Constitutive law of LAIGLE

Summarized:

The model rheological of Laigle allows to analyze the structural mechanics behavior of the rocks. The development of this model of behavior was initiated following the difficulty in correctly apprehending the response of the solid mass during the excavation of an underground cavity, with an aim:

- to define the need and the nature of possible supportings to implement;
- to determine the extent of the ground around a work influenced by the digging.

The placement of this elastoplastic model was mainly focused on the simulation of the behavior post-peak of the rock. It is supposed, accordingly, that there is no hardening of the rock prior to the fracture of this one. That results in a linear elastic behavior to the peak of strength (there can nevertheless be damage of the rock whereas the material is not yet in fracture). The definite plasticity criterion is of type generalized Hoek and Brown and gives an account of the influence of the level of stress on the shear strength. The rise in temperature of the material is associated with a progressive reduction in the properties of cohesion and friction angle accompanied by a change of volume. It is controlled by the plastic strain déviatoire cumulated considered as only variable of hardening.

To facilitate the integration of this model in *Code_Aster*, the model initially developed in the formalism of the principal stresses was rewritten with invariants of stresses on a basis of the model Cambou - Jafari-Sidoroff (CJS). The numerical formulation is implicit compared to the criterion and explicit compared to the flow direction.

The sign convention used for the formulation of the equations, in the frame of this note, is that of the mechanics of the continuums.

Contents

1 Notations

1.1 General information

$\boldsymbol{\sigma}$ indicates the tensor of the effective stresses in small disturbances, noted in the shape of the following vector:

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sqrt{2}\sigma_{12} \\ \sqrt{2}\sigma_{13} \\ \sqrt{2}\sigma_{23} \end{pmatrix}$$

One notes:

$I_1 = \mathrm{tr}(\sigma)$	first tensor invariant of
$\boldsymbol{s} = \boldsymbol{\sigma} - \frac{I_1}{3}\boldsymbol{I}$	the stresses of the stresses déviatoires
$s_{II} = \sqrt{\boldsymbol{s}.\boldsymbol{s}}$	second invariant of the tensor of the stresses déviatoires
σ_1	major principal stress
σ_3	minor principal stress
$\boldsymbol{e} = \boldsymbol{\varepsilon} - \frac{Tr(\varepsilon)}{3}\boldsymbol{I}$	deviator of the strains
$\varepsilon_v = \mathrm{Tr}(\boldsymbol{\varepsilon})$	voluminal strain
$\cos(3\theta) = 2^{1/2}3^{3/2}\frac{det(\boldsymbol{s})}{s_{II}^3}$	θ being the angle of Lode
$\gamma^p = \sqrt{\frac{2}{3}e_{ij}^p e_{ij}^p}$	plastic deviatoric strains cumulated
$\mathbf{n}$	normal of the hypersurface of strain
$\mathbf{G}$	function controlling the evolution of plastic strains and describing the deviative flow direction
$\tilde{\boldsymbol{G}} = \boldsymbol{G} - \frac{\mathrm{Tr}(\boldsymbol{G})}{3}\boldsymbol{I}$	of $\mathbf{G}$
$G = Tr(\boldsymbol{G})$	trace of $\mathbf{G}$
$\tilde{G}_{II} = \sqrt{\tilde{\boldsymbol{G}}.\tilde{\boldsymbol{G}}}$	norm of $\tilde{\mathbf{G}}$
ψ	angle of dilatancy
φ	friction angle
f	surfaces of load

1.2 Parameters of the model

Notation	Description
m	Slope of the criterion in the plane (p',q) for the very strong stresses (function of the mineralogical nature of the rock)
s	Cohesion of the medium. Representative of the damage of the rock.
a	Characterization of the concavity of the criterion, function of the level of deterioration of the rock. It defines the influence of the component of dilatancy in the behavior in the large deformations.
γ_{ult}	Plastic strain déviatoire corresponding to the ultimate criterion
γ_e	plastic Strain déviatoire corresponding to the complete disappearance of cohesion
m_{ult}	Value of m ultimate criterion reached in γ_{ult}
m_e	Value of m intermediate criterion reached in γ_e
a_e	Value of a intermediate criterion reached in γ_e
m_{pic}	Value of m criterion of peak reached with the peak of stress
a_{pic}	Value of a criterion of peak reached with the Exposing peak of
η	stress controlling hardening
σ_c	Compressive strength simple
γ	First parameter regulating dilatancy
ζ	Second parameter regulating dilatancy
γ_{cjs}	Parameter of form of the plasticity criterion in the plane déviatoire
E	Modulus Young
ν	Poisson's ratio
σ_{p1}	Intersection of the intermediate criterion and the criterion of peak
σ_{p2}	Intersection of the intermediate criterion and the ultimate criterion
PA	Atmospheric pressure

2 Introduction

the object of this note is to present the model rheological to analyze the structural mechanics behavior of the rocks, adapted to the simulation of the underground works, introduced into *Code_Aster* and developed by the CIH [bib1]. The finality of this model is of being able to be implemented, in a fast and industrial way in order to answer the principal interrogations that the engineer during the analysis and of the design of an underground cavity is posed. The rheological model must for that remain relatively simple, as well during the identification of the parameters as in its implementation and during interpretation of the results.

2.1 Phenomenology of the behavior of the soils

One of the characteristics of a rock, compared to a soil, that its structural mechanics behavior is, on a beach of important stress, is controlled by cohesion. This cohesion is associated with a cementing of the medium, is induced during the geological history of the solid mass, and is primarily of epitaxic nature. On the contrary, the strength of a soil is more particularly governed by the term of friction and/ or dilatancy. Cohesion, of primarily capillary origin, then affects only for very weak stress states of containment.

This distinction between a soil and a rock is important because it directs the choice and the basic assumptions of the model of behavior.

The principal rheological phenomena associated in this context are the following:

•In the field of the small strains, the response of a rock, in particular under weak states of containment, can be comparable to a linear elastic behavior, slightly depend on the state of the stresses. Non-linearities of the behavior are likely to appear prior to peak of strength, in the case of the tender rocks, for a level of stress of about 70 to 80% of the maximum value. This threshold decreases with the increase in the average pressure for almost cancelling itself when the stress of surconsolidation is reached (course-model). Under very low stresses of containment representative of those reigning near the underground works, these non-linearities are generally low, more especially as cementing is important, and thus the high level of surconsolidation of the rock.

•Dilatancy (increase in volume) is initiated when non-linearities appear on stress-strain curve. This dilatancy increases until there is localization within the sample. At this time, the rate of dilatancy (or the angle of dilatancy ψ) is maximum, for then gradually decreasing and cancelling themselves with the very large deformations.

•The peak of strength is reached for stresses describing a rupture criterion, generally curved in the plane of Mohr or the plane of the major and minor principal stresses. The assumption of a linear criterion of Mohr-Coulomb is thus only one simplifying assumption, tending, for low stresses of containment, to raise the cohesion of the medium.

•Once maximum strength reached, the strength of the rock decreases. This rise in temperature post-peak is all the more fast and important (in intensity) that the stress of containment is low. This decrease is related to a damage more or less localised of the rock, according to the level of containment. Whatever this stress, beyond the peak, **the rock cannot be regarded as continuous any more**. Its behavior is then controlled by the conditions of strain and strength to the level of the zone of localization of the strains.

•The appearance of one or more kinematical discontinuities within the rock is associated with a loss of cohesion. The behavior post-peak is then governed by the conditions of friction and dilatancy along the planes of discontinuity or within a tape of localization of the strains. It comes out from this reasoning that for very large deformations, the behavior of the comparable rock to a “structure”, is only rubbing, and is characterized by an ultimate friction angle ϕ . This angle is an intrinsic data of the material, function of minerals constitutive of the rock. It thus does not depend directly on the conditions of cohesion, and it can especially be regarded as independent of dimensions of the sample.

- When the behavior only becomes rubbing, it is associated with no voluminal strain. Dilatancy was thus cancelled, and does not exist any more with the large deformations.
- The evolution between the strength of peak and the critical condition corresponding to the large deformations, is more or less progressive according to the state of the pressures applied. For a state of null containment (simple compression), the behavior is only controlled by cohesion, and the fracture results in an immediate and brutal loss of any strength. Rise in temperature will be more progressive as the stress of containment increases, to become non-existent beyond of a certain stress of containment limiting the ductile and brittle fields of behavior.

2.2 Context of study and simplifying assumptions of the model

the will to develop a model easy to implement is necessarily accompanied by simplifications, resulting from a compromise between the expected purposes, the conditions of use of the model (quality of the data input, times and cost available…) and the average implemented to ensure these developments. These compromises are primarily the following:

- **A linear elastic behavior** to the peak of strength. This amounts supposing that there is no hardening of the rock prior to the fracture of this one.
- **Only a rupture criterion in shears is retained**. This means that if the rock is crushed in an isotropic way, the behavior remains elastic, and that there are not damage and hardening of the material under this kind of path. During the phases of excavation of an underground work with implementation of a light supporting, the average pressure in the solid mass located in the vicinity can only decrease (or to remain constant in the ideal case of a circular cavity subjected to an isotropic request, for a linear elastic behavior). Plasticization under isotropic stress, that one can find on a Cape-Model or on a model of the Camwood-Clay type did not seem essential to us taking into account the searched purposes, and in the case of an isothermal and short-term request.

During the development of this model, we voluntarily focused ourselves on the study and the simulation of the behavior post-peak of the rock. In this field of behavior, the strength of the material is supposed to be controlled, according to the state of the stresses and the level of damage of the rock, by cohesion, dilatancy or friction.

Cohesion defines the strength of the material as long as this one remains continuous. It is active to the peak of strength, and has only little influence on the behavior softening, unless cohesion is representative of a ductile “adhesive” (case of the soils injected by silicate freezing,…).

As cohesion worsens by damage, dilatancy increases, to reach its maximum value at the time of the loss of continuity of the medium. At this time, under the effect of the shears of induced discontinuity, this dilatancy is degraded gradually and slowly. The rheology of the rock evolves then to a behavior purely rubbing.

3 The model continuous

3.1 Behavior elastic

the elastic behavior is controlled by a linear model, with a constant modulus independent of the stress state. The 2 parameters characterizing this behavior are the elasticity modulus E and the Poisson's ratio ν .

$$\dot{\boldsymbol{s}}=2\mu\left(\dot{\boldsymbol{e}}-\dot{\boldsymbol{e}}^{p}\right) \quad \text{éq 3.1-1}$$

$$\dot{\boldsymbol{I}}_1=3\mathrm{K}\left(\dot{\boldsymbol{\varepsilon}}_{v}-\dot{\boldsymbol{\varepsilon}}_{v}^{p}\right) \quad \text{éq 3.1-2}$$

3.2 Plasticity criterion

the adopted formulation is that of [bib2].

3.2.1 Surface of load

3.2.1.1 Statement of the criterion of Laigle in major and minor stresses

$$f=\left(\frac{\sqrt{\frac{2}{3}}}{\sigma_c}\right)^{\frac{1}{a(\gamma^p)}}\left[\left(|\sigma_1-\sigma_3|\right)\frac{1}{a\left(\gamma^p\right)}-(\sigma_c)\frac{1}{a\left(\gamma^p\right)}\left(\frac{m\left(\gamma^p\right)}{\sigma_c}(-\sigma_3)+s(\gamma^p)\right)\right] \quad \text{éq 3.2.1.1 - 1}$$

