Constitutive law CAM_CLAY

Summarized:

The model of Camwood-Clay one of the elastoplastic models the most known and the most is used in soil mechanics. It is especially adapted to the argillaceous materials. There are several types of models Camwood-Clay, that presented here is most current and is called modified Camwood-Clay. This model is characterized by hammer-hardenable surfaces of load in the shape of ellipses in the diagram of the first two invariants of the stresses. Inside these surfaces of reversibility, the material is elastic nonlinear. There exists moreover, in a point of each ellipse, a critical condition characterized by a variation of volume null. The set of these points constitutes a line separating the zones from dilatancy and contractance of the material as well as the zones of negative and positive hardening. Hardening is governed by only one scalar variable and the normal flow rule is adopted.

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1 Notations

σ indicates the tensor of the effective stresses in small disturbances defined as being the difference between the total stresses and the pressure of water in the case of the water-logged soils, noted in the shape of the following vector:

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sqrt{2}\sigma_{12} \\ \sqrt{2}\sigma_{23} \\ \sqrt{2}\sigma_{31} \end{pmatrix}$$

One notes:

$P=-\frac{1}{3}tr(\sigma)$	**forced deviative**
$s=\sigma+PI$	containment of the stresses
$I_2=\frac{1}{2}tr(s.s)$	second invariant of the stresses
$Q=\sigma_{eq}=\sqrt{3I_2}$	equivalent stress
$\varepsilon=\frac{1}{2}(\nabla u+\nabla^T u)$	total deflection
$\varepsilon=\varepsilon_e+\varepsilon_p+\varepsilon_{th}$	partition of the strains (elastic, plastic, thermal)
$\varepsilon_v=-tr(\varepsilon)+3\alpha(T-T_0)$	**total deflection voluminal**
$\varepsilon_V^p=-tr(\varepsilon^p)$	voluminal plastic strain
$\tilde{\varepsilon}=\varepsilon+\frac{1}{3}\varepsilon_v I$	deviative deviator of
$\tilde{\varepsilon}^e=\tilde{\varepsilon}-\tilde{\varepsilon}^p$	the strains of the elastic strain
$\tilde{\varepsilon}^p=\varepsilon^p+\frac{1}{3}\varepsilon_v^p I$	deviatoric plastic strain
$\varepsilon_{eq}^e=\sqrt{\frac{2}{3}tr(\tilde{\varepsilon}^e.\tilde{\varepsilon}^e)}$	elastic strain equivalent
$\varepsilon_{eq}^p=\sqrt{\frac{2}{3}tr(\tilde{\varepsilon}^p.\tilde{\varepsilon}^p)}$	equivalent plastic strain

e index of the vacuums of the material (ratio of the volume of the pores on the volume of the solid matter constituents)

e_0 initial index of the vacuums

ϕ porosity (ratio of the volume of the pores on total volume)

κ coefficient of swelling (elastic slope in a hydrostatic compression test)

M slope of the right of critical condition

$$k_0=\frac{(1+e_0)}{\kappa}$$

P_{cr} local variable of the model, critical pressure equal to half of the pressure of consolidation P_{cons}

λ coefficient of compressibility (slope plastic in a hydrostatic compression test)

$$k=\frac{(1+e_0)}{(\lambda-\kappa)}$$

μ elastic coefficient of shears (coefficient of Lamé)

f surfaces of load

Λ tensor plastic

I^d multiplier unit of order 2 whose term running is δ_{ij}

I_4^d tensor unit of order 4 whose term running is δ_{ijkl}

2 Introduction

The model described here is the model known as of modified Camwood-Clay. The model initial of Camwood-Clay was developed by the school of soil mechanics of Cambridge in the Sixties. He predicted too important deviatoric strains under weak loading deviatoric, and was modified by Burland and Roscoe in 1968 [bib1].

2.1 Phenomenology of the behavior of the soils

the materials poroplastic such as certain clays are characterized by the following behaviors:

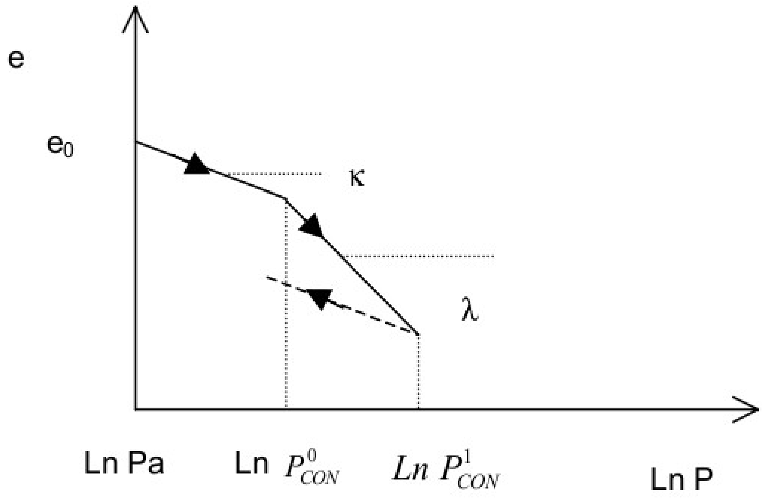
- the strong porosity of these materials causes unrecoverable deformations under hydrostatic loading corresponding to an important reduction of porosity. This mechanism purely contractor is sometimes called "collapse",
- under loading deviatoric, these materials show a contracting phase followed by a phase where the material becomes deformed with constant plastic volume or dilates.

For the two types of loading, the energy blocked in the material evolves according to the number of contact between the grains. For a hydrostatic loading, the number of contact increases, as well as blocked energy, one thus has positive hardening. For a loading deviatoric, the material can become deformed without variation of volume to many intergranular contacts constant. Moreover, one can observe in the tests of the localizations of strains accompanied by a strong dilatancy. In these zones, the number of grains in decreasing contact, there is reduction in blocked energy and thus softening.

These behaviors are highlighted primarily by triaxial compression tests of revolution. These observations bring to apply that there exists a plastic threshold whose evolution is controlled by two mechanisms: one purely contractor associated with the hydrostatic stress, and a mechanism deviatoric controlled by internal friction being held with constant volume and possibly dilating with the approach of the localization.
All the advantage of the Camwood Clay model lies in its faculty to describe these phenomena with a minimum of ingredients and in particular only one surface of load and a hardening associated with only one scalar variable.

2.1 Behavior under hydrostatic compression

During a hydrostatic compression test, the soils present an index of the vacuums which decrease logarithmiquement with the exerted hydrostatic pressure (cf [Figure 2.2-a]). e_0 being the initial index of the vacuums under initial loading. Until a pressure P_{cons}^0 called pressure of consolidation, the behavior is reversible, the slope κ of the diagram $(e, Ln\ P)$ is called elastic coefficient of swelling. P_{cons}^0 corresponds to the maximum pressure which the material during its history underwent. Beyond this preconsolidation, the diagram presents a new slope λ (coefficient of compressibility) more marked and the appearance of unrecoverable deformations. P_{cons}^0 thus corresponds to an evolutionary elastoplastic threshold.



Appear 2.2-a: Hydrostatic test of loading and unloading

Note:

The diagram above corresponds to a set of measurements where the effective stress is stabilized. Indeed, in the process of consolidation of the soils, it is the water contained in the pores which takes again initially the hydrostatic pressure with very little strain, before running out and letting the squelette become deformed. After consolidation of the material and stabilization of the pressure of water, the effective stress (forced total minus pressure of water) is stabilized and deferred on the graph. The behavior models in the saturated porous environments are generally expressed with the effective stresses according to the assumption of Terzaghi.

2.2 Behavior under loading deviatoric

the triaxial compression tests of revolution make it possible to control at the same time the deviatoric component Q and the spherical component P of the loading. According to the ratio of these two components, one observes a plastic behavior purely dilating ($\frac{Q}{P-P_{trac}}>M$) or contracting ($\frac{Q}{P-P_{trac}}<M$), line $Q=M\left(P_{cr}-P_{trac}\right)$ representing all the critical points on surfaces of load where the mechanical state evolves without plastic change of volume. The model basic of Camwood Clay makes the assumption that plastic strain rates are normal on the surface of load f ($\dot{\varepsilon}_v^p=\dot{\Lambda}\frac{\partial f}{\partial P},\dot{\tilde{\varepsilon}}^p=\dot{\Lambda}\frac{\partial f}{\partial Q}$). Moreover, plastic work in an unspecified point of the surface of load is considered equal to plastic work with the critical condition.

3 Camwood Clay model modified

3.1 Assumptions of modelization

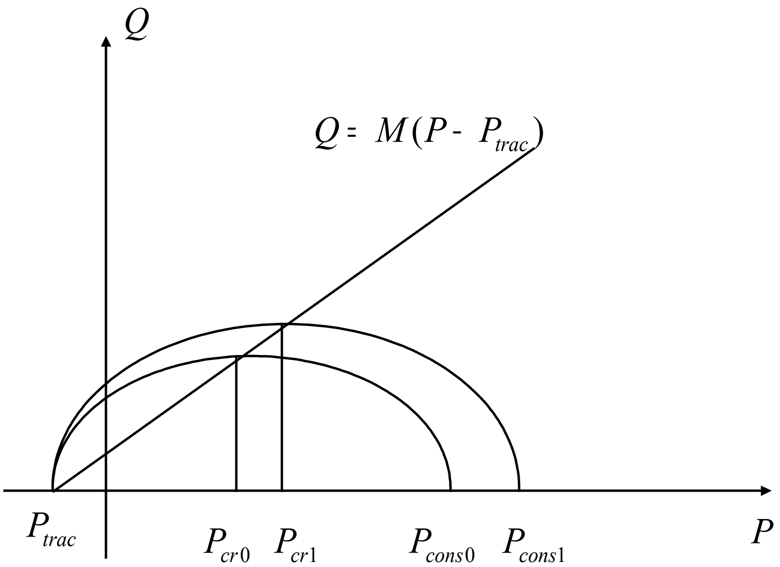
The model is written in small disturbances.
The coefficients of the model do not depend on the temperature.

3.2 Surface of load

the statement of the surface of load is written in the following way:

$$f(P,Q,P_{cr})=Q^2+M^2\left(P-P_{trac}\right)^2-2M^2\left(P-P_{trac}\right)P_{cr} \qquad \leq 0 \qquad \textbf{éq 3.2-1}$$

In the plane (P,Q), the statement represents a family of ellipses, centered on P_{cr} which is related to the pressure of consolidation: $P_{cons}=2P_{cr}-P_{trac}$ (cf [Figure 3.2-a). P_{cr} will be the hardening parameter of the model.



Appear 3.2-a: Family of hammer-hardenable surfaces of load

When $f=0$ and $P-P_{trac}<P_{cr}$ the material is dilating ($\dot{\varepsilon}_v^p<0$) and P_{cr} is decreasing (softening).
When $f=0$ and $P-P_{trac}>P_{cr}$ the material is contacting ($\dot{\varepsilon}_v^p>0$) and P_{cr} is increasing (hardening).

3.3 Elastic model and model of hardening

the assumption of the decoupling of the partly hydrostatic and deviatoric elastic model and the additional assumption are made that the shear modulus is constant.

One thus considers an isotropic elastic model, with a linear deviatoric part and a nonlinear voluminal part:

Déviatoire part :

$$\tilde{\varepsilon}^e = \frac{s}{2\mu} \qquad \textbf{éq 3.3-1}$$

voluminal Part :

$$\dot{\varepsilon}_v^e = -\frac{\dot{e}}{1+e_0} \text{ ou } e = e_0 - \kappa \, Ln\left(\frac{P}{K_{cam}}\right) \text{ si } P < \text{Pconsolidation} \qquad \textbf{éq 3.3-2}$$

the model [éq 3.3-2] is in fact derived from a test oedometric where one measures the variation of the index of the vacuums according to the loading [Figure 2.2-a]. Let us recall that a homogeneous test oedometric consists in increasing the axial effective stress null all while maintaining the strain radial on a cylindrical test-tube.

