Model CJS in géomechanics

Summarized:

One presents here the model CJS which applies to the soil mechanics. One specifies:

- the description of the model,
- the integration of the model in *Code_Aster*,
- the description of the introduced routines.

Contents

1 Notations

the notations used here are the usual notations of the soil mechanics, to which the notations suitable are added for the writing of the parameters of model CJS.

One also gives the correspondence, if it takes place, between the parameters of the model and their notations in Aster.

A	parameter of model	A_CJS
b	parameter of model	B_CJS
c	parameter of model	C_CJS
n	parameter of elastic	model
K	N_CJS bulk modulus	
K_o^e	parameter of the model	
K_o^p	parameter of the model	elastic
G	KP shear modulus	
G_o^e	parameter of the model	
$\boldsymbol{G}^d$	function controlling the evolution of plastic strains the déviatoires	
$\boldsymbol{s}$	deviative of the tensor of the stresses	
I_1	first invariant of the stresses	
p_{co}	pressure of initial criticism	PCO
P_a	pressure of reference of the model	PA
$f^i,\ f^d$	thresholds of the plastic mechanisms isotropic and déviatoire	
Q_{iso}	local variable of the model corresponding to the acceptable limit of déviatoire the tensor	
$\boldsymbol{q}, \boldsymbol{Q}$	plane of the model	
$R, \boldsymbol{X}$	local variables of the model corresponding to the average radius and the center of the surface of load in the plane déviatoire	
R_m	parameter of the model	RM
R_c	parameter of the model	RC
λ^i, λ^d	plastic multipliers of the tensor mechanisms isotropic and	
$\varepsilon, \varepsilon^e, \varepsilon^{ip}, \varepsilon^{dp}$	déviatoire of strains respectively total, elastic, plastic isotropic and plastic déviatoires	
ε_v	voluminal strains	
β	parameter of model	BETA_CJS
γ	parameter of model	GAMMA_CJS
θ	angle of Lode	
φ	function limiting the evolution of $\boldsymbol{X}$	
μ	parameters of model	MU_CJS
Q_{init}	parameter of model	Q_INIT

Note :

Foreword: Contrary for the use of géomechanics, sign convention selected is that of the mechanics of the continuums, i.e the tensions are counted positively. Introduction

2 The model

CJS is an elastoplastic constitutive law adapted to the modelization of the granular materials. It was developed at the Central School of Lyon ([bib1], [bib2], [bib3]).

Version CJS established in Code_Aster *is* a model arranged hierarchically including several levels of complexity. In its most complete statement, the model has two surfaces of load: one is activated by the isotropic requests, the other by the requests déviatoires. The first undergoes an isotropic hardening and the second a mixed hardening (isotropic and kinematical). The elastic model is of hypoelastic type nonlinear. Description

3 of model CJS Partition

3.1 of the strains

the increment of total strain breaks up into three parts, relative to each concerned mechanism: where

$$\dot{\varepsilon}_{ij}=\dot{\varepsilon}^{e}_{ij}+\dot{\varepsilon}^{ip}_{ij}+\dot{\varepsilon}^{dp}_{ij}$$

, $\dot{\varepsilon}^{e}_{ij}$ and $\dot{\varepsilon}^{ip}_{ij}$ are $\dot{\varepsilon}^{dp}_{ij}$ respectively the increments of elastic strain, isotropic plastic strain and plastic strain déviatoire. Elastic

3.2 mechanism

the elastic part of the model is of hypoelastic type, whose general statement is: where

$$\dot{\varepsilon}^{e}_{ij}=\frac{\dot{s}_{ij}}{2\,G}+\frac{\dot{I}_1}{9\,K}\delta_{ij}$$

is I_1 the first invariant of the stresses: , $I_1=tr(\sigma)$ $\mathbf{s}$ tensor of the stresses is the déviatoire part, and where and K are G respectively the bulk modulus and the shear modulus elastics. Those depend on the stress state according to:

$$K=K^{e}_{o}\left(\frac{I_1+Q_{init}}{3\,P_a}\right)^{n}\quad G=G^{e}_{o}\left(\frac{I_1+Q_{init}}{3\,P_a}\right)^{n}$$

K^{e}_{o} , G_o and P_a are n parameters of the model. is P_a a pressure of reference equal to -100 kPa. Isotropic

3.3 plastic mechanism

the surface of corresponding load is f^{i} , within the space of principal stresses, a plane perpendicular to the hydrostatic axis, that is to say: where

$$f^{i}(\sigma,\,Q_{iso})=-\frac{(I_1+Q_{init})}{3}+Q_{iso}$$

is Q_{iso} the thermodynamic force which depends on the local variable according to q : ,

$$\dot{Q}_{iso}=K^{p}\,\dot{q}=K^{p}_{o}\left(\frac{Q_{iso}}{P_a}\right)^{n}\dot{q}$$

K_o^p and P_a are n the parameters of the plastic mechanism déviatoire (and P_a are n identical to those of the elastic mechanism). The normality rule makes it possible to express the evolution of the plastic strain and the variable of hardening according to the evolution of the plastic multiplier: λ^i and

$$\dot{\varepsilon}_{ij}^{ip}=\dot{\lambda}^i \frac{\partial f^i}{\partial \sigma_{ij}}=-\frac{1}{3}\dot{\lambda}^i \delta_{ij} \text{ Taking into account } \dot{q}=-\dot{\lambda}^i \frac{\partial f^i}{\partial Q_{iso}}=-\dot{\lambda}^i$$

the second equation, the model of hardening can be also put in the form: Plastic

$$\dot{Q}_{iso}=-\dot{\lambda}^i K_o^p \left(\frac{Q_{iso}}{P_a}\right)^n$$

3.4 mechanism déviatoire

the surface of load of this second plastic mechanism is a convex surface with ternary symmetry defined by the equation: with

$$f^d(\sigma,\ R,\boldsymbol{X})=q_{II}\, h(\theta_q)+R(\ I_1+Q_{init})$$

. $q_{ij}=s_{ij}-I_1\, X_{ij}$

$$q_{II}=\sqrt{q_{ij}\, q_{ij}}$$

$$h(\theta_q)=\left(1+\gamma\cos(3\theta_q)\right)^{1/6}=\left(1+\gamma \quad \sqrt{54}\,\frac{\det(\boldsymbol{q})}{q_{II}^3}\right)^{1/6}$$

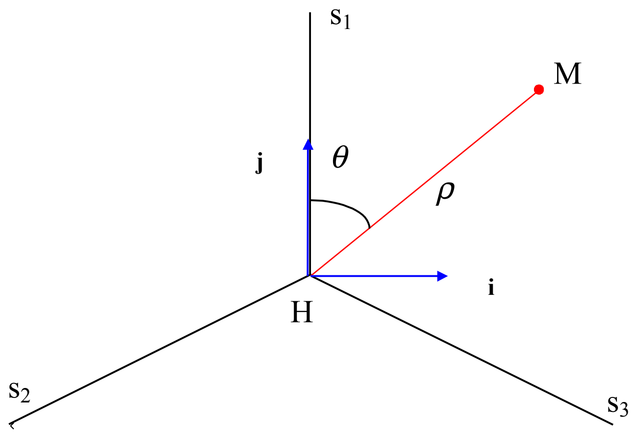
The scalar and R the tensor respectively $\boldsymbol{X}$ represent the average radius and the center of the surface of load in the déviatoire plane. ,

$\boldsymbol{s}$ and $\boldsymbol{q}$ are $\boldsymbol{X}$ tensors déviatoires. is γ a parameter which translates the dissymmetrical behavior of the soils into compression and extension. is θ the angle of Lode. This

surface of load evolves according to two types of hardening: isotropic hardening and kinematic hardening. Note:

The statement of the angle of Lode is found in the following way: In a reference of $(H, \boldsymbol{i}, \boldsymbol{j})$ *the déviatoire plane the vector can* $\mathbf{HM}$ *be given from the distance and* $HM=\rho$ *of the angle from Lode (cf* θ_s *[Figure 3.4-a]). The coordinates of are* $\mathbf{HM}$ *: Appear*

$\mathbf{HM}=(\rho\sin\theta_s, \rho\cos\theta_s)$



3.4-a: Angle of Lode in the déviatoire plane

the principal components of the deviator are thus: ,

$s_1=\rho\,\cos\theta_s$ *and* $s_2=\rho\,\cos\left(\frac{4\,\pi}{3}-\theta_s\right)$ *Consequently* $s_3=\rho\,\cos\left(\frac{2\,\pi}{3}-\theta_s\right)$