3.2.1.2 general Statement

One transforms the preceding statement according to the first invariant and of the deviator of the stresses, by a retiming of the criterion on triaxial in compression, to obtain:

$$f=\left(\frac{g\left(\boldsymbol{s}\right)}{\sigma_c h_c^0}\right)^{\frac{1}{a\left(\gamma^p\right)}}-u\left(\boldsymbol{\sigma},\gamma^p\right)\leq 0 \quad \text{éq 3.2.1.2 - 1}$$

with:

$$h\left(\theta\right)=\left(1+\gamma_{\text{cjs}}\cos(3\theta)\right)^{1/6}=\left(1+\gamma_{\text{cjs}}\sqrt{54}\,\frac{det(\boldsymbol{s})}{s_{II}^3}\right)^{1/6} \quad \text{éq 3.2.1.2 - 2}$$

$$\left|\begin{array}{l} h_c^0=h\left(\theta=\frac{\pi}{3}\right)=\left(1-\gamma_{\text{cjs}}\right)^{1/6} \\ h_t^0=\left(1+\gamma_{\text{cjs}}\right)^{1/6}\end{array}\right.$$

$$g(\boldsymbol{s})=s_{II}h(\theta) \quad \text{éq 3.2.1.2 - 3}$$

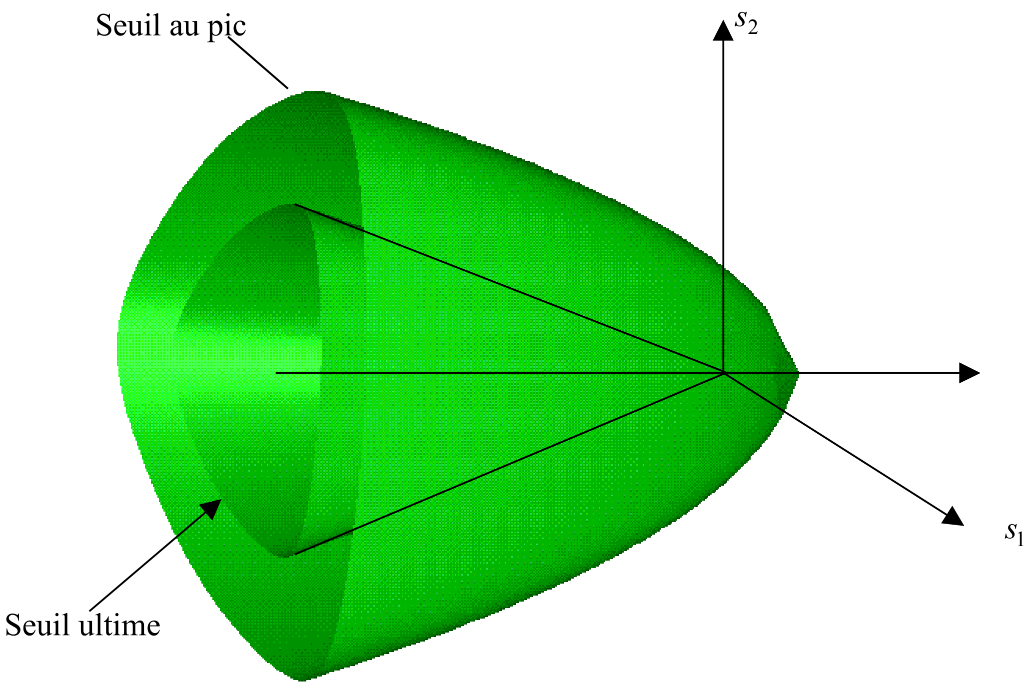
$$u(\boldsymbol{\sigma},\gamma^p)=-\frac{m(\gamma^p)k(\gamma^p)}{\sqrt{6}\sigma_c}\frac{g(\boldsymbol{s})}{h_c^0}-\frac{m(\gamma^p)k(\gamma^p)}{3\sigma_c}I_1+s(\gamma^p).k(\gamma^p) \quad \text{éq 3.2.1.2 - 4}$$

Note::

•One shows [Appendix 1] the equivalence of the two statements
•One shows that a second formulation of the criterion with a retiming on triaxial in compression and extension is possible but we do not choose it. It however is presented to the chapter [§9].

3.2.1.3 Pace of the thresholds

One traces the pace of the thresholds to the criterion of peak and the ultimate criterion.



3.2.2 Hardening

to translate rise in temperature post-peak of the rock one defines models of variations of the parameters m, S and has criterion according to the local variable of hardening γ^p (it is the strain déviatoire plastic cumulated, proportional to the second invariant of the tensor of the strains déviatoires, corresponding to the plastic distortion).

$$s(\gamma^p)=\left(1-\frac{\gamma^p}{\gamma^e}\right) \quad \text{si}\, \gamma^p<\gamma_e$$
$$s(\gamma^p)=0 \quad \text{si}\, \gamma^p\geq\gamma_e$$
éq 3.2.2-1

$\underline{\text{si}\, \gamma^p>\gamma_{\text{ult}}(1-10^{-3})}$ - # one chooses to take an epsilon 10^{-3} to avoid numerical errors # during division by γ_{ult} in the equation [éq 3.2.2-2]

$a=1$

$m=m_{\text{ult}}$

If not

$$\Omega\left(\gamma^{p}\right)=\left(\frac{\gamma^{p}}{\gamma^{e}}\right)^{\eta}\frac{a_{e}-a_{\text{pic}}}{1-a_{e}}\frac{\gamma_{\text{ult}}-\gamma_{e}}{\gamma_{\text{ult}}-\gamma^{p}}$$ éq 3.2.2-2

$$a\left(\gamma^{p}\right)=\frac{a_{\text{pic}}+\Omega\left(\gamma^{p}\right)}{1+\Omega\left(\gamma^{p}\right)}$$ éq 3.2.2-3

$$m\left(\gamma^{p}\right)=\frac{\sigma_{c}}{\sigma_{\text{p1}}}\left[\left(m^{\text{pic}}\frac{\sigma_{\text{p1}}}{\sigma_{c}}+1\right)^{\frac{a_{\text{pic}}}{a\left(\gamma^{p}\right)}}-s\left(\gamma^{p}\right)\right]\quad \text{si}\,\gamma^{p}<\gamma_{e}$$

$$m\left(\gamma^{p}\right)=\frac{\sigma_{c}}{\sigma_{\text{p2}}}\left[\left(m_{e}\frac{\sigma_{\text{p2}}}{\sigma_{c}}\right)^{\frac{a_{\text{pic}}}{a\left(\gamma^{p}\right)}}\right]\quad \text{si}\,\gamma^{p}\geq\gamma_{e}$$ éq 3.2.2-4

$$k\left(\gamma^{p}\right)=\left(\frac{2}{3}\right)^{\frac{1}{2a\left(\gamma^{p}\right)}}$$.éq 3.2.2-5

These laws of evolutions for each of the 3 parameters is dependant from/to each other and observes the conditions of intersection of the criteria during the phase of hardening [bib1].

Note:

The condition of coherence to respect door on the continuity of the parameter m *in* γ_e *:*

$$\lim_{\gamma^{p}\to\gamma_{e}} m\left(\gamma^{p}\right)=\frac{\sigma_{c}}{\sigma_{\text{p1}}}\left[\left(m_{\text{pic}}\frac{\sigma_{\text{p1}}}{\sigma_{c}}+1\right)^{\frac{a_{\text{pic}}}{a\left(\gamma_{p}\right)}}-s\left(\gamma^{p}\right)\right]$$

that is to say:

$$m_{e}=\frac{\sigma_{c}}{\sigma^{\text{p1}}}\left(m_{\text{pic}}\frac{\sigma_{\text{p1}}}{\sigma_{c}}+1\right)^{\frac{a_{\text{pic}}}{a_{e}}}$$ *éq 3.2.2-6*

3.2.3 Model of dilatancy

3.2.3.1 generalized Writing

the model of dilatancy (it is admitted that the value of dilatancy is inversely proportional to that of cohesion) can be generalized while writing:

$$\sin\psi=\sin\left(\psi\left(\alpha^{'}\right)\right)=\gamma\,\frac{\alpha^{'}-m_{\text{ult}}-1}{\zeta\alpha^{'}+m_{\text{ult}}+1}$$ éq 3.2.3.1 - 1

with:

$$\alpha' = \alpha'\left(I_{1,} g(\boldsymbol{s}), \sigma_{t0}\right) = \frac{\tilde{\sigma}_1 - \sigma_{t0}}{\tilde{\sigma}_3 - \sigma_{t0}} \quad \text{éq 3.2.3.1 - 2}$$

$s_3 = \sqrt{\frac{2}{3}} s_{II} \cos(\theta)$; $s_1 = \sqrt{\frac{2}{3}} s_{II} \cos(\theta + \frac{2\pi}{3})$; $s_2 = \sqrt{\frac{2}{3}} s_{II} \cos(\theta - \frac{2\pi}{3})$; where θ is the angle of Lode

$\sigma_1 = \frac{I_1}{3} + s_1$; $\sigma_2 = \frac{I_1}{3} + s_2$; $\sigma_3 = \frac{I_1}{3} + s_3$;

$$\begin{cases} \tilde{\sigma}_1 = \sigma_i \, \text{avec} \, i \, \text{tel que} |\sigma_i| = \max\left(|\sigma_j|, j=1,2,3\right) \\ \tilde{\sigma}_3 = \sigma_i \, \text{avec} \, i \, \text{tel que} |\sigma_i| = \max\left(|\sigma_j|, j=1,2,3\right) \end{cases}$$

Note:

A condition to respect is that the ratio $\frac{\gamma}{\zeta}$ rest lower than 1. In the case of very resistant hard stones, subjected to relatively low stresses of containment, the model of dilatancy can thus tend towards this ratio. If the two parameters are unit one finds the form of the model of Rowe describing the model of dilatancy for non-cohesive soils. This approach amounts preserving the same statement as for a strongly damaged rock, by comparing the effect of cohesion to that of an additional containment of value σ_{t0}.

Characterization of σ_{t0} according to the parameters (has, m, S) characterizing the rock

- Case where $s(\gamma^p) = 0$

  Disappearance of cohesion, one poses $\sigma_{t0} = 0$

- Case where $s(\gamma^p) \neq 0$

$$\sigma_{t0} = \sigma_{t0}\left(\phi_{0,} C_0\right) = 2\mathrm{C}_0 \sqrt{\frac{1 - \sin\phi_0}{1 + \sin\phi_0}} \quad \text{éq 3.2.3.1 - 3}$$

with:

$$\begin{cases} \phi_0 = \phi_0(m, s, a) = 2 \cdot \arctan\left(\sqrt{1 + \mathrm{ams}^{a-1}}\right) - \frac{\pi}{2} \\ C_0 = C_0(m, s, a) = \frac{\sigma_c s^a}{\sqrt{1 + \mathrm{ams}^{a-1}}} \end{cases}$$

3.2.3.2 Determination of the intersection of the intermediate criterion and the ultimate criterion

By writing the continuity of m in γ_{ult} one obtains the following relation:

$$m\left(\gamma_{\text{ult}}\right)=\frac{\sigma_c}{\sigma_{\text{p2}}}\left[\left(m_e\frac{\sigma_{\text{p2}}}{\sigma_c}\right)^{\frac{a_e}{a\left(\gamma_{\text{ult}}\right)}}\right]$$

$$m_{\text{ult}}=\frac{\sigma_c}{\sigma_{\text{p2}}}\left(m_e\frac{\sigma_{\text{p2}}}{\sigma_c}\right)^{\frac{a_e}{a_{\text{ult}}}}$$

$$m_{\text{ult}}=m_e^{a_e}\left(\frac{\sigma_{\text{p2}}}{\sigma_c}\right)^{a_e-1}$$

$$\sigma_{\text{p2}}=\sigma_c\left(\frac{m_{\text{ult}}}{m_e^{a_e}}\right)^{\frac{1}{a_e-1}}$$ éq 3.2.3.2 - 1

3.2.4 Yielding

the adopted formalism is rewritten on the basis of CJS the model [R7.01.13]. When the stresses reach edge of the field of reversibility, of plastic strains develop. To calculate them, there exists a potential function controlling the evolution of the strains and defined by the relation $\dot{\boldsymbol{\varepsilon}}^p=\dot{\lambda}\boldsymbol{G}$ where $\dot{\lambda}$ is the plastic multiplier and

$$\boldsymbol{G}=\frac{\partial f}{\partial\boldsymbol{\sigma}}-\left(\frac{\partial f}{\partial\boldsymbol{\sigma}}\boldsymbol{n}\right)\boldsymbol{n}$$.éq 3.2.4-1

the potential function is obtained from the following kinematical condition:

$$\dot{\varepsilon}_v^p=-\beta'\frac{\boldsymbol{s}.\dot{\boldsymbol{\varepsilon}}^p}{s_{II}}$$ éq 3.2.4-2

the parameter of dilatancy β' is calculated from the angle of dilatancy ψ (defined by [éq 3.2.3.1 - 1]) by the formula:

$$\beta'=\beta'\left(\psi\right)=-\frac{2\sqrt{6}\sin\left(\psi\right)}{3-\sin\left(\psi\right)}$$
$$\beta'=0\,\text{si}\,\gamma^p>\gamma_{\text{ult}}\left(1-10^{-3}\right)$$ **éq 3.2.4-3**

Note::

β' is positive when $\gamma^p=0$ and in compression, then it becomes negative when plasticity develops. It is always negative in tension

It is then possible to seek to express the kinematical condition [éq 3.2.4-2] from a tensor $\mathbf{n}$ in the form:

$$\boldsymbol{n}.\dot{\boldsymbol{\varepsilon}}^p=0$$ éq 3.2.4-4

After decomposition of each term in déviatoire parts and hydrostatics, one finds the statement:

$$\left(n_I s_{\text{ij}}+n_2\delta_{\text{ij}}\right).\left(\dot{e}_{\text{ij}}^p+\frac{1}{3}\dot{\varepsilon}_v^p\delta_{\text{ij}}\right)=n_1 s_{\text{ij}}\dot{e}_{\text{ij}}^p+n_2\dot{\varepsilon}_v^p=0$$

One of deduced the relation $\frac{n_1}{n_2}=\frac{\beta^{'}}{s_{\mathrm{II}}}$ which, added to the condition of standardization of the tensor $\mathbf{n}$, conduit to the statement:

$$\boldsymbol{n}=\frac{\beta_{'}\frac{\boldsymbol{s}}{s_{\mathrm{II}}}+\boldsymbol{I}}{\sqrt{\beta^{'2}+3}}$$ éq 3.2.4-5

the law of evolution of $\dot{\boldsymbol{\varepsilon}}^p$ must be such as the kinematical condition is satisfied. It is thus proposed to take the projection of $\dot{\boldsymbol{\varepsilon}}^p$ on $\mathbf{n}$ (norm of the hypersurface of strain), that is to say:

$$\dot{\boldsymbol{\varepsilon}^p}=\dot{\lambda}\boldsymbol{G}=\dot{\lambda}\left(\frac{\partial f}{\partial\boldsymbol{\sigma}}-\left(\frac{\partial f}{\partial\boldsymbol{\sigma}}\boldsymbol{n}\right)\boldsymbol{n}\right)$$