Note:

The pressures P correspond to tests drained or not. Nevertheless, in modelization with Code_Aster stresses handled in constitutive laws are effective i.e. that one does not take into account the hydrostatic pressure of the fluid which can circulate in the pores, this one being calculated in modelizations THM.

The tests of voluminal loading (cf [Figure 2.2-a]) bring us to the following elastic model:

$$k_0 P + K_{cam} = \left(k_0 P_0 + K_{cam}\right) \exp\left[k_0 (\varepsilon_v^e - \varepsilon_{v0}^e)\right] \text{ avec } k_0 = \frac{(1+e_0)}{\kappa} \qquad \textbf{éq 3.3-3}$$

In the same way, the growth of the surface of load in phase of contractance, its decrease in dilatancy, and the experimental results suggest writing:

$$P_{cr} = P_{cr}^0 \exp\left[k (\varepsilon_v^p - \varepsilon_{v0}^p)\right] \text{ , avec } k = \frac{(1+e_0)}{(\lambda - \kappa)} \qquad \textbf{éq 3.3-4}$$

ε_{v0}^p and e_0 corresponds to the voluminal strain and the index of the initial vacuums, determined by extrapolation of the oedometric curve of the test to the pressure K_{cam} (cf [Figure 2.2-a]).

3.4 Model of yielding

the two plastic variables are the voluminal plastic strain ε_v^p and the tensor deviatoric of plastic strains $\tilde{\varepsilon}^p$. The local variable ε_v^p but is also associated by the strength of hardening P_{cr}. The material standard is not generalized. The flow rule is written:

$$\dot{\varepsilon}^p=\dot{\Lambda}\frac{\partial f}{\partial \sigma}, \ , \dot{\varepsilon}_v^p=-\dot{\Lambda}\frac{\partial F}{\partial P_{cr}} , \qquad \textbf{éq 3.4-1}$$

Λ being the plastic multiplier.

By breaking up the first term, one obtains:

$$\dot{\varepsilon}_v^p=\dot{\Lambda}\frac{\partial f}{\partial P} \qquad \dot{\tilde{\varepsilon}}^p=\dot{\Lambda}\frac{\partial f}{\partial s} \qquad \dot{\varepsilon}_v^p=-\dot{\Lambda}\frac{\partial F}{\partial P_{cr}} \qquad \textbf{éq 3.4-2}$$

knowing that:

$$\underline{\underline{P=-\frac{1}{3}tr(\sigma)}} \ \text{ et } \ \underline{\underline{\varepsilon_v=-tr(\varepsilon)+3\alpha(T-T_0)}} \qquad \textbf{éq 3.4-3}$$

F is the plastic potential associated with the phenomenon of hardening. Let us note that the third part of [éq 3.4-2] is only formal. Indeed, one thus $\dot{\varepsilon}_v^p$ knows by the first relation one knows the evolution of P_{cr}.

3.5 Energy writing and plastic hardening modulus

One is thus in the frame of “not generalized standard materials” (one uses three potentials then: the surface of load f, plastic potential F, and free energy ψ. Even in this configuration less favorable than the traditional frame of the not generalized standard materials, one is ensured to satisfy the second principle with the thermodynamics [bib4]. Using the condition of consistency (expressing that the point representative of the loading “follows” the surface of load) which is written in the following way:

$$df=\frac{\partial f}{\partial P}dP+\frac{\partial f}{\partial Q}dQ+\frac{\partial f}{\partial P_{cr}}dP_{cr}=0 \ , \qquad \textbf{éq 3.5-1}$$

one determines the statement of the plastic multiplier [bib4]:

$$\Lambda=\frac{1}{H_p}\frac{\partial f}{\partial \sigma}d\sigma=-\frac{1}{H_p}\frac{\partial f}{\partial P_{cr}}dP_{cr} \qquad \textbf{éq 3.5-2}$$

with [bib4]:

$$H_p=\frac{\partial f}{\partial P_{cr}}\frac{\partial^2\psi}{\partial {\varepsilon_v^p}^2}\frac{\partial F}{\partial P_{cr}} \ , \ \text{où } H_p \text{ est le module d'écrouissage} \qquad \textbf{éq 3.5-3}$$

the identification of the first and third part of [éq 3.4-2] makes it possible to calculate F which is written:

$$F=-\int \frac{\partial f}{\partial P} dP_{cr}=M^2 P_{cr}\left(P_{cr}-2\mathrm{P}+2\mathrm{P}_{trac}\right) \quad \textbf{éq 3.5-4}$$

notion of hardening being associated with that of blocked energy:

$$P_{cr}=\frac{\partial \psi}{\partial \varepsilon_v^p} \quad \text{donc} \quad dP_{cr}=\frac{\partial^2 \psi}{\partial^2 \varepsilon_v^p} d\varepsilon_v^p \quad \textbf{éq 3.5-5}$$

where ψ is the density of free energy:

$$\psi=\frac{3}{2}\mu\,(\varepsilon_{eq}^e)^2+\frac{P_0}{k_0}\exp(k_0\varepsilon_v^e)+\frac{P_{cr}^0}{k}\exp(k(\varepsilon_v^p-\varepsilon_{v0}^p)) \quad \textbf{éq 3.5-6}$$

By means of them [éq 3.4-2], [éq 3.5-4] and [éq 3.5-6], one can draw according to [éq 3.5-3] the statement from the plastic hardening modulus:

$$H_p=\frac{\partial f}{\partial P_{cr}}\frac{\partial^2 \psi}{\partial {\varepsilon_v^p}^2}\frac{\partial F}{\partial P_{cr}}=4\,kM^4\left(P-P_{trac}\right)P_{cr}\left(P-P_{trac}-P_{cr}\right) \quad \textbf{éq 3.5-7}$$

the hardening modulus is positive in phase of contractance $\left(P-P_{trac}>P_{cr}\right)$ and negative in phase of dilatancy $\left(P-P_{trac}<P_{cr}\right)$. For $P-P_{trac}=P_{cr}$, the behavior is plastic perfect and proceeds with constant plastic volume.

3.6 Incremental relations

the equation [éq 3.4-3] and the condition of consistency give the flow relations:

$$d\varepsilon_v^p=\frac{1}{k}\left[\left(\frac{1}{P_{cr}}-\frac{1}{\left(P-P_{trac}\right)}\right)dP+\frac{Q}{M^2\left(P-P_{trac}\right)P_{cr}}dQ\right] \quad \textbf{éq 3.6-1}$$

$$d\varepsilon_{eq}^p=\frac{1}{k}\left[\frac{Q}{M^2\left(P-P_{trac}\right)P_{cr}}dP+\frac{Q^2}{M^4\left(P-P_{trac}\right)P_{cr}\left(P-P_{trac}-P_{cr}\right)}dQ\right] \quad \textbf{éq 3.6-2}$$

$$d\,\tilde{\varepsilon}^p=d\varepsilon_{eq}^p\frac{3}{2}\frac{s}{Q} \quad \textbf{éq 3.6-3}$$

the rearrangement of [éq 3.6-1] and [éq 3.6-2] led to:

$$\frac{d\varepsilon_{eq}^p}{d\varepsilon_v^p}=\frac{Q}{M^2\left(P-P_{trac}-P_{cr}\right)} \quad \textbf{éq 3.6-4}$$

i.e. with the equation [éq 3.6-3],

$$\frac{d\,\tilde{\varepsilon}^p}{d\varepsilon_v^p}=\frac{3}{2}\frac{s}{M^2\left(P-P_{trac}-P_{cr}\right)} \quad \textbf{éq 3.6-5}$$

Typical case of the critical point:

For $f=0$ et $P-P_{trac}=P_{cr}$: $\dot{P}_{cr}=0 \quad \dot{\varepsilon}_v^p=0$. One from of deduced, by considering the elastic model: $\dot{P}=k_0 P \dot{\varepsilon}_v$. The condition of consistency gives us $\dot{Q}=0$.

3.7 Abstract of the behavior models

Elasticity

$$s=2\mu\tilde{\varepsilon}^e \qquad \text{éq 3.7-1}$$

$$P=P_0\exp(k_0\Delta\varepsilon_v^e)+\frac{K_{cam}}{k_0}\left(\exp\left(k_0\Delta\varepsilon_v^e\right)-1\right) \qquad \text{éq 3.7-2}$$

Plasticity

the criterion: $f(\sigma, P_{cr})=Q^2+M^2(P-P_{trac})^2-2M^2(P-P_{trac})P_{cr}=0$ with $(Q=\sigma_{eq})$

$$\frac{\partial f}{\partial\sigma}=\underbrace{\left(-\frac{1}{3}\frac{\partial f}{\partial P}I^d+\frac{3}{2}\frac{\partial f}{\partial Q}\frac{s}{Q}\right)} \qquad \text{éq 3.7-3}$$

thus:

$$\dot{\tilde{\varepsilon}}^p=3\dot{\Lambda}s \qquad \text{éq 3.7-4}$$

$$\dot{\varepsilon}_v^p=\dot{\Lambda}2M^2\left(P-P_{trac}-P_{cr}\right) \qquad \text{éq 3.7-5}$$

Hardening

$$P_{cr}\left(\varepsilon_v^p\right)=P_{cr0}\exp\left(k\left(\varepsilon_v^p-\varepsilon_v^{p0}\right)\right) \qquad \text{éq 3.7-6}$$

Behavior elastic: So $f<0$ then:

$$\dot{P}_{cr}=0 \qquad \text{éq 3.7-7}$$

$$\dot{\tilde{\varepsilon}}_{eq}^p=0, \dot{\varepsilon}_v^p=0 \qquad \text{éq 3.7-8}$$

$$\dot{s}=2\mu\dot{\tilde{\varepsilon}} \qquad \text{éq 3.7-9}$$

$$\dot{P}=\left(k_0P+K_{cam}\right)\dot{\varepsilon}_v \qquad \text{éq 3.7-10}$$

Behavior elastoplastic: If $f=0$ and $\dot{f}=0$ then:

$$\dot{P}_{cr}\neq 0 \quad ; \quad \dot{P}_{cr}=k\,\dot{\varepsilon}_v^p P_{cr} \qquad \text{éq 3.7-11}$$

$$\dot{\tilde{\varepsilon}}^p=3\,\dot{\Lambda}\,s \quad \text{si} \quad P-P_{trac}\neq P_{cr} \qquad \text{éq 3.7-12}$$

$$\dot{\varepsilon}_v^p=\dot{\Lambda}2M^2\left(P-P_{trac}-P_{cr}\right) \quad \text{si} \quad P-P_{trac}\neq P_{cr} \qquad \text{éq 3.7-13}$$

$$\dot{s}=2\mu\dot{\tilde{\varepsilon}} \qquad \text{éq 3.7-14}$$

$$\dot{P}=\left(k_0P+K_{cam}\right)\dot{\varepsilon}_v \qquad \text{éq 3.7-15}$$

Note:

From the only unknown $\dot{\varepsilon}_v^p$ *, one can deduce the other unknowns* $\dot{\tilde{\varepsilon}}^p$ *and* $\dot{P}_{cr}$ *.*

If $P-P_{trac}=P_{cr}$: $\dot{\varepsilon}_v^p=0$ $\dot{Q}=\dot{P}_{cr}=0,\ \dot{P}=k_0 P\ \dot{\varepsilon}_v$.

1 Numerical integration of the behavior models

1.1 Recall of the problem

For an increment of loading given and a set of variables given (initial field of displacement, stress and local variable), one solves the discretized total system (2.2.2.2 - 1 of [bib3]) which seeks to satisfy the balance equations.

The resolution of this system gives us Δu , therefore $\Delta \varepsilon$. One thus seeks locally, in each Gauss point, the increment of stress and local variable corresponding to $\Delta \varepsilon$ and which satisfy the constitutive law.

The following notations are employed: $A^{-}, A, \Delta A$ for the evaluated quantity at known time T, time $t+\Delta t$ and its increment, respectively. The equations are discretized in an implicit way, expressed according to the unknown variables at time $t+\Delta t$.

