Modelization of the coupling creep/plasticity for the Summarized

concrete

It acts of a model of coupling between the behaviors modelling creep, of type GRANGER and the model of plasticity, either Generals (model of Von Mises with isotropic hardening), or specific with the concrete, of type BETON_DOUBLE_DP, established in the general algorithm of resolution of DYNA/STAT_NON_LINE. The coupling is carried out by a sequence of the resolution of the equations of the two models during an iterative algorithm, which will take place in each Gauss point, with each iteration. It is thus about a “strong” coupling, since the equations of the creep model and model of plasticity are satisfied rigorously at every moment.

This sequence makes it possible to free from the complete development of a true coupled resolution creep/plasticity, by means of the existing models, and without intervening in their resolution, but presents the disadvantage of a cost of higher computation.

Contents

1 Introduction

It acts of a model of coupling between the behaviors modelling creep, of type GRANGER and the model concrete modelling the phenomena of plasticity, either Generals (Von Mises), or specific with the concrete, of type BETON_DOUBLE_DP, established in the general algorithm of resolution of STAT_NON_LINE [R5.03.01].

One uses the existing models with for purpose chaining the resolution of the internal equations of the two models during an iterative algorithm, which takes place in each Gauss point, with each iteration. It is thus about a “strong” coupling, since the balance equations interns creep and plasticity are satisfied rigorously at every moment.

This sequence makes it possible to free from the complete development of a true coupled resolution creep/plasticity, but presents the disadvantage of a cost of higher computation. Indeed, in each Gauss point, and with each iteration, it is necessary a priori to solve several times the balance equations of the plasticity and those of creep, until obtaining the equilibrium between creep and plasticity. But, that makes it possible to couple a creep model, with any model of plasticity, at a moderate development cost. In addition, in the case of the BETON_DOUBLE_DP model , in tension post-peak, the curve of hardening is linear, and the local algorithm of resolution converges in only one iteration. The cost of computation thus remains restricted.

The couplings available by this process are, in the version of Code_Aster relative to this document:

- for the creep model: GRANGER_FP, GRANGER_FP_V, GRANGER_FP_INDT
- for the model of plasticity: ELAS, VMIS_ISOT_LINE, VMIS_ISOT_TRAC, VMIS_ISOT_PUIS, ROUSS_PR, BETON_DOUBLE_DP

Note: the coupling between creep model BETON_UMLV_FP and the model D “damage MAZARS and ENDO_ISOT_BETON is in fact a constitutive law written specifically to take into account the coupling between these two models, which does not concern the principle described in this document. This coupling is described in R7.01.06.

2 Principle of the coupling creep/plasticity

the coupling is carried out during the resolution internal at the Gauss point in the frame of an iterative algorithm in which computations of creep and plasticity are chained.

To carry out such a coupling, one integrates into the local resolution of operators STAT_NON_LINE DYNA_NON_LINE a modulus allowing successively to calculate starting from the same initial state (stresses and local variables, strains and increments of strains) the resolution of creep and plasticity. The principle is the following: during the resolution at the Gauss point, from L” initial state in stresses, local variables and strains, one carries out a computation of creep. The increment of strain of creep calculated during the resolution of creep is then deduced from the increment of deflection total, and provided in argument of the resolution of plasticity. The resolution of plasticity is carried out, starting from the same initial state in stresses, local variables and strains, except the increment of total deflection, to which one removed the increment of strain of creep. When the increment of plastic strain was calculated, after resolution of plasticity, one can correct the value of the strain as starter of the computation of creep, and reiterate computation chained creep/plasticity, until obtaining the convergence of the solution, i.e. the stability of the stresses and the strains of creep and plasticity.

The stresses calculated using the equations of creep are not a priori identical to those of the equations of plasticity. Their equality is not formally written in the equations. But when the equilibrium creep/plasticity is reached, one can show that the stresses in output of creep are quite equal to the stresses in output of plasticity (c.f. paragraph 3.2). As

the coupling is solved by sequence of the resolutions of creep and plasticity during an iterative algorithm, this allowing a weak data-processing development cost, the computation of the tangent matrix is carried out exclusively starting from the model of plasticity, without taking account of creep or of the coupling

creep/plasticity, by measurement of simplification. The strains of creep then constitute a loading for the computation of plasticity. That supposes that the evolution of creep is slower than the evolution of plasticity. This

simplification, all the more relevant as the computation step is judiciously selected, makes it possible to converge towards the solution (total) with a nombre of iterations total slightly more significant than if the tangent matrix is calculated in an exact way, but does not deteriorate of anything result computation (converged). Put

3 in equations Formulation

3.1 of the creep of Granger *in* Code_Aster *(Recall*) We

briefly present in this paragraph the modelization of creep in Code_Aster *, following* the model of Granger *. For* more detail, one will refer to [R7.01.01 and V6.04.142].
The statement of the strain of creep applied to a linear viscoelastic material, by applying the Principle of superposition of Boltzmann for a history of nonconstant loading, $\sigma(t)$ can be written in 1D: represent

$$\varepsilon_{fl}(t)=\int_{\tau=0}^{t} f(t-\tau)\frac{\partial \sigma}{\partial \tau}d\tau = f * \frac{\partial \sigma}{\partial t}$$

$*$ the product of convolution is

$J(t,t_c)=f(t-t_c)$ the creep function, increasing function of and $(t-t_c)$ null for negative $(t-t_c)$. One can show that any linear viscoelastic body can be modelled by a series connection of models of Kelvin and that the creep function can then be put in the form: and

$$f(t)=\sum_{s=1}^{r} J_s\left(1-\exp\left(-\frac{t}{\tau_s}\right)\right)$$

τ_s formula J_s plus coefficients and identified on the experimental curves of creep. One

uses a series connection of models of Kelvin whose coefficients are identified from experimental curves of creeps. It is shown in practice that one with satisfaction reproduces the curves of concrete creep with a series of 8 models.

