Constitutive law of HOEK_BROWN modified

Summarized

This document describes a constitutive law for the rocks. The criterion of plasticization is a parabolic criterion of type Hoek-Brown. He is written in principal stresses. The evolution of plastic strains of “nonassociated” type, is formulated from a criterion of Drucker-Prager whose friction angle evolves with plasticization.

This model is usable in pure mechanics as in coupled modelization thermo hydro mechanics.

In modelization thermo-hydro-mechanics, it can be used in effective stresses or total stresses:

- In the first case, in fact the effective stresses are subjugated to respect the criterion of Hoek and Brown. The strain figures are by means of calculated the surface of Drucker and the friction angle within the space of effective stresses
- In the second case, in fact the total stresses are subjugated to respect the criterion of Hoek and Brown. The strain figures are by means of calculated the surface of Drucker and the friction angle within the space of total stresses

Contents

1 Introduction

This note presents a model of structural mechanics behavior for the rocks. To represent the structural mechanics behavior of the rocks the user of Code_Aster can use the model of Drucker-Prager `DRUCK_PRAGER`, the model of `LAIGLE` or model `HOEK_BROWN` presented here. The model of Drucker-Prager is simplest and furthest away from the real behavior of the rocks. The model of Laigle is most faithful to the physics of the phenomena. Thus the model presented in this note is intermediate between these two models, in term of complexity as in term of representation of reality. It uses rather classical formulations in the medium of géomechanics.

This model is usable in modelizations of mechanics alone or modelizations of the type THM.
In modelization thermo-hydro-mechanics, it can be used in effective stresses or total stresses:

- In the first case, in fact the effective stresses are subjugated to respect the criterion of Hoek and Brown. The strain figures are by means of calculated the surface of Drucker and the friction angle within the space of effective stresses

- In the second case, in fact the total stresses are subjugated to respect the criterion of Hoek and Brown. The strain figures are by means of calculated the surface of Drucker and the friction angle within the space of total stresses

One will find in the reference [1] elements useful for the comprehension of this model
the formulation is of standard nonassociated plasticity:
- The field of elasticity is defined by a criterion of the type Hoek-Brown, whose parameters evolve with the hardening parameter
- the hardening parameter is a combination of the plastic strain of shears and voluminal plastic strain
- plastic strains derive from a criterion of Drucker-Prager whose friction angle evolves with plasticization.

1.1 Characteristics of the model

The model simulates the short-term structural mechanics behavior of the rocks in 4 phases, described in the reference [1]:
1) Elastic phase characterized by a modulus Young and a Poisson's ratio constant
2) elastoplastic Phase with positive hardening which simulates the initiation of a form of damage and its progression towards the fracture of the rock. This phase is modelled by a plasticity criterion of the type Hoek-Brown. This criterion evolves according to the major unrecoverable deformation. The evolution of the unrecoverable deformation is determined by a potential of yielding expressed by a function of the Drucker-Prager type.
3) Elastoplastic phase with negative hardening which represents the behavior post-fracture of the rocks. The rupture criterion is of the type Hoek-Brown. The strain is determined by a potential of yielding nonassociated with Drucker-Prager type.
4) Phase of residual strength characterized by a function of the type Hoek-Brown modified.

2 Notations

2.1 General information

the strains are told positive in extension and the forced are positive for states of tension.

Notation	Description
$I_1 = tr(\boldsymbol{\sigma})$	First Deviative invariant of
$\mathbf{s} = \boldsymbol{\sigma} - \frac{tr(\boldsymbol{\sigma})}{3}\mathbf{I}$	the stresses of the stresses
$s_{II} = \sqrt{\mathbf{s}\,.\,\mathbf{s}}$	Second invariant of the Deviative tensor of the stresses

$\mathbf{e}=\boldsymbol{\varepsilon}-\frac{tr(\boldsymbol{\varepsilon})}{3}\mathbf{I}$	déviatoires of the strains
$\boldsymbol{\varepsilon}_v=tr(\boldsymbol{\varepsilon})$	Traces strains: voluminal strain
$\boldsymbol{\varepsilon}^p$	Tensor of plastic strains
$\boldsymbol{\varepsilon}_v^p=tr(\boldsymbol{\varepsilon}^p)$	the plastic Variation of volume
$\delta\gamma^p=\sqrt{\frac{2}{3}d\,\mathbf{e}^p:d\,\mathbf{e}^p}$	cumulated Plastic strain of shears
σ_1	major Principal stress
σ_3	minor Principal stress ($\sigma_1<\sigma_2<\sigma_3$)
$\mathbf{H}$	Matrix of Hooke
μ	Coefficient of Lamé

2.2 Parameters of the model

Notation	Description
γ	Hardening parameter (defined in paragraph 3.2.3)
S	Represents the state of damage and of fracturing of the rock
m	Parameter of lissage of the model
σ_c	Strength of solid rock without any damage
γ^{rup}	Hardening parameter corresponding when the material breaks
γ^{res}	Hardening parameter corresponding to the beginning of residual strength
$(S\sigma_c^2)^{rup}$	Value of the product $S\sigma_c^2$ to the fracture reached in γ^{rup}
$(S\sigma_c^2)^{end}$	Value of the product $S\sigma_c^2$ to the initiation of damage $(\gamma=0)$
$(m\sigma_c)^{rup}$	Value of the product $m\sigma_c$ to the fracture reached in γ^{rup}
$(m\sigma_c)^{end}$	Value of the product $m\sigma_c$ to the initiation of damage $(\gamma=0)$
E	Modulus Young
ν	Poisson's ratio
β	Characterizes residual strength
ϕ^{end}	Friction angle with the initiation of damage $(\gamma=0)$: optional parameter taken no by default
ϕ^{rup}	Friction angle with the fracture reached in γ^{rup}
ϕ^{res}	Friction angle with the residual strength reached in γ^{res}
α	Parameter of the model characterizing the behavior post-fracture of the continuous

3 Model material

We describe here the model independently owing to the fact that it is used in mechanics alone, or in computations hydro-mechanical in total or effective stress. Thus the notation σ used in the following paragraphs will have to be intreprétée according to use.

3.1 Behavior elastic

the elastic behavior is given by a linear model. The two parameters characterizing this behavior are the modulus D "elasticity E and the Poisson's ratio ν .

3.2 Behavior plastic

the adopted formulation is resulting from the document [1].

3.2.1 Surface of load

$$F(\boldsymbol{\sigma},\gamma)=(\sigma_3-\sigma_1)-\sqrt{-\sigma_3.m\sigma_c+S(\gamma)\sigma_c^2}-b(\gamma).\left(1-\frac{\sigma_3}{\sigma_3^{b-d}}\right)$$

where:

- γ D" hardening is the parameter (defined in paragraph 3.2.3)
- S characterizes the state of damage and fracturing of the rock
- m is a parameter of lissage of the model
- σ_c is the strength of solid rock without any damage $\sigma_c>0$
- b is a function of the variable of hardening to parabolic evolution which characterizes the behavior post-fracture
- σ_3^{b-d} is the intersection between the line $\sigma_1=\alpha\sigma_3$ (α being a parameter of the model) and the criterion at the instant of the failure ($\gamma=\gamma^{rup}$). $\sigma_3^{b-d}>0$
- σ_1 and σ_3 are the principal stresses major and minor: $\sigma_1<\sigma_2<\sigma_3$

3.2.2 Potential of yielding

the potential of yielding is given by a function resulting from the criterion of Drucker-Prager:

$G(\boldsymbol{\sigma},\gamma)=\eta(\gamma)I_1+\sqrt{\frac{3}{2}}s_{II}=\eta(\gamma)I_1+\sigma_{eq}$ with $\eta(\gamma)=\frac{2\sin\phi(\gamma)}{3+\sin\phi(\gamma)}$, $\sigma_{eq}=\sqrt{\frac{3}{2}}s_{II}$ and $\phi(\gamma)$ the friction angle are equivalent.

3.2.3 Hardening parameter γ

the hardening parameter γ which one considers takes the following values:

$\gamma=0$ with the initiation of damage
$\gamma=\gamma^{rup}$ to the fracture
$\gamma=\gamma^{res}$ at the beginning of residual strength

It is defined while being placed in triaxial compression by: $\gamma=\varepsilon_1^p$, and one can show whereas $\gamma^p=|\frac{\varepsilon_v^p}{3}-\gamma|$. One thus has $\gamma=\frac{\varepsilon_v^p}{3}\pm\gamma^p$ who must be positive.