1.2 Computation of the stresses and local variables

the elastic prediction of the deviatoric stress is written:

$$s^e = s^- + 2\mu \Delta \tilde{\varepsilon}$$ **éq 4.2-1**

gold one can always write s at time + as being:

$$s = s^- + 2\mu \Delta \tilde{\varepsilon}^e$$ **éq 4.2-2**

These two equations enable us to deduce s according to s^e :

$$s = s^e - 2\mu \Delta \tilde{\varepsilon} + 2\mu \Delta \tilde{\varepsilon}^e$$ **éq 4.2-3**

$$\text{ou} \quad s = s^e - 2\mu \Delta \tilde{\varepsilon}^p$$ **éq 4.2-4**

While replacing $\Delta \tilde{\varepsilon}^p$ by its statement according to $\Delta \varepsilon_v^p$, one obtains:

$$s = \frac{s^e}{1 + \dfrac{3\mu \Delta \varepsilon_v^p}{M^2 \left(P - P_{trac} - P_{cr}\right)}}$$ **éq 4.2-5**

from where,

$$Q = \frac{Q^e}{1 + \dfrac{3\mu \Delta \varepsilon_v^p}{M^2 \left(P - P_{trac} - P_{cr}\right)}}$$ **éq 4.2-6**

By supposing that k_0 is independent of the temperature, the incremental writing of P is written:

$$P=P^{-}\exp\left[k_0\varepsilon_v^e-k_0\varepsilon_v^{e^-}\right]+\frac{K_{cam}}{k_0}\left(\exp\left[k_0\varepsilon_v^e-k_0\varepsilon_v^{e^-}\right]-1\right) \quad \text{éq 4.2-8}$$

$$P=P^{-}\exp\left[k_0\Delta\varepsilon_v^e\right]+\frac{K_{cam}}{k_0}\left(\exp\left[k_0\Delta\varepsilon_v^e\right]-1\right) \quad \text{éq 4.2-9}$$

$$\Delta P=P^{-}\left(\exp\left[k_0\Delta\varepsilon_v^e\right]-1\right)+\frac{K_{cam}}{k_0}\left(\exp\left[k_0\Delta\varepsilon_v^e\right]-1\right) \quad \text{éq 4.2.10}$$

In the same way one can write the statement of P^e according to P^- :

$$P^e=P^{-}\exp\left[k_0\Delta\varepsilon_v\right]+\frac{K_{cam}}{k_0}\left(\exp\left[k_0\Delta\varepsilon_v\right]-1\right) \quad \text{éq 4.2-11}$$

from where the statement of P at time + is:

$$P=P^e\exp\left[-k_0\Delta\varepsilon_v^p\right]+\frac{K_{cam}}{k_0}\left(\exp\left[-k_0\Delta\varepsilon_v^p\right]-1\right) \quad \text{éq 4.2-12}$$

In the incremental writing of P_{cr} , the coefficient k does not depend on the temperature, one thus finds the statement following:

$$P_{cr}=P_{cr0}\exp\left[k\left(\varepsilon_v^p-\varepsilon_v^{p_0}\right)\right] \quad \text{éq 4.2-13}$$

$$P_{cr}=P_{cr}^{-}\exp\left[k\Delta\varepsilon_v^p\right] \quad \text{éq 4.2-14}$$

$$\Delta P_{cr}=P_{cr}^{-}\left[\exp\left(k\Delta\varepsilon_v^p\right)-1\right] \quad \text{éq 4.2-15}$$

Abstract:

$f\left(s^e, P^e, P_{cr}^-\right)\leq 0$ in this case $\Delta P_{cr}=0$ either $s=s^-+\Delta s=s^e$

$$P=P^e$$

$f\left(s^e, P^e, P_{cr}^-\right)>0$ in this case, $\Delta P_{cr}>0$ $\Delta\tilde{\varepsilon}^p\neq 0$ and $\Delta\varepsilon_v^p\neq 0$

or $s=s^e-2\mu\Delta\tilde{\varepsilon}^p$

$$P=P^e\exp\left[-k_0\Delta\varepsilon_v^p\right]+\frac{K_{cam}}{k_0}\left(\exp\left[-k_0\Delta\varepsilon_v^p\right]-1\right)$$

$$P_{cr}=P_{cr}^{-}\exp\left[k\Delta\varepsilon_v^p\right]$$

Note::

The principal unknown is $\Delta\varepsilon_v^p$.

1.3 Computation of the unknown $\Delta\varepsilon_v^p$

By deferring in the criterion the statements of P and Q according to P^e and of Q^e and by means of the equation [éq 4.2-6]:

$$Q_e^2=-\left[1+\frac{3\mu\Delta\varepsilon_v^p}{M^2(P-P_{trac}-P_{cr})}\right]^2 M^2\left(P-P_{trac}\right)\left(P-P_{trac}-2\mathrm{P}_{cr}\right) \quad \textbf{éq 4.3-1}$$

$$Q_e^2=-M^2\left[1+\frac{3\mu\Delta\varepsilon_v^p}{M^2\left(P_e\exp\left[-k_0\Delta\varepsilon_v^p\right]+\frac{K_{cam}}{k_0}\left(\exp\left[-k_0\Delta\varepsilon_v^p\right]-1\right)-P_{trac}-P_{cr}^-\exp\left[k\Delta\varepsilon_v^p\right]\right)}\right]^2$$
$$\left(P_e\exp\left[-k_0\Delta\varepsilon_v^p\right]+\frac{K_{cam}}{k_0}\left(\exp\left[-k_0\Delta\varepsilon_v^p\right]-1\right)-P_{trac}\right)$$
$$\left(P_e\exp\left[-k_0\Delta\varepsilon_v^p\right]+\frac{K_{cam}}{k_0}\left(\exp\left[-k_0\Delta\varepsilon_v^p\right]-1\right)-P_{trac}-2\mathrm{P}_{cr}^-\exp\left[k\Delta\varepsilon_v^p\right]\right) \quad \textbf{éq 4.3-2}$$

In under following paragraph one determines limits with this function which facilitate the resolution of the equation [éq 4.3-2] with for example the method of the ropes or by the method of Newton.

1.4 Determination of the limits of the function

One poses $\Delta\varepsilon_v^p=x$ the unknown of the problem.
One thus has:

$$P(x)=P^e\exp\left(-k_0x\right)+\frac{K_{cam}}{k_0}\left(\exp\left(-k_0x\right)-1\right) \quad \textbf{éq 4.4-1}$$

$$P_{cr}(x)=P_{cr}^-\exp(kx) \quad \textbf{éq 4.4-2}$$

$$\Lambda(x)=\frac{x}{2\mathrm{M}^2\left(P(x)-P_{trac}-P_{cr}(x)\right)} \quad \textbf{éq 4.4-3}$$

$$Q(x)=\frac{Q^e}{1+6\ \mu\ \Lambda(x)} \quad \textbf{éq 4.4-4}$$

$$f(x)=Q^2(x)+M^2\left(P(x)-P_{trac}\right)^2-2\mathrm{M}^2\left(P(x)-P_{trac}\right)P_{cr}(x)=0 \quad \textbf{éq 4.4-5}$$

At the point $x=0\ ;\ P(0)=P^e\ ;\ P_{cr}(0)=P_{cr}^-\ ;\ \lambda(0)=0\ ;\ Q(0)=Q^e$ éq 4.4-6

$$f(0)=Q^{e^2}+M^2\left(P^e-P_{trac}\right)\left(P^e-P_{trac}-2P_{cr}^-\right) \quad \text{éq 4.4-7}$$

$$f(0)>0$$

With point:

$$P-P_{trac}=P_{cr} \ ; \ \Lambda(x_b)=\infty \ ; \ Q(x_b)=0 \ \text{et} \ f(x_b)=-M^2\left(P-P_{trac}\right)^2 \quad \text{éq 4.4-8}$$

$$f(x_b)<0$$

In $x=0$; $f(0)>0$ and in $x=x_b$; $f(x_b)<0$

One seeks X between 0 and x_b ; to determine it, one writes:

$$P(x_b)-P_{trac}=P_{cr}(x_b)$$

$$\Leftrightarrow P^e\exp(-k_0x_b)+\frac{K_{cam}}{k_0}\exp(-k_0x_b)-P_{cr}^-\exp(kx_b)=\frac{K_{cam}}{k_0}+P_{trac} \quad \text{éq 4.4-9}$$

It is a nonlinear equation in x_b , one makes a restricted development of order 1 to deduce the statement from x_b :

If $P^e-P_{cr}^--P_{trac}=0$; $x_b=0$ and $\Delta\varepsilon_v^p=0$

If $k_0P^e+K_{cam}+kP_{cr}^-\neq 0$; $x_b=\left(\dfrac{P^e-P_{cr}^--P_{trac}}{k_0P^e+K_{cam}+kP_{cr}^-}\right)$

If not one makes a restricted development of order 2 and one finds;

$$\left(P^e-P_{cr}^--P_{trac}\right)-\left(k_0P^e+K_{cam}+kP_{cr}^-\right)x_b+\frac{1}{2}\left(k_0P^e+K_{cam}-kP_{cr}^-\right)x_b^2=0$$

As $k_0P^e+K_{cam}+kP_{cr}^-=0$ then $k_0P^e+K_{cam}-kP_{cr}^-\neq 0$

And one solves

$$\left(P^e-P_{cr}^--P_{trac}\right)+\frac{1}{2}\left(k_0P^e+K_{cam}-kP_{cr}^-\right)x_b^2=0$$

If $P^e-P_{cr}^--P_{trac}=0$; $x_b=0$ and $\Delta\varepsilon_v^p=0$

If not $x_b=\pm\sqrt{\dfrac{-2\left(P^e-P_{cr}^--P_{trac}\right)}{\left(k_0P^e+K_{cam}-kP_{cr}^-\right)}}$

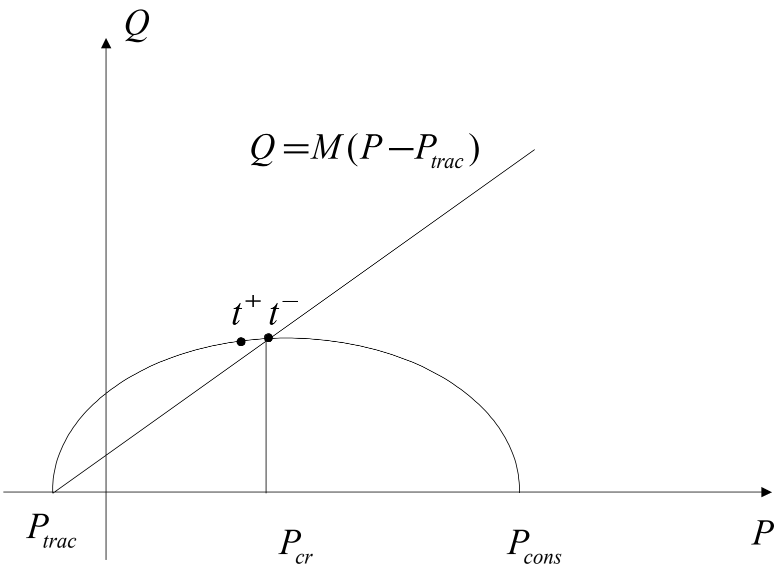
If σ <0 one chooses a value for x_b approximate is $x_b=\dfrac{1}{k_0+k}Log\left(\dfrac{|P^e-P_{trac}|}{P_{cr}^-}\right)$

If not one has the choice between two values of x_b ;
The following test is made:

So $\left(P^e - P_{trac} > P_{cr}^-\right)$ then $x_b = \sqrt{\frac{-2\left(P^e - P_{cr}^- - P_{trac}\right)}{\left(k_0 P^e + K_{cam} - kP_{cr}^-\right)}}$; the solution would be positive; x>0

So $\left(P^e - P_{trac} < P_{cr}^-\right)$ then $x_b = -\sqrt{\frac{-2\left(P^e - P_{cr}^- - P_{trac}\right)}{\left(k_0 P^e + K_{cam} - kP_{cr}^-\right)}}$; the solution would be negative; x<0

1.5 Typical case of the critical point



Appears 4.5-a: Mechanical state around the point criticizes

So at time t^- one reaches the critical condition, then $P_{cr}^+ = P_{cr}^-$, $\Delta\varepsilon_v^p = 0$ and $Q^- = MP^-$. If $f = 0,\ \dot{f} = 0$, then the point (P, Q) at time t^+ moves on the initial ellipse (cf [Figure 4.5-a]). One deduces immediately from the elastic model and the condition $\Delta\varepsilon_v^p = 0$:

$$\Delta P = k_0\ \Delta\ \varepsilon_v P^- \quad \textbf{éq 4.5-1}$$

the criterion being checked at time t^+ , one has by means of [éq 4.5-1]:

$$Q^{+2}=M^2 P^+(2P_{cr}^- - P^+)=M^2(P^- + \Delta P)(P^- - \Delta P)=M^2 P^{-(2)}(1-k_0^2\Delta\varepsilon_v^2)=Q^{-(2)}(1-k_0^2\Delta\varepsilon_v^2)$$
éq 4.5-2

In addition the deviator of the stresses can be written:

$$s=s^e-2\mu\ \Delta\tilde{\varepsilon}^p=s^e-2\mu\ \Lambda\frac{\partial f}{\partial s}=s^e-6\mu\ \Lambda\ s$$ **éq 4.5-3**

One from of deduced:

$$1+6\ \mu\ \Lambda=\frac{Q^e}{Q}\ ,$$ **éq 4.5-4**

and:

$$s=\frac{Q^-\sqrt{(1-k_0^2\Delta\varepsilon_v^2}}{Q^e}\ s^e$$ **éq 4.5-5**

1.6 Abstract

the discretization of the equations and the implicit constitutive law of way leads to the resolution of the equation [éq 4.3-2].