The following creep function is thus used: what $J(t,t_c)=\sum_{s=1}^{s=8} J_s\left(1-\exp\left(-\frac{t-t_c}{\tau_s}\right)\right)$

leads to the statement: By

$$\varepsilon_{fl}(t)=\int_{\tau=0}^{t}\left[\sum_{s=1}^{s=8} J_s\left(1-\exp\left(-\frac{t-\tau}{\tau_s}\right)\right)\right]\frac{\partial \sigma}{\partial \tau}d\tau$$

deriving this statement in the case of an only model from Kelvin (s=1*),* one obtains the following equation easily: who $\tau_s \dot{\varepsilon}_{fl}(t)+\varepsilon_{fl}(t)=J_s\sigma(t)$

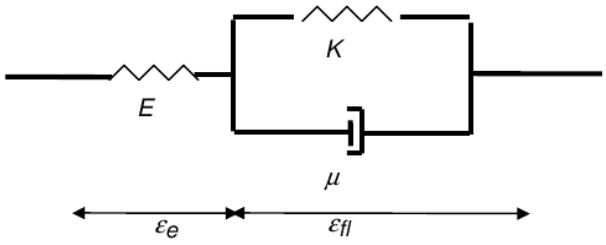
can be also written: that is to say $\sigma(t)=\frac{\tau_s}{J_s}\dot{\varepsilon}_{fl}(t)+\frac{1}{J_s}\varepsilon_{fl}(t)$

still: with $\sigma(t)=\mu\,\dot{\varepsilon}_{fl}(t)+K\,\varepsilon_{fl}(t)$ and $\mu=\frac{\tau_s}{J_s}$ One $K=\frac{1}{J_s}$

can extend this formulation to the case of the development in series of 8 models of Kelvin (or more), with the help of some mathematical developments, which gives: The model

$$\sigma(t)=\sum_{s=1}^{s=8}\left[K_s\varepsilon_{fl}^{s}(t)+m_s\dot{\varepsilon}_{fl}^{s}(t)\right]$$

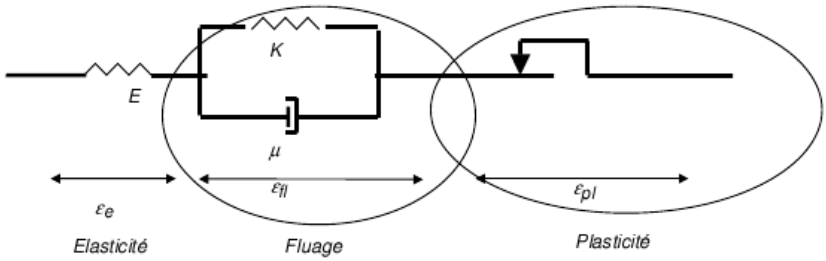
rheological of creep is represented on the figure below, in the case of a single model of Kelvin:



To simplify the presentation of the coupling in the paragraph which follows, one takes account only first term of decomposition in series of Kelvin, and one writes: Moreover $\sigma(t)=\mu\dot{\varepsilon}_{fl}(t)+K\varepsilon_{fl}(t)$
, for the presentation, one supposes and μ constant K , i.e. and $\mu=\mu^{-}$. $K=K^{-}$ *Formulation*

3.2 of the coupling creep/plasticity in 1D

the coupling creep/plasticity corresponds to the rheological model below, where the creep, characterized by the coefficients and K (stiffness μ and viscosity), and plasticity are modelled in series.



The equations governing creep are the following ones: that

$$\sigma=\mu\dot{\varepsilon}_{fl}+K\varepsilon_{fl}$$
$$\varepsilon=\varepsilon_e+\varepsilon_{fl}$$
$$\sigma=E e_{\varepsilon}$$

one can write at time running (the exhibitor "- "indicating L" urgent previous): L"

$$\varepsilon=\varepsilon_e+\varepsilon_{fl}=\varepsilon^{-}+\Delta\varepsilon_e+\Delta\varepsilon_{fl}$$
$$\sigma=\mu\dot{\varepsilon}_{fl}+K\varepsilon_{fl}=\mu\dot{\varepsilon}_{fl}+K\left[\varepsilon-\varepsilon_e\right]$$
$$\sigma=E\varepsilon_e$$

equation governing the model plasticity is the following one: What

$$\sigma=E\left[\varepsilon-\varepsilon_{pl}\right]$$

leads, in the case of the coupling creep/plasticity, with the following system of equations: In

$$\varepsilon=\varepsilon_e+\varepsilon_{fl}+\varepsilon_{pl}=\varepsilon^{-}+\Delta\varepsilon_e+\Delta\varepsilon_{fl}+\Delta\varepsilon_{pl}$$
$$\sigma=\mu\dot{\varepsilon}_{fl}+K\varepsilon_{fl}=\mu\dot{\varepsilon}_{fl}+K\left[\varepsilon-\varepsilon_e-\varepsilon_{pl}\right]$$
$$\sigma=E\varepsilon_e$$
$$\sigma=E\left[\varepsilon-\varepsilon_{pl}-\varepsilon_{fl}\right]$$

the resolution of the equations of behavior at the Gauss point, in the elementary routines of Code_Aster , the equality of the stresses resulting from creep and plasticity is not formally written, which corresponds in fact to

the following system of equations (by noting σ_1 the stress resulting from the resolution of creep and σ_2 the stress resulting from the resolution of plasticity at time running): However

$$\varepsilon=\varepsilon_e+\varepsilon_{fl}+\varepsilon_{pl}=\varepsilon^-+\Delta\varepsilon_e+\Delta\varepsilon_{fl}+\Delta\varepsilon_{pl}$$
$$\sigma_1=\mu\dot{\varepsilon}_{fl}+K\varepsilon_{fl}=\mu\dot{\varepsilon}_{fl}+K\left[\varepsilon-\varepsilon_e-\varepsilon_{pl}\right]$$
$$\sigma_1=E\varepsilon_e$$
$$\sigma_2=E\left[\varepsilon-\varepsilon_{pl}-\varepsilon_{fl}\right]$$