, one a: and $s_{II}=\sqrt{\frac{3}{2}}\,\rho$ *one* $\det(\boldsymbol{s})=\frac{1}{4}\,\rho^3\,\cos\theta_s\,(\cos^2\theta_s-3\,\sin^2\theta_s)=\frac{1}{4}\,\rho^3\,\cos(3\,\theta_s)$

from of then deduced the relation:

$$\cos(3\,\theta_s)=2^{1/2}\,3^{3/2}\,\frac{\det(s)}{s_{II}^3}$$

The angle θ_q *is calculated in the same way. isotropic*

3.4.1 hardening

the isotropic model of hardening is written as follows:

$$\dot{R}=\frac{A\,R_m^2\,\dot{r}}{(R_m+A\,r)^2}$$

The thermodynamic force is R function of which r the evolution is given by: By

$$\dot{r}=-\dot{\lambda}^d\,\frac{\partial f^d}{\partial R}\left(\frac{I_1+Q_{init}}{3\,P_a}\right)^{-1.5}=-\dot{\lambda}^d\,(I_1+Q_{init})\left(\frac{I_1+Q_{init}}{3\,P_a}\right)^{-1.5}$$

direct integration of the model of hardening, it comes: ,

$$R=\frac{A\ R_m\ r}{R_m+A\ r}\text{ that is to say also }r=\frac{R\ R_m}{A\left(R_m-R\right)}$$

the model of hardening can thus also express itself by: with

$$\dot{R}=-\dot{\lambda}^d\ A\left(1-\frac{R}{R_m}\right)^2\left(I_1+Q_{init}\right)\left(\frac{I_1+Q_{init}}{3\ P_a}\right)^{-1.5}=\dot{\lambda}^d G^R\left(\sigma,\ R\right)$$

and $G^R\left(\sigma,\ R\right)=-A\left(1-\frac{R}{R_m}\right)^2\left(I_1+Q_{init}\right)\left(\frac{I_1+Q_{init}}{3\ P_a}\right)^{-1.5}$

where (which R_m is the average radius of the elastic domain in fracture) and are A parameters of the model. é

3.4.2 CROUISSAGE kinematical

the model of kinematic hardening is given by:

$$\dot{X}_{ij}=\frac{1}{b}\ \dot{\alpha}_{ij}$$

The thermodynamic force is $\boldsymbol{X}$ function of the variable whose α nonlinear evolution is given by:

$$\dot{\alpha}_{ij}=-\dot{\lambda}^d\left[dev\left(\frac{\partial f^d}{\partial X_{ij}}\right)-\left(I_1+Q_{init}\right)\varphi\ X_{ij}\right]\left(\frac{I_1+Q_{init}}{3\ P_a}\right)^{-1.5}$$

The term makes it possible $-\left(I_1+Q_{init}\right)\varphi X$ to obtain nonlinear kinematic hardening, representing the limitation of the evolution of the surface of load. By

taking account of, $\frac{\partial f^d}{\partial X_{ij}}=\frac{\partial f^d}{\partial q_{kl}}\ \frac{\partial q_{kl}}{\partial X_{ij}}=-\left(I+Q_{init_1}\right)\frac{\partial f^d}{\partial q_{ij}}$ and while posing: , $Q_{ij}=dev\left(\frac{\partial f^d}{\partial q_{ij}}\right)$ it comes finally for the model from hardening: with

$$\dot{X}_{ij}=\dot{\lambda}^d\ \frac{1}{b}\left(Q_{ij}+\varphi\ X_{ij}\right)\left(I_1+Q_{init}\right)\left(\frac{I_1+Q_{init}}{3\ P_a}\right)^{-1.5}=\dot{\lambda}^d\ G_{ij}^X\left(\sigma,\boldsymbol{X}\right)$$

. $G_{ij}^X\left(\sigma,\boldsymbol{X}\right)=\frac{1}{b}\left(Q_{ij}+\varphi\ X_{ij}\right)\left(I_1+Q_{init}\right)\left(\frac{I_1+Q_{init}}{3\ P_a}\right)^{-1.5}$ where

φ a function which limits the evolution of and $\boldsymbol{X}$ is a parameter of the model.

The tensor $\boldsymbol{Q}$ is calculated according to the formula:

$$Q_{ij}=\frac{1}{h\left(\theta\right)^5}\left[\left(1+\frac{\gamma}{2}\ \cos\left(3\,\theta\right)\right)\frac{q_{ij}}{q_{II}}+\frac{\gamma\sqrt{54}}{6q_{II}^2}dev\left(\frac{\partial\det\left(\boldsymbol{q}\right)}{\partial q_{ij}}\right)\right]$$

The preceding statement is obtained in the following way. One a: where

$$\frac{\partial f^d}{\partial q_{ij}}=h(\theta_q)\frac{\partial q_{II}}{\partial q_{ij}}+q_{II}\frac{\partial h(\theta_q)}{\partial q_{ij}}$$

and $\frac{\partial q_{II}}{\partial q_{ij}}$ $\frac{\partial h(\theta_q)}{\partial q_{ij}}$ are respectively given by: from where

$$\frac{\partial q_{II}}{\partial q_{ij}}=\frac{q_{ij}}{q_{II}}$$

$$\frac{\partial h(\theta_q)}{\partial q_{ij}}=\frac{1}{6\,h(\theta_q)^5}\frac{\partial}{\partial q_{ij}}\left(1+\gamma\sqrt{54}\frac{\det(\boldsymbol{q})}{q_{II}^3}\right)=\frac{-\gamma\cos(3\theta_q)}{2\mathrm{h}(\theta_q)^5}\frac{q_{ij}}{q_{II}^2}+\frac{\gamma\sqrt{54}}{6\mathrm{h}(\theta_q)^5 q_{II}^3}\frac{\partial\det(\boldsymbol{q})}{\partial q_{ij}}$$

$$\frac{\partial f^d}{\partial q_{ij}}=\frac{1}{h(\theta_q)^5}\left[\left(1+\frac{\gamma}{2}\cos(3\theta_q)\right)\frac{q_{ij}}{q_{II}}+\frac{\gamma\sqrt{54}}{6\mathrm{q}_{II}^2}\left(\frac{\partial\det(\boldsymbol{q})}{\partial q_{ij}}\right)\right]$$

the function φ , as for it is given by: where

$$\varphi=\varphi_o\,h(\theta_s)\,Q_{II}$$

and $Q_{II}=\sqrt{Q_{ij}\,Q_{ij}}$. $h(\theta_s)=(1+\gamma\cos(3\theta_s))^{1/6}=\left(1+\gamma\sqrt{54}\frac{\det(\boldsymbol{s})}{s_{II}^3}\right)^{1/6}$ The term φ_o is expressed according to characteristic when the material breaks. Law

3.4.3 of evolution of the plastic mechanism déviatoire In

the granular materials, a variation of volume can occur for a loading déviatoire purely. This variation of volume is related on the discontinuous aspect of the material and the conditions kinematics which result the loading during. This particular phenomenon does not make it possible to define plastic strains the déviatoires from the only normality rule. This is why the plastic mechanism déviatoire is nonassociated. There thus exists a potential function controlling the evolution of the strains:

$$\dot{\varepsilon}_{ij}^{dp}=\dot{\lambda}^d\,G_{ij}^d$$

The potential function is defined from the following kinematical condition: where

$$\dot{\varepsilon}_v^{dp}=-\beta\left(\frac{s_{II}}{s_{II}^c}-1\right)\frac{|s_{ij}\,\dot{e}_{ij}^{dp}|}{s_{II}}$$

is β a parameter of the model and represents s_{II}^c the characteristic stress state. A surface, from form identical to the surface of load within the space of stresses, separates the contracting states from the dilating states. This surface, known as characteristic, has as an equation: where

$$f^c=s_{II}^c\,h(\theta_s)+R_c\,(I_1+Q_{init})$$

is R_c a parameter corresponding to the average radius of this characteristic surface. The kinematical condition can be also put in the form: where

$$\dot{\varepsilon}_v^{dp}+\beta\left(\frac{s_{II}}{s_{II}^c}-1\right)\frac{|s_{ij}\,\dot{e}_{ij}^{dp}|}{s_{II}}=\dot{\varepsilon}_v^{dp}+\beta\left(\frac{s_{II}}{s_{II}^c}-1\right)\frac{|s_{ij}\,\dot{e}_{ij}^{dp}|}{s_{ij}\,\dot{e}_{ij}^{dp}}\frac{s_{ij}\,\dot{e}_{ij}^{dp}}{s_{II}}$$
$$=\dot{\varepsilon}_v^{dp}+\frac{\beta'}{s_{II}}\,s_{ij}\,\dot{e}_{ij}^{dp}$$
$$=\dot{\varepsilon}_v^{dp}+\frac{\beta'}{s_{II}}\,s_{ij}\,\dot{\varepsilon}_{ij}^{dp}=0$$

. $\beta'=\beta\left(\frac{s_{II}}{s_{II}^c}-1\right)signe\left(s_{ij}\,\dot{\varepsilon}_{ij}^{dp}\right)$ It

is then possible to seek to express this kinematical condition from a tensor in n the form: i.e.

$$\dot{\varepsilon}_{ij}^{dp}\,n_{ij}=0$$

, after decomposition of each term in parts déviatoire and hydrostatics: One

$$\dot{\varepsilon}_{ij}^{dp}\,n_{ij}=\left(\dot{e}_{ij}^{dp}+\frac{1}{3}\,\dot{\varepsilon}_v^{dp}\,\delta_{ij}\right)\left(n_1\,s_{ij}+n_2\,\delta_{ij}\right)=n_1\,s_{ij}\,\dot{e}_{ij}^{dp}+n_2\,dt\,\varepsilon_v^{dp}=0$$

from of deduced the relation, $\frac{n_1}{n_2}=\frac{\beta'}{s_{II}}$ which added to the condition of standardization, $\boldsymbol{n}:\boldsymbol{n}=1$ conduit to the statements: and

$$n_1=\frac{\frac{\beta'}{s_{II}}}{\sqrt{\beta'^2+3}}\ ,\ n_2=\frac{1}{\sqrt{\beta'^2+3}}\ \text{is}\ \ n_{ij}=\frac{\beta'\,\frac{s_{ij}}{s_{II}}+\delta_{ij}}{\sqrt{\beta'^2+3}}$$

the law of evolution of must $\dot{\varepsilon}_{ij}^{dp}$ be such as the kinematical condition is satisfied. It is thus proposed to take the projection of on $\dot{\varepsilon}_{ij}^{dp}$ the hypersurface of strain of norm, $\boldsymbol{n}$ that is to say: with

$$\dot{\varepsilon}_{ij}^{dp}=\dot{\lambda}^d\left(\frac{\partial f^d}{\partial\sigma_{ij}}-\left(\frac{\partial f^d}{\partial\sigma_{kl}}\,n_{kl}\right)n_{ij}\right)=\dot{\lambda}^d\,G_{ij}^d$$