3.2.4 Other parameters

the parameters $S\sigma_c^2, m\sigma_c, \phi, b$ vary in the following way according to the hardening parameter γ :

1) $$(m\sigma_c)(\gamma)=\begin{cases}\gamma\dfrac{(m\sigma_c)^{rup}-(m\sigma_c)^{end}}{\gamma^{rup}}+(m\sigma_c)^{end}=p_{m\sigma}\gamma+(m\sigma_c)^{end} & si\quad \gamma\le\gamma^{rup}\\ (m\sigma_c)^{rup} & si\quad \gamma\ge\gamma^{rup}\end{cases}$$

2) $$(S\sigma_c^2)(\gamma)=\begin{cases}\gamma\dfrac{(S\sigma_c^2)^{rup}-(S\sigma_c^2)^{end}}{\gamma^{rup}}+(S\sigma_c^2)^{end}=p_{S\sigma^2}\gamma+(S\sigma_c^2)^{end} & si\quad \gamma\le\gamma^{rup}\\ (S\sigma_c^2)^{rup} & si\quad \gamma\ge\gamma^{rup}\end{cases}$$

3) $$\phi(\gamma)=\begin{cases}\dfrac{\phi^{rup}-\varphi^{end}}{\gamma^{rup}}\gamma+\phi^{end} & \text{si } \gamma\le\gamma^{rup}\\ \dfrac{\phi^{res}-\phi^{rup}}{\gamma^{res}-\gamma^{rup}}\gamma+\dfrac{\phi^{rup}\gamma^{res}-\phi^{res}\gamma^{rup}}{\gamma^{res}-\gamma^{rup}} & \text{si } \gamma^{rup}\le\gamma\le\gamma^{res}\\ \phi^{res} & \text{sinon}\end{cases}$$

4) $$b(\gamma)=\begin{cases}0 & \text{si } \gamma\le\gamma^{rup}\\ a\gamma^2+d\gamma+c & \text{si } \gamma^{rup}\le\gamma\le\gamma^{res}\\ b^{res} & \text{si } \gamma\ge\gamma^{res}\end{cases}$$ where $a=-\dfrac{b^{res}}{(\gamma^{rup}-\gamma^{res})^2}$

$$d=\frac{2b^{res}\gamma^{res}}{(\gamma^{rup}-\gamma^{res})^2},\quad c=\frac{b^{res}\gamma^{rup}(\gamma^{rup}-2\gamma^{res})}{(\gamma^{rup}-\gamma^{res})^2}$$ and the $b^{res}=\beta-\sqrt{(S\sigma_c^2)^{rup}}$

5) $$\sigma_3^{b-d}=\frac{-(m\sigma_c)^{rup}-\sqrt{((m\sigma_c)^{rup})^2+4(1-\alpha)^2(S\sigma_c^2)^{rup}}}{2(1-\alpha)^2}$$

6) coefficients $\alpha,\beta,(S\sigma_c^2)^{rup},(S\sigma_c^2)^{end},(m\sigma_c)^{rup},(m\sigma_c)^{end},\phi^{end},\phi^{rup},\phi^{res}$ are given.

4 Incremental form

One is placed here in the frame of finished increases. The index – indicates a component at the beginning of step of loading and the absence of index a component at the end of the step of loading. The operator Δ indicates the increase in a component.

In pure mechanics or when the model is used in modelization THM in effective stresses, the equations translating the elastic behavior are written:

$$\mathbf{s}=\mathbf{s}^-+2\mu(\Delta\mathbf{e}-\Delta\mathbf{e}^p)=\mathbf{s}^e-2\mu\Delta\mathbf{e}^p \quad \textit{où} \quad \mathbf{s}^e=\mathbf{s}^-+2\mu\Delta\mathbf{e}$$
$$I_1=I_{1^-}+3\mathrm{K}(\Delta\varepsilon_v-\Delta\varepsilon_v^p)=I_1^e-3\mathrm{K}\Delta\varepsilon_v^p \quad \textit{où} \quad I_1^e=I_1^-+3\mathrm{K}\Delta\varepsilon_v$$

When the model is used in modelization THM in total stresses, the tensor of the stresses and the equations translating the elastic behavior are written as follows:

$$\boldsymbol{\sigma}=\mathbf{H}(\varepsilon-\varepsilon^p)+\sigma_p\mathbf{I}$$
$$\boldsymbol{\sigma}^{\mathbf{e}}=\boldsymbol{\sigma}^-+\boldsymbol{H}\Delta\varepsilon+\Delta\sigma_p\mathbf{I}=\sigma'^-+\boldsymbol{H}\Delta\varepsilon+\sigma_pI$$
$$\mathbf{s}=\mathbf{s}^e-2\mu\boldsymbol{\Delta}\mathbf{e}^p \quad \text{où} \quad \mathbf{s}^e=\mathbf{s}^-+2\mu\boldsymbol{\Delta}\mathbf{e}$$
$$I_1=I_1^e-3\mathrm{K}\Delta\varepsilon_v^p \quad \text{où} \quad I_1^e=I_1^-+3\mathrm{K}\Delta\varepsilon_v+3\Delta\sigma_p$$
$$\Delta\sigma_p=-b\left(S\Delta p_{lq}+(1-S)\Delta p_{gz}\right)=b\left(S\Delta p_c-\Delta p_{gz}\right)$$

In addition, the flow rule is written:

$$d\varepsilon_{ij}^p=d\lambda.\frac{\partial G}{\partial\sigma_{ij}}(\sigma,\gamma)=d\lambda\left(\eta(\gamma)\delta_{ij}+\sqrt{\frac{3}{2}}\frac{s_{ij}}{s_{II}}\right)$$

Like $\mathbf{e}^p=\boldsymbol{\varepsilon}^p-\dfrac{tr(\boldsymbol{\varepsilon}^p)}{3}\mathbf{I}$ $\quad\Delta\boldsymbol{\varepsilon}^p=\Delta\mathbf{e}^p+\dfrac{\Delta\varepsilon_v^p}{3}\mathbf{I}$. One from of deduced that $\begin{cases}\Delta\mathbf{e}^p=\dfrac{3}{2}\dfrac{\mathbf{s}}{\sigma_{eq}}\Delta\lambda\\ \Delta\varepsilon_v^p=3\eta(\gamma)\Delta\lambda\end{cases}$, and

consequently, like $\sigma_{eq}^e\mathbf{s}=\sigma_{eq}\mathbf{s}^e$, the following relations are identical for the two aspects of the model:

$$\begin{cases} \mathbf{s}=\mathbf{s}^e-3\mu\dfrac{\mathbf{s}}{\sigma_{eq}}\Delta\lambda=\mathbf{s}^e\left(1-3\mu\dfrac{\Delta\lambda}{\sigma_{eq}^e}\right) \\ I_1=I_1^e-9\mathrm{K}\,\eta(\gamma)\Delta\lambda \\ \sigma_{\mathrm{eq}}=\sigma_{\mathrm{eq}}^e-3\mu\Delta\lambda \end{cases}$$

Lastly $\Delta\gamma^p=\sqrt{\dfrac{2}{3}\Delta\mathbf{e}^p:\Delta\mathbf{e}^p}=\sqrt{\dfrac{2}{3}\Delta\lambda^2\left(\dfrac{3}{2}\dfrac{\mathbf{s}}{\sigma_{eq}}\right):\left(\dfrac{3}{2}\dfrac{\mathbf{s}}{\sigma_{eq}}\right)}=\Delta\lambda$. Thus, while including result obtained in paragraph 3.2.3, one sees that $\Delta\gamma=\Delta\lambda[\eta(\gamma)\pm 1]$ and like $\Delta\gamma,\Delta\lambda\geq 0$, one has necessarily $\boxed{\Delta\gamma=\Delta\lambda[\eta(\gamma)+1]\geq 0.}$

Note: : The computation of $\Delta\lambda$ is the same one in all the formulations.

4.1 Computation of $\Delta\lambda$

One notes $\mathbf{P}$ the transition matrix such as:

$\tilde{\mathbf{P}}.\mathbf{s}^e.\mathbf{P}=\bar{\mathbf{s}}^e$ avec $\bar{\mathbf{s}}^e=diag(s_1^e, s_2^e, s_3^e)$, or

$\tilde{\mathbf{P}}.\boldsymbol{\sigma}^e.\mathbf{P}=\bar{\boldsymbol{\sigma}}^e$ avec $\bar{\boldsymbol{\sigma}}^e=diag(\sigma_1^e,\sigma_2^e,\sigma_3^e)$ et $\sigma_i^e=s_i^e+\dfrac{I_1^e}{3}$

Then:

$$\begin{aligned}\tilde{\mathbf{P}}.\boldsymbol{\sigma}.\mathbf{P}&=\tilde{\mathbf{P}}.\left(\mathbf{s}+\frac{I_1}{3}\mathbf{I}\right).\mathbf{P}=\tilde{\mathbf{P}}.\left(\mathbf{s}^e\left(1-3\mu\frac{\Delta\lambda}{\sigma_{eq}^e}\right)+\frac{1}{3}\left(I_1^e-9\mathrm{K}\,\eta(\gamma)\Delta\lambda\right)\mathbf{I}\right).\mathbf{P}\\ &=\left(1-3\mu\frac{\Delta\lambda}{\sigma_{eq}^e}\right)\bar{\mathbf{s}}^e+\frac{1}{3}\left(I_1^e-9\mathrm{K}\,\eta(\gamma)\Delta\lambda\right)\mathbf{I}\\ &=diag\left(\sigma_1,\sigma_2,\sigma_3\right)\end{aligned}$$

avec $\sigma_i=\left(1-3\mu\dfrac{\Delta\lambda}{\sigma_{eq}^e}\right)s_i^e+\dfrac{1}{3}\left(I_1^e-9\mathrm{K}\,\eta(\gamma)\Delta\lambda\right)=\sigma_i^e-3\Delta\lambda\left(\dfrac{\mu s_i^e}{\sigma_{eq}^e}+K\eta(\gamma)\right)$