One from of also deduced the condition relating to the plastic voluminal strain:

$$\dot{\varepsilon_v^p}=\dot{\lambda}G$$ éq 3.2.4-6

4 Computation of derivatives

4.1 Derived from the criterion

4.1.1 Derived compared to the stresses

4.1.1.1 Derived intermediary compared to the deviator

One leaves: $\frac{\partial g}{\partial s_{\mathrm{ij}}}=h(\theta)\frac{\partial s_{\mathrm{II}}}{\partial s_{\mathrm{ij}}}+s_{\mathrm{II}}\frac{\partial h(\theta)}{\partial s_{\mathrm{ij}}}$

where $\frac{\partial s_{\mathrm{II}}}{\partial s_{\mathrm{ij}}}$ and $\frac{\partial h(\theta)}{s_{\mathrm{ij}}}$ respectively are given by:

$$\frac{s_{\mathrm{II}}}{s_{\mathrm{ij}}}=\frac{s_{\mathrm{ij}}}{s_{\mathrm{II}}}$$

$$\frac{\partial h(\theta)}{\partial s_{\mathrm{ij}}}=\frac{1}{6\mathrm{h}(\theta)_5}\frac{\partial}{\partial s_{\mathrm{ij}}}\left(1+\gamma_{\mathrm{cjs}}\sqrt{54}\frac{det(\underline{\underline{\mathbf{s}}})}{s_{\mathrm{II}}^3}\right)$$

$$=\frac{-\gamma_{\mathrm{cjs}}\cos(3\theta)}{2\mathrm{h}(\theta)^5 s_{\mathrm{II}}^2}s_{\mathrm{ij}}+\frac{\gamma_{\mathrm{cjs}}\sqrt{54}}{6\mathrm{h}(\theta)^5 s_{\mathrm{II}}^3}\left(\frac{\partial\, det(\underline{\underline{\mathbf{s}}})}{\partial s_{\mathrm{ij}}}\right)$$

Finally:

$$\frac{\partial g}{\partial s_{\mathrm{ij}}}=\frac{1}{h(\theta)^5}\left[\left(1+\frac{\gamma_{\mathrm{cjs}}}{2}\cos(3\theta)\right)\frac{s_{\mathrm{ij}}}{s_{\mathrm{II}}}+\frac{\gamma_{\mathrm{cjs}}\sqrt{54}}{6\mathrm{s}_{\mathrm{II}}^2}\left(\frac{\partial\, det(\underline{\underline{\mathbf{s}}})}{\partial s_{\mathrm{ij}}}\right)\right]$$

And consequently:

$$\frac{\partial g}{\partial s_{ij}}=\frac{1}{h(\theta)^5}\left[\left(1+\frac{\gamma_{cjs}}{2}\cos(3\theta)\right)\frac{\mathbf{s}}{s_{II}}+\frac{\gamma_{cjs}\sqrt{54}}{6s_{II}^2}\left(\frac{\partial\, det(\underline{\mathbf{s}})}{\partial s_{ij}}\right)\right] \quad \text{éq 4.1.1.1 - 1}$$

4.1.1.2 intermediate Derivative compared to the stresses

One poses by definition: $Q_{ij}=dev\left(\frac{\partial g}{\partial s_{ij}}\right)$

$$\frac{\partial g}{\partial \sigma_{ij}}=\frac{\partial g}{\partial s_{kl}}\frac{\partial s_{kl}}{\partial \sigma_{ij}}=\left[dev\left(\frac{\partial g}{\partial s_{kl}}\right)+\frac{1}{3}\frac{\partial g}{\partial s_{mm}}\delta_{kl}\right]\left[\delta_{ik}\delta_{jl}-\frac{1}{3}\delta_{ij}\delta_{kl}\right]$$

$$\frac{\partial g}{\partial \sigma_{ij}}=Q_{kl}\delta_{ik}\delta_{jl}-\frac{1}{3}\delta_{ij}Q_{kl}\delta_{kl}+\frac{1}{3}\frac{\partial g}{\partial q_{mm}}\left[\delta_{ik}\delta_{jl}\delta_{kl}-\frac{1}{3}\delta_{ij}\delta_{kl}\delta_{kl}\right]$$

$$\frac{\partial g}{\partial \sigma_{ij}}=Q_{ij}$$

It is then enough to take the deviatoric part $\frac{\partial g}{\partial s_{ij}}$ to obtain:

$$\frac{\partial g}{\partial \sigma_{ij}}=Q_{ij}=dev\left(\frac{\partial g}{\partial s_{ij}}\right)=\frac{1}{h(\theta)^5}\left[\left(1+\frac{\gamma_{cjs}}{2}\cos(3\theta)\right)\frac{s_{ij}}{s_{II}}+\frac{\gamma_{cjs}\sqrt{54}}{6\,s_{II}^2}dev\left(\frac{\partial\, det(\underline{\mathbf{s}})}{\partial s_{ij}}\right)\right]$$

And consequently:

$$\mathbf{Q}=\frac{\partial g}{\partial \boldsymbol{\sigma}}=\frac{1}{h(\theta)^5}\left[\left(1+\frac{\gamma_{cjs}}{2}\cos(3\theta)\right)\frac{\mathbf{s}}{s_{II}}+\frac{\gamma_{cjs}\sqrt{54}}{6\,s_{II}^2}dev\left(\frac{\partial\, det(\underline{\mathbf{s}})}{\partial s}\right)\right] \quad \text{éq 4.1.1.2 - 1}$$

4.1.1.3 final Statement of derivative of the criterion compared to the stresses

the derivative of the criterion compared to the stresses is then:

$$\frac{\partial f}{\partial \sigma}=\frac{1}{a(\gamma^p)}\frac{1}{\sigma_c h_c^0}^{\frac{1}{a(\gamma^p)}}(g)\,r^{\frac{1-a(\gamma^p)}{a(\gamma^p)}}\mathbf{Q}-\frac{\partial u}{\partial \boldsymbol{\sigma}} \quad \text{éq 4.1.1.3 - 1}$$

with

$$\frac{\partial u}{\partial \boldsymbol{\sigma}}=-\frac{m(\gamma^p)k(\gamma^p)}{\sigma_c}\left(\frac{1}{\sqrt{6}h_c^0}\mathbf{Q}+\frac{1}{3}\mathbf{I}\right) \quad \text{éq 4.1.1.3 - 2}$$

4.1.2 Derivative compared to the variable of hardening

$$\frac{\partial f}{\partial \gamma^p}=-\left(\frac{1}{a(\gamma^p)}\right)^2\left(\frac{g(\mathbf{s})}{\sigma_c h_c^0}\right)^{\frac{1}{a(\gamma^p)}}\log\left(\frac{g(\mathbf{s})}{\sigma_c h_c^0}\right).\frac{\partial a}{\partial \gamma^p}-\frac{\partial u}{\partial \gamma^p} \quad \text{éq 4.1.2-1}$$

with

$$\frac{\partial u}{\partial \gamma^p}=-\frac{1}{\sqrt{6}\sigma_c}\frac{\partial(km)}{\partial \gamma^p}(\gamma^p)\frac{g}{h_c^0}-\frac{1}{3\sigma_c}\frac{\partial(km)}{\partial \gamma^p}(\gamma^p)I_1+\frac{\partial(ks)}{\partial \gamma^p}(\gamma^p) \quad \text{éq 4.1.2-2}$$

4.2 total Derivative of the criterion compared to the plastic multiplier

Let us consider the function:

$$f^{*}(\Delta\lambda)=f\left(\boldsymbol{s}^{e}-2\mu\,\Delta\lambda\,\tilde{\boldsymbol{G}},\ \boldsymbol{I}_{1}^{e}-3\mathrm{K}\,\Delta\lambda\,G,\ \gamma^{p^{-}}+\Delta\lambda\sqrt{\frac{2}{3}}\tilde{G}_{\mathrm{II}}\right) \quad \text{éq 4.2-1}$$

Where $\mathbf{G}$ is a fixed tensor independent of. $\Delta\lambda$ It is of this function of which we seek the zero to find the stress state:

$$\frac{\partial f^{*}}{\partial\Delta\lambda}=-\frac{\partial f}{\partial\boldsymbol{\sigma}}.\left(2\mu\tilde{\boldsymbol{G}}+KG\boldsymbol{I}\right)+\frac{\partial f}{\partial\gamma^{p}}\sqrt{\frac{2}{3}}\tilde{G}_{\mathrm{II}} \quad \text{éq 4.2-2}$$

4.3 Derivatives of the parameters compared to the variable of hardening

$$\begin{cases}\dfrac{\partial s}{\partial\gamma^{p}}=-\dfrac{1}{\gamma_{e}} & si\,\gamma^{p}<\gamma_{e}\\[2mm] \dfrac{\partial s}{\partial\gamma^{p}}=0 & si\,\gamma^{p}\geq\gamma_{e}\end{cases} \quad \text{éq 4.3-1}$$

$$\begin{cases}\dfrac{\partial m}{\partial s}=-\dfrac{\sigma_{c}}{\sigma_{\mathrm{p1}}} & si\,\gamma^{p}<\gamma_{e}\\[2mm] \dfrac{\partial m}{\partial s}=0 & si\,\gamma^{p}\geq\gamma_{e}\end{cases} \quad \text{éq 4.3-2}$$

$$\begin{cases}\dfrac{\partial m}{\partial a}=-\dfrac{\sigma_{c}}{\sigma_{\mathrm{p1}}}\log\left(m_{\mathrm{pic}}\dfrac{\sigma_{\mathrm{p1}}}{\sigma_{c}}+1\right)\dfrac{a_{\mathrm{pic}}}{a^{2}}\left(m_{\mathrm{pic}}\dfrac{\sigma_{\mathrm{p1}}}{\sigma_{c}}+1\right)^{\frac{a_{\mathrm{pic}}}{a}} & \mathrm{si}\,\gamma^{p}<\gamma_{e}\\[2mm] \dfrac{\partial m}{\partial a}=-\dfrac{\sigma_{c}}{\sigma_{\mathrm{p2}}}\log\left(m_{e}\dfrac{\sigma_{\mathrm{p2}}}{\sigma_{c}}\right)\dfrac{a_{e}}{a^{2}}\left(m_{e}\dfrac{\sigma_{\mathrm{p2}}}{\sigma_{c}}\right)^{\frac{a_{\mathrm{pic}}}{a}} & \mathrm{si}\,\gamma^{p}<\gamma_{e}\end{cases} \quad \text{éq 4.3-3}$$

$$\frac{\partial\Omega}{\partial\gamma^{p}}=\frac{\left(\gamma_{\mathrm{ult}}-\gamma_{e}\right)}{\left(\gamma_{e}\right)^{\eta}}\frac{a_{e}-a_{\mathrm{pic}}}{1-a_{e}}\left(\frac{\eta}{\gamma_{\mathrm{ult}}-\gamma^{p}}\left(\gamma^{p}\right)^{\eta-1}+\left(\gamma^{p}\right)^{\eta}\frac{1}{\left(\gamma_{\mathrm{ult}}-\gamma^{p}\right)^{2}}\right) \quad \text{éq 4.3-4}$$

$$\frac{\partial a}{\partial\boldsymbol{\Omega}}=\frac{1-a_{\mathrm{pic}}}{\left(1+\Omega\right)^{2}} \quad \text{éq 4.3-5}$$

$$\begin{aligned}&\frac{\partial m}{\partial\gamma^{p}}=\frac{\partial m}{\partial a}\frac{\partial a}{\partial\gamma^{p}}+\frac{\partial m}{\partial s}\frac{\partial s}{\partial\gamma^{p}} && \mathrm{si}\,\gamma^{p}<\gamma_{e}\\ &\frac{\partial m}{\partial\gamma^{p}}=\frac{\partial m}{\partial a}\frac{\partial a}{\partial\gamma^{p}} && \mathrm{si}\,\gamma_{\mathrm{ult}}\left(1-10^{-3}\right)>\gamma^{p}\geq\gamma_{e}\\ &\frac{\partial m}{\partial\gamma^{p}}=0 && \mathrm{si}\,\gamma_{\mathrm{ult}}\left(1-10^{-3}\right)<\gamma^{p}\end{aligned} \quad \text{éq 4.3-6}$$

$$
\begin{aligned}
&\frac{\partial k}{\partial \gamma^p} = -\left(\frac{2}{3}\right)^{\frac{1}{2a}} \log\left(\frac{2}{3}\right) \frac{1}{2a^2} \frac{\partial a}{\partial \gamma^p} \quad \gamma_{\text{ult}}\left(1-10^{-3}\right) > \gamma^p \\
&\frac{\partial k}{\partial \gamma^p} = 0 \quad \text{sinon}
\end{aligned}
$$

éq 4.3-7

5 tangent Operator of velocity

the condition

$$\dot{f}=0 \quad \text{éq 5-1}$$

is written:

$$\dot{f}=\frac{\partial f}{\partial \sigma_{ij}}\dot{\sigma}_{ij}+\frac{\partial f}{\partial \gamma^p}\dot{\gamma}^p=0$$