. $G_{ij}^d=\frac{\partial f^d}{\partial\sigma_{ij}}-\left(\frac{\partial f^d}{\partial\sigma_{kl}}\,n_{kl}\right)n_{ij}$ In addition

, for the computation of the potential, one can note that: Rough

$$\frac{\partial f^d}{\partial\sigma_{ij}}=\frac{\partial f^d}{\partial q_{kl}}\,\frac{\partial q_{kl}}{\partial\sigma_{ij}}+R\,\delta_{ij}$$
$$=\left[dev\left(\frac{\partial f^d}{\partial q_{kl}}\right)+\frac{1}{3}\,\frac{\partial f^d}{\partial q_{mm}}\,\delta_{kl}\right]\left[\delta_{ik}\,\delta_{jl}-\delta_{ij}\left(\frac{1}{3}\,\delta_{kl}+X_{kl}\right)\right]+R\,\delta_{ij}$$
$$=Q_{kl}\,\delta_{ik}\,\delta_{jl}-\delta_{ij}\left(\frac{1}{3}\,Q_{kl}\,\delta_{kl}+Q_{kl}\,X_{kl}\right)+\frac{1}{3}\,\frac{\partial f^d}{\partial q_{mm}}\left[\delta_{ik}\,\delta_{jl}\,\delta_{kl}-\delta_{ij}\left(\frac{1}{3}\,\delta_{kl}\,\delta_{kl}+\delta_{kl}\,X_{kl}\right)\right]+R\,\delta_{ij}$$
$$=Q_{ij}-\left(Q_{kl}\,X_{kl}-R\right)\delta_{ij}$$

3.4.4 surface

the state of fracture results from the nonlinear nature of the models of hardening and the existence of limiting values associated with the variables of hardening and R . $\boldsymbol{X}$ The limit of, R noted, R_m is reached when tends r towards the infinite one. The limit of X_{ij} is reached when becomes $\dot{X}_{ij}$ null. Under
these conditions: and

$$Q_{ij}=\varphi\ X_{ij} \text{ with } Q_{II}=\varphi\ X_{II\,\mathbf{lim}} \Rightarrow X_{II\,\mathbf{lim}}=\frac{1}{\varphi_o\ h(\theta_s)}$$

the state of fracture one thus has [Figure 3.4.4-a]: By

$$q_{II}=\frac{s_{II}+I_1\ X_{II\,\mathbf{lim}}\ \cos\alpha}{\cos(\theta_s-\theta_q)}$$

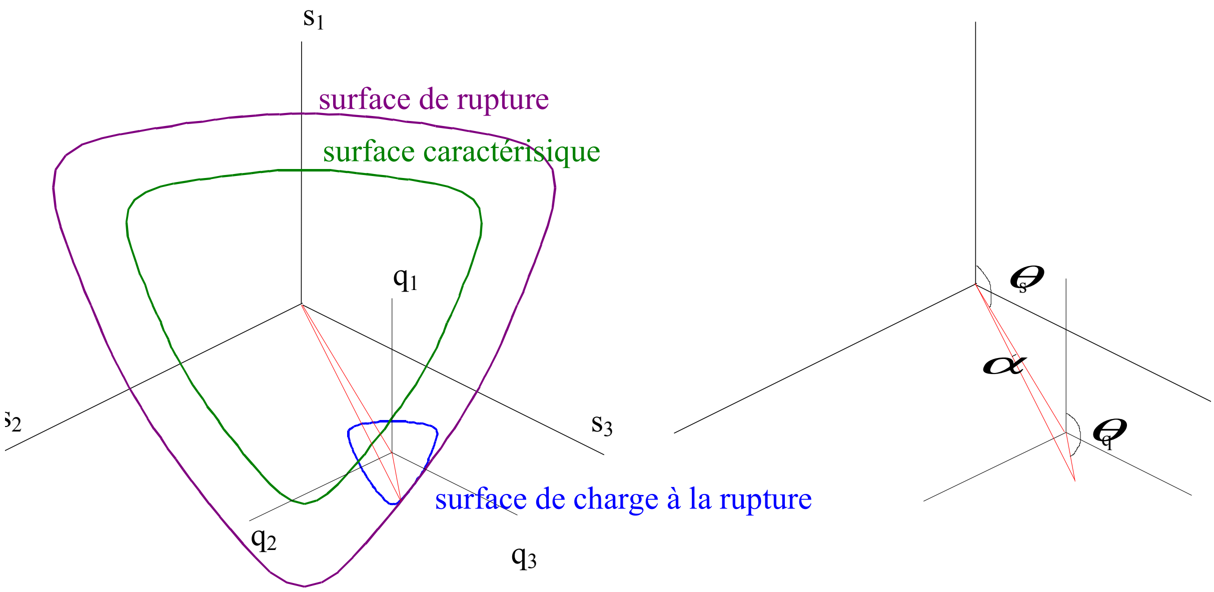
replacing this statement and the value of in R fracture, in the equation of the surface of breaking load, one obtains the equation of a limiting envelope for surfaces of load: with

$$f^r=s_{II}\ h(\theta_s)+R_r\ (I_1+Q_{init})=0$$

, $R_r=\frac{\cos\alpha}{\varphi_o}+\frac{h(\theta_s)}{h(\theta_q)}\ R_m\ \cos(\theta_s-\theta_q)$ average radius of the envelope, which is determined starting from the mechanical characteristics when the material breaks. The value of can φ_o then be deduced about it: with

$$\varphi_o=\frac{\cos\alpha}{R_r-\frac{h(\theta_s)}{h(\theta_q)}\ R_m\ \cos(\theta_s-\theta_q)}$$

Figure $\cos\alpha=\frac{q_{II}^2-s_{II}^2-(I_1\ X_{II})^2}{2\ s_{II}\ I_1\ X_{II}}$



3.4.4-a: Representation of the rough surfaces, characteristic and of load in the déviatoire plane In addition

, is R_r related to the maximum friction angle and depends on the average constraint and the relative density. To take into account the dependence of the maximum friction angle according to the average constraint and of the relative density, one considers the relation: where

$$R_r = R_c + \mu \ \ln\left(\frac{3\ p_c}{I_1 + Q_{init}}\right)$$

and R_c are μ parameters of the model. is p_c the average constraint criticizes, i.e. the minimal average constraint (it is negative with our sign convention) known by the material during its history. It depends on the initial relative density according to the classical notion critical line in the plane: $(e,\ \ln|p|)$ where

$$p_c = p_{co}\ \exp(-c\ \varepsilon_v)$$

is p_{co} the initial critical pressure and is $1/c$ the slope of the right of critical condition in the plane. $(|\varepsilon_v|,\ \ln|p|)$ Hierarchisation

3.5 of the summary model

3.5.1 Description of three levels CJS From

the complete description of the model given above, one deduces three levels from increasing complexity whose characteristics are summarized in the following table: Elastic

	mechanism plastic	Mechanism isotropic plastic	Mechanism linear déviatoire
CJS 1	not	activated activated	, perfect plasticity CJS
2 nonlinear	activated	activated	, isotropic hardening CJS
3 nonlinear	activated	activated	, kinematic hardening Table

3.5.1-1: The various mechanisms used by the various levels of model CJS Assessment

3.5.2 of parameters CJS In addition

, one can also summarize the correspondence between the various levels of the model and the parameters associated with each one from them: CJS

	n	K_o^e	G_o^e	K^p	γ	β	R_c	A	b	R_m	μ	p_{co}	c	p_a
1 formulates		×	×		×	×				×				×
2 formulates	×	×	×	×	×	×	×	×		×				×
3 formulates	×	×	×	×	×	×	×		×	×	×	×	×	×

3.5.2-1: Assessment of the various parameters according to levels CJS In

Code_Aster , the elastic parameters of model CJS (and K_o^e) G_o are directly taken into account in the elastic characteristics of the material, i.e. through the Poisson's ratio and Young modulus E . NU In

Code_Aster , the user explicitly does not indicate selected level CJS qu "it. C" is indeed the choice of the various parameters which determines the corresponding level We have to summarize the following logical tests which are integrated in the code: so

- then $n=0$ level CJS1, if
- (and $n\neq 0$) $A\neq 0$ then level CJS2, if
- (and $n\neq 0$) $A=0$ then level CJS3. Note:

> *The user must fix the value of equal P_a to according to $-100\,kPa$ the selected units. Moreover, for CJS3, the value of must p_{co} be negative. Correspondence*