, with the equilibrium of the coupling creep/plasticity, there is well equality of the strain of creep and the plastic strain in the equations of the model of creep of Granger, and in the equations of plasticity, which implies by modifying the equation governing plasticity: then

$$\varepsilon_e=\varepsilon-\varepsilon_{pl}-\varepsilon_{fl}$$

from where $\sigma_2=E\left[\varepsilon-\varepsilon_{pl}-\varepsilon_{fl}\right]=E\varepsilon_e$

that is to say $\sigma_2=E\varepsilon_e=\sigma_1$

the equality of the stresses calculated during the resolution of creep, with the stresses calculated during the resolution of plasticity: It $\sigma_2=\sigma_1$

is the partition of the strains in the form, $\varepsilon=\varepsilon_e+\varepsilon_{pl}+\varepsilon_{fl}$ and the identity of the Young moduli of the models of creep and plasticity which are at the origin of this result. Indeed, the same Young modulus intervenes in the model of creep and the model of plasticity, and led, with the identity of the elastic strain, with the equality of the stresses. Note:

In a more general way, this kind of algorithm requires that the two models calculate in an identical way the elastic stress. Under

this assumption, a generalization of this connection with the models whose viscous and plastic strains are additive is possible [1]. One will write then:

$$\begin{pmatrix}\sigma_1\\ \varepsilon_{fl}\end{pmatrix}=F_1\left(\varepsilon-\varepsilon_{pl};\sigma^-;\varepsilon_{pl}^-,\ldots\right)$$
$$\begin{pmatrix}\sigma_2\\ \varepsilon_{pl}\end{pmatrix}=F_2\left(\varepsilon-\varepsilon_{fl};\sigma^-;\varepsilon_{fl}^-,\ldots\right)$$

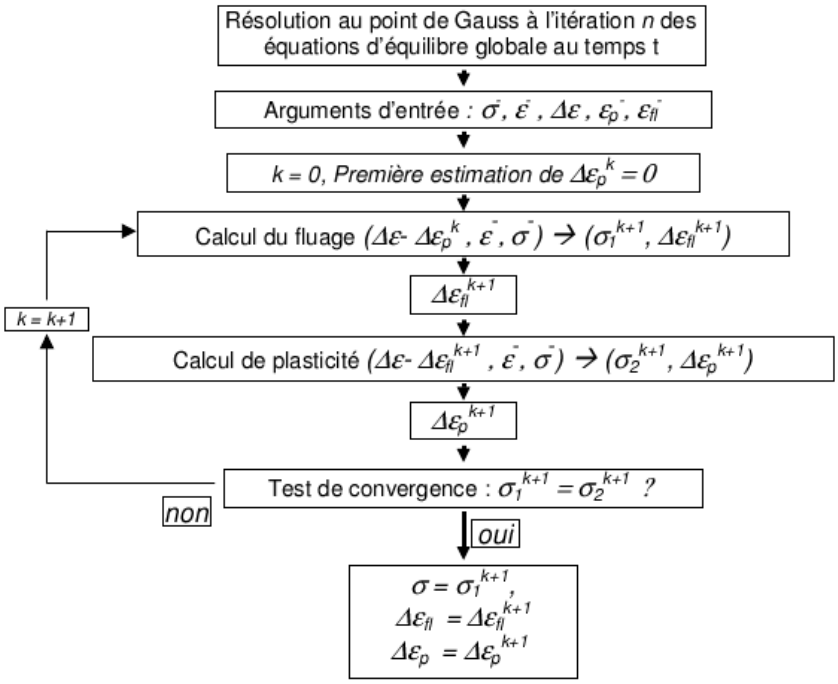
A theoretical study of the convergence of such a model, under the assumption that the two behaviors fit in the frame of the generalized standard mediums, is described in appendix.

Other possibilities of coupling, relating to in particular the models D “damage and of plasticity exist [7]. In particular coupling BETON_UMLV_FP /ENDO_ISOT_BETON is described in [R7.01.06]. Algorithm