From the statement of the cumulated plastic deviatoric strain and $\gamma^p=\sqrt{\frac{2}{3}e^p_{ij}e^p_{ij}}$ relation $\dot{\boldsymbol{e}}^{\boldsymbol{p}}=\dot{\lambda}\tilde{\boldsymbol{G}}$, one finds the condition then:

$$\dot{f}=\frac{\partial f}{\partial \sigma_{ij}}\dot{\sigma}_{ij}+\frac{\partial f}{\partial \gamma^p}\sqrt{\frac{2}{3}}\dot{\lambda}\tilde{G}_{II}=0$$

What gives us for the plastic multiplier:

$$\dot{\lambda}=\frac{-\frac{\partial f}{\partial \sigma_{ij}}\dot{\sigma}_{ij}}{\sqrt{\frac{2}{3}}\frac{\partial f}{\partial \gamma^p}\tilde{G}_{II}}$$

By then considering the relation forced/strains:

$$\frac{\partial f}{\partial \sigma_{ij}}\dot{\sigma}_{ij}=\frac{\partial f}{\partial \sigma_{ij}}D_{ijkl}\dot{\varepsilon}_{kl}=\frac{\partial f}{\partial \sigma_{ij}}D_{ijkl}=\frac{\partial f}{\partial \sigma_{ij}}D_{ijkl}\dot{\varepsilon}_{kl}-\dot{\lambda}\frac{\partial f}{\partial \sigma_{ij}}D_{ijkl}G_{kl}$$

and by deferring it in the statement of $\dot{\lambda}$ one can write:

$$\dot{\lambda}=-\frac{\frac{\partial f}{\partial \sigma_{ij}}D_{ijkl}\dot{\varepsilon}_{kl}-\dot{\lambda}\frac{\partial f}{\partial \sigma_{ij}}D_{ijkl}G_{kl}}{\sqrt{\frac{2}{3}}\frac{\partial f}{\partial \gamma^p}\tilde{G}_{II}}$$

That is to say:

$$\dot{\lambda}=-\frac{\frac{\partial f}{\partial \sigma_{ij}}D_{ijkl}\dot{\varepsilon}_{kl}}{\sqrt{\frac{2}{3}}\frac{\partial f}{\partial \gamma^p}\tilde{G}_{II}-\frac{\partial f}{\partial \sigma_{ij}}D_{ijkl}G_{kl}} \quad \text{éq 5-2}$$

While deferring this result in the statement of $\dot{\sigma}_{ij}$ one finds:

$$\dot{\sigma}_{ab}=D_{abcd}\left(\dot{\varepsilon}_{cd}+\frac{\frac{\partial f}{\partial \sigma_{ij}}D_{ijkl}\dot{\varepsilon}_{kl}}{\sqrt{\frac{2}{3}}\frac{\partial f}{\partial \gamma^p}\tilde{G}_{II}-\frac{\partial f}{\partial \sigma_{ij}}D_{ijkl}G_{kl}}G_{cd}\right) \quad \text{éq 5-3}$$

6 Digital processing adapted to the nonregular models

the law of evolution of the plastic mechanism, defined in the chapter [§3], must satisfy the kinematical condition [éq 3.2.4-2]. The projection suggested on the norm of the hypersurface of strain can lead to a "NON-solution" which results in a failure of the digital processing (see the graphic interpretation of the chapter [§ 6.1.3.3]). One proposes in this chapter to define rules of projection allowing to manage the models known as "NON-regular" in their imposing projection called "to the top of the cone".

Moreover, as for other behavior models, one adds the possibility of cutting out locally (with Gauss points) time step to facilitate numerical integration.

6.1 Projection at the top of the cone

6.1.1 Definition of the jetting angle

One is placed in this chapter in the frame of finished increase. The equations translating the elastic behavior are written:

$$\boldsymbol{s}=\boldsymbol{s}^{-}+2\mu\left(\Delta\boldsymbol{e}-\Delta\boldsymbol{r}^{p}\right)=\boldsymbol{s}^{e}-2\mu\,\Delta\boldsymbol{e}^{p} \qquad \text{éq 6.1.1-1}$$

$$I_1=I_1^{-}+3\mathrm{K}\left(\Delta\varepsilon_v-\Delta\varepsilon_v^{p}\right)=I_1^{e}-\Delta\varepsilon_v^{p} \qquad \text{éq 6.1.1-2}$$

One can also express the kinematical condition starting from the tensor $\mathbf{n}$ (cf paragraph [§3.2.4]):

$$\boldsymbol{n}.\,\Delta\boldsymbol{\varepsilon}^{p}=0 \qquad \text{éq 6.1.1-3}$$

By deferring the two equations translating the elastic behavior in the preceding statement one finds:

$$\Delta\boldsymbol{e}^{p}=\frac{1}{2\mu}\left(\boldsymbol{s}^{e}-\boldsymbol{s}\right) \qquad \text{éq 6.1.1-4}$$

$$\Delta\varepsilon_v^{p}=\frac{1}{3\mathrm{K}}\left(I_1^{e}-I_1\right) \qquad \text{éq 6.1.1-5}$$

One then expresses the kinematical condition by the following relation:

$$\boldsymbol{n}.\left(\frac{1}{2\mu}\left(\boldsymbol{s}^{e}-\boldsymbol{s}\right)+\frac{1}{3\mathrm{K}}\left(\frac{1}{3\mathrm{K}}\left(I_1^{e}-I_1\right)\boldsymbol{I}\right)\right)=0 \text{ with } \boldsymbol{n}=\frac{\beta'\dfrac{\boldsymbol{s}}{s_{\mathrm{II}}}+\boldsymbol{I}}{\sqrt{\beta'^2+3}}$$

Is by combining the two preceding relations where $\mathbf{n}$ the norm of the hypersurface of strain indicates:

$$\frac{1}{2\mu}\frac{\beta'\dfrac{\boldsymbol{s}}{s_{\mathrm{II}}}+\boldsymbol{I}}{\sqrt{\beta'^2+3}}\left(\boldsymbol{s}^{e}-\boldsymbol{s}\right)+\frac{1}{9\mathrm{K}}\left(I_1^{e}-I_1\right).\,\mathrm{Tr}\left(\boldsymbol{n}\right)=0$$

$$\frac{1}{2\mu}\beta'\frac{\boldsymbol{s}.\left(\boldsymbol{s}^{e}-\boldsymbol{s}\right)}{s_{\mathrm{II}}}+\frac{1}{3\mathrm{K}}\left(I_1^{e}-I_1\right)$$

This last equation defines the point $\left(I_1, \mathbf{s}\right)$ as a projection of the point $\left(I_1^e, \mathbf{s}^e\right)$ on the criterion. The point $\left(I_1, s_{II}\right)$ will be the oblique projection of the point $\left(I_1^e, s_{II}^e\right)$, projection whose direction varies with θ . One can give the chart of it of the chapter [§ 6.1.3.3].

The preceding relation can then be rewritten as follows:

$$I_1^e - I_1 = -\beta' \frac{3\mathrm{K}}{2\mu} \frac{\boldsymbol{s}.\left(\boldsymbol{s}^e - \boldsymbol{s}\right)}{s_{\mathrm{II}}} \quad \text{éq 6.1.1-6}$$

One then defines the jetting angle φ_s by the relation:

$$\cos\varphi_s = \frac{\boldsymbol{s}.\left(\boldsymbol{s}^e - \boldsymbol{s}\right)}{s_{\mathrm{II}} \sqrt{\left(\boldsymbol{s}^e - \boldsymbol{s}\right)\left(\boldsymbol{s}^e - \boldsymbol{s}\right)}} \quad \text{éq 6.1.1-7}$$

By deferring the definition of the angle φ_s in the relation of projection one finds the relation:

$$\frac{I_1^e - I_1}{\sqrt{\left(\boldsymbol{s}^e - \boldsymbol{s}\right)\left(\boldsymbol{s}^e - \boldsymbol{s}\right)}} = -\beta' \frac{3\mathrm{K}}{2\mu} \cos\varphi_s \quad \text{éq 6.1.1-8}$$

6.1.2 Existence of projection

the principle of this paragraph is to discuss on question the existence the angle φ_s such as projection the point $\left(I_1^e, \boldsymbol{s}^e\right)$ always belongs to the surface of load. These problems appear essential for projections around the top of the surface of load, in other words when $\boldsymbol{s} \rightarrow \mathbf{0}$. There is by definition the relation:

$$\cos\varphi_s = \frac{\boldsymbol{s}.\left(\boldsymbol{s}^e - \boldsymbol{s}\right)}{s_{\mathrm{II}} \sqrt{\left(\boldsymbol{s}^e - \boldsymbol{s}\right)\left(\boldsymbol{s}^e - \boldsymbol{s}\right)}} = \frac{\boldsymbol{s}.\left(\boldsymbol{s}^e - \boldsymbol{s}\right)}{s_{\mathrm{II}} \left\|\boldsymbol{s}^e - \boldsymbol{s}\right\|} \quad \text{éq 6.1.2-1}$$

By combining this equation with the statement: $\boldsymbol{s} = \boldsymbol{s}^e - 2\mu\, \Delta \boldsymbol{e}^p = \boldsymbol{s}^e - 2\mu\, \Delta\lambda\, \tilde{\boldsymbol{G}}$

One obtains:

$$\cos\varphi_s = \frac{\boldsymbol{s}.\tilde{\boldsymbol{G}}}{s_{\mathrm{II}} \tilde{G}_{II}} \quad \text{éq 6.1.2-2}$$

One seeks an estimate of $\cos\varphi_s$.

Stage 1: estimate of $\dfrac{\mathbf{s}.\tilde{\mathbf{G}}}{s_{II}}$

One is placed in this paragraph under the conditions: $\boldsymbol{s} \rightarrow \mathbf{0}$ and $f = 0$.

By definition of $\tilde{\mathbf{G}}$ and $\mathbf{G}$ one a: $\tilde{\boldsymbol{G}}.\boldsymbol{s} = \left(\boldsymbol{G} - \frac{\mathrm{Tr}\left(\boldsymbol{G}\right)}{3}\boldsymbol{I}\right).\boldsymbol{s} = \boldsymbol{G}.\boldsymbol{s} = \left(\frac{\partial f}{\partial \boldsymbol{\sigma}} - \left(\frac{\partial f}{\partial \boldsymbol{\sigma}}\boldsymbol{n}\right)\boldsymbol{n}\right).\boldsymbol{s}$

For preoccupations with a simplification of computation one brings back the resolution of f to the resolution of the equation:

$$f=\left(\frac{g(\boldsymbol{s})}{\sigma_c h_c^0}\right)^{\frac{1}{a(\gamma^p)}}-u(\boldsymbol{\sigma},\gamma^p)=0\Rightarrow f_2=\left(\frac{g(\boldsymbol{s})}{\sigma_c h_c^0}\right)-u(\boldsymbol{\sigma},\gamma^p)^{a(\gamma^p)}=0 \qquad 6.1.2.3$$

By derivative of this new function one finds the relation:

$$\frac{\partial f_2}{\partial\boldsymbol{\sigma}}=\left(\frac{1}{\sigma_c h_c^0}\right)\frac{\partial g}{\partial\boldsymbol{\sigma}}-a(\gamma^p)u(\boldsymbol{\sigma},\gamma^p)^{a(\gamma^p)-1}\frac{\partial u}{\partial\boldsymbol{\sigma}}=\left(\frac{1}{\sigma_c h_c^0}\right)\boldsymbol{Q}-a(\gamma^p)u(\boldsymbol{\sigma},\gamma^p)^{a(\gamma^p)-1}\frac{\partial u}{\partial\boldsymbol{\sigma}}$$

with: $\frac{\partial u}{\partial\boldsymbol{\sigma}}=-\frac{m(\gamma^p)k(\gamma^p)}{\sigma_c}\left(\frac{1}{\sqrt{6}h_c^0}\boldsymbol{Q}+\frac{1}{3}\boldsymbol{I}\right)$

Who gives after simplification:

$$\frac{\partial f_2}{\partial\boldsymbol{\sigma}}=A\boldsymbol{Q}+B\boldsymbol{I} \qquad \text{éq 6.1.2.4}$$

Where:

$$\begin{cases}A=\frac{1}{\sigma_c h_c^0}\left(1+\frac{a(\gamma^p)m(\gamma^p)k(\gamma^p)}{\sqrt{6}}u(\sigma,\gamma^p)^{a(\gamma^p)-1}\right)\\ B=\frac{a(\gamma^p)m(\gamma^p)k(\gamma^p)}{3\sigma_c h_c^0}u(\boldsymbol{\sigma},\gamma^p)^{a(\gamma^p)-1}\end{cases} \qquad \text{éq 6.1.2.5}$$

One has as follows: $\frac{\partial f_2}{\partial\boldsymbol{\sigma}}.\boldsymbol{n}=(A\boldsymbol{Q}+b\boldsymbol{I})\frac{\beta'\frac{\boldsymbol{s}}{s_{\mathrm{II}}}+\boldsymbol{I}}{\sqrt{\beta'^2+3}}=\frac{\beta'}{\sqrt{\beta'^2+3}}\frac{A}{s_{\mathrm{II}}}\boldsymbol{Q.s}+\frac{3\mathrm{B}}{\sqrt{\beta'^2+3}}$