If $P^- \neq P_{cr}^-$, then one solves the equation [éq 4.3-2] whose unknown is $\Delta\varepsilon_v^p$.

One deduces then:

$$P_{cr}=P_{cr^-}\exp(k\Delta\varepsilon_v^p),$$
$$P=P^e\exp\left[-k_0\,\Delta\varepsilon_v^p\right]+\frac{K_{cam}}{k_0}\left(\exp\left[-k_0\,\Delta\varepsilon_v^p\right]-1\right),$$
$$\text{puis}\ \ s=\frac{s^e}{1+\dfrac{3\mu\Delta\varepsilon_v^p}{M^2\left(P-P_{trac}-P_{cr}\right)}}$$
éq 4.6-1

One deduces finally:

$$\Delta\tilde{\varepsilon}^p=\frac{3}{2}\frac{\Delta\varepsilon_v^p}{M^2(P-P_{trac}-P_{cr})}s$$ **éq 4.6-2**

At the critical point:

$$\Delta\varepsilon_v^p=0, P_{cr}=P_{cr}^-$$ **éq 4.6-3**

In this point, it has no evolution of hardening there, on the other hand the stress state can continue to evolve either in contractance, or in dilatancy (the tangent with the criterion is horizontal). The new stress state moves on the surface of load of the preceding state.

2 Tangent operator

If the option is: RIGI_MECA_TANG , option used at the time of the prediction, the tangent operator calculated in each Gauss point is known as of velocity:

$$\dot{\sigma}_{ij}=D^{elp}_{ijkl}\dot{\varepsilon}_{kl}$$

In this case, D^{elp}_{ijkl} is calculated starting from the not discretized equations.

If the option is: FULL_MECA , option used when one and the reactualizes the tangent matrix with each iteration by updating the stresses local variables:

$$d\sigma_{ij}=A_{ijkl}\,d\varepsilon_{kl}$$

In this case, A_{ijkl} is calculated starting from the implicitly discretized equations.

2.1 Nonlinear elastic tangent operator

the elastic relation of velocity is written:

$$\dot{\sigma}_{ij}=-\dot{P}\,\delta_{ij}+\dot{s}_{ij}=\left(k_0 P+K_{cam}\right)tr\,\{\,\dot{\varepsilon}\;\delta_{ij}+2\mu\;\dot{\tilde{\varepsilon}}\,\} \qquad \textbf{éq 5.1-1}$$

$$\dot{\sigma}_{ij}=(k_0 P+K_{cam}-\frac{2}{3}\mu)tr\,\{\,\dot{\varepsilon}\;\delta_{ij}+2\mu\;\dot{\varepsilon}_{ij}\,\} \qquad \textbf{éq 5.1-2}$$

the tangent operator in elasticity of the model noted Cam_Clay D^e is thus deduced from the following matric writing:

$$\begin{Bmatrix}\dot{\sigma}_{11}\\ \dot{\sigma}_{22}\\ \dot{\sigma}_{33}\\ \sqrt{2}\,\dot{\sigma}_{12}\\ \sqrt{2}\,\dot{\sigma}_{23}\\ \sqrt{2}\,\dot{\sigma}_{31}\end{Bmatrix}=\underbrace{\begin{bmatrix}k_0P+K_{cam}+\frac{4}{3}\mu & k_0P+K_{cam}-\frac{2}{3}\mu & k_0P+K_{cam}-\frac{2}{3}\mu & 0 & 0 & 0\\ k_0P+K_{cam}-\frac{2}{3}\mu & k_0P+K_{cam}+\frac{4}{3}\mu & k_0P+K_{cam}-\frac{2}{3}\mu & 0 & 0 & 0\\ k_0P+K_{cam}-\frac{2}{3}\mu & k_0P+K_{cam}-\frac{2}{3}\mu & k_0P+K_{cam}+\frac{4}{3}\mu & 0 & 0 & 0\\ 0 & 0 & 0 & 2\mu & 0 & 0\\ 0 & 0 & 0 & 0 & 2\mu & 0\\ 0 & 0 & 0 & 0 & 0 & 2\mu\end{bmatrix}}_{D^e}\begin{Bmatrix}\dot{\varepsilon}_{11}\\ \dot{\varepsilon}_{22}\\ \dot{\varepsilon}_{33}\\ \sqrt{2}\,\dot{\varepsilon}_{12}\\ \sqrt{2}\,\dot{\varepsilon}_{23}\\ \sqrt{2}\,\dot{\varepsilon}_{31}\end{Bmatrix} \qquad \textbf{éq 5.1-3}$$

2.2 plastic tangent Operator of velocity. Option RIGI_MECA_TANG

the total tangent operator is in this case K_{i-1} (option RIGI_MECA_TANG called with the first iteration of a new increment of load) starting from the results known at time t_{i-1} [bib3].
If the tensor of the stresses with t_{i-1} is on the border of the field of elasticity, the condition is written: $\dot{f}=0$ who must be checked jointly with the condition $f=0$. If the tensor of the stresses with t_{i-1} is inside the field $f<0$, then the tangent operator is the operator of elasticity.

$$\dot{f}=\left(\frac{\partial f}{\partial \sigma}\right)\dot{\sigma}+\frac{\partial f}{\partial P_{cr}}\dot{P}_{cr}=0$$ **éq 5.2-1**

like $\dot{P}_{cr}=\frac{\partial P_{cr}}{\partial \varepsilon_v^p}\dot{\varepsilon}_v^p$, then:

$$\dot{f}=\left(\frac{\partial f}{\partial \sigma}\right)\dot{\sigma}+\frac{\partial f}{\partial P_{cr}}\frac{\partial P_{cr}}{\partial \varepsilon_v^p}\dot{\varepsilon}_v^p=0$$ **éq 5.2-2**

In addition $\dot{\varepsilon}^e=\dot{\varepsilon}-\dot{\varepsilon}^p$

thus:

$$D^{e^{-1}}\dot{\sigma}=\dot{\varepsilon}-\dot{\Lambda}\frac{\partial f}{\partial \sigma} ,$$ **éq 5.2-3**

i.e.:

$$\dot{\sigma}_{ij}=D^e_{ijkl}\dot{\varepsilon}_{kl}-\dot{\Lambda}D^e_{ijkl}\left(\frac{\partial f}{\partial \sigma}\right)_{kl}$$ **éq 5.2-4**

the plastic hardening modulus is written according to the equation [éq 3.5-7] and by means of the flow rule:

$$H_p=\frac{\partial f}{\partial P_{cr}}\frac{\partial P_{cr}}{\partial \varepsilon_v^p}\frac{\partial F}{\partial P_{cr}}=-\frac{1}{\dot{\Lambda}}\frac{\partial f}{\partial P_{cr}}\frac{\partial P_{cr}}{\partial \varepsilon_v^p}\dot{\varepsilon}_v^p$$ **éq the 5.2-5**

equations [éq 5.2-1] and [éq 5.2-5] give:

$$\left(\frac{\partial f}{\partial \sigma}\right)_{ij}\dot{\sigma}_{ij}-\dot{\Lambda}H_p=0$$ **éq 5.2-6**

the multiplication of the equation [éq 5.2-4] by $\left(\frac{\partial f}{\partial \sigma}\right)_{ij}$ gives:

$$\left(\frac{\partial f}{\partial \sigma}\right)_{ij}\dot{\sigma}_{ij}=\left(\frac{\partial f}{\partial \sigma}\right)_{ij}D^e_{ijkl}\dot{\varepsilon}-\dot{\Lambda}\left(\frac{\partial f}{\partial \sigma}\right)_{ij}D^e_{ijkl}\left(\frac{\partial f}{\partial \sigma}\right)_{kl}$$ **éq the 5.2-7**

two preceding equations make it possible to find:

$$H_p \dot{\Lambda} = \left(\frac{\partial f}{\partial \sigma}\right)_{ij} D^e_{ijkl} \dot{\varepsilon}_{kl} - \dot{\Lambda} \left(\frac{\partial f}{\partial \sigma}\right)_{ij} D^e_{ijkl} \left(\frac{\partial f}{\partial \sigma}\right)_{kl}$$ **éq 5.2-8**

from where the statement of the plastic multiplier:

$$\dot{\Lambda} = \frac{\left(\frac{\partial f}{\partial \sigma}\right)_{ij} D^e_{ijkl} \dot{\varepsilon}_{kl}}{\left(\frac{\partial f}{\partial \sigma}\right)_{ij} D^e_{ijkl} \left(\frac{\partial f}{\partial \sigma}\right)_{kl} + H_p}$$ **éq 5.2-9**

Is H the definite elastoplastic modulus like:

$$H = \left(\frac{\partial f}{\partial \sigma}\right)_{ij} D^e_{ijkl} \left(\frac{\partial f}{\partial \sigma}\right)_{kl} + H_p$$ **éq 5.2-10**

the plastic multiplier is written:

$$\dot{\Lambda} = \frac{\left(\frac{\partial f}{\partial \sigma}\right)_{ij} D^e_{ijkl} \dot{\varepsilon}_{kl}}{H}$$ **éq 5.2-11**

While replacing $\dot{\Lambda}$ by its statement in the equation [éq 5.2-4], one obtains:

$$\dot{\sigma}_{ij} = D^e_{ijkl} \dot{\varepsilon}_{kl} - \frac{1}{H} \left[\left(\frac{\partial f}{\partial \sigma}\right)_{mn} D^e_{mnop} \dot{\varepsilon}_{op} \right] . D^e_{ijkl} \left(\frac{\partial f}{\partial \sigma}\right)_{kl}$$ **éq 5.2-12**

One from of thus deduced the elastoplastic operator $D^{elp} = D^e - D^p$:

$$\dot{\sigma}_{ij} = \underbrace{\left[D^e_{ijkl} - \frac{1}{H} \left(\frac{\partial f}{\partial \sigma}\right)_{op} D^e_{ijop} \, D^e_{mnkl} \left(\frac{\partial f}{\partial \sigma}\right)_{mn} \right]}_{D^{elp}} \dot{\varepsilon}_{kl}$$ **éq 5.2-13**

with,

$$D^p_{ijkl} = \frac{1}{H} \left(\frac{\partial f}{\partial \sigma}\right)_{op} D^e_{ijop} \, D^e_{mnkl} \left(\frac{\partial f}{\partial \sigma}\right)_{mn}$$ **éq 5.2-14**