4 D" integration of the coupling creep/plasticity Algorithm

4.1 with Gauss points

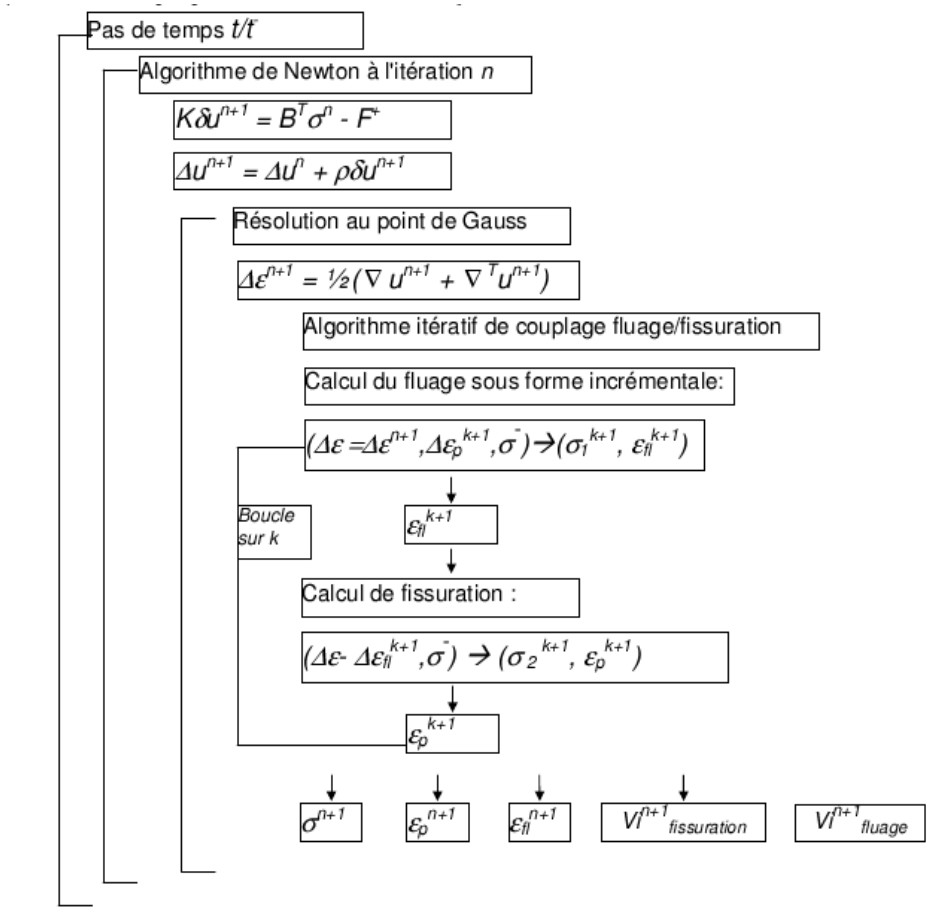
to introduce the coupling creep/plasticity into Code_Aster *, one* adopts the following approach: an iterative algorithm is used in which, at each Gauss point, one chains creep with plasticity. One solves the equation of creep to each iteration using (k) an estimated value of the increment of plastic strain, that one removes with the increment of total deflection. One thus obtains an estimated value of the increment of strain of creep, which one removes with the total deflection for the resolution of plasticity to the iteration. k One thus obtains a new estimate of the increment of plastic strain, which makes it possible to carry out the computation of creep to the iteration. $k+1$ One
carries out in an iterative way the sequence creep/plasticity, until obtaining a stable solution, i.e. a variation in relative value of the stress lower than a convergence criterion. Insofar as



the resolution of creep in Code_Aster is programmed in an incremental way, simplification consists in replacing by $\Delta\varepsilon$ in $\Delta\varepsilon - \Delta\varepsilon_{pl}^{k}$ the arguments of call of the resolution of creep. There is thus no intervention to make in the modulus of resolution of creep. It is the same of the resolution of plasticity, where one replaces by $\Delta\varepsilon$ in $\Delta\varepsilon - \Delta\varepsilon_{fl}^{k}$ the arguments of call of the resolution of plasticity. Top-level flowchart

4.2 One

can outline the flow chart of the resolution in the following way: Note:



indicate Vi the local variables The computation

creep aims only to provide the strain of creep which intervenes like an additional loading of the computation of plasticity. The stresses calculated during the resolution of creep neither are stored in memory, nor used for the computation of plasticity. With the equilibrium creep/plasticity, they are equal to the stresses resulting from the computation of plasticity.

The local variables of the computation of creep are on the other hand preserved of one iteration at the other because the computation of creep to the iteration requires n the knowledge of the local variables to the iteration. $n-1$ These variables also make it possible to calculate with each time step, the strain of creep whose increment is used for the computation of plasticity. It is the same of the local variables of model plasticity. Processing

4.3 of the local variables

the simultaneous resolution of the two models, creep and plasticity, requires a processing particular of the local variables, similar to what is made in environment THM (cf [R7.01.10]): the dimension of the table of the local variables is the sum of the numbers of variables of each model. They are transmitted to the routines of lower level the necessary information to recover these variables at every moment computation. For

the office plurality of two coupled mechanical models, the nombre total of local variables is stored in the card of behavior, as well as the name of the models.

The local variables of the various models follow one another in the table of the local variables: in first that which relates to creep, then that relating to plasticity. Management

4.4 of the behaviors and taking into account of the recutting of time step

the sequence of the constitutive laws requires the computation of the increment of strain of creep starting from the local variables of the model, which is then removed increment of total deflection before call of another model. One cannot thus permute the behaviors.

The first model to be coupled must be a creep model of the type GRANGER_FP (model of Granger *) or* GRANGER_FP_V (model of Granger *with* aging) or GRANGER_FP_INDT (model of Granger *independent* of the temperature). The order of the constitutive laws defined by the user must respect this logic.

The second model must be a nonlinear model of mechanics, type BETON_DOUBLE_DP , or a more general way, an elastic model ELAS , or elastoplastic VMIS _ISOT_*.

The number of models of the coupling is restricted to two, the first being a creep model, and nonthe respect of these choices of modelization led to a “Fatal” stop.