And consequently:

$$\begin{aligned}\tilde{\boldsymbol{G}}.\boldsymbol{s}&=\left(\frac{\partial f}{\partial\boldsymbol{\sigma}}-\left(\frac{\partial f}{\partial\boldsymbol{\sigma}}\boldsymbol{n}\right)\boldsymbol{n}\right).\boldsymbol{s}\\ &=\left(A\boldsymbol{Q}+B\boldsymbol{I}-\left(\frac{\beta'}{\sqrt{\beta'^2+3}}\frac{A}{s_{\mathrm{II}}}\boldsymbol{Q.s}+\frac{3\mathrm{B}}{\sqrt{\beta'^2+3}}\right)\frac{\beta'\frac{\boldsymbol{s}}{s_{\mathrm{II}}}+\boldsymbol{I}}{\sqrt{\beta'^2+3}}\right)\\ &=\frac{3\mathrm{A}}{\beta'^2+3}\boldsymbol{Q.s}-\frac{3\mathrm{B}\beta'}{\beta'^2+3}s_{\mathrm{II}}\end{aligned}$$

From where it is deduced that:

$$\frac{\tilde{\boldsymbol{G}}.\boldsymbol{s}}{s_{\mathrm{II}}}=\frac{3\mathrm{A}}{\beta'^2+3}\frac{\boldsymbol{Q.s}}{s_{\mathrm{II}}}-\frac{3\mathrm{B}\beta'}{\beta'^2+3} \qquad \text{éq 6.1.2.6}$$

By definition of $\mathbf{Q}$ one a:

$$\boldsymbol{Q}\,.\,\boldsymbol{s}=\mathrm{dev}\left(\frac{\partial g}{\partial \boldsymbol{s}}\right).\,\boldsymbol{s}=\frac{1}{h\left(\theta\right)^{5}}\left[\left(1+\frac{\gamma_{\mathrm{cjs}}}{2}\cos\left(3\theta\right)\right)\frac{\boldsymbol{s}}{s_{\mathrm{II}}}+\frac{\gamma_{\mathrm{cjs}}\sqrt{54}}{6\,s_{\mathrm{II}}^{2}}\mathrm{dev}\left(\right)\right].\,\boldsymbol{s}$$

$$=\frac{1}{h\left(\theta\right)^{5}}\left(1+\frac{\gamma_{\mathrm{cjs}}}{2}\cos\left(3\theta\right)\right)s_{\mathrm{II}}$$

$$=h\left(\theta\right)s_{\mathrm{II}}$$

One expresses finally:

$$\frac{\tilde{\boldsymbol{G}}\,.\,\boldsymbol{s}}{s_{\mathrm{II}}}=\frac{3\mathrm{A}}{\beta'^{2}+3}h\left(\theta\right)-\frac{3\beta'}{\beta'^{2}+3} \qquad \text{éq 6.1.2.7}$$

When $\boldsymbol{s}\rightarrow\boldsymbol{0}$ then $u\left(\boldsymbol{\sigma},\gamma^{p}\right)\rightarrow 0 \text{ et } A\rightarrow\frac{1}{\sigma_{c}h_{c}^{0}},\,B\rightarrow 0$

And thus:

When $\boldsymbol{s}\rightarrow\boldsymbol{0}$ then $\frac{\tilde{\boldsymbol{G}}\,.\,\boldsymbol{s}}{s_{\mathrm{II}}}\underset{\boldsymbol{s}\rightarrow\boldsymbol{0}}{\rightarrow}\frac{3\mathrm{h}\left(\theta\right)}{\sigma_{c}h_{c}^{0}\left(\beta'^{2}+3\right)}$ éq 6.1.2.8

Stage 2: estimate of $\tilde{G}_{II}$

One is placed in this paragraph under the conditions: $\boldsymbol{s}\rightarrow\boldsymbol{0},\,A\rightarrow\frac{1}{\sigma_{c}h_{c}^{0}},\,b\rightarrow 0$

$$\tilde{\boldsymbol{G}}=\left(\frac{\partial f}{\partial\boldsymbol{\sigma}}-\left(\frac{\partial f}{\partial\boldsymbol{\sigma}}\boldsymbol{n}\right)\boldsymbol{n}\right)$$

$$\left(\frac{1}{\sigma_{c}h_{c}^{0}}\boldsymbol{Q}+B\,\boldsymbol{I}-\left(\frac{\beta'}{\sqrt{\beta'^{2}+3}}\frac{1}{\sigma_{c}h_{c}^{0}s_{\mathrm{II}}}\boldsymbol{Q.}\,\boldsymbol{s}+\frac{3\mathrm{B}}{\sqrt{\beta'^{2}+3}}\right)\frac{\beta'\frac{\boldsymbol{s}}{s_{\mathrm{II}}}+\boldsymbol{I}}{\sqrt{\beta'^{2}+3}}\right)$$

$$\frac{1}{\sigma_{c}h_{c}^{0}}\boldsymbol{Q}-\frac{\beta'^{2}h\left(\theta\right)}{\left(\beta'^{2}+3\right)\sigma_{c}h_{c}^{0}s_{\mathrm{II}}}\boldsymbol{s}$$

$$\tilde{G}_{\mathrm{II}}^{2}\,\tilde{\boldsymbol{G}}\,.\,\tilde{\boldsymbol{G}}=\frac{Q_{\mathrm{II}}^{2}}{\left(\sigma_{c}h_{c}^{0}\right)^{2}}+\frac{\beta'^{4}h^{2}\left(\theta\right)s_{\mathrm{II}}^{2}}{\left(\beta'^{2}+3\right)^{2}\left(\sigma_{c}h_{c}^{0}\right)_{2}s_{\mathrm{II}}^{2}}-2\frac{\beta'^{2}h^{2}\left(\theta\right)}{\left(\beta'^{2}+3\right)\left(\sigma_{c}h_{c}^{0}\right)^{2}}$$

$$\frac{1}{\sigma_{c}h_{c}^{02}}\left(Q_{\mathrm{II}}^{2}-\frac{\beta'^{2}\left(\beta'^{2}+6\right)h^{2}\left(\theta\right)}{\left(\beta'^{2}+3\right)^{2}}\right)$$

It is shown [Appendix 2] that:

$$Q_{\mathrm{II}}^{2}=\frac{1}{h\left(\theta\right)^{10}}\left[\left(1+\frac{\gamma_{\mathrm{cjs}}}{2}\cos\left(3\theta\right)\right)_{2}+\frac{\gamma_{\mathrm{cjs}}^{2}}{4}+\gamma_{\mathrm{cjs}}\cos\left(3\theta\right)\left(1+\frac{\gamma_{\mathrm{cjs}}}{2}\cos\left(3\theta\right)\right)\right] \qquad \text{éq 6.1.2.9}$$

and thus like $h(\theta)=\left(1+\cos(3\theta)\right)^{1/6}$:

$$\tilde{G}_{\mathrm{II}}^{2}=\frac{1}{\sigma_c h_c^{22}}\left(\frac{1}{h(\theta)^{10}}\left[\left(\frac{1}{2}+\frac{h(\theta)^6}{2}\right)^2+\frac{\gamma_{\mathrm{cjs}}{}^2}{4}+\left(h(\theta)^6-1\right)\left(\frac{1}{2}+\frac{h(\theta)^6}{2}\right)\right]-\frac{\beta'^2\left(\beta'^2+6\right)h^2(\theta)}{\left(\beta'^2+3\right)^2}\right)$$

$$\tilde{G}_2^{\mathrm{II}}=\frac{1}{\left(\sigma_c h_c^0\right)^2}\left(\frac{3\mathrm{h}(\theta)^2}{4}+\frac{1}{2h(\theta)^4}+\frac{\gamma_{\mathrm{cjs}}^2-1}{4\mathrm{h}(\theta)^{10}}-\frac{\beta'^2\left(\beta'^2+6\right)h^2(\theta)}{\beta'^2+3^2}\right)$$

$$\tilde{G}_{\mathrm{II}}^{2}=\left(\frac{h(\theta)}{\sigma_c h_c^0}\right)^2\left[\frac{1}{2\mathrm{h}(\theta)^6}+\frac{\gamma_{\mathrm{cjs}}^2-1}{4h(\theta)^{12}}+\left(\frac{3}{\beta'^2+3}\right)^2-\frac{1}{4}\right]$$

And consequently:

$$\tilde{G}_{\mathrm{II}}=\left(\frac{h(\theta)}{\sigma_c h_c^0}\right)\sqrt{\left(\frac{3}{\beta'^2+3}\right)^2-\frac{1}{4}+\frac{1}{2\mathrm{h}(\theta)^6}+\frac{\gamma_{\mathrm{cjs}}^2-1}{4\mathrm{h}(\theta)^{12}}} \qquad \text{éq 6.1.2.10}$$

Stage 3: estimate of $\cos(\varphi_s)$

One deduces from the two paragraphs precedent the statement of the following jetting angle:
When $\boldsymbol{s}\rightarrow\mathbf{0}$ then:

$$\cos\varphi_s tward \frac{3}{\left(\beta'^2+3\right)\sqrt{\frac{3}{\beta'^2+3}^2-\frac{1}{4}+\frac{1}{2\left(1+\gamma_{\mathrm{cjs}}\cos(3\theta)\right)}+\frac{\gamma_{\mathrm{cjs}}^2-1}{4\left(1+\gamma_{\mathrm{cjs}}\cos(3\theta)\right)^2}}} \qquad \text{éq 6.1.2.11}$$

One notices that φ_s depends on the angle of Lode θ , and that consequently limit of the jetting angle when $\boldsymbol{s}\rightarrow\mathbf{0}$ does not exist. However a framing of $\cos\varphi_s$ enables us to determine a zone of projection at the top a priori (demonstration of the framing in [Appendix 3]):

$$\cos\varphi_s^{\min}\leq\cos\varphi_s\leq\cos\varphi_s^{\max}$$

with:

$$\begin{cases}\cos\varphi_s^{\min}=\dfrac{3}{\left(\beta'^2+3\right)\sqrt{\left(\dfrac{3}{\beta'^2+3}\right)^2+\dfrac{\gamma_{\mathrm{cjs}}{}^2}{4\left(1-\gamma_{\mathrm{cjs}}^2\right)}}}\\ \cos\varphi_s^{\max}=1\end{cases} \qquad \textbf{éq 6.1.2.12}$$

6.1.3 Rules of projection

One calls I_1^0 the intersection of the field of reversibility with the hydrostatic axis. One obtains:

$$I_1^0=\frac{3\sigma_c . s(\gamma_p)}{m(\gamma_p)} \quad \text{éq 6.1.3-1}$$

While deferring I_1^0 and the framing of $\cos\varphi_s$, when $\boldsymbol{s}\rightarrow\boldsymbol{0}$, in the relation $\frac{I_1^e-I_1}{\sqrt{(\boldsymbol{s}^e-\boldsymbol{s})(\boldsymbol{s}^e-\boldsymbol{s})}}=-\beta'\frac{3\mathrm{K}}{2\mu}\cos\varphi_s$, one from of deduced the following rules of projection according to the sign from the parameter of dilatancy β' , and for values of I_1^e and s_{II}^e data.

6.1.3.1 Case where the parameter of dilatancy is negative

So $\frac{I_1^e-I_1^0}{s_{\mathrm{II}}^e}<-\beta'\frac{3\mathrm{K}}{2\mu}\cos\varphi_s^{\min}$ then projection will be regular;

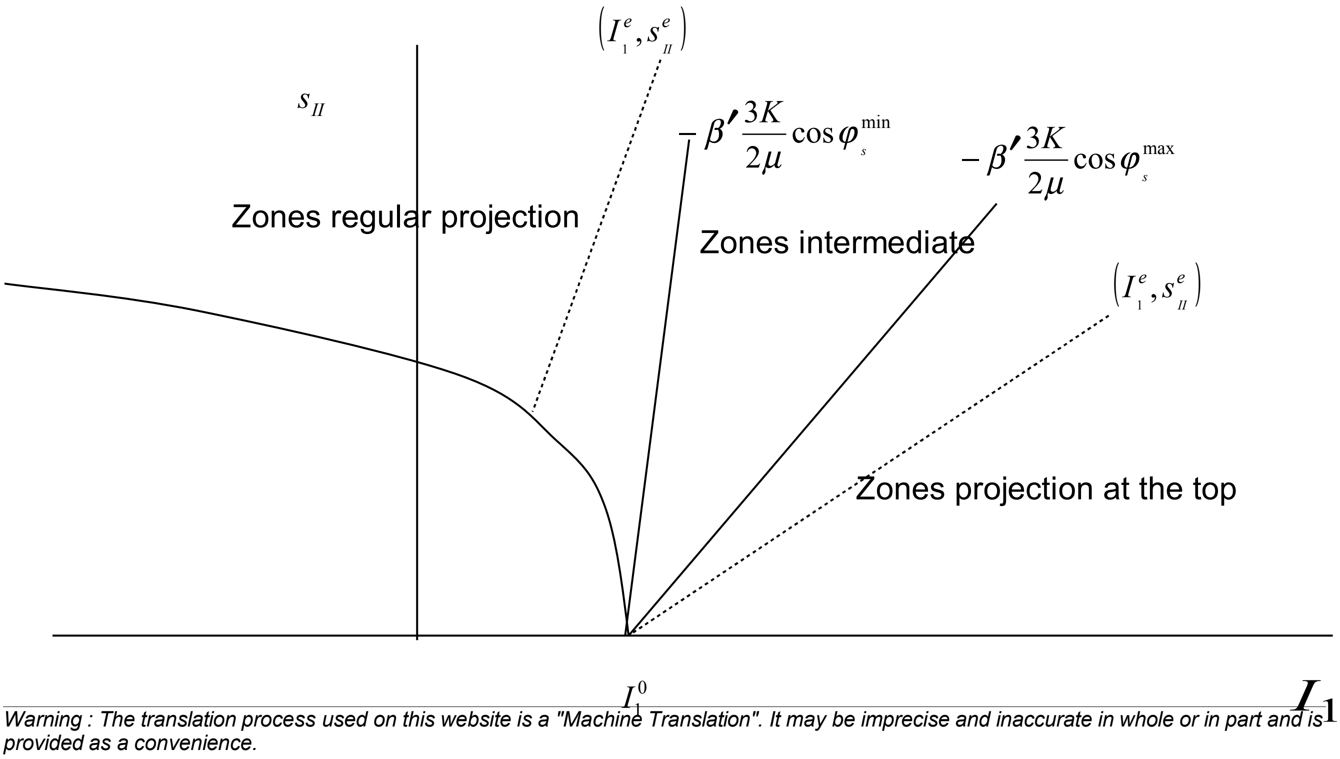
So $\frac{I_1^e-I_1^0}{s_{\mathrm{II}}^e}>-\beta'\frac{3\mathrm{K}}{2\mu}\cos\varphi_s^{\max}$ then projection will be at the top.