3.5.3 with cohesion and the friction angle

the mechanics of the soils have the habit to use the notions of cohesion Cohesion, c friction angle and φ angle of dilatancy: . ψ These parameters are used in the model of Mohr Coulomb.
Level 1 of model CJS makes it possible to find a behavior very close by making the following choice to parameters: Integration

$$\left(\frac{1-\gamma}{1+\gamma}\right)^{1/6}=\frac{3-\sin(\varphi)}{3+\sin(\varphi)}$$

$$R_m=\frac{2\sqrt{\frac{2}{3}}\sin(\varphi)(1-\gamma)^{1/6}}{3-\sin(\varphi)}$$

$$Q_{init}=-3\,c\,\text{cotan}(\varphi)$$

$$\beta=\frac{-2\sqrt{6}\sin\psi}{3-\sin\psi}$$

4 of model CJS We

detail below the integration of model CJS according to or the activated mechanisms: nonlinear

- elastic, nonlinear
- elastic and nonlinear and plastic
- elastic plastic isotropic déviatoire elastic
- nonlinear, plastic isotropic and plastic déviatoire. In

each case, the goal is to calculate, starting from the fields known with the state less, ε^- and σ^- of the increment of strain, $\Delta\varepsilon$ the new stress state. σ^+ In

the sequence of computations, one starts by making the assumption that only the nonlinear elastic mechanism intervenes. An elastic prediction is thus carried out. This prediction is then used to compute: the loading functions and f^i , f^d one seeks to know if one goes then beyond the thresholds: if

- and $f^i\leq 0$, $f^d\leq 0$ the elastic prediction is regarded as new stress state, if

- and $f^i > 0$, $f^d \leq 0$ one makes the integration of the mechanisms elastic nonlinear and plastic isotropic, if
- and $f^i \leq 0$, $f^d > 0$ one makes the integration of the mechanisms elastic nonlinear and plastic déviatoire, if
- and $f^i > 0$, $f^d > 0$ one makes the integration of the mechanisms elastic nonlinear, plastic isotropic and plastic déviatoire. In

output of elastoplastic computation, when only one plastic threshold was initially exceeded, one recomputes each loading function. Indeed, it is possible that while seeking to bring back itself on one of the thresholds, one then exceeds the other threshold not activated initially by the elastic prediction. In this case, one solves then by integrating all the mechanisms. Choices

4.1 of the local variables

the variables, q and r are α equivalent to the associated thermodynamic forces, Q_{iso} and R . $\boldsymbol{X}$ For this reason and since their geometrical meaning is more obvious, we will retain like local variables for the integration of model CJS, the quantities, Q_{iso} and R . $\boldsymbol{X}$ In addition
, we add to the number of the local variables:

- the sign of the product $s_{ij}\,\varepsilon_{ij}^{dp}$
- the elastic or elastoplastic state of the material, while noting: 0
  •: elastic state 1
  •: elastoplastic state, isotropic plastic mechanism 2
  •: elastoplastic state, plastic mechanism déviatoire 3
  •: elastoplastic state, plastic mechanisms isotropic and déviatoire Finally

, the local variables are stored in a vector VI `in` the following order: Index

of local variable CJS		1 CJS	2 CJS	3 3D
2D	CJS	1 CJS	2 CJS	3 1
1	2	$Q_{iso}=\infty$	Q_{iso}	Q_{iso}
2	3	$R=R_m$	R	$R=R_m$
3	0	0	4	X_{11}
4	0	0	5	X_{22}
5	0	0	6	X_{33}
6	0	0	7	$\sqrt{2}\,X_{12}$
□	0	0	8	$\sqrt{2}\,X_{13}$
□	0	0	9	$\sqrt{2}\,X_{23}$
7	10	$\frac{q_{II}\,h(\theta_q)}{\lvert R_m(I_1+Q_{init})\rvert}$	$\frac{q_{II}\,h(\theta_q)}{\lvert R(I_1+Q_{init})\rvert}$	$\frac{q_{II}\,h(\theta_q)}{\lvert R_m(I_1+Q_{init})\rvert}$
8	11		$\frac{R}{R_m}$	$\frac{X_{II}}{X_{II}^{\lim}}$
9	12		$\lvert\frac{3Q}{I_1+Q_{init}}\rvert$	$\lvert\frac{3Q}{I_1+Q_{init}}\rvert$
10	Nombre of iterations	interns Nombre of iterations	interns Nombre of iterations	interns 13
11	local	test reaches local	test reached local	test reached 14
12	nbre	of recutting nbre	of recutting nbre	of recutting 15
13	16	$signe(s_{ij}\varepsilon_{ij}^{dp})$	$signe(s_{ij}\varepsilon_{ij}^{dp})$	$signe(s_{ij}\varepsilon_{ij}^{dp})$
14	0,1,2,3	state of material 0,1,2,3	state of material 0,1,2,3	state of the material Integration

4.2 of the nonlinear elastic mechanism In

the elastic case, the new stress state, σ^+ check simply:

$$\sigma_{ij}^{+}=\sigma_{ij}^{-}+D_{ijkl}(\sigma^{+})\,\Delta\varepsilon_{kl}$$

The dependence of the nonlinear elasticity tensor according to the stress state is summarized in fact with: where

$$D_{ijkl}(\sigma^{+})=D_{ijkl}^{lineaire}\left(\frac{I_1^{+}+Q_{init}}{3\,P_a}\right)^{n}$$

is $D_{ijkl}^{lineaire}$ the classical isotropic linear elasticity tensor, obtained from and K_o^e or G_o by equivalence from E `and` Nu `. Of`
this relation, it is deduced in particular that the first invariant of the stresses satisfied: This

$$I_1^{+}-I_1^{-}-3\,K_o^e\left(\frac{I_1^{+}+Q_{init}}{3\,P_a}\right)^{n}tr(\Delta\varepsilon)=0$$

nonlinear equation is solved by a secant method for CJS2 and CJS3, by differentiating the cases following the sign from. $tr(\Delta\varepsilon)$ With regard to the model CJS1, for which the parameter is n null, the explicit resolution is immediate, since one has then In

$$I_1^+ = I_1^- + 3\, K_o^e\, tr(\Delta\varepsilon)$$

the general case, the knowledge of and I_1^+ thus of the term allows $\left(\frac{I_1^+ + Q_{init}}{3\, P_a}\right)^n$ to define the nonlinear operator of elasticity. $D_{ijkl}(\sigma^+)$ Obtaining the new stress state is then direct. Isotropic

4.3 integration of the mechanisms elastic nonlinear and plastic In this case,

the new stress state, σ^+ checks:

$$\sigma_{ij}^+ = \sigma_{ij}^- + D_{ijkl}(\sigma^+)\left(\Delta\varepsilon_{kl} - \Delta\varepsilon_{kl}^{ip}\right)$$

Being given the simple form, plastic strains isotropic plastic mechanism:

$$\Delta\varepsilon_{ij}^{ip} = -\frac{1}{3}\Delta\lambda^i \delta_{ij}$$

the nonlinear system to solve is composed of: :

- LE_{ij} the elastic model state: : $\sigma_{ij}^+ - \sigma_{ij}^- - D_{ijkl}(\sigma^+)\left(\Delta\varepsilon_{kl} + \frac{1}{3}\Delta\lambda^i\delta_{kl}\right) = 0$
- LQ the model of hardening of the local variable: Q_{iso} : $Q_{iso}^+ - Q_{iso}^- - \Delta\lambda^i\, G^{Q_{iso}}(Q_{iso}^+) = 0$
- FI the equation of the isotropic surface of load: Schematically $-\frac{I_1^+ + Q_{init}}{3} + Q_{iso}^+ = 0$

, one thus seeks to solve the system, $R(Y)=0$ where the unknown Y is given by and $Y = (\sigma_{ij}^+,\, Q_{iso}^+,\, \Delta\lambda^i)$ where. $R = (LE_{ij},\, LQ,\, FI)$ The resolution of $R(Y)=0$ is done by the method of Newton: initialization

- and computation of a solution of test iterations
- of Newton: resolution of test $\frac{DR}{DY}(Y^p)\; DY^{p+1} = -R(Y^p)$
- of convergence: if convergence; $Y = Y^p$ if not and $Y^{p+1} = Y^p + DY^{p+1}$ We $p = p+1$

detail these three stages below. Initialization

4.3.1 and solution of test We

take simply for, $Y^0 = (\sigma_{ij}^0,\, Q_{iso}^0,\, \Delta\lambda^{i^0})$ the following values: :

$\sigma_{ij}^0 = \sigma_{ij}^{elas}$ stresses given by the elastic prediction:
$Q_{iso}^0 = Q_{iso}^-$ local variable with T:
$\Delta\lambda^{i^0} = 0$ plastic multiplier no one Contrary

to the other elastoplastic mechanisms, one does not calculate here a solution of test. Iterations

4.3.2 of Newton

the resolution of naturally $\frac{DR}{DY}\left(Y^p\right)\ DY^{p+1}=-R\left(Y^p\right)$ requires the computation of derivatives of, LE_{ij} and LQ compared to FI each component of. Y One a: with

$$\frac{DR}{DY}=\begin{bmatrix}\frac{\partial LE_{ij}}{\partial\sigma_{kl}} & \frac{\partial LE_{ij}}{\partial Q_{iso}} & \frac{\partial LE_{ij}}{\partial\Delta\lambda^i}\\ \frac{\partial LQ}{\partial\sigma_{kl}} & \frac{\partial LQ}{\partial Q_{iso}} & \frac{\partial LQ}{\partial\Delta\lambda^i}\\ \frac{\partial FI}{\partial\sigma_{kl}} & \frac{\partial FI}{\partial Q_{iso}} & \frac{\partial FI}{\partial\Delta\lambda^i}\end{bmatrix}$$