Thus, if $\left(1-3\mu\dfrac{\Delta\lambda}{\sigma_{eq}^e}\right)>0$, i.e. $s_{II}>0$, them σ_i are ordered like s_i^e . If $\left(1-3\mu\dfrac{\Delta\lambda}{\sigma_{eq}^e}\right)=0$, i.e. $s_{II}=0$, are σ_i to them all equal to $\dfrac{1}{3}\left(I_1^e-9\mathrm{K}\,\eta\Delta\lambda\right)=\dfrac{1}{3}\left(I_1^e-3\mathrm{K}\,\eta\dfrac{\sigma_{eq}^e}{\mu}\right)$. One can thus write the criterion F according to $\Delta\lambda$ ou $\Delta\gamma$ only, s_i^e being given them and being ordered, while taking:

$$\sigma_3-\sigma_1=\left(1-3\mu\frac{\Delta\lambda}{\sigma_{eq}^e}\right)(s_3^e-s_1^e)$$

$$\sigma_3=s_3^e\left(1-3\mu\frac{\Delta\lambda}{\sigma_{eq}^e}\right)+\frac{1}{3}\left(I_1^e-9\mathrm{K}\,\eta(\gamma)\Delta\lambda\right)$$

One then obtains a nonlinear function $F(\Delta\lambda)$ solved by an algorithm of Newton, given by:

$$F(\Delta\gamma)=0=(s_3^e-s_1^e)\left[1-\frac{3\mu}{\sigma_{eq}^e}h(\Delta\gamma)\Delta\gamma\right]-b(\Delta\gamma)\left[1-\frac{1}{\sigma_3^{b-d}}\left(s_3^e+\frac{I_1^e}{3}-g(\Delta\gamma)\Delta\gamma\right)\right]$$
$$-\left(S(\Delta\gamma)\sigma_c^2(\Delta\gamma)-m(\Delta\gamma)\sigma_c(\Delta\gamma)\left[s_3^e+\frac{I_1^e}{3}-g(\Delta\gamma)\Delta\gamma\right]\right)^{\frac{1}{2}} \quad \textbf{Eq 3.1}$$

where one noted:

¿

$$h(\Delta\gamma)=\frac{1}{\eta+1}=\frac{3+\sin\phi(\gamma^-+\Delta\gamma)}{3(1+\sin\phi(\gamma^-+\Delta\gamma))} \quad \text{et} \quad g(\Delta\gamma)=\frac{3+\sin\phi(\gamma^-+\Delta\gamma)}{3(1+\sin\phi(\gamma^-+\Delta\gamma))}\left(\frac{6K\sin\phi(\gamma^-+\Delta\gamma)}{3+\sin\phi(\gamma^-+\Delta\gamma)}+\frac{3\mu s_3^e}{\sigma_{eq}^e}\right)$$

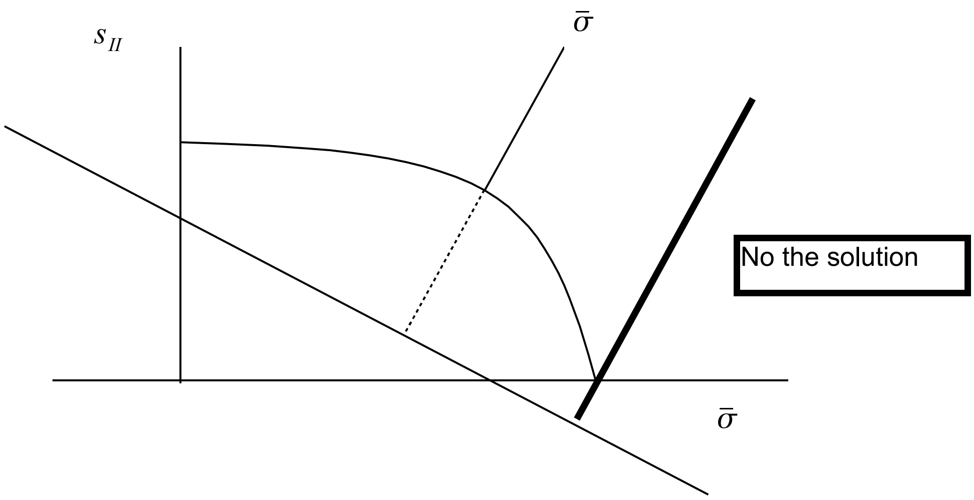
In practice, one will apply the algorithm of Newton to the function

$$\bar{F}(\Delta\gamma)=\left[(\sigma_3-\sigma_1)-b\left(1-\frac{\sigma_3}{\sigma_3^{b-d}}\right)\right]^2-\left(S\sigma_c^2-m\sigma_c\sigma_3\right)$$
$$=\left[(s_3^e-s_1^e)\left[1-\frac{3\mu}{\sigma_{eq}^e}h(\Delta\gamma)\Delta\gamma\right]-b(\Delta\gamma)\left[1-\frac{1}{\sigma_3^{b-d}}\left(s_3^e+\frac{I_1^e}{3}-g(\Delta\gamma)\Delta\gamma\right)\right]\right]^2$$
$$-S(\Delta\gamma)\sigma_c^2(\Delta\gamma)+m(\Delta\gamma)\sigma_c(\Delta\gamma)\left[s_3^e+\frac{I_1^e}{3}-g(\Delta\gamma)\Delta\gamma\right]$$

in order to free oneself from the difficulties related to the sign of the element under the root at the time as of iterations. The derivative from $\bar{F}$ ratio with $\Delta\gamma$ is given in Appendix 2.

4.2 Existence of the solution

the principle of the analytical resolution consists in determining the point $(I_1, \mathbf{s})$ like the projection of the point $(I_1^e, \mathbf{s}^e)$ on the surface of load compared to the potential of yielding:



But, the solution must observe the condition $s_{II}>0$, i.e. $F(\bar{\sigma},\gamma)=(\bar{\sigma}_3-\bar{\sigma}_1)-\sqrt{-\bar{\sigma}_3.m\sigma_c+S\sigma_c^2}-b.\left(1-\frac{\bar{\sigma}_3}{\sigma_3^{b-d}}\right)$: it is thus seen that there exists a zone in which the problem does not admit a solution, which corresponds to $\frac{\partial F}{\partial\bar{\sigma}_i}=\delta_{i3}$-.

5 Computation of derivatives

5.1 Derived from the criterion compared to the stresses

One a: $\frac{\partial F}{\partial\boldsymbol{\sigma}}=\frac{\partial F}{\partial\bar{\boldsymbol{\sigma}}}\frac{\partial\bar{\boldsymbol{\sigma}}}{\partial\boldsymbol{\sigma}}$ where $\bar{\boldsymbol{\sigma}}$ is the tensor of the stresses expressed in the base of the eigenvectors. If the eigenvalues are ordered in $\bar{\boldsymbol{\sigma}}$, the criterion F will be written:

$$P(\sigma)=\left(X_1(\sigma)\quad X_2(\sigma)\quad X_3(\sigma)\right)\quad F(\bar{\boldsymbol{\sigma}},\gamma)=(\bar{\boldsymbol{\sigma}}_3-\bar{\boldsymbol{\sigma}}_1)-\sqrt{-\bar{\boldsymbol{\sigma}}_3.m\boldsymbol{\sigma}_c+S\boldsymbol{\sigma}_c^2}-b.\left(1-\frac{\bar{\boldsymbol{\sigma}}_3}{\boldsymbol{\sigma}_3^{\bar{b}-d}}\right)$$

and $X_2(\sigma)\neq X_3(\sigma)$ $\frac{\partial F}{\partial\bar{\boldsymbol{\sigma}}_i}=\delta_{i3}-\delta_{i1}+\frac{1}{2}\delta_{i3}\boldsymbol{\sigma}_c m[-\bar{\boldsymbol{\sigma}}_3.m\boldsymbol{\sigma}_c+S\boldsymbol{\sigma}_c^2]^{\frac{1}{2}}+b\frac{\delta_{i3}}{\boldsymbol{\sigma}_3^{b-d}}$.

5.2 Derived from the tensor of the stresses compared to the principal stresses

One can show (see Appendix 1) that:

$$\left|\begin{array}{l}\text{Si } \tilde{\boldsymbol{P}}(\sigma).\sigma.P(\sigma)=\bar{\sigma}\\ \quad \text{où } P(\sigma)=\text{matrice de passage (matrice des vecteurs propres)}\\ \quad \text{et } \bar{\boldsymbol{\sigma}}=\text{ matrice diagonale des valeurs propres de } \sigma\\ \text{alors } \frac{\partial\bar{\sigma}_k}{\partial\sigma_{ij}}=P_{ik}P_{jk}\text{(sans sommation sur les indices)}\end{array}\right.$$

5.2.1 Typical case of multiple eigenvalues

In the typical case where several of the principal stresses are equal, for example $\sigma_2=\sigma_3$, the result preceding one will apply to the fields $\sigma_2<\sigma_3$ and $\sigma_2>\sigma_3$. One will thus have, in the first field, $\mathbf{P}(\boldsymbol{\sigma})=\left(\mathbf{X}_1(\boldsymbol{\sigma})\quad\mathbf{X}_2(\boldsymbol{\sigma})\quad\mathbf{X}_3(\boldsymbol{\sigma})\right)$ where $\mathbf{X}_2\neq\mathbf{X}_3$ and, in the second field $\mathbf{P}(\boldsymbol{\sigma})=\left(\bar{\mathbf{X}}_1(\boldsymbol{\sigma})\quad\bar{\mathbf{X}}_2(\boldsymbol{\sigma})\quad\bar{\mathbf{X}}_3(\boldsymbol{\sigma})\right)$. Thus, when $\sigma_2-\sigma_3\rightarrow0^-$ (resp. $\sigma_2-\sigma_3\rightarrow0^+$), the transition matrix will tend towards $\mathbf{P}^-=\left(\mathbf{X}_1\quad\mathbf{X}_2\quad\mathbf{X}_3\right)$ (resp. towards $\mathbf{P}^+=\left(\mathbf{X}_1\quad\mathbf{X}_3\quad\mathbf{X}_2\right)$) with $\mathbf{X}_2\neq\mathbf{X}_3$, vectors $(\mathbf{X}_2,\mathbf{X}_3)$ defining the clean subspace associated with $\sigma_2=\sigma_3$. It is thus seen that the tensor $\frac{\partial\bar{\boldsymbol{\sigma}}}{\partial\boldsymbol{\sigma}}$ is not defined in a single way in this point.