6.1.3.2 Case where the parameter of dilatancy is positive

So $\frac{I_1^e-I_1^0}{s_{\mathrm{II}}^e}<-\beta'\frac{3\mathrm{K}}{2\mu}\cos\varphi_s^{\max}$ then projection will be regular;

So $\frac{I_1^e-I_1^0}{s_{\mathrm{II}}^e}>-\beta'\frac{3\mathrm{K}}{2\mu}\cos\varphi_s^{\min}$ then projection will be at the top.

6.1.3.3 Graphic interpretation



6.1.3.4 flow Equations

In the intermediate zone one solves the equations corresponding to a regular projection. If this resolution does not give a solution one then solves the flow equations of projection at the top.

In the case of projection at the top there are the relations:

$$\mathbf{s}=\mathbf{0} \quad \text{éq 6.1.3.4 - 1}$$

$$I_1^0=\frac{3\sigma_c . s\left(\gamma^p\right)}{m\left(\gamma^p\right)} \quad \text{éq 6.1.3.4 - 2}$$

$$\Delta\gamma^p=\frac{1}{2\mu}\sqrt{\frac{2}{3}}s_{\mathrm{II}}^e \quad \text{éq 6.1.3.4 - 3}$$

6.2 local Recutting of time step

As for other behavior models (the model CJS for example) one added the possibility for the model of LAIGLE of redécouper locally (with Gauss points) time step in order to facilitating numerical integration. This possibility is managed by operand ITER_INTE_PAS of key word CONVERGENCE of operator STAT_NON_LINE. If the value of ITER_INTE_PAS (itepas) is worth 0,1 or – 1 it N `has no recutting there (note: 0 are the value by default). If itepas is positive recutting is systematic, if it is negative recutting is taken into account only in the event of nonnumerical convergence.

Recutting consists in carrying out the integration of the plastic mechanism with an increment of strain whose components correspond to the components of the initial increment of strain divided by the absolute value of itepas (cf Doc. STAT_NON_LINE [U4.51.03]).

7 The local variables

For implementation the data-processing we retained the 4 following local variables:

7.1 V1: the plastic strain déviatoire cumulated

the variable of hardening γ^p is proportional to the second invariant of the tensor of the strains déviatoires.

$$\gamma^p = \sqrt{\frac{2}{3} e_{ij}^p e_{ij}^p}$$

$$\text{with } e_{ij}^p = \varepsilon_{ij}^p - \frac{\mathrm{tr}\left(\varepsilon_{ij}^p\right)}{3} \delta_{ij}$$

7.2 V2: the cumulated plastic voluminal strain

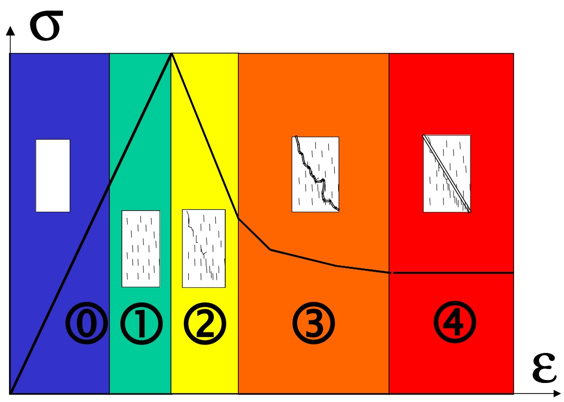
the plastic voluminal strain is defined by the relation presented to the paragraph [§3.2.4] on the law of evolution of the plastic mechanism: $\dot{\varepsilon}_v^p = \dot{\lambda} G$

7.3 V3: the fields of behavior of the rock

Five fields of behavior, numbered from 0 to 4 (cf appears), are identified to make it possible to have a relatively simple representation of the state of damage of the rock, since the intact rock to the rock in a residual state. These fields are function of the cumulated plastic strain déviatoire γ^p and the stress state. Each increment of number of field defines the transition in a field of higher damage.

- If the deviator is lower than 70% of the deviator of peak, then the material is in field 0;
- If not:
  - So $\gamma^p = 0$ then the material is in field 1;
    1) So $0 < \gamma^p < \gamma^e$ then the material is in field 2;
  - So $\gamma_e < \gamma^p < \gamma_{ult}$ then the material is in field 3;
  - So $\gamma^p > \gamma_{ult}$ then the material is in field 4.

Domaine	Etat de la roche
0	Intacte
1	Endommagement pré-pic
2	Endommagement post-pic
3	Fissurée
4	Fracturée



7.4 V4: the state of plasticization

It is an internal indicator with *the Code_Aster*. It is worth 0 if the point of gauss is in elastic load or discharge, and is worth 1 if the point of gauss is in plastic load.

8 Detailed presentation of the algorithm

One retains an implicit formulation compared to the criterion and clarifies flow direction compared to the: the criterion will have to be checked at the end of the step, whereas the flow direction is that calculated at the beginning of the step (and thus the value of dilatancy will be also that calculated at the beginning of time step).

One places oneself in a material point, and one considers that are given:

- The tensor of increase in strains $\Delta\varepsilon$ from where one deduces Δe and; $\Delta\varepsilon_v$
- Stresses at the beginning of the step from where $\boldsymbol{\sigma}^-$ one deduces $\boldsymbol{s}^-$ and I_1^- ;
- Values of the local variables at the beginning of time step (only the cumulated plastic strain γ^{p^-} is necessary).

It is a question of calculating:

- Stresses at the end of time step $\boldsymbol{\sigma}$;
- Local variables in the end of time step (γ^p ε_v^p , fields of behavior);
- The tangent behavior at the end of the step: $\dfrac{\partial\boldsymbol{\sigma}}{\partial\boldsymbol{\varepsilon}}$

8.1 Computation of the elastic solution

$$\Delta\varepsilon^e=\Delta\varepsilon^- -\alpha\Delta T$$
$$\boldsymbol{s}^e=\boldsymbol{s}^- +2\mu\Delta\boldsymbol{e}$$
$$I_¿^e=I_1^- +3\mathrm{K}\Delta\varepsilon_v$$

8.2 Computation of the elastic criterion

Computation of $g^e=s_{II}^e h\left(\theta^e\right)$

Computation of $m^-=m\left(\gamma^{p^-}\right)$ $s^-=s\left(\gamma^{p^-}\right)$, $a^-=a\left(\gamma^{p^-}\right)$ and $k^-=k\left(a^-\right)$

Computation of $u^e=-\dfrac{m^- k^-}{\sqrt{6}\sigma_c}\dfrac{g^e}{h_c^0}-\dfrac{m^- k^-}{3\sigma_c}I_1^e+s^-.k^-$

Computation of $f^e=\left(\dfrac{g^e}{\sigma_c h_c^0}\right)^{\frac{1}{a^-}}-u^e$

8.3 Algorithm

If $f^e > 0$

Computation of:

$$I_1^{0-} = \frac{3\sigma_c . s^-}{m^-}; \quad g^- = g\left(\boldsymbol{s}^-\right)$$

$$\phi_0^- = \phi_0\left(m^-, s^-, a^-\right); \quad C_0^- = C_0\left(m^-, s^-, a^-\right); \quad \sigma_0^- = \sigma_{t0}\left(\phi_0^-, C_0^-\right)$$

$$\alpha'^- = \alpha'\left(I_1^-, g^-, \sigma_{t0}^-\right); \quad \psi^- = \psi\left(\alpha'^-\right); \quad \beta'^- = \beta'\left(\psi^-\right)$$

Computation a priori of projection at the top

$\boldsymbol{s} = \boldsymbol{0}$; Computation of $\gamma^p = \gamma^{p^-} + \frac{1}{2\mu}\sqrt{\frac{2}{3}} s_{\mathrm{II}}^e = \gamma^{p^{\mathrm{sommet}}}$ and of $I_1 = \frac{3\sigma_c . s\left(\gamma^p\right)}{m\left(\gamma^p\right)} = I_1^{\mathrm{sommet}}$.

If $\begin{cases} \left(I_1^e - I_1^{\mathrm{sommet}}\right) < -\frac{3\mathrm{K}}{2\mu}\beta' s_n^e \cos\varphi_s^{\max}; \mathrm{si}\ \beta'^- < 0 \\ \left(I_1^e - I_1^{\mathrm{sommet}}\right) < -\frac{3\mathrm{K}}{2\mu}\beta' s_n^e \cos\varphi_s^{\min}; \mathrm{si}\ \beta'^- \geq 0 \end{cases}$

projection at the top is not retained a priori. The regular solution is calculated.

$$\boldsymbol{Q}^- = \begin{cases} \boldsymbol{Q}\left(\boldsymbol{\sigma}^-\right) \mathrm{si}\ \boldsymbol{\sigma}^- \neq 0 \\ \boldsymbol{Q}\left(\boldsymbol{\sigma}^{\mathbf{e}}\right) \mathrm{si}\ \boldsymbol{\sigma}^- = 0 \end{cases} \quad \boldsymbol{n}^f = \begin{cases} \boldsymbol{n}\left(\beta'^-, \boldsymbol{\sigma}^-\right) \mathrm{si}\ \boldsymbol{\sigma}^- \neq 0 \\ \boldsymbol{n}\left(\beta'^{\mathrm{e}}, \boldsymbol{\sigma}^{\mathbf{e}}\right) \mathrm{si}\ \boldsymbol{\sigma}^- = 0 \end{cases} \quad \boldsymbol{G}^f = \begin{cases} \boldsymbol{G}\left(\beta'^-, \boldsymbol{\sigma}^-\right) \mathrm{si}\ \boldsymbol{\sigma}^- \neq 0 \\ \boldsymbol{G}\left(\beta'^{\mathrm{e}}, \boldsymbol{\sigma}^{\mathbf{e}}\right) \mathrm{si}\ \boldsymbol{\sigma}^- = 0 \end{cases}$$

If $\gamma^{p^-} = 0$

Initialization $\Delta\lambda^0 = 0; \gamma^{p^0} = \gamma^{p^-}; \boldsymbol{s}^{\mathbf{0}} = \boldsymbol{s}^{\boldsymbol{e}}; I_1^0 = I_1^e; f^0 = f^e$

And $\begin{cases} \Delta\gamma^{p^1} = \frac{1}{10}\max|\Delta\varepsilon_{\mathrm{ij}}^e| \\ \delta\lambda^{p^1} = \frac{\Delta\gamma^{p^1}}{\tilde{G}_{\mathrm{II}}^{f_b}}\sqrt{\frac{2}{3}} \end{cases}$

If not

Computation of the increase in the plastic multiplier $\Delta\lambda$ by Newton:

