varepsilon_y}{\varepsilon_R}\left(\frac{1}{2} - \nu\right)$$ éq 3.2-6

For the computation of p_D , one considers that the rate of triaxiality to the threshold of damage is identical to that of the fracture:

$$p_D = 0{,}8\, p_R$$ éq 3.2-7

4 numerical Resolution

4.1 Integration of the model

the integration of the model is carried out in two stages, first of all an elastic prediction of the stress:

1st stage: elasticity
$$\begin{cases} \dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p \\ \dot{\sigma} = E\,\dot{\varepsilon}^e \\ \dot{\varepsilon}^p = 0 \\ \dot{p} = 0 \\ \dot{D} = 0 \end{cases}$$ éq 4.1-1

Let us notice in particular that the damage D does not intervene in the elastic relation stress-strain (not of coefficient $(1-D)$ in the moduli of elasticity).

The second stage consists of an implicit actualization of the local variables of the model by corrections made to total deflection null. In this stage, one makes the iteration between plasticity and the damage.

$$\begin{cases} \dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^p = \mathbf{0} \\ \dot{\boldsymbol{\sigma}} = \boldsymbol{E}\,\dot{\boldsymbol{\varepsilon}}^p \\ \dot{\boldsymbol{\varepsilon}}^p = \dot{\lambda}\frac{\partial f}{\partial \boldsymbol{\sigma}} \\ \dot{r} = -\dot{\lambda}\frac{\partial f}{\partial R} \end{cases}$$ éq 4.1-2

the statement of the plastic multiplier is obtained by linearizing the function threshold compared to the local variables and by imposing the respect of the criterion on convergence:

$$f = f^- + \frac{\partial f^-}{\partial \boldsymbol{\sigma}}\left(\boldsymbol{\sigma} - \boldsymbol{\sigma}^-\right) + \frac{\partial f^-}{\partial R}\left(R - R^-\right) = 0$$ éq 4.1-3

Let us consider the discretization of the equations of relaxation:

$$\begin{cases} \boldsymbol{\sigma} - \boldsymbol{\sigma}^{-} = -\Delta\lambda\,\mathbf{E}\dfrac{\partial f^{-}}{\partial \boldsymbol{\sigma}} \\ R - R^{-} = R^{'}(r)\Delta r \\ \Delta p = \dfrac{\Delta\lambda}{1-D^{-}} \\ R^{'}(p) = \dfrac{K}{m}\left[\dfrac{\boldsymbol{\sigma}_{eq}}{(1-D^{-})K} - \dfrac{\sigma_y}{K}\right]^{(1-m)} \end{cases}$$ éq 4.1-4

the statement of the plastic multiplier is obtained while replacing [éq 4.1-3] in [éq 4.1-4], ($\Delta r = \Delta\lambda$ and for D fixed at D^{-} , $R'(p) = R'(r)$)

$$\Delta\lambda = \frac{f^{-}}{\dfrac{\partial f^{-}}{\partial \boldsymbol{\sigma}}\mathbf{E}\dfrac{\partial \varphi^{-}}{\partial \boldsymbol{\sigma}} - \dfrac{\partial f^{-}}{\partial R}\dfrac{K}{m}\left[\dfrac{\boldsymbol{\sigma}_{eq}}{K(1-D^{-})} - \dfrac{\sigma_y}{K}\right]^{(1-m)}}$$ éq 4.1 - the 5

forms of various derivatives of surface threshold and the potential of dissipation are given in the following equations:

$$\frac{\partial f}{\partial \boldsymbol{\sigma}} = \frac{3}{2}\frac{\boldsymbol{\sigma}'}{\boldsymbol{\sigma}_{eq}(1-D)}$$
$$\frac{\partial \varphi}{\partial \boldsymbol{\sigma}} = \frac{3}{2}\frac{\boldsymbol{\sigma}'}{\boldsymbol{\sigma}_{eq}(1-D)}$$ éq 4.1 - 6
$$\frac{\partial f}{\partial R} = -1$$

In the case 1D:

$$\frac{\partial f}{\partial \boldsymbol{\sigma}} = \frac{\partial \phi}{\partial \boldsymbol{\sigma}} = \frac{\mathrm{sgn}(\boldsymbol{\sigma})}{1-D}$$ éq 4.1-7 B

the value obtained for the plastic multiplier with each integration is reinjected in the equations of relaxation until convergence (return of the stress on surface threshold). The criterion selected consists in stopping when the value of the threshold to the following iteration became sufficiently small compared to σ_y :

$$|f| \le tol.\sigma_y$$ éq 4.1-8

For a direct use of the properties of normality of surfaces thresholds, computation to be carried out with each iteration is reduced only to the computation of the multiplier of plasticity allowing the corrections so necessary (computation step of Jacobian as for the methods of Newton) [bib1], [bib6].

The following stage is the damage which takes into account the variation of the variable of damage. The function threshold of the damage is the following one:

$$f_{end} = p - p_D$$ éq 4.1 - 9

if $f_{end} > 0$ the damage is calculated then one turns over to the correction of plasticity:

$$D = \frac{D_C}{p_R - p_D}(p - p_D)$$ éq 4.1-10

Note:: in practice, D evolves even if $p > p_R$, one limits D to 0,99 maximum. D_c by the key word D_CORR of `DEFI_MATERIAU/CORR_ACIER is defined` the same approach for convergence is retained:

$$|f| \leq tol.\sigma_y \qquad \text{éq 4.1 - 11}$$

One will refer to reference [bib1] and to [bib6] for all the details concerning this method and this algorithm employed.