Moreover, the vector $\frac{\partial\sigma_3}{\partial\sigma_{ij}}=\frac{\partial\sigma_2}{\partial\sigma_{ij}}$ is defined only from one of the two vectors $\mathbf{X}_2$ or $\mathbf{X}_3$ (it is equal to $\mathbf{P}_{i3}\mathbf{P}_{j3}$ or $\mathbf{P}_{i2}\mathbf{P}_{j2}$), and thus corresponds makes some only with only one of two directional derivatives. This remark applies in the same way to $\frac{\partial s_3^e}{\partial \sigma_{ij}}$ for the computation of $\frac{\partial \sigma}{\partial \varepsilon}$ $\frac{\partial \lambda}{\partial \varepsilon_{ij}}$ in the coherent tangent matrix (see paragraph 6).

5.3 Derived from the criterion compared to the variable of hardening

$$\frac{\partial F}{\partial \gamma}=-\frac{1}{2}\left(-\frac{\partial (m\sigma_c)}{\partial \gamma}\sigma_3+\frac{\partial (S\sigma_c^2)}{\partial \gamma}\right)\left[-\sigma_3.m\sigma_c+S\sigma_c^2\right]^{-\frac{1}{2}}-\frac{\partial b}{\partial \gamma}.\left(1-\frac{\sigma_3}{\sigma_3^{b-d}}\right)$$

$$=\begin{cases}-\frac{1}{2}\left(-p_{m\sigma}\sigma_3+p_{S\sigma^2}\right)\left[-\sigma_3.m\sigma_c+S\sigma_c^2\right]^{-\frac{1}{2}} & \text{si } \gamma<\gamma^{rup} \\ -(2a\gamma+d).\left(1-\frac{\sigma_3}{\sigma_3^{b-d}}\right) & \text{si } \gamma^{rup}<\gamma<=\gamma^{res} \\ 0 & \text{si } \gamma>=\gamma^{res}\end{cases}$$

$\frac{\partial \sigma'}{\partial \varepsilon}$

5.4 Derived from the parameters compared to the variable of hardening

[1] $$\frac{\partial (m\sigma_c)}{\partial \gamma}(\gamma)=\begin{cases}\frac{(m\sigma_c)^{rup}-(m\sigma_c)^{end}}{\gamma^{rup}}=p_{m\sigma} & si \quad \gamma<\gamma^{rup} \\ 0 & si \quad \gamma>\gamma^{rup}\end{cases}$$

[2] $$\frac{\partial (S\sigma_c^2)}{\partial \gamma}(\gamma)=\begin{cases}\frac{(S\sigma_c^2)^{rup}-(S\sigma_c^2)^{end}}{\gamma^{rup}}=p_{S\sigma^2} & si \quad \gamma<\gamma^{rup} \\ 0 & si \quad \gamma>\gamma^{rup}\end{cases}$$

[3] $$\frac{\partial \varphi}{\partial \gamma}(\gamma)=\begin{cases}\frac{\phi^{rup}-\phi^{end}}{\gamma^{rup}} & \text{si } \gamma\leq\gamma^{rup} \\ \frac{\phi^{res}-\phi^{rup}}{\gamma^{res}-\gamma^{rup}} & \text{si } \gamma^{rup}\leq\gamma\leq\gamma^{res} \\ 0 & \text{sinon}\end{cases}$$ $\frac{\partial \sigma}{\partial p_g}$

[4] $$\frac{\partial b}{\partial \gamma}(\gamma)=\begin{cases}0 & \text{si } \gamma<\gamma^{rup} \\ 2a\gamma+d & \text{si } \gamma^{rup}<\gamma\leq\gamma^{res} \\ 0 & \text{si } \gamma\geq\gamma^{res}\end{cases}$$ $\frac{\partial \sigma}{\partial p_c}$

6 Computation of the coherent tangent operator

In pure mechanics or modelization THM with the model Hoek-Brown used according to its first aspect, the tensor of the stresses represents the tensor of the effective stresses which depends only on the tensor of the strains. The mechanical constitutive law provides thus only derivative $\frac{\partial \sigma}{\partial \varepsilon}$ in pure mechanics noted $\frac{\partial \sigma'}{\partial \varepsilon}$ in modelization thermo hydro mechanics.

The computation of the quantity $\frac{\partial \sigma}{\partial \varepsilon}$ $\frac{\partial \sigma}{\partial p_c}=\frac{\partial \sigma}{\partial \sigma_p}\frac{\partial \sigma_p}{\partial p_c}$ **of the coherent tangent matrix is the same one for the two aspects of the model.** The detail of this derivative is presented under paragraph 6.1.

In modelization THM with the model Hoek-Brown used according to its second aspect, the tensor of the stresses representing the tensor of the total stresses, the variation of the tensor of the stresses depends on the variation of the tensor of the strains $\frac{\partial \sigma}{\partial \varepsilon}$, of the variation of the gas pressure $\frac{\partial \boldsymbol{\sigma}}{\partial p_g}$ as well as variation of the capillary pressure $\frac{\partial \boldsymbol{\sigma}}{\partial p_c}$.
The gas pressure and the capillary pressure always do not intervene in computation but according to the case of simulation. In the code, one will thus calculate rather $\frac{\partial \boldsymbol{\sigma}}{\partial p_p}$ then $\frac{\partial \sigma}{\partial p_g}=\frac{\partial \sigma}{\partial \sigma_p}\frac{\partial \sigma_p}{\partial p_g}$ and. $\frac{\partial \sigma_p}{\partial p_c}$ and $\frac{\partial \sigma_p}{\partial p_g}$ depend on hydraulic simulation, and are independent of the mechanical model (3).

6.1 Computation of $\frac{\partial \sigma}{\partial \varepsilon}$

One seeks to calculate the coherent matrix: $\frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{\varepsilon}}=\frac{\partial \mathbf{s}}{\partial \boldsymbol{\varepsilon}}+\frac{1}{3}\frac{\partial I_1}{\partial \boldsymbol{\varepsilon}}\mathbf{I}$ $\frac{\partial s_i^e}{\partial \varepsilon_{pq}}$. However:

$$\frac{\partial \mathbf{s}}{\partial \boldsymbol{\varepsilon}}=\frac{\partial \mathbf{s}^e}{\partial \boldsymbol{\varepsilon}}\left(1-\frac{3\mu}{\sigma_{eq}^e}\Delta\lambda\right)+\frac{3\mu\Delta\lambda}{(\sigma_{eq}^e)^2}\mathbf{s}^e\frac{\partial \sigma_{eq}^e}{\partial \boldsymbol{\varepsilon}}-\frac{3\mu}{\sigma_{eq}^e}\mathbf{s}^e\frac{\partial \Delta\lambda}{\partial \boldsymbol{\varepsilon}}$$

$$\frac{\partial I_1}{\partial \boldsymbol{\varepsilon}}=\frac{\partial I_1^e}{\partial \boldsymbol{\varepsilon}}-9\mathrm{K}\left(\frac{\partial \eta}{\partial \Delta\lambda}\frac{\partial \Delta\lambda}{\partial \boldsymbol{\varepsilon}}\Delta\lambda+\eta\frac{\partial \Delta\lambda}{\partial \boldsymbol{\varepsilon}}\right)$$

$$\tilde{P}.s^e.P=\bar{s}^e$$

$$\frac{\partial s_{ij}^e}{\partial \varepsilon_{pq}}=2\mu\left(\delta_{ip}\delta_{jq}-\frac{1}{3}\delta_{ij}\delta_{pq}\right)$$

$$\frac{\partial I_1^e}{\partial \varepsilon_{pq}}=3\mathrm{K}\,\delta_{pq}$$

with:

$$\frac{\partial \sigma_{eq}^e}{\partial \varepsilon_{pq}}=\sqrt{\frac{3}{2}}\frac{\partial s_{II}^e}{\partial \varepsilon_{pq}}=\frac{3}{2\sigma_{eq}^e}\sum_{i,j}s_{ij}^e\frac{\partial s_{ij}^e}{\partial \varepsilon_{pq}}=\frac{3}{2\sigma_{eq}^e}2\mu\sum_{i,j}s_{ij}^e\left(\delta_{ip}\delta_{jq}-\frac{1}{3}\delta_{ij}\delta_{pq}\right)=\frac{3\mu}{\sigma_{eq}^e}s_{pq}^e$$

$$\frac{\partial \eta}{\partial \Delta\lambda}=\frac{\partial \eta}{\partial \Delta\gamma}\frac{\partial \Delta\gamma}{\partial \Delta\lambda}=(\eta+1)\frac{\partial \eta}{\partial \Delta\gamma}$$

$$\bar{s}^e=diag(s_1^e,s_2^e,s_3^e)$$

from where:

$$\frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{\varepsilon}}=\frac{\partial \mathbf{s}^e}{\partial \boldsymbol{\varepsilon}}\left(1-\frac{3\mu}{\sigma_{eq}^e}\Delta\lambda\right)+\frac{9\mu^2\Delta\lambda}{(\sigma_{eq}^e)^3}\mathbf{s}^e.\mathbf{s}^e-\frac{3\mu}{\sigma_{eq}^e}\mathbf{s}^e.\frac{\partial \Delta\lambda}{\partial \boldsymbol{\varepsilon}}+\frac{1}{3}\frac{\partial I_1^e}{\partial \boldsymbol{\varepsilon}}-3\mathrm{K}\left(\frac{\partial \eta}{\partial \Delta\lambda}\frac{\partial \Delta\lambda}{\partial \boldsymbol{\varepsilon}}\Delta\lambda+\eta\frac{\partial \Delta\lambda}{\partial \boldsymbol{\varepsilon}}\right)$$