Initialization $\Delta\lambda^0 = 0; \gamma^{p^0} = \gamma^{p^-}; \boldsymbol{s}^{\mathbf{0}} = \boldsymbol{s}^{\boldsymbol{e}}; I_1^0 = I_1^e; f^0 = f^e$

$$\frac{\partial u^0}{\partial\boldsymbol{\sigma}} = \frac{\partial u^-}{\partial\boldsymbol{\sigma}} = -\frac{m^-}{\sqrt{6}\sigma_c}\frac{k^-}{h_c^0}\boldsymbol{Q}^- - k^-\frac{m^-}{3\sigma_c}\boldsymbol{I}$$

$$\frac{\partial u^0}{\partial\gamma^p} = -\frac{1}{\sqrt{6}\sigma_c}\frac{\partial(km)}{\gamma^p}\left(\gamma^{p^-}\right)\frac{g^e}{h_c^0} - \frac{1}{3\sigma_c}\frac{\partial(km)}{\partial\gamma^p}\left(\gamma^{p^-}\right)I_1^e + \frac{\partial(ks)}{\partial\gamma^p}\left(\gamma^{p^-}\right)$$

$$\frac{\partial f^0}{\partial\boldsymbol{\sigma}} = \frac{1}{a^-}\left(\frac{1}{\sigma_c h_c^0}\right)^{\frac{1}{a^-}}\left(g^e\right)\frac{1-a^-}{a^-}\boldsymbol{Q}^- - \frac{\partial u^0}{\partial\boldsymbol{\sigma}} \neq \frac{\partial f^-}{\partial\boldsymbol{\sigma}}$$

$$\frac{\partial f^0}{\partial\gamma^p} = -\left(\frac{1}{a^-}\right)^2\left(\frac{g^e}{\sigma_c h_c^0}\right)^{\frac{1}{a^-}}\log\left(\frac{g^e}{\sigma_c h_c^0}\right)\frac{\partial a}{\partial\gamma^p}\left(\gamma^{p^-}\right) - \frac{\partial u^0}{\partial\gamma^p} \neq \frac{\partial f^-}{\partial\gamma^p}$$

$$\frac{\partial f^{0^*}}{\partial \Delta\lambda}=-\frac{\partial f^0}{\partial \boldsymbol{\sigma}}.\left(2\mu\tilde{\boldsymbol{G}}^f+KG^f\boldsymbol{I}\right)+\frac{\partial f^0}{\partial \gamma^p}\sqrt{\frac{2}{3}}\tilde{G}^f_{\mathrm{II}}$$

Buckles iterations N

$$\frac{\partial f^n}{\partial \Delta\lambda}\delta\lambda^{n+1}=-f^n$$

$$\Delta\lambda^{n+1}=\Delta\lambda^n+\delta\lambda^{n+1}$$

$$\Delta\gamma^{p^{n+1}}=\Delta\gamma^{n+1}\sqrt{\frac{2}{3}}\tilde{G}^f_{\mathrm{II}};\quad \Delta\varepsilon_v^p=\Delta\lambda^{n+1}G^f$$

$$\boldsymbol{s}^{\boldsymbol{n+1}}=\boldsymbol{s}^{\boldsymbol{e}}-2\mu\Delta\lambda^{p^{n+1}}\tilde{\boldsymbol{G}}^f\quad;\quad I_1^{n+1}=I_1^e-3\mathrm{K}\Delta\lambda^{p^{n+1}}G^f$$

So $\Delta\gamma^{p^{n+1}}<0$ No convergence

Computation $\boldsymbol{Q}^{\boldsymbol{n+1}}$

$$g^{n+1}=g\left(\boldsymbol{s}^{\boldsymbol{n+1}}\right);\ m^{n+1}=m\left(\gamma^{p^{n+1}}\right);\ s^{n+1}=s\left(\gamma^{p^{n+1}}\right);\ a^{n+1}=a\left(\gamma^{p^{n+1}}\right);\ k^{n+1}=k\left(a^{n+1}\right)$$

$$u_{n+1}=-\frac{m^{n+1}k^{n+1}}{\sqrt{6}\sigma_c}\frac{g^{n+1}}{h_c^0}-\frac{m^{n+1}k^{n+1}}{3\sigma_c}I_1^{n+1}+s^{n+1}.k^{n+1}$$

$$f^{n+1}=\left(\frac{g^{n+1}}{\sigma_c h_c^0}\right)^{\frac{1}{a^{n+1}}}-u^{n+1}$$

$$\frac{\partial u^{n+1}}{\partial\sigma}=-\frac{m}{\sqrt{6}\sigma_c}\frac{k^{n+1}}{h_c^0}\boldsymbol{Q}^{\boldsymbol{n+1}}-k^{n+1}\frac{m^{n+1}}{3\sigma_c}\boldsymbol{I}$$

$$\frac{\partial u}{\partial\gamma^p}^{n+1}=-\frac{1}{\sqrt{6}\sigma_c}\frac{\partial(km)}{\partial\gamma^p}\left(\gamma^{p^{n+1}}\right)\frac{g^{n+1}}{h_c^0}-\frac{1}{3\sigma_c}\frac{\partial(km)}{\partial\gamma^p}\left(\gamma^{p^{n+1}}\right)I_1^{n+1}+\frac{\partial(ks)}{\partial\gamma^p}\left(\gamma^{p^{n+1}}\right)$$

$$\frac{\partial f^{n+1}}{\partial\boldsymbol{\sigma}}=\frac{1}{a^{n+1}}\left(\frac{A}{\sigma_c h_c^0}\right)^{\frac{1}{a^{n+1}}}\left(g^{n+1}\right)^{\frac{A-a^{n+1}}{a^{n+1}}}\boldsymbol{Q}^{\boldsymbol{n+1}}-\frac{\partial u^{n+1}}{\partial\boldsymbol{\sigma}}$$

$$\frac{\partial f^{n+1}}{\partial\gamma^p}=-\left(\frac{1}{a^{n+1}}\right)^2\left(\frac{g^{n+1}}{\sigma_c h_c^0}\right)^{\frac{1}{a^{n+1}}}\log\left(\frac{g^{n+1}}{\sigma_c h_c^0}\right)\frac{\partial\alpha}{\partial\gamma^p}\left(\gamma^{p^{n+1}}\right)-\frac{\partial u^{n+1}}{\partial\gamma^p}$$

$$\frac{\partial f^{n+1^*}}{\partial\Delta\lambda}=-\frac{\partial f^{n+1}}{\partial\boldsymbol{\sigma}}.\left(2\mu\tilde{\boldsymbol{G}}^f+KG^f\boldsymbol{I}\right)+\frac{\partial f^{n+1}}{\partial\gamma^p}\sqrt{\frac{2}{3}}\tilde{G}^f_{\mathrm{II}}$$

If $\left|f^{n+1}/\sigma_c\right|>\varepsilon_{\mathrm{prec}}$

n=n+1

If N > nbre ite internal max

If $\begin{cases}\left(I_1^e-I_1^{\mathrm{sommet}}\right)>-\dfrac{3\mathrm{K}}{2\mu}\beta' s_{\mathrm{II}}^e\cos\varphi_s^{\min};\text{si }\beta'<0\\ \left(I_1^e-I_1^{\mathrm{sommet}}\right)>-\dfrac{3\mathrm{K}}{2\mu}\beta' s_{\mathrm{II}}^e\cos\varphi_s^{\max};\text{si }\beta'\geq 0\end{cases}$

One retains projection at the top: $\boldsymbol{s}=\boldsymbol{0}\,;\,I_1=I_1^{\mathrm{sommet}}\,;\gamma^p=\gamma^{p^{\mathrm{sommet}}}$

If not

No convergence

If not

No convergence

If not

| Convergence
If FULL_MECA
Computation of:

$$\frac{\partial \boldsymbol{\sigma}^{n+1}}{\partial \boldsymbol{\varepsilon}} = \boldsymbol{H} + \frac{\boldsymbol{H}\,.\,\boldsymbol{G}^{f}\,.\left(\frac{\partial f^{n+1}}{\partial \boldsymbol{\sigma}}\right)_{T}\boldsymbol{H}}{\sqrt{\frac{2}{3}}\frac{\partial f^{n+1}}{\partial \gamma^{p}}\tilde{G}^{f}_{\mathrm{II}} - \left(\frac{\partial f^{n+1}}{\partial \boldsymbol{\sigma}}\right)^{T}\boldsymbol{H}\,\boldsymbol{G}^{f}}$$

Mechanical symmetrization:

$$\frac{\partial \boldsymbol{\sigma}^{\boldsymbol{n+1}}_{\mathbf{sym}}}{\partial \boldsymbol{\varepsilon}} = \frac{1}{2}\left(\frac{\partial \boldsymbol{\sigma}^{\boldsymbol{n+1}}}{\partial \boldsymbol{\varepsilon}} + \frac{\partial \boldsymbol{\sigma}^{\boldsymbol{n+1}^{T}}}{\partial \boldsymbol{\varepsilon}}\right)$$

9 Alternative on the statement of the plasticity criterion

In this proposal alternative, one expresses the plasticity criterion according to the first invariant and of the deviator of the stresses, by a retiming on triaxial in compression and extension by the following relations:

9.1 General formulation

$$f = \left(\frac{s_{\mathrm{II}}}{\sigma_c}\right)^{\frac{1}{a(\gamma^p)}} - u(\boldsymbol{\sigma}, \gamma^p) \leq 0 \qquad \text{éq 9.1-1}$$

Where the statement of $u(\boldsymbol{\sigma}, \gamma^p)$ is:

If $\gamma_{\mathrm{cjs}} \neq 0$

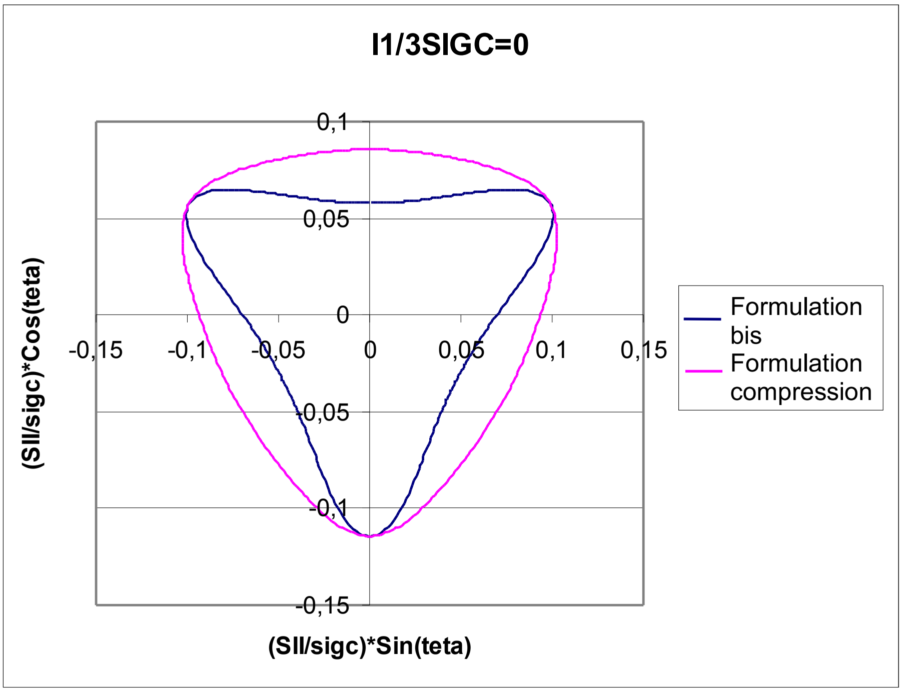
$$u(\boldsymbol{\sigma}, \gamma^p) = -\frac{m(\gamma^p)k(\gamma_p)}{\sqrt{6}\sigma_c}\left(\frac{h(\theta) + h_t^0 - 2\mathrm{h}_c^0}{h_t^0 - h_c^0}\right) - \frac{m(\gamma^p)k(\gamma^p)}{3\sigma_c} I_1 + s(\gamma^p).k(\gamma^p) \qquad \text{éq 9.1-2}$$

If $\gamma_{\mathrm{cjs}} = 0$

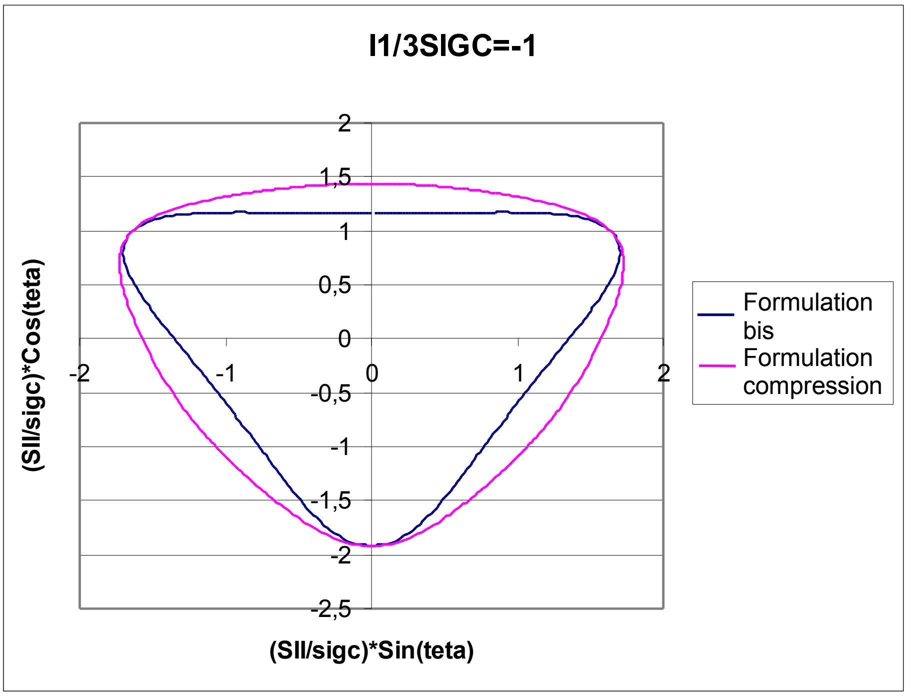
$$u(\boldsymbol{\sigma}, \gamma^p) = -\frac{m(\gamma^p)k(\gamma_p)}{\sqrt{6}\sigma_c}\left(\frac{3}{2} + \frac{1}{2}\cos(3\theta)\right) - \frac{m(\gamma^p)k(\gamma^p)}{3\sigma_c} I_1 + s(\gamma^p).k(\gamma^p) \qquad \text{éq 9.1-3}$$

9.2 Pace of the thresholds

One is placed in the case where $\gamma_{\mathrm{cjs}} = 0.7\,;\, m = 21\,;\, s = 1\,;\, a = 1$, then one traces the pace of the thresholds in the plane perpendicular to the hydrostatic axis (known as plane π), one standardizes compared to and one σ_c considers the two values of containments such as $I_1 = 0$ [Figure 9.2-a] and $I_1 = -3\sigma_c$ [Figure 9.2 - B].