Computation of H :

$$\left(\frac{\partial f}{\partial \sigma}\right)_{ij} = -\frac{2}{3} M^2 \left(P - P_{trac} - P_{cr}\right) \delta_{ij} + 3 s_{ij} \ , \qquad \text{éq 5.2-15}$$

which is written in vectorial notation:

$$\begin{bmatrix} -\frac{2}{3} M^2 \left(P - P_{trac} - P_{cr}\right) + 3 s_{11} \\ -\frac{2}{3} M^2 \left(P - P_{trac} - P_{cr}\right) + 3 s_{22} \\ -\frac{2}{3} M^2 \left(P - P_{trac} - P_{cr}\right) + 3 s_{33} \\ 3\sqrt{2}\, s_{12} \\ 3\sqrt{2}\, s_{23} \\ 3\sqrt{2}\, s_{31} \end{bmatrix} \qquad \text{éq 5.2-16}$$

from where the statement of:

$$D^e_{ijkl}\left(\frac{\partial f}{\partial \sigma}\right)_{kl} : \begin{bmatrix} -2 k_0 M^2 \left(P - P_{trac}\right)\left(P - P_{trac} - P_{cr}\right) + 6 \mu s_{11} \\ -2 k_0 M^2 \left(P - P_{trac}\right)\left(P - P_{trac} - P_{cr}\right) + 6 \mu s_{22} \\ -2 k_0 M^2 \left(P - P_{trac}\right)\left(P - P_{trac} - P_{cr}\right) + 6 \mu s_{33} \\ 6\mu \sqrt{2}\, s_{12} \\ 6\mu \sqrt{2}\, s_{23} \\ 6\mu \sqrt{2}\, s_{31} \end{bmatrix} \qquad \text{éq 5.2-17}$$

and

$$\left(\frac{\partial f}{\partial \sigma}\right)_{ij} D^e_{ijkl}\left(\frac{\partial f}{\partial \sigma}\right)_{kl} = 4 k_0 M^4 \left(P - P_{trac}\right)\left(P - P_{trac} - P_{cr}\right)^2 + 12 \mu Q^2 \qquad \textbf{where}$$

$$12 \mu Q^2 = 18 \mu \, tr\left(s \, . \, s\right) \qquad \text{éq 5.2-18}$$

According to the equations [éq 3.5-7] and [éq 5.2-18], one can deduce the statement from H :

$$H = 4 M^4 \left(P - P_{trac}\right)\left(P - P_{trac} - P_{cr}\right)\left(k_0 \left(P - P_{trac} - P_{cr}\right) + k P_{cr}\right) + 12 \mu Q^2 \qquad \text{éq 5.2-19}$$

While posing:

$$A_{ij} = -2 k_0 M^2 \left(P - P_{trac}\right)\left(P - P_{trac} - P_{cr}\right) \delta_{ij} + 6 \mu s_{ij} \ , \qquad \text{éq 5.2-20}$$

one can write the following symmetric plastic matrix:

$$D^p=\frac{1}{H}\begin{bmatrix} A_{11}^2 & A_{11}A_{22} & A_{11}A_{33} & 6\sqrt{2}\,\mu A_{11}s_{12} & 6\sqrt{2}\,\mu A_{11}s_{23} & 6\sqrt{2}\,\mu A_{11}s_{31} \\ . & A_{22}^2 & A_{22}A_{33} & 6\sqrt{2}\,\mu A_{22}s_{12} & 6\sqrt{2}\,\mu A_{22}s_{23} & 6\sqrt{2}\,\mu A_{22}s_{31} \\ . & . & A_{33}^2 & 6\sqrt{2}\,\mu A_{33}s_{12} & 6\sqrt{2}\,\mu A_{33}s_{23} & 6\sqrt{2}\,\mu A_{33}s_{31} \\ . & . & . & 36\,\mu^2 s_{12}^2 & 36\,\mu^2 s_{12}s_{23} & 36\,\mu^2 s_{12}s_{31} \\ . & . & . & . & 36\,\mu^2 s_{23}^2 & 36\,\mu^2 s_{23}s_{31} \\ . & . & . & . & . & 36\,\mu^2 s_{31}^2 \end{bmatrix}$$ **éq 5.2-21**

2.3 tangent Operator into implicit. Option FULL_MECA

the coherent tangent operator of option `FULL_MECA` is calculated like the tangent operator of velocity for the current stress state.

Nevertheless, of the theoretical elements allowing to calculate it are given in appendix, in the paragraph . A to note, that the equations present in the appendix suppose that the criterion passes by a stress state null, P_{trac} and K_{cam} were not introduced yet there. It is necessary to think of taking them into account and with the need to reactivate these equations for the coherent tangent operator.

3 Materials parameters and local variables

3.1 Materials parameters

the parameters E and ν compulsory under key word ELAS are not used by model CAM_CLAY. Key word ELAS can of this fact avoided being if the user does not need to inform α or ρ .

The data specific to the Cam_Clay model are:

- The elastic modulus of shears μ ,
- the critical slope M ,
- porosity associated with a pressure initial and related to the initial index of the vacuums: $n=\frac{e_0}{1+e_0}$
- Initial compressibility K_{cam} ,
- pressure of tolerated tension P_{trac} , (must be negative)
- the elastic coefficient of swelling: κ (which leads to k_0),
- the coefficient of plastic compressibility: λ (which leads to k),
- the initial critical pressure P_{cr0} such as $P_{cr0}-P_{trac}$ is equal to half of the pressure of consolidation,

Notices 1 :

The number of data is relatively low, which makes very simple the model. One of the most visible limitations of the model is the assumption of the alignment of the critical points on a line of slope M . This is besides the statement of the concept of internal friction. One can also interpret the quantity M by connecting it to the internal friction angle of Coulomb by the relation: $\sin\ \varphi=\frac{3M}{6+M}$. However it is known that for very cohesive materials, this angle varies when the average constraint decreases. It is noted besides that for a chock of M on a triaxial compression test to a certain average constraint, one simulates well with this model the triaxial ones realized with a average constraint step too different but one cannot correctly consider the bearings plastic for a broad range of confining pressure (cf [bib2]). It is thus necessary to readjust M for several beaches of average constraint.

Notice 2:

The increase in stresses is connected to the voluminal increase in the strains according to one or the other of the constitutive laws:

With Cam_Clay:

$$\Delta P=\left(k_0 P^{-}+K_{cam}\right)\Delta\varepsilon_v$$

$tr(\Delta\sigma)=3\left(k_0 P^{-}+K_{cam}\right)\Delta\varepsilon_v$ with $k_0=\frac{1+e_0}{\kappa}$ where $e_0=\frac{n}{1-n}$; n is porosity and it is a material characteristic.

In elasticity:

$$tr(\Delta\sigma)=\frac{E}{(1-2\nu)}tr(\Delta\varepsilon)=3\mathrm{K}\Delta\varepsilon_v$$

The analogy between the hydrostatic part of Cam_Clay and linear elasticity **in an initial state** makes it possible to write:

$$\frac{\left(1+e_0\right)P^{-}}{\kappa}+K_{cam}=\frac{E}{3\left(1-2\nu\right)}$$

E and ν are not data materials but rather μ the shear modulus: $\mu=\frac{E}{2\left(1+\nu\right)}$

What amounts writing the following equality while eliminating E :

$$\frac{\left(1+e_0\right)P^{-}}{\kappa}+K_{cam}=\frac{2\mu}{3}\frac{\left(1+\nu\right)}{\left(1-2\nu\right)} \quad \text{or} \quad \frac{\left(1+\nu\right)}{\left(1-2\nu\right)}=\frac{3\left(1+e_0\right)P^{-}+3\mathrm{K}_{cam}\kappa}{2\mu\kappa}$$

and one finds the statement of ν :

$$\nu=\frac{3(1+e_0)P^{-}+3K_{cam}\kappa-2\mu\kappa}{6(1+e_0)P^{-}+6K_{cam}\kappa-2\mu\kappa}$$

to the starting of computation, P^{-} corresponds to the initial stress field.

one can then deduce E from ν : $E=2\mu(1+\nu)$

the following conditions are to be checked:

$$0<\nu=\frac{3\left(1+e_0\right)P^{-}+3\mathrm{K}_{cam}\kappa-2\mu\kappa}{6\left(1+e_0\right)P^{-}+6\mathrm{K}_{cam}\kappa+2\mu\kappa}\leq 0.5 \quad \text{and} \quad E>0$$

if one or the other of the two conditions is not satisfied, an alarm message informs the user of nonthe coherence of the provided parameters.

Notice 3:

*Si P_{trac} is given null:

two possibilities for K_{cam} :

1 - K_{cam} positive (of the null initial stresses are allowed)

2 - K_{cam} no one (the stresses should absolutely be initialized)

*Si P_{trac} is given negative:
only one possibility for K_{cam} :
* K_{cam} positive as the relation should be satisfied $k_0 P_{trac} + K_{cam} > 0$
(one cannot initialize the stresses and zero give a value to K_{cam})

3.1 Local variables

V_1 : critical pressure P_{cr}
V_2 : plastic state
V_3 : stress of containment P
V_4 : equivalent stress Q
V_5 : voluminal plastic strain ε_v^p
V_6 : equivalent plastic strain ε_{eq}^p
V_7 : index of the vacuums e

4 Implemented of a computation with model CAM_CLAY

4.1 Initialization of computation

In the model CAM_CLAY, the nonlinear elastic model reveals a hydrostatic stress for a voluminal strain null [éq 3.3-3].

The user adopts one of the two following choices:

- To give to the material parameter K_{cam} which represents an initial compressibility a positive **value**. The computation can be done with a stress state initial null.

- To give to the material parameter K_{cam} which represents an initial compressibility zero **a value**. To initialize the stress state according to one in the two different ways:

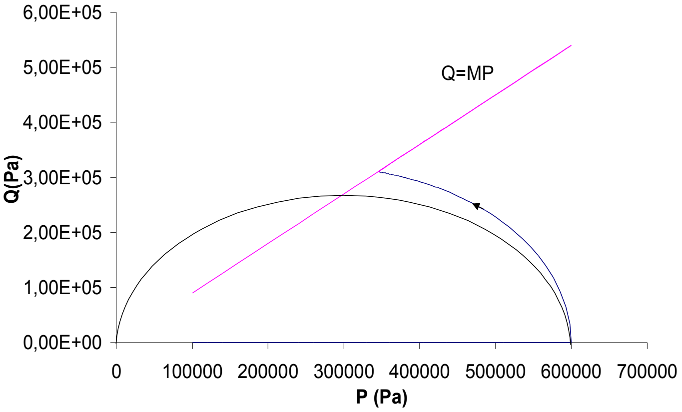
To carry out a linear elastic design by affecting boundary conditions such as the stress field in structure is a uniform hydrostatic compression. One extracts from this computation the stress field with Gauss points. This stress field is regarded as the initial state of the hydrostatic stress necessary to model CAM_CLAY in STAT_NON_LINE computation using the model CAM_CLAY.

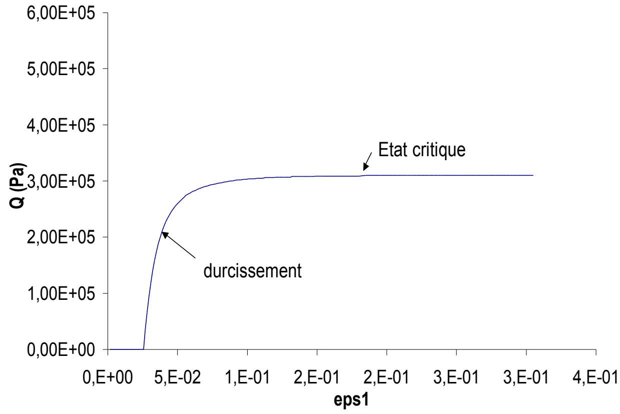
To use operator CREA_CHAMP to create with operation "AFFE" a hydrostatic stress field with Gauss points, the stress in this case is given of negative sign (convention *Aster* for compressions) and constitutes the initial state in the STAT_NON_LINE according to.