P

To compute: $\frac{\partial \Delta\lambda}{\partial \varepsilon_{pq}}$, the equation is used $\dot{F}(\Delta\lambda)=0$ $\frac{\partial s^e}{\partial \varepsilon_{pq}}=P.\frac{\partial \bar{s}^e}{\partial \varepsilon_{pq}}.\tilde{P}$. One obtains:

$$
\frac{\partial\Delta\lambda}{\partial\varepsilon_{pq}}
\left|
\begin{array}{l}
-(s_3^e-s_1^e)\frac{3\mu}{\sigma_{eq}^e}-\frac{\partial b}{\Delta\lambda}\left[1+\frac{1}{\sigma_3^{b-d}}\left[s_3^e+\frac{I_1^e}{3}-\left(3K\eta+s_3^e\frac{3\mu}{\sigma_{eq}^e}\right)\Delta\lambda\right]\right]\\
+\frac{b}{\sigma_3^{b-d}}\left(\frac{\partial\eta}{\partial\Delta\gamma}3K(\eta+1)\Delta\lambda+\left(3K\eta+s_3^e\frac{3\mu}{\sigma_{eq}^e}\right)\right)\\
-\frac{1}{2}\left(\begin{array}{l}\frac{\partial(S\sigma_c^2)}{\partial\Delta\lambda}-\frac{\partial(m\sigma_c)}{\partial\Delta\lambda}\left(s_3^e+\frac{I_1^e}{3}-\left(3K\eta+s_3^e\frac{3\mu}{\sigma_{eq}^e}\right)\Delta\lambda\right)\\
+\sigma_c m\left(\frac{\partial\eta}{\partial\Delta\gamma}3K(\eta+1)\Delta\lambda+\left(3K\eta+s_3^e\frac{3\mu}{\sigma_{eq}^e}\right)\right)\end{array}\right)\times\\
\left(S\sigma_c^2-m\sigma_c\left(s_3^e+\frac{I_1^e}{3}-\left(3K\eta=s_3^e\frac{3\mu}{\sigma_{eq}^e}\Delta\lambda\right)\right)\right)^{-\frac{1}{2}}\\
=-\left(\frac{\partial s_3^e}{\partial\varepsilon_{pq}}-\frac{\partial s_1^e}{s_3^e}\right)\left[1-\frac{3\mu}{\sigma_{eq}^e}\Delta\lambda\right]-(s_3^e-s_1^e)\frac{3\mu}{(\sigma_{eq}^e)^2}\frac{\partial\sigma_{eq}^e}{\partial\varepsilon_{pq}}\Delta\lambda\\
+\left(\frac{\partial s_3^e}{\partial\varepsilon_{pq}}+\frac{1}{3}\frac{\partial I_1^e}{\partial\varepsilon_{pq}}-\left(-\frac{1}{\sigma_{eq}^e}\frac{\partial\sigma_{eq}^e}{\partial\varepsilon_{pq}}s_3^e+\frac{\partial s_3^e}{\partial\varepsilon_{pq}}\right)\frac{3\mu}{\sigma_{eq}^e}\right)\times\\
\left(\frac{b}{\sigma_3^{b-d}}-\frac{1}{2}\sigma_c m\left(S\sigma_c^2-m\sigma_c\left(s_3^e+\frac{I_1^e}{3}-\left(3K\eta+s_3^e\frac{3\mu}{\sigma_{eq}^e}\right)\Delta\lambda\right)\right)^{-\frac{1}{2}}\right)
\end{array}
\right|
$$

It thus remains us to calculate $\frac{\partial s_i^e}{\partial\varepsilon_{pq}}$. By taking again the notations of the paragraph 4.1, one a: $\tilde{\mathbf{P}}.\mathbf{s}^e.\mathbf{P}=\bar{\mathbf{s}}^e$ where $\bar{\mathbf{s}}^e=diag(s_1^e,s_2^e,s_3^e)$ and $\mathbf{P}$ is the matrix of the associated eigenvectors. Consequently $\frac{\partial\mathbf{s}^e}{\partial\varepsilon_{pq}}=\tilde{\mathbf{P}}.\frac{\partial\bar{s}^e}{\partial\varepsilon_{pq}}.\mathbf{P}+\frac{\partial\tilde{\mathbf{P}}}{\partial\varepsilon_{pq}}.\bar{s}^e.\mathbf{P}+\tilde{\mathbf{P}}.\bar{s}^e.\frac{\partial\mathbf{P}}{\partial\varepsilon_{pq}}$.

By taking again the same reasoning that in Appendix 1, one can show that $\frac{\partial\mathbf{s}^e}{\partial\varepsilon_{pq}}=\mathbf{P}.\frac{\partial\bar{\mathbf{s}}^e}{\partial\varepsilon_{pq}}.\tilde{\mathbf{P}}$ and finally: . $\tilde{\mathbf{P}}.\frac{\partial\mathbf{s}^e}{\partial\varepsilon_{pq}}.\mathbf{P}=\frac{\partial\bar{\mathbf{s}}^e}{\partial\varepsilon_{pq}}$

6.2 Typical case of multiple eigenvalues for $\mathbf{s}^e$

If the matrix $\mathbf{s}^e$ has multiple eigenvalues, the remarks passed in paragraph 5.2.1 apply. In the case $\mathbf{s}_3^e=\mathbf{s}_2^e$ for example, the vector $\frac{\partial s_3^e}{\partial\varepsilon_{pq}}$ thus takes into account only one of two directional derivatives of $\mathbf{s}_3^e = \mathbf{s}_2^e$. From where the idea to write $s_2^e = s_3^e = \frac{1}{2}(s_2^e + s_3^e)$ and thus $\frac{\partial s_2^e}{\partial \varepsilon_{ij}} = \frac{\partial s_3^e}{\partial \varepsilon_{ij}} = \frac{1}{2}\left(\frac{\partial s_2^e}{\partial \varepsilon_{ij}} + \frac{\partial s_3^e}{\partial \varepsilon_{ij}}\right)$.

6.3 Computation of $\frac{\partial \sigma}{\partial \sigma_p}$

1) Elasticity:

One a: $\boldsymbol{\sigma} = \boldsymbol{\sigma}^e = \mathbf{H}\,\boldsymbol{\varepsilon} + \sigma_p \mathbf{I}$ and consequently $\frac{\partial \boldsymbol{\sigma}}{\partial \sigma_p} = \mathbf{I}$.

2) Plasticity:

One has: $\frac{\partial \boldsymbol{\sigma}}{\partial \sigma_p} = \frac{\partial \mathbf{s}}{\partial \sigma_p} + \frac{1}{3}\frac{\partial I_1}{\partial \sigma_p}\mathbf{I}$ with:

$$\frac{\partial \mathbf{s}}{\partial \sigma_p} = \frac{\partial \mathbf{s}^e}{\partial \sigma_p}\left(1 - \frac{3\mu}{\sigma_{eq}^e}\Delta\lambda\right) + \frac{3\mu\Delta\lambda}{(\sigma_{eq}^e)^2}\mathbf{s}^e\frac{\partial \sigma_{eq}^e}{\partial \sigma_p} - \frac{3\mu}{\sigma_{eq}^e}\mathbf{s}^e\frac{\partial \Delta\lambda}{\partial \sigma_p}$$

$$\frac{\partial I_1}{\partial \sigma_p} = \frac{\partial I_1^e}{\partial \sigma_p} - 9\mathrm{K}\left(\frac{\partial \eta}{\partial \Delta\lambda}\frac{\partial \Delta\lambda}{\partial \sigma_p}\Delta\lambda + \eta\frac{\partial \Delta\lambda}{\partial \sigma_p}\right)$$