Appear 9.2-a: Pace of the thresholds for a null containment



Appears 9.2-b: Pace of the thresholds for a null containment in compression

One notes in these charts that the (a) formulation has the disadvantage of taking a nonconvex form in the plane π .

10 Functionalities and checking

the constitutive law can be defined by the key word LAIGLE (command STAT_NON_LINE, key word factor COMP_INCR). It is associated with material LAIGLE (command DEFI_MATERIAU).

Model LAIGLE is checked by the cases following tests:

SSNV158	[V6.04.158]	triaxial Compression test drained with the model of Laigle
WTNV101	[V7.31.101]	triaxial Compression test not drained with the model of Laigle and hydraulic coupling

11 Bibliography

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12 Description of the versions of the document

Version Aster	**Author (S) Organization (S)**	**Description of the modifications**
7.4	R.Fernandes, C.Chavant EDF-R&D/AMA	initial Text

Annexe 1Retiming of the criterion on the triaxial one in compression

By taking the general statement of the criterion under the conditions of triaxial in compression, one finds:

$$f=\left(\frac{g(\mathbf{s})}{\sigma_c h_c^0}\right)^{\frac{1}{a(\gamma^p)}}-\left(-\frac{m(\gamma^p)k(\gamma^p)}{\sqrt{6}\sigma_c}\frac{g(\mathbf{s})}{h_c^0}-\frac{m(\gamma^p)k(\gamma^p)}{3\sigma_c}I_1+s(\gamma_p).k(\gamma^p)\right)$$

$$=\left(\frac{\sqrt{\frac{2}{3}}\left|\sigma_1-\sigma_3\right|.h}{\sigma_c h_c^0}\right)^{\frac{1}{a(\gamma^p)}}+\frac{1}{\sigma_c}\left(\frac{m(\gamma^p)k(\gamma^p)}{\sigma_c\sqrt{6}}\frac{\sqrt{\frac{2}{3}}\left|\sigma_1-\sigma_3\right|.h}{\sigma_c h_c^0}+\frac{m(\gamma^p)k(\gamma^p)}{3\sigma_c}(\sigma_1+2\sigma_3)-s(\gamma^p).k(\gamma^p)\right)$$

$$=\left(\frac{\sqrt{\frac{2}{3}}\left|\sigma_1-\sigma_3\right|}{\sigma_c h_c^0}\right)^{\frac{1}{a(\gamma^p)}}+\left(\frac{m(\gamma^p)k(\gamma^p)}{\sigma_c\sqrt{6}}\sqrt{\frac{2}{3}}\left|\sigma_1-\sigma_3\right|+\frac{m(\gamma^p)k(\gamma^p)}{3\sigma_c}(\sigma_1+2\sigma_3)-s(\gamma^p).k(\gamma^p)\right)$$

$$=\left(\frac{\sqrt{\frac{2}{3}}}{\sigma_c}\right)^{\frac{1}{a(\gamma^p)}}\left(\left|\sigma_1-\sigma_3\right|\right)^{\frac{1}{a(\gamma^p)}}+\left(\frac{m(\gamma^p).k(\gamma^p)}{3\sigma_c}\left|\sigma_1-\sigma_3\right|+\frac{m(\gamma^p).k(\gamma^p)}{3\sigma_c}(\sigma_1+2\sigma_3)-s(\gamma^p).k(\gamma^p)\right)$$

$$=\left(\frac{\sqrt{\frac{2}{3}}}{\sigma_c}\right)^{\frac{1}{a(\gamma^p)}}\left(\left|\sigma_1-\sigma_3\right|\right)^{\frac{1}{a(\gamma^p)}}+\left(\frac{m(\gamma^p).k(\gamma^p)}{3\sigma_c}(\sigma_3-\sigma_1)+\frac{m(\gamma^p).k(\gamma^p)}{3\sigma_c}(\sigma_1+2\sigma_3)-s(\gamma^p).k(\gamma^p)\right)$$

$$=\left(\frac{\sqrt{\frac{2}{3}}}{\sigma_c}\right)^{\frac{1}{a(\gamma^p)}}\left(\left|\sigma_1-\sigma_3\right|\right)^{\frac{1}{a(\gamma^p)}}+\left(\frac{m(\gamma^p).k(\gamma^p)}{\sigma_c}(\sigma_3)-s(\gamma^p).k(\gamma^p)\right)$$

$$=\left(\frac{\sqrt{\frac{2}{3}}}{\sigma_c}\right)^{\frac{1}{a(\gamma^p)}}\left(\left|\sigma_1-\sigma_3\right|\right)^{\frac{1}{a(\gamma^p)}}-\sqrt{\frac{2}{3}}^{\frac{1}{a(\gamma^p)}}\left(\frac{m(\gamma^p)}{\sigma_c}(-\sigma_3)+s(\gamma^p)\right)$$

$$=\left(\frac{\sqrt{\frac{2}{3}}}{\sigma_c}\right)^{\frac{1}{a(\gamma^p)}}\left[\left(\left|\sigma_1-\sigma_3\right|\right)^{\frac{1}{a(\gamma^p)}}-(\sigma_c)^{\frac{1}{a(\gamma^p)}}\left(\frac{m(\gamma^p)}{\sigma_c}(-\sigma_3)+s(\gamma^p)\right)\right]$$

Annexe 2Standardization of Q

$$\boldsymbol{Q}=\frac{1}{h(\theta)^5}\left[\left(1+\frac{\gamma_{\mathrm{cjs}}}{2}\cos(3\theta)\right)\frac{\boldsymbol{s}}{s_{\mathrm{II}}}+\frac{\gamma_{\mathrm{cjs}}\sqrt{54}}{6.s_{\mathrm{II}}^2}\mathrm{dev}\left(\frac{\partial\det(\underline{\boldsymbol{s}})}{\partial\boldsymbol{s}}\right)\right]$$

One poses $\boldsymbol{t}=\frac{\partial\det(\underline{\boldsymbol{s}})}{\partial\boldsymbol{s}}$ and $\boldsymbol{t}=\mathrm{dev}\left(\frac{\partial\det(\underline{\boldsymbol{s}})}{\partial\boldsymbol{s}}\right)$ (cf document de Référence CJS R7.01.13)

$$Q_{\mathrm{II}}^2=\boldsymbol{Q}.\boldsymbol{Q}=\frac{1}{h(\theta)^{10}}\left[1+\frac{\gamma_{\mathrm{cjs}}}{2}\cos(3\theta)^2+\frac{3}{2}.\frac{\gamma_{\mathrm{cjs}}{}^2}{s_{\mathrm{II}4}}\boldsymbol{t}^{\boldsymbol{d}}.\boldsymbol{t}^{\boldsymbol{d}}+\frac{\gamma_{\mathrm{cjs}}\sqrt{54}}{3.s_{\mathrm{II}}^3}\left(1+\frac{\gamma_{\mathrm{cjs}}}{2}\cos(3\theta)\right)\boldsymbol{s}.\boldsymbol{t}^{\boldsymbol{d}}\right]$$

to evaluate this statement, one places oneself if $\boldsymbol{s}$ is diagonal by preoccupations with a simplification of computations.

As follows: $\boldsymbol{s}=\begin{bmatrix}s_1\\s_2\\s_3\\0\\0\\0\end{bmatrix}$ and $\boldsymbol{t}^{\boldsymbol{d}}=\frac{1}{3}\begin{bmatrix}2s_2s_3-s_1s_2-s_1s_3\\2s_1s_3-s_1s_2-s_2s_3\\2s_1s_2-s_1s_3-s_2s_3\\0\\0\\0\end{bmatrix}$

By means of the property of $\boldsymbol{s}$: , it $\boldsymbol{s}=s_1+s_2+s_3$ is shown that $s_{\mathrm{II}}^4=\left(s_1^2s_2^2+s_1^2s_3^2+s_2^2s_3^2\right)$ and consequently:

$$\boldsymbol{t}^{\boldsymbol{d}}.\boldsymbol{t}^{\boldsymbol{d}}=\frac{1}{9}\begin{vmatrix}2s_2s_3-s_1s_2-s_1s_3\\2s_1s_3-s_1s_2-s_2s_3\\2s_1s_2-s_1s_3-s_2s_3\\0\\0\\0\end{vmatrix}\begin{vmatrix}2s_2s_3-s_1s_2-s_1s_3\\2s_1s_3-s_1s_2-s_2s_3\\2s_1s_2-s_1s_3-s_2s_3\\0\\0\\0\end{vmatrix}=\frac{s_{\mathrm{II}}{}^4}{6}$$

One also shows from the property $s_1+s_2+s_3=0$ that $s_1^3s_2^3s_3^3=3s_1s_2s_3=3.\det(\boldsymbol{s})$ and consequently:

$$\frac{\gamma_{\mathrm{cjs}}.\sqrt{54}}{3.s_{\mathrm{II}}^3}\boldsymbol{s}.\boldsymbol{t}^{\boldsymbol{d}}=\frac{\gamma_{\mathrm{cjs}}.\sqrt{54}}{9.s_{\mathrm{II}}^3}\begin{vmatrix}s_1\\s_2\\s_3\\0\\0\\0\end{vmatrix}.\begin{vmatrix}2s_2s_3-s_1s_2-s_1s_3\\2s_1s_3-s_1s_2-s_2s_3\\2s_1s_2-s_1s_3-s_2s_3\\0\\0\\0\end{vmatrix}=\frac{\gamma_{\mathrm{cjs}}.\sqrt{54}}{s_{\mathrm{II}}^3}\det(\boldsymbol{s})=\gamma_s.\cos(3\theta)$$

One from of deduced as follows:

$$Q_{\mathrm{II}}^2=\frac{1}{h(\theta)^{10}}\left[\left(1+\frac{\gamma_{\mathrm{cjs}}}{2}\cos(3\theta)\right)^2+\frac{{\gamma_{\mathrm{cjs}}}^2}{4}+\gamma_{\mathrm{cjs}}\cos(3\theta)\left(1+\frac{\gamma_{\mathrm{cjs}}}{2}\cos(3\theta)\right)\right]$$

Annexe 3Framing of the jetting angle

One recalls that
$$\cos\varphi_s \underset{s\to 0}{\rightarrow} \frac{3}{(\beta'^2+3)\sqrt{\left(\frac{3}{\beta'^2+3}\right)^2-\frac{1}{4}+\frac{1}{2(1+\gamma_{\mathrm{cjs}}\cos(3\theta))}+\frac{\gamma_{\mathrm{cjs}}^2-1}{4(1+\gamma_{\mathrm{cjs}}\cos(3\theta))^2}}}$$

One poses: $X(\psi)=\frac{1}{2(1+\gamma_{\mathrm{cjs}}\cos(\psi))}+\frac{\gamma_{\mathrm{cjs}}^2-1}{4(1+\gamma_{\mathrm{cjs}}\cos(\psi))^2}$ where $\psi\in[\,0{,}2\pi[$

It is noted that: $X(-\psi)=X(\psi)$, the function X being even one restricts the interval of study at $\psi\in[\,0{,}2\pi[$.

The resolution of $\frac{dX}{d\psi}=0$ gives $\frac{\gamma_{\mathrm{cjs}}\sin(\psi)}{2(1+\gamma_{\mathrm{cjs}}\cos(\psi))^3}\cdot\gamma_{\mathrm{cjs.}}(\gamma_{\mathrm{cjs}}+\cos(3\psi))=0$

One from of deduced that the limits lower and higher of the function X are:

$$\begin{cases} X(\psi=0)=\frac{1}{4} \\ X(\psi_{\mathrm{cjs}})=\frac{1}{4(1-\gamma_{\mathrm{cjs}}^2)} \text{ où } \psi_{\mathrm{cjs}} \text{ est tel que } \cos(\psi_{\mathrm{cjs}})=-\gamma_{\mathrm{cjs}} \end{cases}$$

One can thus give the framing of $\cos\varphi_s$ following: $\cos\varphi_s^{\min}\le co\varphi_s\le\cos\varphi_s^{\max}$ with:

$$\begin{cases} \cos\varphi_s^{\min}=\frac{3}{(\beta'^2+3)\sqrt{\left(\frac{3}{\beta'^2+3}\right)^2+\frac{{\gamma_{\mathrm{cjs}}}^2}{4(1-\gamma_{\mathrm{cjs}}^2)}}} \\ \cos\varphi_s^{\max}=1 \end{cases}$$