The modulus of resolution of the behavior coupled creep/plasticity is presented in the form of a new model. Currently

, the creep model is not integrated into the automatic recutting of time step of Code_Aster *, because of* an explicit resolution, contrary to the models of plasticity. During the resolution coupling, the model of plasticity profits from the automatic recutting of time step, according to same conventions as the recutting applied to the not coupled models of plasticity (systematic recutting or only in the event of nonconvergence, or not of recutting). Convergence criterion

4.4.1

the sequence creep/plasticity is carried out in a loop, in the modulus of coupling, until convergence of the solution. When the increment of strain of creep and the increment of strain of plasticity are stable from one iteration to another, the stresses resulting from the computation of creep are equal to the stresses resulting from the computation of plasticity. One could establish a convergence criterion in strain, but it is more logical than the convergence criterion is calculated starting from the stresses, which constitute the argument of output of elementary computations to Gauss points (with the local variables). Finally
, the convergence criterion of the coupling is form: by

$$\frac{\|\sigma_1\|-\|\sigma_2\|}{|\sigma_1|}<\text{tolérance}$$

noting σ_1 the stress resulting from the resolution of creep and σ_2 the stress resulting from the resolution of plasticity at current time. It is thus about a relative criterion in stress.

The maximum number of iterations of coupling creep/plasticity is defined by the parameter giving the maximum number of iterations of local resolution ITER_INTE_MAXI . In the same way, the convergence criterion of the coupling, defined by the relative residue in stress is defined by the maximum residue of local resolutions RESI_INTE_RELA . These two parameters thus make it possible to manage the convergence of each of the two local laws, and the convergence of the coupling. Increment

4.4.2 of strain of creep The computation

of the increment of strain of creep is calculated in the modulus of resolution of the coupling. Indeed, it is not stored in the local variables. From
the local variables corresponding to creep, one can recompute in a simple way, the strain of creep at times running and precedent, and thus deduce the increment from it from strain of creep to be removed with the increment of total deflection. (One will refer to the documentation of reference of the behavior model of Granger, for further information [R7.01.01 and V6.04.142]). In the frame of

a monodimensional statement, the strain of creep of the model of Granger is the following one: with

$$\varepsilon_{fl}^{1D}(t_n)=\sum_{s=1}^{8}\varepsilon_{fl}^{s}(t_n) \quad =J\cdot A_n^0-\sum_{s=1}^{8}A_n^s \qquad \text{and } J=\sum_s J_s$$

where

$$\begin{cases} A_{n+1}^0=A_n^0+\Delta(S)_{n+1} \\ A_{n+1}^s=A_n^s\cdot\exp(-\dfrac{\Delta t_{n+1}}{\tau_s})+\Delta(S)_{n+1}\cdot\dfrac{\tau_s}{\Delta t_{n+1}}\cdot J_s\cdot(1-\exp-\dfrac{\Delta t_{n+1}}{\tau_s}) \end{cases}$$

$$S=\sigma\,.\,h\,.\,\frac{T-248}{45}$$

Δt_n the increment of time between times represents and t_n . t_{n+1} and

J_s are τ_S coefficients of decomposition in series of model of Kelvin.

The coefficients and $J\cdot A_n^0$ are A_n^s $_{(s=1,8)}$ by way of the same tensors dimension than the strain tensor, whose various components are stored as local variables of the model. In
3D, the classical assumption consists in supposing the existence of a Poisson's ratio of creep, constant and equal to the elastic Poisson's ratio. This Poisson's ratio of creep is not taken into account in the formulation 1D:

. $\varepsilon_{fl}^{1D}(t_n)=J\cdot A_n^0-\sum_{s=1}^{8}A_n^s$ It

is thus necessary to multiply the strain tensor obtained starting from the local variables by the matrix:

$$H_0=\begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1+\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 1+\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 1+\nu \end{bmatrix}$$

to obtain the strain of creep 3D. That is to say

(There $\varepsilon_{fl}(t_n)=H_0\cdot\varepsilon_{fl}^{1D}(t_n)=H_0\cdot\left(J\cdot A_n^0-\sum_{s=1}^{8}A_n^s\right)$

is a similar formulation in 2D). One

transmits then like increment of strain as starter of the resolution of plasticity, the increment of total deflection, to which one removed the increment of strain of creep calculated by difference of the strains of creep to times running and precedent. Increment

4.4.3 of plastic strain The computation

of the increment of plastic strain is calculated in the modulus of resolution of the coupling. Indeed, it is not stored in the local variables.

The increment of plastic strain is obtained simply by shrinkage of the increment of strain of creep and the increment of elastic strain to the increment of total deflection. Bibliography

5 P.

1) of Bonnières: Preliminary study for the EL 2000-135: generalization of concept KIT_DDI of coupling of constitutive laws. CR MN 2001/010 of 1/15/2001 I.

2) EKELAND, R. TEMAM. “Convex Analysis and variational problems” Bordered, Paris (1974) D.G.

3) LUENBERGER. “ Introduction to linear and nonlinear programming” Addison-Wesley publishing company (1973) P.

4) MIALON. “Convex Optimization” Course of DEA. Tunis 1991. P.

5) MIALON. “Elements of analysis and numerical resolution of the relations of elastoplasticity” EDF - Bulletin of the Management of the Studies and RecherchesSérie C Mathematical, Data processing, n°3, 1986, p. 57-89. B.

6) Waxed: Development of coupling creep/cracking in the Code _*Aster*© - Specifications. Ratio CS SI/311-1/420AL0/RAP/00.019 Version 1.1 D.