However:

$$\frac{\partial \mathbf{s}^e}{\partial \sigma_p} = 2\mu\frac{\partial \Delta\mathbf{e}}{\partial \sigma_p} = 0$$

$$\frac{\partial I_1^e}{\partial \sigma_p} = 3\mathrm{K}\frac{\partial \Delta\varepsilon_v}{\partial \sigma_p} + 3 = 3$$

$$\frac{\partial \sigma_{eq}^e}{\partial \varepsilon_{pq}} = \sqrt{\frac{3}{2}}\frac{\partial s_{II}^e}{\partial \varepsilon_{pq}} = 0$$

$$\frac{\partial \eta}{\partial \Delta\lambda} = \frac{\partial \eta}{\partial \Delta\gamma}\frac{\partial \Delta\gamma}{\partial \Delta\lambda} = (\eta + 1)\frac{\partial \eta}{\partial \Delta\gamma}$$

from where: $\frac{\partial \boldsymbol{\sigma}}{\partial \sigma_p} = -\frac{3\mu}{\sigma_{eq}^e}\mathbf{s}^e.\frac{\partial \Delta\lambda}{\partial \sigma_p} + \mathbf{I} - 3\mathrm{K}\left(\frac{\partial \eta}{\partial \Delta\lambda}\frac{\partial \Delta\lambda}{\partial \sigma_p}\Delta\lambda + \eta\frac{\partial \Delta\lambda}{\partial \sigma_p}\right)\mathbf{I}$

To compute: $\frac{\partial \Delta\lambda}{\partial \sigma_p}$, the equation is used $\dot{F}(\Delta\lambda) = 0$. One obtains:

$$\frac{\partial \Delta\lambda}{\partial \sigma_p}\left[\begin{array}{c}\frac{3\mu}{\sigma_{eq}^e}(s_3^e - s_1^e) + \frac{\partial B}{\partial \Delta\lambda}\left(1 - \frac{\sigma_3}{\sigma_3^{b-d}}\right) + 3\frac{B}{\sigma_3^{b-d}}\left(\frac{\mu s_3^e}{\sigma_{eq}^e} + K\eta + K\frac{\partial \eta}{\partial \Delta\lambda}\Delta\lambda\right)\\ + \left(\frac{\partial S\sigma_c^2}{\partial \Delta\lambda} - \frac{\partial m\sigma_c}{\partial \Delta\lambda}\sigma_3 + 3\mathrm{m}\sigma_c\left[\frac{\mu s_3^e}{\sigma_{eq}^e} + K\eta + K\frac{\partial \eta}{\partial \Delta\lambda}\Delta\lambda\right]\right) \times \left(S\sigma_c^2 - \sigma_3.m\sigma_c\right)^{-1/2}\end{array}\right]$$

$$= \frac{B}{\sigma_3^{b-d}} + \frac{m\sigma_c}{2\sqrt{S\sigma_c^2 - \sigma_3.m\sigma_c}}$$

3)In the tangent operator it is the derivative $\frac{\partial \boldsymbol{\sigma}'}{\partial \sigma_p}=\frac{\partial \boldsymbol{\sigma}}{\partial \sigma_p}-\mathbf{I}$ which must be returned.

7 Computation of the tangent operator of velocity

the tangent operator of velocity is given as an indication. In the programming, one calculates the operator corresponding to `RIGI_MECA` by the same formulas that those providing `FULL_MECA` in which one poses: $\Delta\lambda=0$, H and $\gamma^{+}=\gamma^{-}$.

<u>If the stress tensor represents the effective stress</u> :

The condition $\dot{F}=0$ is written: $\dot{F}=\frac{\partial F}{\partial \boldsymbol{\sigma}}\dot{\boldsymbol{\sigma}}+\frac{\partial F}{\partial \gamma}\dot{\gamma}=0$. This thus gives us:

$$\dot{\gamma}=-\frac{\sum_{ij}\frac{\partial F}{\partial \sigma_{ij}}\dot{\sigma}_{ij}}{\frac{\partial F}{\partial \gamma}}$$

However, one has $\dot{\boldsymbol{\sigma}}=\mathbf{H}(\dot{\boldsymbol{\varepsilon}}-\dot{\boldsymbol{\varepsilon}}^p)$ where $\mathbf{H}$ is the matrix of Hooke. Moreover $\dot{\boldsymbol{\varepsilon}}^p=\dot{\lambda}\frac{\partial G}{\partial \boldsymbol{\sigma}}=\frac{\dot{\gamma}}{(\eta(\gamma)+1)}\frac{\partial G}{\partial \boldsymbol{\sigma}}$, from where

$$\dot{\gamma}=-\frac{\sum_{ijkl}\frac{\partial F}{\partial \sigma_{ij}}H_{ijkl}\dot{\varepsilon}_{kl}}{\frac{\partial F}{\partial \gamma}-\frac{1}{\eta(\gamma)+1}\sum_{ijkl}\frac{\partial F}{\partial \sigma_{ij}}H_{ijkl}\frac{\partial G}{\partial \sigma_{kl}}} \qquad \text{avec} \quad \frac{\partial G}{\partial \sigma_{ij}}=\eta(\gamma)\delta_{ij}+\sqrt{\frac{3}{2}}\frac{s_{ij}}{s_{II}},$$

and finally

$$\dot{\sigma}_{ab}=H_{abcd}\left(\dot{\varepsilon}_{cd}-\dot{\lambda}\frac{\partial G}{\partial \sigma_{cd}}\right)=\sum_{cd}D_{abcd}\dot{\varepsilon}_{cd}$$
$$\text{où} \quad D_{abcd}=H_{abcd}+\frac{\left(\sum_{kl}H_{abkl}\frac{\partial G}{\partial \sigma_{kl}}\right)\left(\sum_{ij}\frac{\partial F}{\partial \sigma_{ij}}H_{ijcd}\right)}{(\eta(\gamma)+1)\frac{\partial F}{\partial \gamma}-\sum_{pqmn}\frac{\partial F}{\partial \sigma_{pq}}H_{pqmn}\frac{\partial G}{\partial \sigma_{mn}}}.$$

<u>If the stress tensor represents the total stress</u> :

One has $\dot{\boldsymbol{\sigma}}=H(\dot{\boldsymbol{\varepsilon}}-\dot{\boldsymbol{\varepsilon}}^p)+\dot{\sigma}_p\mathbf{I}$ where $\mathbf{H}$ is the matrix of Hooke. In subscripted notation, one can write:

$$\dot{\sigma}_{ij}=H_{ijkl}\left(\dot{\varepsilon}_{kl}-\dot{\varepsilon}^p_{kl}\right)+\dot{\sigma}_p\delta_{ij}$$

Moreover $\dot{\boldsymbol{\varepsilon}}^p=\dot{\lambda}\frac{\partial G}{\partial \boldsymbol{\sigma}}=\frac{\dot{\gamma}}{(\eta(\gamma)+1)}\frac{\partial G}{\partial \boldsymbol{\sigma}}$, the condition $\dot{F}=0$ is written: $\dot{F}=\frac{\partial F}{\partial \sigma}\dot{\sigma}+\frac{\partial F}{\partial \gamma}\dot{\gamma}=0$. This thus gives us:

$$\dot{\gamma}=-\frac{\frac{\partial F}{\partial \sigma_{ij}}\dot{\sigma}_{ij}}{\frac{\partial F}{\partial \gamma}}$$

while replacing $\dot{\sigma}_{ij}$ and $\dot{\varepsilon}_{kl}^{p}$ by their statements one can deduce:

$$\dot{\gamma}\frac{\partial F}{\partial \gamma}=-\frac{\partial F}{\partial \sigma_{ij}}H_{ijkl}\left(\dot{\varepsilon}_{kl}-\frac{\dot{\gamma}}{\left(\eta\left(\gamma\right)+1\right)}\frac{\partial G}{\partial \sigma_{kl}}\right)-\frac{\partial F}{\partial \sigma_{ij}}\dot{\sigma}_{p}\delta_{ij}$$

and then the statement of $\dot{\gamma}$:

$$\dot{\gamma}=-\frac{\frac{\partial F}{\partial \sigma_{ij}}H_{ijkl}\dot{\varepsilon}_{kl}}{\frac{\partial F}{\partial \sigma_{ij}}-\frac{1}{\left(\eta\left(\gamma\right)+1\right)}\frac{\partial F}{\partial \sigma_{ij}}H_{ijkl}\frac{\partial G}{\partial \sigma_{kl}}}-\frac{\frac{\partial F}{\partial \sigma_{ij}}\dot{\sigma}_{p}\delta_{ij}}{\frac{\partial F}{\partial \sigma_{ij}}-\frac{1}{\left(\eta\left(\gamma\right)+1\right)}\frac{\partial F}{\partial \sigma_{ij}}H_{ijkl}\frac{\partial G}{\partial \sigma_{kl}}}$$

$$\text{avec}\quad \frac{\partial G}{\partial \sigma_{ij}}=\eta(\gamma)\delta_{ij}+\sqrt{\frac{3}{2}}\frac{s_{ij}}{s_{II}},$$

and finally

$$\dot{\sigma}_{ab}=H_{abcd}\left(\dot{\varepsilon}_{cd}-\dot{\lambda}\frac{\partial G}{\partial \sigma_{cd}}\right)+\dot{\sigma}_{p}\delta_{ab}=D_{abcd}\dot{\varepsilon}_{cd}+E_{ab}\dot{\sigma}_{p}$$