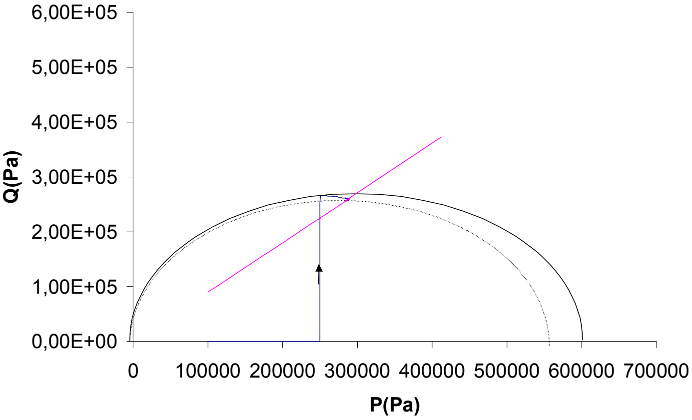
4.1 Examples of results obtained on triaxial compression tests

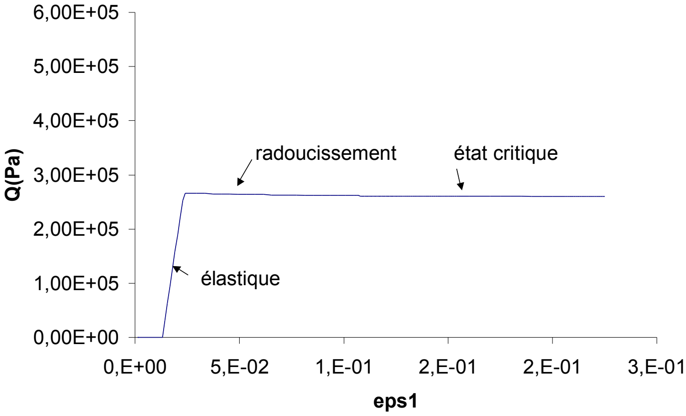
the following figures show triaxial ways of loading with evolutions of the axial strain according to the deviator Q . They resulting from numerical computations carried out with CAM_CLAY are the model established in *Code_Aster*. These test were carried out by means of a modelization of the type KIT_HM in not drained condition (this condition easily enables us to charge in a purely deviatoric way, the hydrostatic part of the loading being taken again by the pressure of water). The shapes of the curves obtained numerically with *Code_Aster* completely comparable to the schematic curves are presented in the paper of Charlez [bib2].

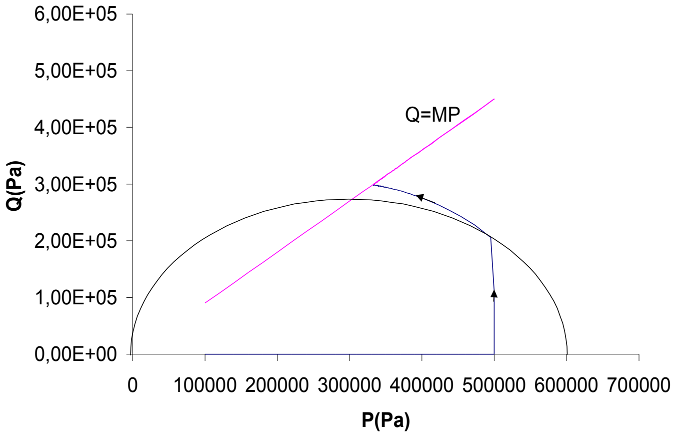
In the first test, the material is normally consolidated, i.e. the starting hydrostatic pressure is equal to the pressure of consolidation (in this case 6.10^5 Pa). Hardening (positive) starts at the beginning of the deviatoric phase, without preliminary elastic phase. Hardening continues to a bearing of perfect plasticity when the critical point is reached ($Q=MP$).
As for the three other tests, the deviatoric phase starts for a value of the average effective stress lower than the pressure of consolidation, the material is of this surconsolidé fact.
If P is higher than P_{cr} equal to $3.10^5 Pa$, the specific point of the loading cuts the surface of load before the critical line. There will be thus three specific phases: an elastic phase, a contracting plastic phase then a perfect plastic phase.
In the case where $P=P_{cr}$, the behavior is plastic perfect right after the elastic phase.
In the case where P is lower than P_{cr} , the point representative of the loading cuts the critical line before the surface of load which it reaches during a purely elastic way. In this configuration, the behavior is lenitive and dilating and blocked energy decreases. The point representative of the loading joined then the critical condition where the material will enter in perfect plasticity.
Behavior `CAM_CLAY` cannot produce a behavior continuement contractor/dilating. The point representative of the loading is obliged to pass by the critical condition where all the hardening parameters (plastic voluminal strain, critical pressure, blocked energy) become steady [bib2].


Q=MP
Q(Pa)
P (Pa)


Etat critique
durcissement
Q (Pa)
eps1


Q(Pa)
P(Pa)


radoucissement
état critique
élastique
Q(Pa)
eps1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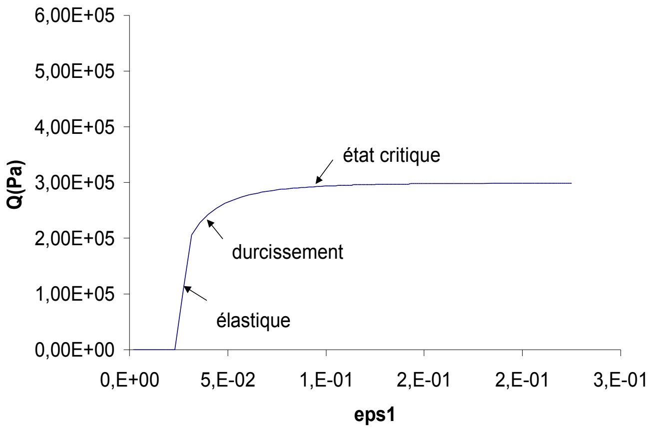
Q(Pa)
P(Pa)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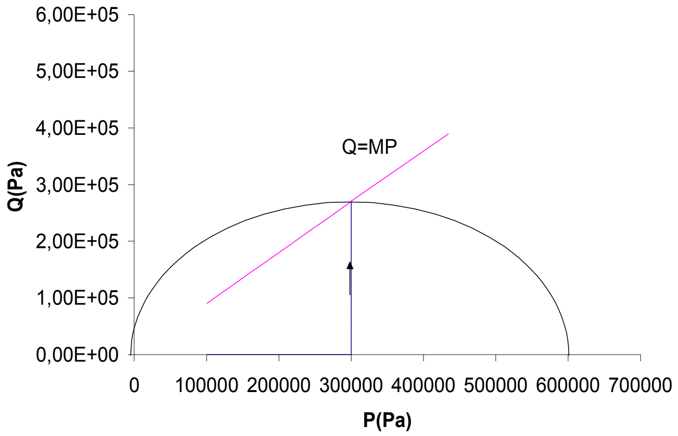
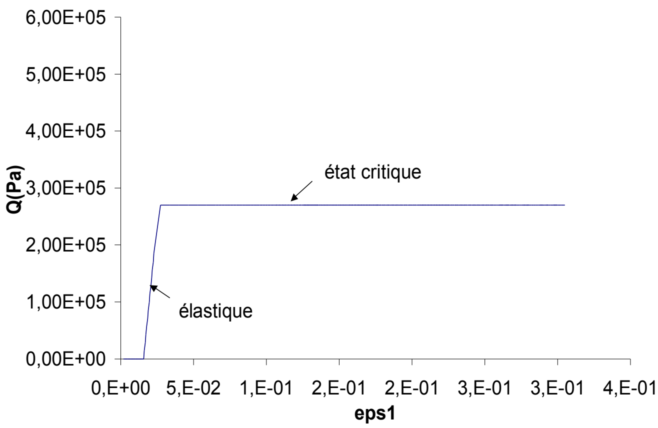

Q=MP
Q(Pa)
P(Pa)


état critique
élastique
Q(Pa)
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5 Appendix: Tangent operator into implicit. Option FULL_MECA

We present in this appendix of the elements of computation of the coherent tangent operator.

5.1 General case

5.1.1 Processing of the deviatoric part

One considers here that the variation of loading is purely deviatoric $(\delta P=0)$.
The increment of the deviatoric stress is written in the form:

$$\Delta s_{ij}=2\mu\left(\Delta\tilde{\varepsilon}_{ij}-\Delta\tilde{\varepsilon}_{ij}^{p}\right) \quad \textbf{éq 8.1.1-1}$$

Around the point of equilibrium $\left(\sigma^{-}+\Delta\sigma\right)$, one considers a variation δs of the deviatoric part of the stress:

$$\delta s_{kl}=2\mu\left(\delta\tilde{\varepsilon}_{kl}-\delta\tilde{\varepsilon}_{kl}^{p}\right) \quad \textbf{éq 8.1.1-2}$$

Computation of $\delta\tilde{\varepsilon}_{kl}^{p}$:

It is known that:

$$\Delta\tilde{\varepsilon}_{kl}^{p}=3\Lambda\ s_{kl} \quad \textbf{éq 8.1.1-3}$$

By deriving this deviatoric equation compared to the forced, one obtains:

$$\delta\tilde{\varepsilon}_{kl}^{p}=3\ \delta\ \Lambda\ s_{kl}+3\ \Lambda\ \delta\ s_{kl} \quad \textbf{éq 8.1.1-4}$$

Computation of $\delta\Lambda$:
One a:

$$\begin{aligned}\Lambda&=\frac{1}{H_p}\left[\left(\frac{\partial f}{\partial\sigma}\right)_{mn}\Delta\sigma_{mn}\right]=\frac{1}{H_p}\left[\left(\frac{\partial f}{\partial s}\right)_{mn}\Delta s_{mn}+\frac{\partial f}{\partial P}\Delta P\right]\\&=\frac{1}{H_p}\left[3\mathrm{s}_{mn}\Delta s_{mn}+2\mathrm{M}^2\left(P-P_{cr}\right)\Delta P\right]\end{aligned} \quad \textbf{éq 8.1.1-5}$$

If one considers only the evolution of the deviatoric part of σ $(\delta P=0)$, then:

$$\delta\ (\Lambda\ H_p)=\delta\ \Lambda H_p+\Lambda\delta H_p=\left[3\delta\mathrm{s}_{mn}\Delta s_{mn}+3\mathrm{s}_{mn}\delta s_{mn}\right]-2\mathrm{M}^2\,\Delta P\delta P_{cr} \quad \textbf{éq 8.1.1-6}$$

Gold: $\delta\ P_{cr}=kP_{cr}\ \delta\ \varepsilon_v^P$.