: Test

$$\frac{\partial LE_{ij}}{\partial\sigma_{kl}\delta}=\delta_{ik}\delta_{jl}-\frac{\partial D_{ijmn}}{\partial\sigma_{kl}}\left(\Delta\epsilon_{mn}+\frac{1}{3}\Delta\lambda^i\delta_{mn}\right)=\delta_{ik}\delta_{jl}-D_{ijmn}^{lineaire}\left(\Delta\epsilon_{mn}+\frac{1}{3}\Delta\lambda^i\delta_{mn}\right)\frac{n}{3\ P_a}\left(\frac{I_1+Q_{init}}{3\ P_a}\right)^{n-\delta}\delta_{kl}$$

$$\frac{\partial LE_{ij}}{\partial Q_{iso}}=0$$

$$\frac{\partial LE_{ij}}{\partial\Delta\lambda^i}=-\frac{1}{3}\ D_{ijmn}\delta_{\ mn}$$

$$\frac{\partial LQ}{\partial\sigma_{kl}}=0$$

$$\frac{\partial LQ}{\partial Q_{iso}}=1-\Delta\lambda^i\frac{\partial G^{Q_{iso}}}{\partial Q_{iso}}=1+\Delta\lambda^i\frac{n\ K_o^p}{P_a}\left(\frac{Q_{iso}}{P_a}\right)^{n-1}$$

$$\frac{\partial LQ}{\partial\Delta\lambda^i}=-\ G^{Q_{iso}}$$

$$\frac{\partial FI}{\partial\sigma_{kl}}=-\frac{1}{3}\delta_{\ kl}$$

$$\frac{\partial FI}{\partial Q_{iso}}=1$$

$$\frac{\partial FI}{\partial\Delta\lambda^i}=0$$

4.3.3 of convergence

the iterations of Newton are continued as much as the relative error remains $\frac{\|DY^{p+1}\|}{\|Y^{p+1}-Y^0\|}$ higher than the tolerance allowed by the user and defined by key word RESI_INTE_RELA . The norm used here is the vectorial norm: . $\|x\|=\sqrt{\sum_i x_i^2}$ Integration

4.4 of the mechanisms elastic nonlinear and plastic déviatoire In this case,

the new stress state, σ^+ checks:

$$\sigma_{ij}^+=\sigma_{ij}^-+D_{ijkl}(\sigma^+)\left(\Delta\epsilon_{kl}-\Delta\epsilon_{kl}^{dp}\right)$$

Plastic strains of the plastic mechanism déviatoire is given by the potential: G^d One

$$\Delta\epsilon_{ij}^{dp}=\Delta\lambda^d G_{ij}^d$$

from of deduced that the nonlinear system to solve is composed of: :

- LE_{ij} the elastic model state: : $\sigma_{ij}^+-\sigma_{ij}^--D_{ijkl}(\sigma^+)\left(\Delta\epsilon_{kl}-\Delta\lambda^d G_{kl}^d(\sigma^+,\ R^+,\boldsymbol{X}^+)\right)=0$
- LR the model of hardening of the variable: R : $R^+-R^--\Delta^d\ G^R(\sigma^+,\ R^+)=0$
- LX_{ij} the model of hardening of the variable: X_{ij} : $X_{ij}^+-X_{ij}^--\Delta\lambda^d\ G^X(\sigma^+,\boldsymbol{X}^+)=0$
- FD the equation of the surface of load déviatoire: As $q_{II}^+\ h(\theta_q^+)+R^+(I_1^++Q_{init})=0$

in the preceding paragraph one solves by the method of Newton the system, $R(Y)=0$ where the unknown Y is given by and $Y=(\sigma_{ij}^+,\ R^+,\ X_{ij}^+,\ \Delta\lambda^d)$ where. $R=(LE_{ij},\ LR,\ LX_{ij},\ FD)$ Initialization

4.4.1 and solution of test From

the state at the moment T, $(\sigma_{ij}^-,\ R^-,\ X_{ij}^-)$ we seek a solution of test which brings us closer to the final solution. For that we solve the following equation: with

$$f^d\left(\sigma_{ij}^-+D_{ijkl}^-\left(\Delta\epsilon_{kl}-\Delta\lambda^d\ G_{kl}^{d-}\right),\ R^-+\Delta\lambda^d\ G^{R-},\ X_{ij}^-+\Delta\lambda^d\ G_{ij}^{X-}\right)=0$$

$D_{ijkl}^-=D_{ijkl}(\sigma^-)$ $G_{kl}^{d-}=G_{kl}^d(\sigma^-,\ R^-,\boldsymbol{X}^-)$, $G^{R-}=G^R(\sigma^-,\ R^-)$ and $G_{ij}^{X-}=G_{ij}^X(\sigma^-,\boldsymbol{X}^-)$ where the unknown is the plastic multiplier, by $\Delta\lambda^d$ only one iteration of Newton, i.e. finally of we let us have: that is to say

$$\frac{\partial f^d}{\partial\Delta\lambda^d}_{|\Delta\lambda^d=0}\Delta\lambda^d=-f^d_{|\Delta\lambda^d=0}\quad\text{still with}\quad\Delta\lambda^d=-\frac{f^d_{|\Delta\lambda^d=0}}{\frac{\partial f^d}{\partial\Delta\lambda^d}_{|\Delta\lambda^d=0}}$$

: Moreover

$$\frac{\partial f^d}{\partial\Delta\lambda^d}=h(\theta_q)\frac{\partial q_{II}}{\partial\Delta\lambda^d}+q_{II}\frac{\partial h(\theta_q)}{\partial\Delta\lambda^d}+(I_1+Q_{init})\frac{\partial R}{\partial\Delta\lambda^d}+R\frac{\partial I_1}{\partial\Delta\lambda^d}$$

, one

a: then $I_1=I_1^-+3\ K^-\left(tr(\Delta\epsilon)-\Delta\lambda^d\ tr(\boldsymbol{G}^{d-})\right)$: one $\frac{\partial I_1}{\partial\Delta\lambda^d}=-3\ K^-\ tr(\boldsymbol{G}^{d-})$

a: then $R=R^-+\Delta\lambda^d\ G^{R-}$: one $\frac{\partial R}{\partial\Delta\lambda^d}=G^{R-}$

a: then

$$q_{ij}=\sigma^{-}_{ij}+D^{-}_{ijkl}\left(\Delta\varepsilon_{kl}-\Delta\lambda^{d}G^{d-}_{kl}\right)-\left[I^{-}_{1}+3K^{-}\left(tr(\Delta\varepsilon)-\Delta\lambda^{d}tr\left(G^{d-}\right)\right)\right]\left[\frac{1}{3}\delta_{ij}+X^{-}_{ij}+\Delta\lambda^{d}G^{X-}_{ij}\right]$$

: one $\left.\dfrac{\partial q_{ij}}{\partial\Delta\lambda^{d}}\right|_{\Delta\lambda^{d}=0}=-D^{-}_{ijkl}G^{d-}_{kl}+3K^{-}tr\left(G^{d-}\right)\left(\dfrac{1}{3}\delta_{ij}+X^{-}_{ij}\right)-G^{X-}_{ij}\left(I^{-}_{1}+3K^{-}tr\left(\Delta\boldsymbol{\varepsilon}\right)\right)$

a: and $\dfrac{\partial q_{II}}{\partial\Delta\lambda^{d}}=\dfrac{\partial q_{II}}{\partial q_{ij}}\dfrac{\partial q_{ij}}{\partial\Delta\lambda^{d}}=\dfrac{q_{ij}}{q_{II}}\dfrac{\partial q_{ij}}{\partial\Delta\lambda^{d}}$ Ultimately $\dfrac{\partial h(\theta_{q})}{\partial\Delta\lambda^{d}}=\dfrac{\partial h(\theta_{q})}{\partial q_{ij}}\dfrac{\partial q_{ij}}{\partial\Delta\lambda^{d}}$

, we takes for the solution of test: , $Y^{0}=\left(\sigma^{0}_{ij},R^{0},X^{0}_{ij},\Delta\lambda^{d0}\right)$ with the following values: :

$\Delta\lambda^{d0}$ the value found according to the preceding formulation. Iterations

$$\sigma^{0}_{ij}=\sigma^{-}_{ij}+D^{-}_{ijkl}\left(\Delta\varepsilon_{kl}-\Delta\lambda^{d0}G^{d-}_{kl}\right)$$
$$R^{0}=R^{-}+\Delta\lambda^{d0}G^{R-}$$
$$X^{0}_{ij}=X^{-}_{ij}+\Delta\lambda^{d0}G^{X-}_{ij}$$

4.4.2 of Newton

$\dfrac{DR}{DY}$ is given here by: with

$$\frac{DR}{DY}=\begin{bmatrix}\dfrac{\partial LE_{ij}}{\partial\sigma_{kl}} & \dfrac{\partial LE_{ij}}{\partial R} & \dfrac{\partial LE_{ij}}{\partial X_{ij}} & \dfrac{\partial LE_{ij}}{\partial\Delta\lambda^{d}}\\ \dfrac{\partial LR}{\partial\sigma_{kl}} & \dfrac{\partial LR}{\partial R} & \dfrac{\partial LR}{\partial X_{ij}} & \dfrac{\partial LR}{\partial\Delta\lambda^{d}}\\ \dfrac{\partial LX_{ij}}{\partial\sigma_{kl}} & \dfrac{\partial LX_{ij}}{\partial R} & \dfrac{\partial LX_{ij}}{\partial X_{ij}} & \dfrac{\partial LX_{ij}}{\partial\Delta\lambda^{d}}\\ \dfrac{\partial FD}{\partial\sigma_{kl}} & \dfrac{\partial FD}{\partial R} & \dfrac{\partial FD}{\partial X_{ij}} & \dfrac{\partial FD}{\partial\Delta\lambda^{d}}\end{bmatrix}$$