4.2 Computation of the tangent matrix

One seeks to calculate the tangent matrixes (continuous and consistent) for the plastic part and the part of damage. Options RIGI_MECA_TANG and FULL_MECA of the tangent matrix are calculated for the finite element 1D and 3D:

4.2.1 Finite element 1D

In the plastic range:

$$\delta \varepsilon^p = \delta \lambda = \frac{\delta f^-}{E + \frac{K}{m} p^{\frac{1}{m}-1}} = \frac{E \delta \varepsilon}{E + \frac{K}{m} p^{\frac{1}{m}-1}} \qquad \text{éq 4.2.1- 1}$$

$$\frac{\partial \sigma}{\partial \Delta \varepsilon} = E(\delta \varepsilon - \delta \varepsilon^p) = \frac{\frac{K}{m} p^{\frac{1}{m}-1}}{1 + \frac{K}{mE} p^{\frac{1}{m}-1}} \qquad \text{éq 4.2.1-2}$$

In the field of damage:

$$\frac{\delta \sigma}{1-D} + \frac{\sigma}{(1-D)^2} \delta D - R'(p) \delta p = -\delta f^-$$

$$\frac{-E}{(1-D)^2} \delta \lambda + \frac{\sigma}{(1-D)^2} \frac{D_c}{p_R - p_D} \frac{\delta \lambda}{(1-D)} - \frac{K}{m} p^{\frac{1}{m}-1} \frac{\delta \lambda}{(1-D)} = -\delta f^- = \frac{E}{(1-D)} \delta \varepsilon$$

$$\frac{\partial \sigma}{\partial \Delta \varepsilon} = E(\delta \varepsilon - \delta \varepsilon^p) = E \frac{\frac{K}{m}(1-D) p^{\frac{1}{m}-1} - \frac{\sigma}{(1-D)} \frac{D_c}{p_R - p_D}}{E + \frac{K}{m}(1-D) p^{\frac{1}{m}-1} - \frac{\sigma}{(1-D)} \frac{D_c}{p_R - p_D}}$$

4.2.2 Finite element 3D

the absence of damage in the plastic part makes it possible to use the tangent matrix calculated for elastoplastic behavior with linear and isotropic kinematic hardening nonlinear [bib7]:

$$\frac{\partial \boldsymbol{\sigma}}{\Delta \boldsymbol{\varepsilon}} = \lambda^* \vec{\mathbf{1}} \otimes \vec{\mathbf{1}} + 2\mu^* \mathbf{Id} - \xi \frac{9\mu^2}{H(p)} \left(1 - \frac{R'(p).\Delta p}{(R(p)+\sigma_y)} \right) \frac{1}{R'(p) + 3\mu} \left(\frac{\boldsymbol{\sigma}^{dev}}{(R(p)+\sigma_y)} \otimes \frac{\boldsymbol{\sigma}^{dev}}{(R(p)+\sigma_y)} \right) \qquad \text{éq 4.2.2-1}$$

with $\lambda^* = K - \frac{2\mu}{3\mathrm{H}(\Delta p)}$ $\quad 2\mu^* = \frac{2\mu}{H(\Delta p)}$ $\quad H(\Delta p) = 1 + \frac{3\mu \xi.\Delta p}{(R(p)+\sigma_y)}$

the initial tangent matrix, used by option RIGI_MECA_TANG is obtained by adopting the behavior of the preceding step (elastic or plastic, meant by local variable ξ being worth 0 or 1) and while taking $\Delta p = 0$ in the equation **éq 4.2.2-2** .

In taking into account the presence of the damage one obtains the following tangent matrix:

$$\frac{\partial \sigma}{\partial \boldsymbol{\Delta} \varepsilon}=\lambda^* \vec{\mathbf{1}} \otimes \vec{\mathbf{1}}+2\mu^* \mathbf{Id}-\xi \frac{9\mu^2}{H(p)}\left(1-\frac{\Delta p\left((1-D)R'(p)-R(p)D'(p)\right)}{(1-D)\left(R(p)-\sigma_y\right)}\right)\frac{1}{\left(3\mu+(1-D)R'(p)-R(p)D'(p)\right)}\left(\frac{\boldsymbol{\sigma}^{dev}}{(1-D)\left(R(p)+\sigma_y\right)}\otimes\frac{\boldsymbol{\sigma}^{dev}}{(1-D)\left(R(p)+\sigma_y\right)}\right)$$

éq 4.2.2-2

with $\lambda^*=K-\frac{2\mu}{3\mathrm{H}(\Delta p)}$ $2\mu^*=\frac{2\mu}{H(\Delta p)}$ $H(\Delta p)=1+\frac{3\mu\xi.\Delta p}{(1-D)\left(R(p)-\sigma_y\right)}$

for the option FULL_MECA : $\boldsymbol{\sigma}^{dev}=\tilde{\boldsymbol{\sigma}}$

for the option RIGI_MECA_TANG : $\boldsymbol{\sigma}^{dev}=\tilde{\boldsymbol{\sigma}}$

4.3 Stored local variables

We indicate in the table according to the local variables stored in each Gauss point for the model of the steel subjected to corrosion:

Physical	local variable Meaning
V1	p : equivalent plastic strain
V2	D : variable of damage
V3	Indicator of plasticity (0 so elastic, 1 if plasticized i.e. as soon as *p* is not null)

5 Bibliography

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6 Checking

constitutive law `CORR_ACIER` is checked by the case following test:

SSNL127	Traction test with the model `CORR_ACIER`	[V6.02.127]

7 Description of the versions of the document

Version Aster	**Author (S) Organization (S)**	**Description of the modifications**
12/21/11	L.DAVENNE (NECS)	Correction of the matrix 1D
04/08/11	*I.PETRE-LAZAR, A. OUGLOVA (EDF* - R&D/MMC) L.DAVENNE, B.ZUBER (NECS)	initial Text