Comme $\Delta\varepsilon_v^p=2\Lambda\mathrm{M}^2(P-P_{cr})$, on a $\delta\varepsilon_V^p=2\delta\Lambda\mathrm{M}^2(P-P_{cr})-2\mathrm{M}^2\,\Lambda\delta P_{cr}$, **éq 8.1.1-7**

From where:

$$2\delta\ \Lambda M^2(P-P_{cr})=\left[\frac{1}{kP_{cr}}+2\Lambda\mathrm{M}^2\right]\delta P_{cr}. \quad \textbf{éq 8.1.1-8}$$

In addition,

$$H_p = 4\,kM^4 PP_{cr}(P - P_{cr}) \text{ et } \delta\, H_p = 4\,kM^4 P(P - 2\mathrm{P}_{cr})\delta\, P_{cr}. \quad \textbf{éq 8.1.1-9}$$

By injecting this last equation in the equation [éq 5.3.1-6], one obtains:

$$\delta\, \Lambda H_p + \left[4\Lambda\, kM^4 P(P - 2\mathrm{P}_{cr}) + 2\mathrm{M}^2 \Delta P\right]\delta\, P_{cr} = \left[3\delta \mathrm{s}_{mn} \Delta s_{mn} + 3\mathrm{s}_{mn}\delta s_{mn}\right] \quad \textbf{éq 8.1.1-10}$$

By means of the relation [éq 5.3.1-8], it comes then:

$$\delta\, \Lambda = \frac{\left[3\delta \mathrm{s}_{mn} \Delta s_{mn} + 3\mathrm{s}_{mn}\delta s_{mn}\right]}{(H_p + A)} \quad \textbf{éq 8.1.1-11}$$

with $A = \left[4\mathrm{k}\ \Lambda M^4 P(P - 2\mathrm{P}_{cr}) + 2\mathrm{M}^2 \Delta P\right]\left[\dfrac{M^2(P - P_{cr})}{\dfrac{1}{2kP_{cr}} + \Lambda M^2}\right]$

One then obtains immediately the variation of the deviatoric part of the plastic strain:

$$\delta\varepsilon_{kl}^p = \frac{9}{(H_p + A)}(\Delta s_{mn}\delta s_{mn}s_{kl} + s_{mn}\delta s_{mn}s_{kl}) + \frac{9}{H_p}s_{mn}\Delta s_{mn}\delta s_{kl} + \frac{6}{H_p}M^2(P - P_{cr})\Delta P\delta s_{kl}$$

éq 8.1.1-12

δs_{ij} is written then:

$$\delta s_{ij} = 2\,\mu\delta\,\{\,\tilde{\varepsilon}_{ij} - \frac{18\,\mu}{(H_p + A)}\left[\left(\Delta s_{kl}\, s_{ij}\,\delta s_{kl} + s_{kl}\, s_{ij}\,\delta s_{kl}\right)\right] - \frac{18\,\mu}{H_p}s_{kl}\,\Delta s_{kl}\,\delta s_{ij} - \frac{12\,\mu}{H_p}M^2\left(P - P_{cr}\right)\Delta P\delta s_{ij}$$

éq 8.1.1-13

which becomes by separating the terms in variation from stresses and the term in variation of total deflection:

éq 8.1.1-14

or in tensorial writing:

$$\left\{I_4^d\left(1 + \frac{12\,\mu}{H_p}M^2\left(P - P_{cr}\right)\Delta P + \frac{18\,\mu}{H_p}\Delta s : s\right) + \frac{18\,\mu}{(H_p + A)}(s + \Delta s)\otimes s\right\}\delta s = 2\,\mu\delta\,\{\,\tilde{\varepsilon}\,\} \quad \textbf{q8.1.1-15}$$

that one can still write by symmetrizing the tensor $(s + \Delta s)\otimes s$:

$$\left\{I_4^d\left(1 + \frac{12\,\mu}{H_p}M^2\left(P - P_{cr}\right)\Delta P + \frac{18\,\mu}{H_p}\Delta s : s\right) + \frac{18\,\mu}{(H_p + A)}\aleph\right\}\delta s = 2\,\mu\delta\,\{\,\tilde{\varepsilon}\,\} \quad \textbf{éq 8.1.1-16}$$

with: $\aleph = \dfrac{1}{2}\left[((s + \Delta s)\otimes s) + (s\otimes(s + \Delta s))^T\right]$

Computation of $\aleph$, while posing: $T_{ij} = s_{ij} + \Delta s_{ij}$

$$T\otimes s=\begin{bmatrix} T_{11}s_{11} & T_{11}s_{22} & T_{11}s_{33} & \sqrt{2}T_{11}s_{12} & \sqrt{2}T_{11}s_{23} & \sqrt{2}T_{11}s_{31}\\ T_{22}s_{11} & T_{22}s_{22} & T_{22}s_{33} & \sqrt{2}T_{22}s_{12} & \sqrt{2}T_{22}s_{23} & \sqrt{2}T_{22}s_{31}\\ T_{33}s_{11} & T_{33}s_{22} & T_{33}s_{33} & \sqrt{2}T_{33}s_{12} & \sqrt{2}T_{33}s_{23} & \sqrt{2}T_{33}s_{31}\\ \sqrt{2}T_{12}s_{11} & \sqrt{2}T_{12}s_{22} & \sqrt{2}T_{12}s_{33} & 2\mathrm{T}_{12}s_{12} & 2\mathrm{T}_{12}s_{23} & 2\mathrm{T}_{12}s_{31}\\ \sqrt{2}T_{23}s_{11} & \sqrt{2}T_{23}s_{22} & \sqrt{2}T_{23}s_{33} & 2\mathrm{T}_{23}s_{12} & 2\mathrm{T}_{23}s_{23} & 2\mathrm{T}_{23}s_{31}\\ \sqrt{2}T_{31}s_{11} & \sqrt{2}T_{31}s_{22} & \sqrt{2}T_{31}s_{33} & T_{31}s_{12} & 2\mathrm{T}_{31}s_{23} & 2\mathrm{T}_{31}s_{31} \end{bmatrix}$$ **éq 8.1.1-17**

$$\aleph=\frac{1}{2}\left[(T\otimes s)+(T\otimes s)^{T}\right]$$ **éq 8.1.1-18**

Is:

$$C=\left\{I_4^d\left(\frac{1}{2\mu}+\frac{6}{H_p}M^2\left(P-P_{cr}\right)\Delta P+\frac{9}{H_p}\Delta s:s\right)+\frac{9}{(H_p+A)}\aleph\right\}$$ **éq 8.1.1-19**

one poses:

$$c=\frac{9}{H_p}\left(\Delta s:s\right)$$ **éq 8.1.1-20**

and

$$d=\frac{6}{H_p}M^2\left(P-P_{cr}\right)\Delta P$$ **éq 8.1.1-21**

the symmetric matrix C of dimensions (6,6) is too large to be presented whole, one breaks up it into 4 parts C_1 C_2 , C_3 and C_4 :

$$C=\begin{bmatrix} C_1 & C_2\\ C_3 & C_4 \end{bmatrix}$$

with

$$C_1=\begin{bmatrix} \frac{1}{2\mu}+c+d+\frac{9}{(H_p+A)}s_{11}T_{11} & \frac{9}{2(H_p+A)}(T_{11}s_{22}+T_{22}s_{11}) & \frac{9}{2(H_p+A)}(T_{11}s_{33}+T_{33}s_{11}) \\ \frac{9}{2(H_p+A)}(T_{22}s_{11}+T_{11}s_{22}) & \frac{1}{2\mu}+c+d+\frac{9}{(H_p+A)}T_{22}s_{22} & \frac{9}{2(H_p+A)}(T_{22}s_{33}+T_{33}s_{22}) \\ \frac{9}{2(H_p+A)}(T_{33}s_{11}+T_{11}s_{33}) & \frac{9}{2(H_p+A)}(T_{22}s_{33}+T_{33}s_{22}) & \frac{1}{2\mu}+c+d+\frac{9}{(H_p+A)}T_{33}s_{33} \end{bmatrix}$$

éq 8.1.1-22

$$C_2=\begin{bmatrix} \frac{9\sqrt{2}}{2(H_p+A)}(T_{11}s_{12}+s_{11}T_{12}) & \frac{9\sqrt{2}}{2(H_p+A)}(T_{11}s_{23}+s_{11}T_{23}) & \frac{9\sqrt{2}}{2(H_p+A)}(T_{11}s_{13}+s_{11}T_{13}) \\ \frac{9\sqrt{2}}{2(H_p+A)}(T_{22}s_{12}+s_{22}T_{12}) & \frac{9\sqrt{2}}{2(H_p+A)}(T_{22}s_{23}+s_{22}T_{23}) & \frac{9\sqrt{2}}{2(H_p+A)}(T_{22}s_{13}+s_{22}T_{13}) \\ \frac{9\sqrt{2}}{2(H_p+A)}(T_{33}s_{12}+s_{33}T_{12}) & \frac{9\sqrt{2}}{2(H_p+A)}(T_{33}s_{23}+s_{33}T_{23}) & \frac{9\sqrt{2}}{2(H_p+A)}(T_{33}s_{13}+s_{33}T_{13}) \end{bmatrix}$$

éq 8.1.1-23

$$C_3=C_2$$

éq 8.1.1-24

$$C_4=\begin{bmatrix} \frac{1}{2\mu}+c+d+\frac{18}{(H_p+A)}s_{12}T_{12} & \frac{9}{(H_p+A)}(T_{12}s_{23}+T_{23}s_{12}) & \frac{9}{(H_p+A)}(T_{12}s_{23}+T_{23}s_{12}) \\ \frac{9}{(H_p+A)}(T_{23}s_{12}+T_{12}s_{23}) & \frac{1}{2\mu}+c+d+\frac{18}{(H_p+A)}T_{23}s_{23} & \frac{9}{(H_p+A)}(T_{23}s_{13}+T_{13}s_{23}) \\ \frac{9}{(H_p+A)}(T_{13}s_{12}+T_{12}s_{13}) & \frac{9}{(H_p+A)}(T_{13}s_{23}+T_{23}s_{13}) & \frac{1}{2\mu}+c+d+\frac{18}{(H_p+A)}T_{13}s_{13} \end{bmatrix}$$

éq 8.1.1-25

Computation of the rate of variation of volume:

$$\Delta\varepsilon_v^p=2\mathrm{M}^2\Lambda(P-P_{cr}),\quad \delta\varepsilon_v^p=2\mathrm{M}^2\delta\Lambda(P-P_{cr})-2\mathrm{M}^2\Lambda\delta\ P_{cr}=B\delta\ \Lambda=\frac{3\mathrm{B}}{(H_p+A)}(s+\Delta s).\delta\ s$$

éq 8.1.1-26

with: $$B=2\mathrm{M}^2(P-P_{cr})-2\mathrm{M}^2\Lambda\frac{M^2(P-P_{cr})}{\frac{1}{2kP_{cr}}+M^2\Lambda}.$$

éq 8.1.1-27

or by means of [éq 5.3.1-11]

$$\delta\varepsilon_v^p=\frac{3\mathrm{B}}{(H_p+A)}(s+\Delta s).\delta\ s$$

éq 8.1.1-28

One thus has:

$$\delta\ \varepsilon_{ij}=(C_{ijkl}-\frac{B}{(H_p+A)}(s+\Delta s)_{kl}\delta_{ij})\ \delta\ s_{kl}$$

éq 8.1.1-29

5.1.2 Processing of the hydrostatic part

One considers now that the variation of loading is purely spherical ($\delta\ s=0$).
The increment of P is written in the form:

$$\Delta P=P^{-}\exp\left(k_0\,\Delta\varepsilon_v^e\right)-P^{-}$$ **éq 8.1.2-1**

the derivative of this equation gives:

$$\delta P=k_0 P\left(\delta\varepsilon_v-\delta\varepsilon_v^p\right)$$ **éq 8.1.2-2**

Computation of $\delta\varepsilon_v^p$:

It is known that:

$$\Delta\varepsilon_v^p=\Lambda 2M^2\left(P-P_{cr}\right)$$ **éq 8.1.2-3**

By differentiating this equation, one obtains:

$$\delta\varepsilon_v^p=2\mathrm{M}^2\left(\delta\ \Lambda\left(P-P_{cr}\right)+\Lambda\left(\delta P-\delta P_{cr}\right)\right)$$ **éq 8.1.2-4**

One knows the statement of Λ :

$$\Lambda=\frac{2\mathrm{M}^2\left(P-P_{cr}\right)\Delta P+3\mathrm{s}\Delta\mathrm{s}}{H_p}=\frac{b}{H_p}$$ **éq 8.1.2-5**

by posing

$$b=2\mathrm{M}^2\left(P-P_{cr}\right)\Delta P+3\mathrm{s}\Delta\mathrm{s}$$ **éq 8.1.2-6**

While differentiating $\Delta\Lambda$, it comes:

$$\delta\Lambda=\frac{2\mathrm{M}^2}{H_p}\left[\left(P-P_{cr}\right)\delta P+\left(\delta P-\delta P_{cr}\right)\Delta P\right]-\frac{4kM^4 b}{H_p^2}\left[\delta\,PP_{cr}\left(2\mathrm{P}-P_{cr}\right)+\delta P_{cr}P\left(P-2\mathrm{P}_{cr}\right)\right]$$