7) Markovic “numerical platform of the elastoplastic couplings endommageables” CR-AMA/07.049 Description

6 of the versions of the document Version

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7 : convergence of the method of coupling Notations

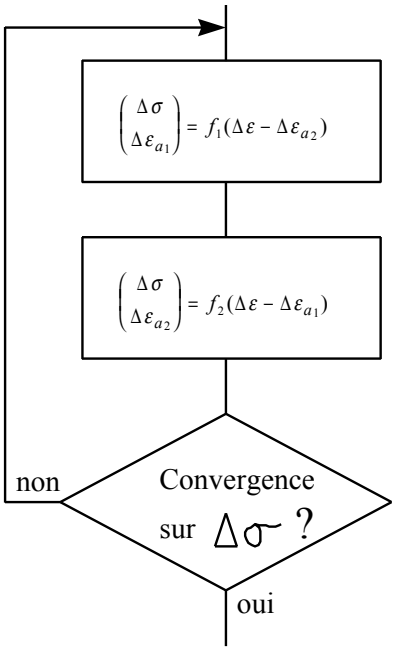
7.1 and assumptions In

this paragraph and the following, we will consider two nonlinear constitutive laws and $L1$ $L2$, dissipative associated with the same elasticity tensor. H The integration of the model consists Li , at each Gauss point, on the one hand with calculating the increment of stress and Ds the increment of unelastic strain according to $\Delta\varepsilon_{a_i}$ an increment of total deflection and $\Delta\varepsilon$ quantities at the beginning of time step (known) and on the other hand with calculating at the point the contribution $\Delta\varepsilon$ to the tangent operator. One $\left(\frac{\partial\Delta\sigma}{\partial\Delta\varepsilon}\right)_{\Delta\varepsilon}^{L_i}$ can describe in a synthetic way the “local incremental problem” which the integration of the model constitutes by introducing L_i the operator defined f_i by: One can

$$\begin{pmatrix}\Delta\sigma\\ \Delta\varepsilon_{a_i}\end{pmatrix}=f_i(\Delta\varepsilon)$$

then define the constitutive law resulting L from the coupling from and. $L1$ Its $L2$ unelastic strain is worth ε_a : and $\varepsilon_a=\varepsilon_{a_1}+\varepsilon_{a_2}$ the relation of elasticity associated with is written L : The integration $\sigma=H(\varepsilon-\varepsilon_{a_1}-\varepsilon_{a_2})$

of the model is carried out L by the successive resolutions of the “local incrémentaux problems” following (models and) L_1 in L_2 a loop until convergence on ($\Delta\sigma$ the tangent operator for $\left(\frac{\partial\Delta\sigma}{\partial\Delta\varepsilon}\right)_{\Delta\varepsilon}^{L}$ the model is calculated L in paragraph 3): The theoretical



approach of this kind of coupling comprises two shutters: The study

- of the convergence of the loop above the study
- of the possibility of putting the total incremental problem (that the structural analysis with the model constitutes) in L the form of a problem of minimization having a single solution. Writing

7.2 of the total incremental problem in the form of a minimization We

suppose that the two constitutive laws and S L_1 "register L_2 in the frame of the generalized standard mediums. In this case, the "local incremental problem" that constitutes L" integration of the model, described L_i by the equality: where

$$\begin{pmatrix} \Delta\sigma \\ \Delta\varepsilon_{a_i} \end{pmatrix} = f_i(\Delta\varepsilon - \Delta\varepsilon_{a_j}) \qquad (E_i)$$

and, $i \in [1,2]$ can $j = 3 - i$ be also written in the form: where

$$\Delta\varepsilon_{a_i} = Arg\,\underset{e_i}{Min}\left[\phi(\varepsilon + \Delta\varepsilon - \Delta\varepsilon_{a_j}, \varepsilon_{a_i} + \varepsilon_{a_j} + e_i) + G_i(e_i)\right] \qquad (P_i)$$

is L φ "elastic strain energy: and where

$$\varphi(\varepsilon, \varepsilon_a) = \frac{1}{2} H(\varepsilon - \varepsilon_a) : (\varepsilon - \varepsilon_a)$$

is G_i the "complementary potential" of the model. We L_i define below this potential from G_i , stored f_i share of L" free energy and function of the local variables, and a_i the potential of dissipation, function D_i unelastic strainrate and $\dot{\varepsilon}_{a_i}$ velocities of evolution of the local variables. With $\dot{\alpha}_i$ an incremental formulation, one a: the vector

$$G_i(e_i) = \underset{\Delta\alpha}{Min}\left[\varphi_i(\alpha_i + \Delta\alpha) + D_i(e_i, \Delta\alpha)\right]$$

of the local variables a_i of the model at the beginning L_i of time step thus intervenes like parameter in the statement of. For G_i each value of this parameter, a_i the function is positive G_i and convex. Note:

If there are no local variables, one has simply One supposes $G_i = D_i$

moreover than one of the two complementary potentials, or, G_1 is G_2 strictly convex and coercif. While posing

, and $\Delta\varepsilon_{a_1} = \bar{x}$, $\Delta\varepsilon_{a_2} = \bar{y}$ $F(x, y) = \varphi(\varepsilon + \Delta\varepsilon, \varepsilon_{a_1} + \varepsilon_{a_2} + x + y)$ the system consisted the equations and is (P_1) equivalent (P_2) to: (S) and

$$\begin{cases} \bar{x} = Arg\,\underset{x}{Min}\left[F(x, \bar{y}) + G_1(x)\right] \\ \bar{y} = Arg\,\underset{y}{Min}\left[F(\bar{x}, y) + G_2(y)\right] \end{cases}$$ being

G_1 G_2 convex and being F Cake-differentiable convex, one has according to the property of [4 $P7$] (which rises from proposal 2.2 of chapter II from [2]): The fact

$$(S)\Leftrightarrow(\bar{x},\bar{y})=Arg\underset{(x,y)}{Min}\left[F(x,y)+G_1(x)+G_2(y)\right]$$

of finding a solution exact with the system consisted the equations and is (E_1) thus (E_2) equivalent to find the minimum of function continuous, coercive and strictly convex. According to proposal 1.2 of chapter II of [2], this minimum exists and is single. The local

resolution of the model resulting L from the coupling from and is L_1 thus L_2 equivalent to the problem of minimization above. Consequently, is L a standard constitutive law generalized with J like elastic strain energy and G complementary potential, where is defined G by: with

$$G(\dot{\varepsilon}_a)=G_1(\dot{\varepsilon}_{a_1})+G_2(\dot{\varepsilon}_{a_2})$$