: In addition

$$\frac{\partial LE_{\mathbf{ij}}}{\partial\sigma_{kl}}=\delta_{ik}\delta_{jl}-D^{lineaire}_{ijmn}\left(\Delta\varepsilon_{mn}-\Delta\lambda^{d}G^{d}_{mn}\right)\frac{n}{3P_{a}}\left(\frac{I_{1}+Q_{init}}{3P_{a}}\right)^{n-1}\delta_{kl}+D_{ijmn}\Delta\lambda^{d}\frac{\partial G^{d}_{mn}}{\partial\sigma_{kl}}$$

$$\frac{\partial LE_{\mathbf{ij}}}{\partial R}=D_{ijmn}\Delta\lambda^{d}\frac{\partial G^{d}_{mn}}{\partial R}$$

$$\frac{\partial LE_{\mathbf{ij}}}{\partial X_{kl}}=D_{ijmn}\Delta\lambda^{d}\frac{\partial G^{d}_{mn}}{\partial X_{kl}}$$

$$\frac{\partial LE_{\mathbf{ij}}}{\partial\Delta\lambda^{d}}=D_{ijmn}G^{d}_{mn}$$

$$\frac{\partial LR}{\partial\sigma_{kl}}=-\Delta\lambda^{d}\frac{\partial G^{R}}{\partial\sigma_{kl}}=-\Delta\lambda^{d}\frac{A}{2}\left(1-\frac{R}{R_{m}}\right)^{2}\left(\frac{I_{1}+Q_{init}}{3P_{a}}\right)^{-1,5}\delta_{kl}$$

$$\frac{\partial LR}{\partial R}=1-\Delta\ \lambda^d\ \frac{\partial G^R}{\partial R}=1-\Delta\ \lambda^d\ \frac{2\ A}{R_m}\left(1-\frac{R}{R_m}\right)\left(I_1+Q_{init}\right)\left(\frac{I_1+Q_{init}}{3\ P_a}\right)^{-1,5}$$

$$\frac{\partial LR}{\partial X_{kl}}=0$$

$$\frac{\partial LR}{\partial \Delta\ \lambda^d}=-G^R$$

$$\frac{\partial LX_{ij}}{\partial \sigma_{kl}}=-\Delta\ \lambda^d\ \frac{\partial G_{ij}^X}{\partial \sigma_{kl}}$$

$$\frac{\partial LX_{ij}}{\partial R}=0$$

$$\frac{\partial LX_{ij}}{\partial X_{kl}}=\delta_{ik}\ \delta_{jl}-\Delta\ \lambda^d\ \frac{\partial G_{ij}^X}{\partial X_{kl}}$$

$$\frac{\partial LX_{ij}}{\partial \Delta\ \lambda^d}=-G_{ij}^X$$

$$\frac{\partial FD}{\partial \sigma_{kl}}=\frac{\partial f^d}{\partial \sigma_{kl}}=Q_{kl}-\left(Q_{mn}\ X_{mn}-R\right)\ \delta_{kl}$$

$$\frac{\partial FD}{\partial R}=I_1$$

$$\frac{\partial FD}{\partial X_{kl}}=\frac{\partial f^d}{\partial X_{kl}}$$

$$\frac{\partial FD}{\partial \Delta\ \lambda^d}=0$$

, the computation of the terms $\frac{\partial G_{mn}^d}{\partial \sigma_{kl}}$ $\frac{\partial G_{mn}^d}{\partial R}$ $\frac{\partial G_{mn}^d}{\partial X_{kl}}$, $\frac{\partial G_{ij}^X}{\partial \sigma_{kl}}$ and $\frac{\partial G_{ij}^X}{\partial X_{kl}}$ is $\frac{\partial f^d}{\partial X_{kl}}$ detailed Ci - after, as well as the computation of useful intermediate terms: computation

- of: $\frac{\partial f^d}{\partial X_{kl}}$ One

$$\begin{aligned}\frac{\partial f^d}{\partial X_{kl}}&=q_{II}\ \frac{\partial h\left(\theta_q\right)}{\partial X_{kl}}+h\left(\theta_q\right)\ \frac{\partial q_{II}}{\partial X_{kl}}\\&=q_{II}\ \frac{\partial h\left(\theta_q\right)}{\partial q_{mn}}\ \frac{\partial q_{mn}}{\partial X_{kl}}+h\left(\theta_q\right)\ \frac{\partial q_{II}}{\partial q_{mn}}\ \frac{\partial q_{mn}}{\partial X_{kl}}\\&=q_{II}\ \frac{\partial h\left(\theta_q\right)}{\partial q_{mn}}\ \frac{\partial q_{mn}}{\partial X_{kl}}+h\left(\theta_q\right)\ \frac{q_{mn}}{q_{II}}\ \frac{\partial q_{mn}}{\partial X_{kl}}\\&=\left(q_{II}\ \frac{\partial h\left(\theta_q\right)}{\partial q_{mn}}+h\left(\theta_q\right)\ \frac{q_{mn}}{q_{II}}\right)\frac{\partial q_{mn}}{\partial X_{kl}}\\&=-I_1\left(q_{II}\ \frac{\partial h\left(\theta_q\right)}{\partial q_{mn}}+h\left(\theta_q\right)\ \frac{q_{mn}}{q_{II}}\right)\delta_{mk}\ \delta_{nl}\end{aligned}$$

$$=-I_1\left(\frac{\partial f^d}{\partial q_{kl}}\right)$$

will notice for the continuation that: computation

$$dev\left(\frac{\partial f^d}{\partial X_{kl}}\right)=-I_1\ Q_{kl}$$

- of: $\dfrac{\partial\left(\dfrac{\partial f^d}{\partial\sigma_{ij}}\right)}{\partial\sigma_{kl}}$ computation

$$\frac{\partial\left(\frac{\partial f^d}{\partial\sigma_{ij}}\right)}{\partial\sigma_{kl}}=\frac{\partial\left(\frac{\partial f^d}{\partial\sigma_{ij}}\right)}{\partial q_{mn}}\ \frac{\partial q_{mn}}{\partial\sigma_{kl}}$$

$$=\left(\frac{\partial\left(Q_{ij}-\left(Q_{rs}\ X_{rs}-R\right)\ \delta_{ij}\right)}{\partial q_{mn}}\right)\frac{\partial q_{mn}}{\partial\sigma_{kl}}$$

$$=\left(\frac{\partial Q_{ij}}{\partial q_{mn}}-\left(\frac{\partial Q_{rs}}{\partial q_{mn}}\ X_{rs}\right)\ \delta_{ij}\right)\frac{\partial q_{mn}}{\partial\sigma_{kl}}$$

$$=\left(\frac{\partial Q_{ij}}{\partial q_{mn}}-\left(\frac{\partial Q_{rs}}{\partial q_{mn}}\ X_{rs}\right)\delta_{ij}\right)\left(\delta_{mk}\ \delta_{nl}-\delta_{kl}\left(\frac{\delta_{mn}}{3}+X_{mn}\right)\right)$$

- of: $\dfrac{\partial Q_{ij}}{\partial q_{mn}}$ As a preliminary

  , one definite the tensor and t his déviatoire part while t^d posing: and

  $t_{ij}=\dfrac{\partial\det(q)}{\partial q_{ij}}$ One $t_{ij}^d=dev\left(\dfrac{\partial\det(q)}{\partial q_{ij}}\right)$

  has as follows:

$$t=\begin{bmatrix}t_{11}\\t_{22}\\t_{33}\\t_{12}\\t_{13}\\t_{23}\end{bmatrix}=\begin{bmatrix}q_{22}\ q_{33}-q_{23}\ q_{23}\\q_{11}\ q_{33}-q_{13}\ q_{13}\\q_{11}\ q_{22}-q_{12}\ q_{12}\\q_{13}\ q_{23}-q_{12}\ q_{33}\\q_{12}\ q_{23}-q_{13}\ q_{22}\\q_{12}\ q_{13}-q_{23}\ q_{11}\end{bmatrix}$$

$$\frac{\partial Q_{ij}}{\partial q_{mn}}=\frac{-5}{h(\theta_q)^6}\left[\left(1+\frac{\gamma}{2}\cos(3\theta_q)\right)\frac{q_{ij}}{q_{II}}+\frac{\gamma\sqrt{54}}{6\,q_{II}^2}dev(t_{ij})\right]\frac{\partial(h(\theta_q))}{\partial q_{mn}}$$

$$+\frac{1}{h(\theta_q)^5}\left(1+\frac{\gamma}{2}\cos(3\theta_q)\right)\frac{\partial\left(\frac{q_{ij}}{q_{II}}\right)}{\partial q_{mn}}+\frac{1}{h(\theta_q)^5}\frac{\gamma}{2}\frac{q_{ij}}{q_{II}}\frac{\partial\cos(3\theta_q)}{\partial q_{mn}}+\frac{1}{h(\theta_q)^5}\frac{\sqrt{54}\,\gamma}{6}\frac{\partial\left(\frac{t_{ij}^d}{q_{II}^2}\right)}{\partial q_{mn}}$$

$$=\frac{-5}{h(\theta_q)^6}\left[\left(1+\frac{\gamma}{2}\cos(3\theta_q)\right)\frac{q_{ij}}{q_{II}}+\frac{\gamma\sqrt{54}}{6\,q_{II}^2}dev(t_{ij})\right]\frac{\partial(h(\theta_q))}{\partial q_{mn}}+\frac{1}{h(\theta_q)^5}\left(1+\frac{\gamma}{2}\cos(3\theta_q)\right)\left(\frac{\delta_{im}\delta_{jn}}{q_{II}}-\frac{q_{ij}q_{mn}}{q_{II}^3}\right)$$

$$+\frac{1}{h(\theta_q)^5}\frac{\gamma}{2}\frac{q_{ij}\sqrt{54}}{q_{II}^4}\left(t_{mn}-3\frac{\det q}{q_{II}^2}q_{mn}\right)+\frac{1}{h(\theta_q)^5}\frac{\gamma}{6}\frac{\sqrt{54}}{q_{II}^2}\left(\frac{\partial t_{ij}^d}{\partial q_{mn}}-2t_{ij}^d\frac{q_{mn}}{q_{II}^2}\right)$$