$$\dot{\sigma}_{ab}=H_{abcd}\left(\dot{\varepsilon}_{cd}+\frac{\frac{\partial F}{\partial \sigma_{ij}}H_{ijkl}\dot{\varepsilon}_{kl}\frac{\partial G}{\partial \sigma_{cd}}}{\left(\eta\left(\gamma\right)+1\right)\frac{\partial F}{\partial \gamma}-\frac{\partial F}{\partial \sigma_{ij}}H_{ijkl}\frac{\partial G}{\partial \sigma_{kl}}}+\frac{\frac{\partial F}{\partial \sigma_{ij}}\delta_{ij}\frac{\partial G}{\partial \sigma_{cd}}\dot{\sigma}_{p}}{\left(\eta\left(\gamma\right)+1\right)\frac{\partial F}{\partial \gamma}-\frac{\partial F}{\partial \sigma_{ij}}H_{ijkl}\frac{\partial G}{\partial \sigma_{kl}}}\right)+\dot{\sigma}_{p}\delta_{ab}$$

où

$$D_{abcd}=H_{abcd}+\frac{H_{abkl}\frac{\partial G}{\partial \sigma_{kl}}\frac{\partial F}{\partial \sigma_{ij}}H_{ijcd}}{(\eta(\gamma)+1)\frac{\partial F}{\partial \gamma}-\frac{\partial F}{\partial \sigma_{ij}}H_{ijkl}\frac{\partial G}{\partial \sigma_{kl}}}$$

et

$$E_{ab}=\frac{H_{abcd}\frac{\partial F}{\partial \sigma_{ij}}\delta_{ij}\frac{\partial G}{\partial \sigma_{cd}}}{(\eta(\gamma)+1)\frac{\partial F}{\partial \gamma}-\frac{\partial F}{\partial \sigma_{ij}}H_{ijkl}\frac{\partial G}{\partial \sigma_{kl}}}+\delta_{ab}$$

8 Algorithm

8.1 Local variables

the constitutive law of Hoek-Brown modified is governed by the three following local variables:

1) *the hardening parameter* γ corresponding to the major unrecoverable deformation.
2) *the cumulated plastic voluminal strain* ε_v^p whose law of evolution is given par. $d\varepsilon_v^p = 3\eta\, d\lambda = \dfrac{3\eta}{\eta+1} d\gamma$
3) *the state of plasticization* ; it is worth 0 if the Gauss point is in elastic load or discharge, and 1 if the Gauss point is in plastic load.

8.2 Algorithm

One retains an implicit formulation compared to the criterion and the flow direction. One places oneself in a material point and one supposes known with t^- :

- The tensor of increase in strains $\Delta\boldsymbol{\varepsilon}$ from where one deduces $\Delta\mathbf{e}$, $\Delta\varepsilon_v$
- the stresses at the beginning of time step $\boldsymbol{\sigma}^-$ which one deduces $\mathbf{s}^-, I_1^-$
- the value from the local variables γ^- and ε_v^{p-} at the beginning of time step which give us $(S\sigma_c^2)^-, (m\sigma_c)^-, b^-, \phi^-$

the goal of the algorithm is then to calculate:

- Stresses in the end of time step σ
- the local variables in the end of time step
- the tangent behavior at the end of time step: $\dfrac{\partial\boldsymbol{\sigma}}{\partial\boldsymbol{\varepsilon}}$ if the model is in effective stresses
- the tangent behavior at the end of time step: $\dfrac{\partial\boldsymbol{\sigma}}{\partial\boldsymbol{\varepsilon}}$ and $\dfrac{\partial\boldsymbol{\sigma}}{\partial\sigma_p}$ if the model is in total stresses.

Algorithm:

Computation of the elastic solution :

If the model is in effective stresses:

$$\begin{cases} \mathbf{s}^e = \mathbf{s}^- + 2\mu\Delta\mathbf{e} \\ I_1^e = I_1^- + 3\mathrm{K}\Delta\varepsilon_v \end{cases}$$

If the model is in total stresses:

$$\begin{cases} \mathbf{s}^e = \mathbf{s}^- + 2\mu\Delta\mathbf{e} \\ I_1^e = I_1^- + 3\mathrm{K}\Delta\varepsilon_v + 3\Delta\sigma_p \end{cases}$$

Computation of the elastic criterion $F(\boldsymbol{\sigma}^e, \gamma^-)$. If $S\sigma_c^2 - \sigma_3^e . m\sigma_c < 0$, the function F is not defined in the point $(\boldsymbol{\sigma}^e, \gamma^-)$. It is considered whereas one is in the plastic case.

Resolution : computation of $\boldsymbol{\sigma}, \gamma$

If $F(\boldsymbol{\sigma}^e, \gamma^-) \le 0$, then $\Delta\boldsymbol{\varepsilon} = \Delta\boldsymbol{\varepsilon}^e, \gamma^p = \gamma^{p^-}, \Delta\boldsymbol{\sigma} = \mathbf{H}\Delta\boldsymbol{\varepsilon}$.

If not, one seeks $\boldsymbol{\sigma}, \gamma$ such as $F(\boldsymbol{\sigma}, \gamma) \le 0$, which amounts seeking $\Delta\gamma$ such as $F(\Delta\gamma) = \bar{F}(\Delta\gamma) = 0$. This problem is solved by means of a method of Newton on $\bar{F}$.

<u>Algorithm of Newton:</u>

Initialization: $\Delta\gamma^0 = 0$

After each iteration:

- if $\Delta\gamma^{n+1} \le 0$, there were not convergence: one subdivides time step

- if $\Delta\gamma^{n+1} \le \frac{\sigma_{eq}^{e}}{3\mu[\eta(\Delta\gamma^{n+1})+1]}$, there is no solution (see paragraph 4.2): one subdivides time step

the Put up to date one of the variables : stresses, local variables

Computation of the coherent tangent matrix $\frac{\partial\boldsymbol{\sigma}}{\partial\varepsilon}$ *if the model is in effective stresses and* $\frac{\partial\boldsymbol{\sigma}}{\partial\varepsilon}$ $\frac{\partial\boldsymbol{\sigma}}{\partial\sigma_p}$ *if the model is in total stresses for the option* `RIGI_MECA_TANG` or `FULL_MECA` .

9 Bibliography

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•François Laigle Model conceptual for the development of constitutive laws adapted to the design of the underground works. Thesis Central School of Lyon

10 Functionalities and checking

the constitutive law can be defined by the key word HOEK_BROWN (command STAT_NON_LINE, key word factor COMP_INCR). It is associated with material HOEK_BROWN (command DEFI_MATERIAU).

Model HOEK_BROWN is checked by the cases following tests:

SSNA116	[V6.01.116]	triaxial Compression test with the model of Hoek-Brown modified in axisymmetric
SSNV184	[V6.04.184]	triaxial Compression test with the model of Hoek-Brown modified
WTNV128	[V7.31.128]	triaxial Compression test not drained with the model of Hoek-Brown modified in effective stresses
WTNV129	[V7.31.129]	triaxial Compression test not drained with the model of Hoek-Brown modified in total stresses

11 Description of the versions of the document

Version Aster	**Author (S) Organization (S)**	**Description of the modifications**
8.5	C.Chavant, J.El-Gharib EDF-R&D/AMA V.Gervais, CS	initial Text

Annexe 1Derivative of the principal stresses

Is $\boldsymbol{\sigma}$ a symmetric tensor and $\boldsymbol{\sigma}_d$ this tensor in the base which it diagonalized. Let us indicate by $\boldsymbol{P}(\boldsymbol{\sigma})$ the transition matrix which diagonalized the tensor σ : $\boldsymbol{\sigma}:\boldsymbol{\sigma}=\mathbf{P}(\boldsymbol{\sigma}).\boldsymbol{\sigma}_d.\tilde{\mathbf{P}}(\boldsymbol{\sigma})$. In indicielle writing, we adopt following convention for the matric writings:

$$M^{i\leftarrow ligne}_{j\leftarrow colonne}$$

so that the matric product is written: $(\mathbf{A}.\mathbf{B})^i_j=A^i_m\mathbf{B}^m_j$ with the rule of summation of the repeated indices.
Then there is the relation:

$$\boxed{\frac{\partial\sigma_d}{\partial\sigma^i_j}=\tilde{\mathbf{P}}(\boldsymbol{\sigma}).\frac{\partial\sigma}{\partial\sigma^i_j}.\mathbf{P}(\boldsymbol{\sigma})}$$

$\frac{\partial\sigma_{d_k}}{\partial\sigma^i_j}=P^i_kP^j_k$ or in indicielle form **<u>without summation on the index</u>** k

Demonstration:
In what follows, we will note σ an unspecified component of the tensor $\boldsymbol{\sigma}$ without specifying the indices of them when they do not play any part.
One has $\boldsymbol{\sigma}=\mathbf{P}(\boldsymbol{\sigma}).\boldsymbol{\sigma}_d.\tilde{\mathbf{P}}(\boldsymbol{\sigma})$, and consequently:

1) $$\frac{\partial\boldsymbol{\sigma}_d}{\partial\sigma}=\tilde{\mathbf{P}}(\boldsymbol{\sigma}).\frac{\partial\boldsymbol{\sigma}}{\partial\sigma}.\mathbf{P}(\boldsymbol{\sigma})+\frac{\partial\tilde{\mathbf{P}}(\boldsymbol{\sigma})}{\partial\sigma}.\boldsymbol{\sigma}.\mathbf{P}(\boldsymbol{\sigma})+\tilde{\mathbf{P}}(\boldsymbol{\sigma}).\boldsymbol{\sigma}.\frac{\mathbf{P}(\boldsymbol{\sigma})}{\partial\sigma}$$

2) .