. While $\varepsilon_a=\varepsilon_{a_1}+\varepsilon_{a_2}$ proceeding

in the same way as in the §3 of [5], one can thus put the total incremental problem (which the structural analysis with the model constitutes) in L the form of a problem of minimization having a single solution. Convergence

7.3 of the algorithm It should

now be proven that the algorithm describes above converges well. One notes

the result $z_n=(x_n,y_n)=(\Delta\varepsilon_a^{n_1},\Delta\varepsilon_a^{n_2})$ resulting one from the iteration and one n poses: One calculates

$$K(z)=K(x,y)=F(x,y)+G_1(x)+G_2(y)$$

according to z_{n+1} in the following way z_n : One notes

$$x_{n+1}=Arg\underset{x}{Min}\,K(x,y_n)$$
$$y_{n+1}=Arg\underset{y}{Min}\,K(x_{n+1},y)$$

:

$$\bar{z}=(\bar{x},\bar{y})=Arg\underset{(x,y)}{Min}\,K(x,y)$$

To show the convergence of the continuation towards (z_n) , we $\bar{z}$ will use the theorem of total convergence of the §6.5 of [3]. One considers

the algorithm A which from given z_0 generates the continuation defined (z_n) above: The group

$$z_{n+1}=A(z_n)$$

solution formula Γ the singleton {}. Let us check $\bar{z}$

the assumptions of this theorem: H1:

all the points z_n are contained in a compact unit. One S has

by construction: However,

$$\forall n \in \mathbb{N}, K(z_n) \leq K(z_0)$$

is coercive K thus with $K(z) \underset{\|z\| \to +\infty}{\to} +\infty$ formula $\|z\|^2 = x_{eq}^2 + y_{eq}^2$)

:

$\forall R>0 \quad \exists M>0 \quad \forall z \quad \|z\| > M \Rightarrow K(z) > R$
$\forall R>0$, $\exists M>0$ While $\forall z$ taking $K(z) \leq R \Rightarrow \|z\| \leq M$

, one $R = K(z_0)$ thus has: Thus
$\forall n \in \mathbb{N}, \|z_n\| \leq M$

all the points z_n are contained in a limited unit, therefore in compact since the vector space is of finished size, therefore the assumption is satisfied. H2. There

exists a function of descent for Z formula $\Gamma \quad A$

Z be continuous and check the two following properties: . if
, then $z \neq \bar{z}$. if $Z(A(z)) < Z(z)$
, then $z = \bar{z}$ One takes $Z(A(z)) \leq Z(z)$

is quite $Z = K$

K continuous and one a:.

If $K(A(\bar{z})) = K(\bar{z})$

, then $z \neq \bar{z}$ such as $A(z) = z' = (x', y')$: If one

$$x' = Arg \underset{x}{Min} K(x, y)$$
$$y' = Arg \underset{y}{Min} K(x', y)$$

has, then $x' \neq x$ being K continuous, coercive and strictly convex, the minimum is single x' and one A. But,
$K(x', y) < K(x, y)$ one thus has $K(x', y') \leq K(x', y)$. If $K(z') < K(z)$ one

has, then $x' = x$ one has (if not $y' \neq y$, one would have). being $z = \bar{z}$

K continuous, coercive and strictly convex, the minimum is single y' and one a: thus

$K(x', y') < K(x', y) et K(x', y) = K(x, y)$ One thus $K(x', y') < K(x, y)$

has in both cases: There thus

$K(A(z)) < K(z)$

exists well a function of descent for and. Γ A The assumption is satisfied. H3. L

"algorithm is closed A for any N" not belonging z to formula Γ

to check this third assumption, we will adopt the same method as that used with the §7.8 of [3] to show the total convergence of an algorithm of cyclic descent by coordinate (cyclic coordinate descent algorithm). Let us consider

the two following algorithms: who with

- C^1 associates (x, y) all the points with (X, y) unspecified X which with
- C^2 associate (x, y) all the points with (x, Y) unspecified Y One in addition

considers the algorithm M of minimization on each one of these two sets (which with an element of the unit associates the point carrying out the minimum on the unit). The algorithm is then A the composition of 4 algorithms: The algorithms

$$A = M C^2 M C^1$$

and are C^1 C^2 continuous and is closed M . As all the points are contained in a compact unit (cf S the first assumption), one of deduced according to corollary 1 from the §6.5 with [3] which is closed A , therefore the assumption is satisfied. The three

assumptions of the theorem of total convergence being checked, we thus showed result according to: The limit

of any convergent under-continuation of is (z_n) . However, $\bar{z}$ we (P)

also showed that all the points z_n are contained in compact. Then S let us suppose that (z_n) does not converge towards, $\bar{z}$ one a: One can

$$\exists \varepsilon > 0, \forall N \in \mathbb{N}, \exists n = \psi(N) > N, \|z_n - \bar{z}\| > \varepsilon$$

build a under-continuation formulates $(z_{b(n)})$ by: One has

$$b(0) = 0$$
$$\forall n \in \mathbb{N}, \beta(n+1) = \psi(\beta(n))$$

then: However, L $\forall n \in \mathbb{N}, \left\| z_{\beta(n)} - \bar{z} \right\| \leq M$