The statement of $\frac{\partial t_{ij}^d}{\partial q_{mn}}$ is clarified as follows:

$$\frac{\partial t^d}{\partial q_{11}}=\begin{bmatrix}-\frac{1}{3}(q_{22}+q_{33})\\ \frac{1}{3}(-q_{22}+2\,q_{33})\\ \frac{1}{3}(2\,q_{22}-q_{33})\\ 0\\ 0\\ -q_{23}\end{bmatrix}\quad \frac{\partial t^d}{\partial q_{22}}=\begin{bmatrix}\frac{1}{3}(-q_{11}+2\,q_{33})\\ -\frac{1}{3}(q_{11}+q_{33})\\ \frac{1}{3}(2\,q_{11}-q_{33})\\ 0\\ -q_{13}\\ 0\end{bmatrix}\quad \frac{\partial t^d}{\partial q_{33}}=\begin{bmatrix}\frac{1}{3}(-q_{11}+2\,q_{22})\\ \frac{1}{3}(2\,q_{11}-q_{22})\\ -\frac{1}{3}(q_{11}+q_{22})\\ -q_{12}\\ 0\\ 0\end{bmatrix}$$

$$\frac{\partial t^d}{\partial q_{12}}=\begin{bmatrix}\frac{2}{3}q_{12}\\ \frac{2}{3}q_{12}\\ -\frac{4}{3}q_{12}\\ -q_{33}\\ q_{23}\\ q_{13}\end{bmatrix},\quad \frac{\partial t^d}{\partial q_{13}}=\begin{bmatrix}\frac{2}{3}q_{13}\\ -\frac{4}{3}q_{13}\\ \frac{2}{3}q_{13}\\ q_{23}\\ -q_{22}\\ q_{12}\end{bmatrix}\ \text{computation}\ \frac{\partial t^d}{\partial q_{23}}=\begin{bmatrix}-\frac{4}{3}q_{23}\\ \frac{2}{3}q_{23}\\ \frac{2}{3}q_{23}\\ q_{13}\\ q_{12}\\ -q_{11}\end{bmatrix}$$

- of: $\frac{\partial G_{mn}^d}{\partial\sigma_{kl}}$ One
  definite a:

$$\frac{\partial G_{mn}^d}{\partial\sigma_{kl}}=\frac{\partial\left(\frac{\partial f^d}{\partial\sigma_{mn}}\right)}{\partial\sigma_{kl}}-\left(\frac{\partial f^d}{\partial\sigma_{rs}}n_{rs}\right)\frac{\partial n_{mn}}{\partial\sigma_{kl}}-\left(\frac{\partial\left(\frac{\partial f^d}{\partial\sigma_{rs}}\right)}{\partial\sigma_{kl}}n_{rs}+\frac{\partial f^d}{\partial\sigma_{rs}}\frac{\partial n_{rs}}{\partial\sigma_{kl}}\right)n_{mn}$$

One the tensor by $\tilde{\boldsymbol{n}}$ i.e. $\tilde{n}_{ij}=\beta'\dfrac{s_{ij}}{s_{II}}+\delta_{ij}$

that $\boldsymbol{n}$ is then given by with $n_{ij}=\dfrac{\tilde{n}_{ij}}{\tilde{n}_{II}}$ In practice $\tilde{n}_{II}=\sqrt{\beta'^2+3}$

, for the computation of, β' one uses instead of $\Delta\varepsilon_{ij}$, $\Delta\varepsilon_{ij}^{dp}$ i.e. one a: One

$$\beta'=\beta\left(\frac{s_{II}}{s_{II}^c}-1\right)signe\left(s_{ij}\Delta\varepsilon_{ij}\right)$$

has then for: $\dfrac{\partial G_{mn}^d}{\partial\sigma_{kl}}$ with

$$\frac{\partial G_{mn}^d}{\partial\sigma_{kl}}=\frac{\partial\left(\dfrac{\partial f^d}{\partial\sigma_{mn}}\right)}{\partial\sigma_{kl}}-\left(\frac{\partial\left(\dfrac{\partial f^d}{\partial\sigma_{rs}}\right)}{\partial\sigma_{kl}}n_{rs}\right)n_{mn}-\left(\frac{\partial f^d}{\partial\sigma_{rs}}\frac{\partial\tilde{n}_{rs}}{\partial\sigma_{kl}}\right)\frac{\tilde{n}_{mn}}{\tilde{n}_{II}^2}-\left(\frac{\partial f^d}{\partial\sigma_{rs}}\tilde{n}_{rs}\right)\frac{\partial\tilde{n}_{mn}}{\partial\sigma_{kl}}\frac{1}{\tilde{n}_{II}^2}-\left(\frac{\partial f^d}{\partial\sigma_{rs}}\tilde{n}_{rs}\right)\tilde{n}_{mn}\frac{\partial\left(\dfrac{1}{\tilde{n}_{II}^2}\right)}{\partial\sigma_{kl}}$$

: computation

$$\frac{\partial\left(\dfrac{1}{\tilde{n}_{II}^2}\right)}{\partial\sigma_{kl}}=\frac{\partial\left(\dfrac{1}{\left(\beta'^2+3\right)}\right)}{\partial\sigma_{kl}}=-\frac{1}{\left(\beta'^2+3\right)^2}\frac{\partial\left(\beta'^2\right)}{\partial\sigma_{kl}}=-\frac{2\beta^2\left(\dfrac{s_{II}}{s_{II}^c}-1\right)}{\left(\beta'^2+3\right)^2}\frac{\partial\left(\dfrac{s_{II}}{s_{II}^c}\right)}{\partial\sigma_{kl}}$$

- of: $\dfrac{\partial\left(\dfrac{s_{II}}{s_{II}^c}\right)}{\partial\sigma_{kl}}$ computation

$$\frac{\partial\left(\dfrac{s_{II}}{s_{II}^c}\right)}{\partial\sigma_{kl}}=\frac{1}{s_{II}^c}\frac{\partial\left(s_{II}\right)}{\partial\sigma_{kl}}-\frac{s_{II}}{s_{II}^{c^2}}\frac{\partial\left(s_{II}^c\right)}{\partial\sigma_{kl}}$$

$$=\frac{1}{s_{II}^c}\frac{\partial\left(s_{II}\right)}{\partial s_{mn}}\frac{\partial s_{mn}}{\partial\sigma_{kl}}-\frac{s_{II}}{s_{II}^{c^2}}\frac{\partial\left(-\dfrac{R_c\left(I_1+Q_{init}\right)}{h\left(\theta_s\right)}\right)}{\partial\sigma_{kl}}$$

$$=\frac{1}{s_{II}^c}\frac{s_{mn}}{s_{II}}\left(\delta_{mk}\delta_{nl}-\frac{1}{3}\delta_{mn}\delta_{kl}\right)-\frac{s_{II}}{s_{II}^{c^2}}\left(-\frac{R_c}{h\left(\theta_s\right)}\frac{\partial I_1}{\partial\sigma_{kl}}+\frac{R_c\left(I_1+Q_{init}\right)}{h\left(\theta_s\right)^2}\frac{\partial h\left(\theta_s\right)}{\partial\sigma_{kl}}\right)$$

$$=\frac{1}{s_{II}^c}\frac{s_{mn}}{s_{II}}\left(\delta_{mk}\delta_{nl}-\frac{1}{3}\delta_{mn}\delta_{kl}\right)-\frac{s_{II}}{s_{II}^{c^2}}\left(-\frac{R_c}{h\left(\theta_s\right)}\delta_{kl}+\frac{R_c\left(I_1+Q_{init}\right)}{h\left(\theta_s\right)^2}\frac{\partial h\left(\theta_s\right)}{\partial s_{rs}}\frac{\partial s_{rs}}{\partial\sigma_{kl}}\right)$$