éq 8.1.2-7

One seeks the statement of δP_{cr} according to $\delta\Lambda$:

There is

$$\delta P_{cr}=kP_{cr}\,\delta\varepsilon_v^p$$ **éq 8.1.2-8**

One can write:

$$\frac{\delta P_{cr}}{kP_{cr}}=\delta\Lambda 2M^2\left(P-P_{cr}\right)+\Lambda 2M^2\left(\delta P-\delta P_{cr}\right)$$ **éq 8.1.2-9**

$$\delta P_{cr}\left(\frac{1+\Lambda 2M^2 kP_{cr}}{kP_{cr}}\right)=\delta\Lambda 2M^2\left(P-P_{cr}\right)+\Lambda 2M^2\delta P$$ **éq 8.1.2-10**

$$\delta P_{cr}=\left(\frac{2\mathrm{M}^2\left(P-P_{cr}\right)kP_{cr}}{1+2\,kP_{cr}\Lambda M^2}\right)\delta\Lambda+\left(\frac{2\Lambda\mathrm{M}^2 kP_{cr}}{1+2\,kP_{cr}\Lambda M^2}\right)\delta P$$ **éq 8.1.2-11**

One poses

$$c=\frac{2\mathrm{M}^2 kP_{cr}\left(P-P_{cr}\right)}{\left[1+2\mathrm{M}^2 kP_{cr}\Lambda\right]}\ ,$$ **éq 8.1.2-12**

$$a=\frac{2\mathrm{M}^2 kP_{cr}\Lambda}{\left[1+2\mathrm{M}^2 kP_{cr}\Lambda\right]}$$ **éq 8.1.2-13**

One has then:

$$\delta P_{cr}=a\delta P+c\delta\Lambda$$ **éq 8.1.2-14**

By replacing the statement of δP_{cr} in $\delta\Lambda$ [éq 5.3.2-7], one finds:

$$\delta\Lambda=\left[2\mathrm{M}^2\left(P-P_{cr}\right)\delta P+2\mathrm{M}^2\left(\delta P-c\delta\Lambda-a\delta P\right)\Delta P\right].\frac{1}{H_p}$$
$$-\frac{4\,kM^4 b}{H_p^2}\left[\delta\,PP_{cr}\left(2\mathrm{P}-P_{cr}\right)+\left(c\delta\Lambda+a\delta P\right)P\left(P-2\mathrm{P}_{cr}\right)\right]$$ **éq 8.1.2-15**

By gathering the terms in $\delta\Lambda$ and those in δP , one finds:

$$\delta\Lambda=\frac{f}{e}\delta P$$ **éq 8.1.2-16**

with,

$$f=\frac{1}{H_p}\left[2\mathrm{M}^2\left(P-P_{cr}\right)+2\mathrm{M}^2\Delta P-2\,aM^2\Delta P\right]$$
$$-\frac{4\,kM^4 b}{H_p^2}\left[\left(2\mathrm{P}-P_{cr}\right)P_{cr}+aP\left(P-2\mathrm{P}_{cr}\right)\right]$$ **éq 8.1.2-17**

$$e=1+\frac{2\,cM^2\Delta P}{H_p}+\frac{4\,bckM^4}{H_p^2}P\left(P-2\mathrm{P}_{cr}\right)$$ **éq 8.1.2-18**

the statement of $\delta\varepsilon_v^p$ thus becomes:

$$\delta\varepsilon_v^p=2\mathrm{M}^2\left[\Lambda-a\Lambda-\Lambda c\frac{f}{e}+\frac{f}{e}\left(P-P_{cr}\right)\right]\delta P$$ **éq 8.1.2-19**

from where the statement of $\delta\varepsilon_v$ according to δP :

$$\delta P = \frac{k_0 P}{G} \delta \varepsilon_v$$ **éq 8.1.2-20**

$$G = 1 + 2\mathrm{M}^2 k_0 P \left(\Lambda - a\Lambda - \Lambda \frac{f}{e} c + \frac{f}{e} \left(P - P_{cr} \right) \right)$$ **éq 8.1.2-21**

Calculus of the variation of deviatoric strain:

$$\delta\ \tilde{\varepsilon}_{ij} = \delta\ \tilde{\varepsilon}^p = 3\delta\ \Lambda\ s = 3 \frac{f}{e}\ \delta\ P s_{ij}$$ **éq 8.1.2-22**

One thus has finally:

$$\delta \varepsilon_{ij} = F_{ij} \delta P$$ **éq 8.1.2-23**

with

$$F = \frac{3\mathrm{f}}{e} s - \frac{G}{3\mathrm{k}_0 P} 1^d$$ **éq 8.1.2-24**

5.1.3 tangent Operator

the tangent operator connects the variation of total stress to the variation of total deflection. Since the increment of the total deflection under loading deviatoric is written:

$$\delta\ \varepsilon_{ij} = (C_{ijkl} - \frac{B}{(H_p + A)} (s + \Delta s)_{kl} \delta_{ij})\ D^1_{klmn} \delta\ \sigma_{mn}\ ,$$ **éq 8.1.3-1**

with:

$$D^1 = \begin{bmatrix} 2/3 & -1/3 & -1/3 & 0 & 0 & 0 \\ -1/3 & 2/3 & -1/3 & 0 & 0 & 0 \\ -1/3 & -1/3 & 2/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$ **éq 8.1.3-2**

projection in space deviatoric,

and that under spherical loading one a:

$$\delta\ \varepsilon_{ij} = F_{ij}\ D^2_{kl}\ \delta\ \sigma_{kl}$$ **éq 8.1.3-3**

with:

$$D^2 = \begin{bmatrix} -1/3 \\ -1/3 \\ -1/3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$ **éq 8.1.3-4**

hydrostatic projection, one has then:

$$\delta \sigma_{ij} = A_{ijkl} \delta \varepsilon_{kl}$$ **éq 8.1.3-5**

with:

$$A_{ijkl}=\left[(C_{ijmn}-\frac{B}{(H_p+A)}(s+\Delta s)_{mn}\delta_{ij})\ D_{mnkl}1+F_{ij}D^2_{kl}\right]^{-1}$$ **éq 8.1.3-6**

the discretized tangent operator.

5.2 Tangent operator at the point criticizes

If the point of load is at the critical point $\left(P=P_{cr}\right)$, the general statement of the tangent operator is not more valid. This appears in particular by divide by 0 (see the equations of [§ 5.3.1]). One details in what follows the coherent tangent operator to the critical point while passing as for the general case by partly deviatoric and partly hydrostatic decomposition.

5.2.1 Processing of the deviatoric part

Let us recall that at the critical point, the statements of the plastic multiplier Λ and its derivative $\delta\Lambda$ are written in the following way:

$$\Lambda=\left(\frac{Q^e}{Q}-1\right)/6\mu \quad \text{and} \quad \delta\Lambda=\frac{\delta Q^e}{6\mu Q}-\frac{Q^e\,\delta Q}{6\mu Q^2}$$ **éq 8.2.1-1**

with,

$$\delta Q^e=\frac{3}{2}\frac{s^e\,\delta s^e}{Q^e} \quad \text{and} \quad \delta Q=\frac{3}{2}\frac{s\delta s}{Q}$$ **éq 8.2.1-2**

from where the statement of $\delta\Lambda$:

$$\delta\Lambda=\frac{1}{6\mu}\frac{3}{2}\left[\frac{s^e\,\delta s^e}{Q^e Q}-\frac{Q^e\,s\delta s}{Q^3}\right]$$ **éq 8.2.1-3**

Let us point out in the same way the statement of δs :

$$\delta s_{ij}=2\mu\left(\delta\tilde{\varepsilon}_{ij}-3\delta\Lambda s_{ij}-3\Lambda\delta s_{ij}\right)$$

While replacing Λ and $\delta\Lambda$ by their statements, one can write:

$$\delta s_{ij}=2\mu\delta\ \{\ \tilde{\varepsilon}_{ij}-\frac{3}{2}\frac{s^e_{kl}\,\delta s^e_{kl}}{Q^e Q}s_{ij}+\frac{3}{2}\frac{Q^e}{Q^3}s_{kl}\,\delta s_{kl}\,s_{ij}-\left(\frac{Q^e}{Q}-1\right)\delta s_{ij}\ ¿$$ **éq 8.2.1-4**

$$\delta s_{kl}\left[\delta_{ijkl}+\frac{Q^e}{Q}\delta_{ijkl}-\delta_{ijkl}-\frac{3}{2}\frac{Q^e}{Q^3}s_{kl}\,.\,s_{ij}\right]=2\mu\left[\delta_{ijkl}-\frac{3}{2}\frac{s^e_{kl}\cdot s_{ij}}{Q^e Q}\right]\delta\tilde{\varepsilon}_{kl}$$ **éq 8.2.1-5**

or in tensorial writing:

$$\delta s\underbrace{\left[\frac{Q^e}{Q}I^d_4-\frac{3}{2}\frac{Q^e}{Q^3}s\otimes s\right]}_{G}=2\mu\underbrace{\left[I^d_4-\frac{3}{2}\frac{s^e\otimes s}{Q^e Q}\right]}_{H}\delta\tilde{\varepsilon}$$ **éq 8.2.1-6**

As δs does not depend on $\delta\varepsilon_v$, one can confuse $\delta\tilde{\varepsilon}$ with $\delta\varepsilon$.

By means of the tensor of projection within the space of deviatoric stresses D^1 [éq 5.3.3-2], one can write:

$$\delta\varepsilon = \frac{D^1 . G . H^{-1}}{2\mu} . \delta\sigma$$ **éq 8.2.1-7**

5.2.2 Processing of the hydrostatic part

In tensorial writing, one has the following relation:

$$I^d \, \delta P = k_0 \, P \, \delta\varepsilon_v$$.éq **8.2.2-1**

according to the equation [éq 5.3.2-2] with $\delta\varepsilon_v^p = 0$ at the critical point.

As δP does not depend on $\delta\,\tilde{\varepsilon}$ then one can confuse $\delta\,\tilde{\varepsilon}$ with $\delta\varepsilon$.

$$I^d \, \delta P = k_0 P \, \delta\varepsilon$$ **éq 8.2.2-2**

By means of the tensor of projection within the space of hydrostatic stresses D^2 [éq 5.3.3-4], one can write:

$$\delta\varepsilon = \frac{I^d}{k_0 \, P} D^2 \, \delta\sigma$$ **éq 8.2.2-3**

5.2.3 tangent Operator

By combining the contributions of the two parts deviatoric and hydrostatic, one finds the writing of the tangent operator who connects the variation of the total stress to the variation of the total deflection at the critical point:

$$\delta\varepsilon = \left[\frac{D^1 . G . H^{-1}}{2\mu} + \frac{I^d}{k_0 \, P} D^2 \right] . \delta\sigma$$

or

$$\delta\sigma_{ij} = A_{ijkl} \, \delta\varepsilon_{kl}$$ **éq 8.2.2-4**

with

$$A_{ijkl} = \left[\frac{D^1 . G . H^{-1}}{2\mu} + \frac{I^d}{k_0 P} D^2 \right]^{-1}$$ **éq 8.2.2-5**

6 Bibliography

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1 Checking

constitutive law `CAM_CLAY` is checked by the following tests:

SSNP136	Test of foundation slipping by with the model of `CAM_CLAY`	[V6.03.136]
SSNV160	Test hydrostatic with model `CAM_CLAY`	[V6.04.160]
SSNV202	Test œdometric drained with the model of `CAM_CLAY`	[V6.04.202]
WTNV122	triaxial Compression test not drained with model `CAM_CLAY`	[V7.31.122]

2 Description of the versions of the document

Version Aster	**Author (S) Organization (S)**	**Description of the modifications**
6.4	J.EL-GHARIB, G.DEBRUYNE EDF-R&D/AMA	initial Text
7.3	J.El-Gharib, EDF-R&D/AMA	tangent Operator for the point criticizes
9.4	J.El-Gharib, EDF-R&D/AMA	Modification tangent operator, addition of local variables, cf files REX 10585 and 10700