"together is compact $(S - B(\bar{z}, e))$, therefore the continuation admits $(z_{b(n)})$ a convergent under-continuation which D" after the property above (P) converges towards, which $\bar{z}$ is absurd. One thus

obtains the result required one: The continuation

converges (z_n) towards. Computation $\bar{z}$

7.4 of the tangent operator By noting

the increment $\Delta\varepsilon$ of total deflection (as starter for the model resulting L from the coupling from and) L_1, one L_2 can explicitly calculate the tangent operator of evaluated L at the point formulates $\Delta\varepsilon$ whom one notes: formulate $\left(\frac{\partial\Delta\sigma}{\partial\Delta\varepsilon}\right)^L_{\Delta\varepsilon}$

$$\left(\frac{\partial\Delta\sigma}{\partial\Delta\varepsilon}\right)^L_{\Delta\varepsilon}=\left(\frac{\partial\Delta\sigma}{\partial\Delta\varepsilon}\right)^{L_1}_{\Delta\varepsilon-\Delta\varepsilon_{a_2}}\frac{\partial(\Delta\varepsilon-\Delta\varepsilon_{a_2})}{\partial\Delta\varepsilon}$$

$$\left(\frac{\partial\Delta\sigma}{\partial\Delta\varepsilon}\right)^L_{\Delta\varepsilon}=\left(\frac{\partial\Delta\sigma}{\partial\Delta\varepsilon}\right)^{L_1}_{\Delta\varepsilon-\Delta\varepsilon_{a_2}}\left(H^{-1}\left(\frac{\partial\Delta\sigma}{\partial\Delta\varepsilon}\right)^L_{\Delta\varepsilon}+\frac{\partial\Delta\varepsilon_{a_1}}{\partial\Delta\varepsilon}\right)\qquad(3.1)$$

them $\Delta\varepsilon_{a_i}$ from the convergence of the loop described in paragraph 7, with 7as starter $\Delta\varepsilon$. However, one also has, by symmetry on indices 1 and 2: formulate

$$\left(\frac{\partial\Delta\sigma}{\partial\Delta\varepsilon}\right)^L_{\Delta\varepsilon}=\left(\frac{\partial\Delta\sigma}{\partial\Delta\varepsilon}\right)^{L_2}_{\Delta\varepsilon-\Delta\varepsilon_{a_1}}\frac{\partial(\Delta\varepsilon-\Delta\varepsilon_{a_1})}{\partial\Delta\varepsilon}$$

$$\left(\frac{\partial\Delta\sigma}{\partial\Delta\varepsilon}\right)^L_{\Delta\varepsilon}=\left(\frac{\partial\Delta\sigma}{\partial\Delta\varepsilon}\right)^{L_2}_{\Delta\varepsilon-\Delta\varepsilon_{a_1}}\left(I-\frac{\partial\Delta\varepsilon_{a_1}}{\partial\Delta\varepsilon}\right)$$

thus calculate formula $\left(\frac{\partial\Delta\varepsilon_{a_1}}{\partial\Delta\varepsilon}\right)$

$$\frac{\partial\Delta\varepsilon_{a_1}}{\partial\Delta\varepsilon}=I-\left[\left(\frac{\partial\Delta\sigma}{\partial\Delta\varepsilon}\right)^{L_2}_{\Delta\varepsilon-\Delta\varepsilon_{a_1}}\right]^{-1}\left(\frac{\partial\Delta\sigma}{\partial\Delta\varepsilon}\right)^L_{\Delta\varepsilon}$$

this statement into the equation (3.1), one obtains: formulate

$$\left(\frac{\partial\Delta\sigma}{\partial\Delta\varepsilon}\right)^L_{\Delta\varepsilon}=\left(\frac{\partial\Delta\sigma}{\partial\Delta\varepsilon}\right)^{L_1}_{\Delta\varepsilon-\Delta\varepsilon_{a_2}}\left(H^{-1}\left(\frac{\partial\Delta\sigma}{\partial\Delta\varepsilon}\right)^L_{\Delta\varepsilon}+I-\left[\left(\frac{\partial\Delta\sigma}{\partial\Delta\varepsilon}\right)^{L_2}_{\Delta\varepsilon-\Delta\varepsilon_{a_1}}\right]^{-1}\left(\frac{\partial\Delta\sigma}{\partial\Delta\varepsilon}\right)^L_{\Delta\varepsilon}\right)$$

: formulate

$$\left[I-\left(\frac{\partial\Delta\sigma}{\partial\Delta\varepsilon}\right)^{L_1}_{\Delta\varepsilon-\Delta\varepsilon_{a_2}}\left(H^{-1}-\left[\left(\frac{\partial\Delta\sigma}{\partial\Delta\varepsilon}\right)^{L_2}_{\Delta\varepsilon-\Delta\varepsilon_{a_1}}\right]^{-1}\right)\right]\left(\frac{\partial\Delta\sigma}{\partial\Delta\varepsilon}\right)^L_{\Delta\varepsilon}=\left(\frac{\partial\Delta\sigma}{\partial\Delta\varepsilon}\right)^{L_1}_{\Delta\varepsilon-\Delta\varepsilon_{a_2}}$$

has finally:

$$\boxed{\left(\frac{\partial\Delta\sigma}{\partial\Delta\varepsilon}\right)^L_{\Delta\varepsilon}=\left[\left[\left(\frac{\partial\Delta\sigma}{\partial\Delta\varepsilon}\right)^{L_1}_{\Delta\varepsilon-\Delta\varepsilon_{a_2}}\right]^{-1}+\left[\left(\frac{\partial\Delta\sigma}{\partial\Delta\varepsilon}\right)^{L_2}_{\Delta\varepsilon-\Delta\varepsilon_{a_1}}\right]^{-1}-H^{-1}\right]^{-1}}$$