By deferring in the last two terms the equality $\boldsymbol{\sigma}=\mathbf{P}(\boldsymbol{\sigma}).\boldsymbol{\sigma}_d.\tilde{\mathbf{P}}(\boldsymbol{\sigma})$, one obtains:

1) $$\frac{\partial\boldsymbol{\sigma}_d}{\partial\sigma}=\tilde{\mathbf{P}}(\boldsymbol{\sigma}).\frac{\partial\boldsymbol{\sigma}}{\partial\sigma}.\mathbf{P}(\boldsymbol{\sigma})+\frac{\partial\mathbf{P}(\boldsymbol{\sigma})}{\partial\sigma}.\tilde{\mathbf{P}}(\boldsymbol{\sigma}).\boldsymbol{\sigma}_d.\mathbf{P}(\boldsymbol{\sigma}).\tilde{\mathbf{P}}(\boldsymbol{\sigma})+\tilde{\mathbf{P}}(\boldsymbol{\sigma}).\mathbf{P}(\boldsymbol{\sigma}).\boldsymbol{\sigma}_d.\tilde{\mathbf{P}}(\boldsymbol{\sigma}).\frac{\partial\mathbf{P}(\boldsymbol{\sigma})}{\partial\sigma}$$

i.e.:

1) $$\frac{\partial\boldsymbol{\sigma}_d}{\partial\sigma}=\tilde{\mathbf{P}}(\boldsymbol{\sigma}).\frac{\partial\boldsymbol{\sigma}}{\partial\sigma}.\mathbf{P}(\boldsymbol{\sigma})+\frac{\partial\tilde{\mathbf{P}}(\boldsymbol{\sigma})}{\partial\sigma}.\mathbf{P}(\boldsymbol{\sigma}).\boldsymbol{\sigma}_d+\boldsymbol{\sigma}_d.\tilde{\mathbf{P}}(\boldsymbol{\sigma}).\frac{\partial\mathbf{P}(\boldsymbol{\sigma})}{\partial\sigma}$$

In matric writing, this is written:

$$\frac{\partial\boldsymbol{\sigma}^i_d}{\partial\sigma}=\left(\tilde{\mathbf{P}}(\boldsymbol{\sigma}).\frac{\partial\boldsymbol{\sigma}}{\partial\sigma}.\mathbf{P}(\boldsymbol{\sigma})+\frac{\partial\tilde{\mathbf{P}}(\boldsymbol{\sigma})}{\partial\sigma}.\mathbf{P}(\boldsymbol{\sigma}).\boldsymbol{\sigma}_d+\boldsymbol{\sigma}_d.\tilde{\mathbf{P}}(\boldsymbol{\sigma}).\frac{\partial\mathbf{P}(\boldsymbol{\sigma})}{\partial\sigma}\right)^i_i$$

Let us show that the sum of the last two terms of this statement is null:

- $$\left(\frac{\partial\tilde{\mathbf{P}}(\boldsymbol{\sigma})}{\partial\sigma}.\mathbf{P}(\boldsymbol{\sigma}).\boldsymbol{\sigma}_d+\boldsymbol{\sigma}_d.\tilde{\mathbf{P}}(\boldsymbol{\sigma}).\frac{\partial\mathbf{P}(\boldsymbol{\sigma})}{\partial\sigma}\right)^i_j=\partial\tilde{P}^i_m.P^m_l.\sigma_{d^l_j}+\sigma_{d^i_m}.\tilde{P}^m_l.\partial P^l_j$$

where ∂a indicates $\frac{\partial a}{\partial\sigma}$ so D `to reduce the writing.

We write whereas $i=j$ and that only the terms $\sigma^p_{d_p}$ are non-zero.
One obtains:

$$\left(\frac{\partial\tilde{\mathbf{P}}(\boldsymbol{\sigma})}{\partial\sigma}.\mathbf{P}(\boldsymbol{\sigma}).\boldsymbol{\sigma}_d+\boldsymbol{\sigma}_d.\tilde{\mathbf{P}}(\boldsymbol{\sigma}).\frac{\partial\mathbf{P}(\boldsymbol{\sigma})}{\partial\sigma}\right)^i_i=\partial\tilde{P}^i_m.P^m_i.\sigma_{d^i_i}+\sigma_{d^i_i}.\tilde{P}^i_l.\partial P^l_i$$

$$=\left(\partial\tilde{P}^i_m.P^m_i+\tilde{P}^i_m.\partial P^m_i\right)\sigma_{d^i_i}\underline{\text{sans sommation sur l'indice } i}$$

who is clearly null since $\tilde{\mathbf{P}}.\mathbf{P}=\mathbf{I}$. From where finally $\frac{\partial\sigma_{d_k}}{\partial\sigma^i_j}=\frac{\partial\sigma_{d^k_k}}{\partial\sigma^i_j}=P^i_kP^j_k$.

Annexe 2 Derived from the function $\bar{F}(\Delta\lambda)$

$$\frac{\partial h}{\partial\Delta\gamma}=\frac{\partial}{\partial\Delta\gamma}\left(\frac{3+\sin\phi(\gamma^-+\Delta\gamma)}{3(1+\sin\phi(\gamma^-+\Delta\gamma))}\right)=\begin{cases}-\dfrac{2\dfrac{\partial\phi}{\partial\gamma}\cos\phi(\gamma)}{3(1+\sin\phi(\gamma))^2} & \text{si } \gamma<\gamma^{res}\\ 0 & \text{sinon}\end{cases}$$

$$\frac{\partial g}{\partial\Delta\gamma}=\frac{\partial}{\partial\Delta\gamma}\left[\frac{3+\sin\phi(\gamma^-+\Delta\gamma)}{3(1+\sin\phi(\gamma^-+\Delta\gamma))}\left(\frac{6\text{K}\sin\phi(\gamma^-+\Delta\gamma)}{3+\sin\phi(\gamma^-+\Delta\gamma)}+\frac{3mus_3^e}{\sigma_{eq}^e}\right)\right]$$

$$=\frac{6\text{K}\dfrac{\partial\phi}{\partial\gamma}\cos\phi(\gamma)}{(3+\sin\phi(\gamma))(1+\sin\phi(\gamma))}-\frac{2\dfrac{\partial\phi}{\partial\gamma}\cos\phi(\gamma)}{3(1+\sin\phi(\gamma))^2}\left(\frac{6\text{K}\sin\phi(\gamma)}{3+\sin\phi(\gamma)}+\frac{3mus_3^e}{\sigma_{eq}^e}\right)$$

$$\frac{\partial\bar{F}}{\partial\Delta\gamma}=2\left[-(s_3^e-s_1^e)\frac{3\mu}{\sigma_{eq}^e}\left(\frac{\partial h}{\partial\Delta\gamma}\Delta\gamma+h\right)-\frac{\partial b}{\partial\Delta\gamma}\left(1-\frac{1}{\sigma_3^{b-d}}\left[s_3^e+\frac{I_1^e}{3}-g\Delta\gamma\right]\right)-\frac{b}{\sigma_3^{b-d}}\left(\frac{\partial g}{\partial\Delta\gamma}\Delta\gamma+g\right)\right]$$

$$\times\left[(s_3^e-s_1^e)\left[1-\frac{3\mu}{\sigma_{eq}^e}h\Delta\gamma\right]-b\left[1-\frac{1}{\sigma_3^{b-d}}\left(s_3^e+\frac{I_1^e}{3}-g\Delta\gamma\right)\right]\right]$$

$$-\left(\frac{\partial(S\sigma_c^2)}{\partial\Delta\gamma}-\frac{\partial(m\sigma_c)}{\partial\Delta\gamma}\left(s_3^e+\frac{I_1^e}{3}-g\Delta\gamma\right)+\sigma_c m\left(\frac{\partial g}{\partial\Delta\gamma}\Delta\gamma+g\right)\right)$$

Annexe 3Computation from derivatives $\frac{\partial\sigma_p}{\partial p_g}$ and $\frac{\partial\sigma_p}{\partial p_c}$

- `LIQU_SATU (PRE1=` p_{lq} `):` $\frac{\partial\sigma_p}{\partial p_c}=-\frac{\partial\sigma_p}{\partial p_{lq}}=bS$
- `LIQU_GAZ_ATM (PRE1=-` p_{lq} `):` $\frac{\partial\sigma_p}{\partial p_c}=-\frac{\partial\sigma_p}{\partial p_{lq}}=bS$
- `GAZ (PRE1=` p_g `) :` $\frac{\partial\sigma_p}{\partial p_g}=-b(1-S)$
- `LIQU_VAPE_GAZ (PRE1=` p_c `, PRE2=` p_g `):` $\frac{\partial\sigma_p}{\partial p_g}=-b$, $\frac{\partial\sigma_p}{\partial p_c}=bS$
- `LIQU_GAZ (PRE1=` p_c `, PRE2=` p_g `):` $\frac{\partial\sigma_p}{\partial p_g}=-b$, $\frac{\partial\sigma_p}{\partial p_c}=bS$
- `LIQU_VAPE (PRE1=` p_{lq} `):` $\frac{\partial\sigma_p}{\partial p_c}=-\frac{\partial\sigma_p}{\partial p_{lq}}=bS$
- `LIQU_AD_GAZ_VAPE (PRE1=` p_c `, PRE2=` p_g `) :` $\frac{\partial\sigma_p}{\partial p_g}=-b$, $\frac{\partial\sigma_p}{\partial p_c}=bS$