- of: $\frac{\partial \tilde{n}_{mn}}{\partial \sigma_{kl}}$ computation

$$\frac{\partial \tilde{n}_{mn}}{\partial \sigma_{kl}}=\beta \left(\frac{1}{s_{II}^{c}}-\frac{1}{s_{II}}\right) signe\left(s_{ij}\ \Delta \varepsilon_{ij}\right) \frac{\partial s_{mn}}{\partial \sigma_{kl}}+\beta\ signe\left(s_{ij}\ \Delta \varepsilon_{ij}\right) s_{mn} \left(\frac{\partial\left(\frac{1}{s_{II}^{c}}\right)}{\partial \sigma_{kl}}-\frac{\partial\left(\frac{1}{s_{II}}\right)}{\partial \sigma_{kl}}\right)$$

$$=\beta \left(\frac{1}{s_{II}^{c}}-\frac{1}{s_{II}}\right) signe\left(s_{ij}\ \Delta \varepsilon_{ij}\right) \left(\delta_{mk}\ \delta_{nl}-\frac{1}{3}\ \delta_{mn}\ \delta_{kl}\right)+\beta\ signe\left(s_{ij}\ \Delta \varepsilon_{ij}\right) s_{mn} \left(\frac{1}{s_{II}^{2}} \frac{\partial\left(s_{II}\right)}{\partial \sigma_{kl}}-\frac{1}{s_{II}^{c^2}} \frac{\partial s_{II}^{c}}{\partial \sigma_{kl}}\right)$$

- of: $\frac{\partial G_{mn}^{d}}{\partial R}$ computation

$$\frac{\partial G_{mn}^{d}}{\partial R}=\frac{\partial\left(\frac{\partial f^{d}}{\partial \sigma_{mn}}\right)}{\partial R}-\left(\frac{\partial f^{d}}{\partial \sigma_{rs}}\ n_{rs}\right) \frac{\partial n_{mn}}{\partial R}-\left(\frac{\partial\left(\frac{\partial f^{d}}{\partial \sigma_{rs}}\right)}{\partial R}\ n_{rs}+\frac{\partial f^{d}}{\partial \sigma_{rs}} \frac{\partial n_{rs}}{\partial R}\right) n_{mn}$$

$$=\frac{\partial\left(\frac{\partial f^{d}}{\partial \sigma_{mn}}\right)}{\partial R}-\left(\frac{\partial\left(\frac{\partial f^{d}}{\partial \sigma_{rs}}\right)}{\partial R}\ n_{rs}\right) n_{mn}$$

$$=\delta_{mn}-\left(\delta_{rs}\ n_{rs}\right) n_{mn}$$

$$=\frac{\beta'^{2}\ \delta_{mn}-3\ \beta' \frac{s_{mn}}{s_{II}}}{\beta'^{2}+3}$$

- of: $\frac{\partial G_{mn}^{d}}{\partial X_{kl}}$ computation

$$\frac{\partial G_{mn}^{d}}{\partial X_{kl}}=\frac{\partial\left(\frac{\partial f^{d}}{\partial \sigma_{mn}}\right)}{\partial X_{kl}}-\left(\frac{\partial f^{d}}{\partial \sigma_{rs}}\ n_{rs}\right) \frac{\partial n_{mn}}{\partial X_{kl}}-\left(\frac{\partial\left(\frac{\partial f^{d}}{\partial \sigma_{rs}}\right)}{\partial X_{kl}}\ n_{rs}+\frac{\partial f^{d}}{\partial \sigma_{rs}} \frac{\partial n_{rs}}{\partial X_{kl}}\right) n_{mn}$$

$$=\frac{\partial\left(\frac{\partial f^{d}}{\partial \sigma_{mn}}\right)}{\partial X_{kl}}-\left(\frac{\partial\left(\frac{\partial f^{d}}{\partial \sigma_{rs}}\right)}{\partial X_{kl}}\ n_{rs}\right) n_{mn}$$

- of: $\frac{\partial\left(\frac{\partial f^{d}}{\partial \sigma_{mn}}\right)}{\partial X_{kl}}$ computation

$$\frac{\partial\left(\frac{\partial f^{d}}{\partial \sigma_{mn}}\right)}{\partial X_{kl}}=\frac{\partial Q_{mn}}{\partial X_{kl}}-\left(\frac{\partial Q_{rs}}{\partial X_{kl}}\ X_{rs}+Q_{rs} \frac{\partial X_{rs}}{\partial X_{kl}}\right) \delta_{mn}$$

$$=\frac{\partial Q_{mn}}{\partial q_{ij}}\frac{\partial q_{ij}}{\partial X_{kl}}-\left(\left(\frac{\partial Q_{rs}}{\partial q_{ij}}\frac{\partial q_{ij}}{\partial X_{kl}}\right)X_{rs}+Q_{rs}\,\delta_{kr}\,\delta_{ls}\right)\delta_{mn}$$

$$=-I_1\frac{\partial Q_{mn}}{\partial q_{ij}}\delta_{ik}\,\delta_{jl}-\left(\left(-I_1\frac{\partial Q_{rs}}{\partial q_{ij}}\delta_{ik}\,\delta_{jl}\right)X_{rs}+Q_{rs}\,\delta_{kr}\,\delta_{ls}\right)\delta_{mn}$$

- of: $\frac{\partial G_{ij}^X}{\partial \sigma_{kl}}$ computation

$$\frac{\partial G_{ij}^X}{\partial \sigma_{kl}}=-\frac{1}{2\,b}\left(Q_{ij}+\varphi\,X_{ij}\right)\left(\frac{I_1+Q_{init}}{3\,P_a}\right)^{-1,5}\frac{\partial I_1}{\partial \sigma_{kl}}+\frac{1}{b}\left(\frac{\partial Q_{ij}}{\partial \sigma_{kl}}+\frac{\partial \varphi}{\partial \sigma_{kl}}X_{ij}\right)\left(I_1+Q_{init}\right)\left(\frac{I_1+Q_{init}}{3\,P_a}\right)^{-1,5}$$

$$=-\frac{1}{2\,b}\left(Q_{ij}+\varphi\,X_{ij}\right)\left(\frac{I_1+Q_{init}}{3\,P_a}\right)^{-1,5}\delta_{kl}+\frac{1}{b}\frac{\partial Q_{ij}}{\partial q_{mn}}\frac{\partial q_{mn}}{\partial \sigma_{kl}}\left(I_1+Q_{init}\right)\left(\frac{I_1+Q_{init}}{3\,P_a}\right)^{-1,5}$$

$$+\frac{1}{b}\left(h(\theta_s)\,Q_{II}\frac{\partial \varphi_o}{\partial \sigma_{kl}}+\varphi_o\,Q_{II}\frac{\partial h(\theta_s)}{\partial \sigma_{kl}}+\varphi_o\,h(\theta_s)\frac{\partial Q_{II}}{\partial \sigma_{kl}}\right)X_{ij}\left(I_1+Q_{init}\right)\left(\frac{I_1+Q_{init}}{3\,P_a}\right)^{-1,5}$$

- of: $\frac{\partial h(\theta_s)}{\partial \sigma_{kl}}$ computation

$$\frac{\partial h(\theta_s)}{\partial \sigma_{kl}}=\frac{\partial h(\theta_s)}{\partial s_{mn}}\frac{\partial s_{mn}}{\partial \sigma_{kl}}$$

$$=\left(\frac{\gamma\sqrt{54}}{6\,h(\theta_s)^5\,q_{II}^3}t_{mn}-\frac{\gamma\cos(3\,\theta_q)}{2\mathrm{h}(\theta_s)^5\,q_{II}^2}s_{mn}\right)\left(\delta_{mk}\,\delta_{nl}-\frac{1}{3}\delta_{mn}\,\delta_{kl}\right)$$

- of: $\frac{\partial Q_{II}}{\partial \sigma_{kl}}$ computation

$$\frac{\partial Q_{II}}{\partial \sigma_{kl}}=\left(\frac{\partial Q_{II}}{\partial Q_{rs}}\frac{\partial Q_{rs}}{\partial q_{mn}}\right)\frac{\partial q_{mn}}{\partial \sigma_{kl}}$$

$$=\left(\frac{Q_{rs}}{Q_{II}}\frac{\partial Q_{rs}}{\partial q_{mn}}\right)\left(\delta_{mk}\,\delta_{nl}-\delta_{mn}\left(\frac{1}{3}\delta_{kl}+X_{kl}\right)\right)$$

- of: $\frac{\partial \varphi_o}{\partial \sigma_{kl}}$ with

$$\frac{\partial \varphi_o}{\partial \sigma_{kl}}=\frac{1}{R_r-\frac{h(\theta_s)}{h(\theta_q)}R_m\cos(\theta_s-\theta_q)}\frac{\partial \cos\alpha}{\partial \sigma_{kl}}$$

$$-\cos\alpha\frac{\left[\frac{\partial R_r}{\partial \sigma_{kl}}-\frac{1}{h(\theta_q)}R_m\cos(\theta_s-\theta_q)\frac{\partial h(\theta_s)}{\partial \sigma_{kl}}+\frac{h(\theta_s)}{h(\theta_q)^2}R_m\cos(\theta_s-\theta_q)\frac{\partial h(\theta_s)}{\partial \sigma_{kl}}-\frac{h(\theta_s)}{h(\theta_q)}R_m\frac{\partial\cos(\theta_s-\theta_q)}{\partial \sigma_{kl}}\right]}{\left[R_r-\frac{h(\theta_s)}{h(\theta_q)}R_m\cos(\theta_s-\theta_q)\right]^2}$$

: computation

$$\frac{\partial \cos\alpha}{\partial \sigma_{kl}} = \frac{1}{2\ s_{II}\ I_1\ X_{II}} \left(2\ q_{II} \frac{\partial q_{II}}{\partial \sigma_{kl}} - 2\ I_1\ X_{II}^2 \frac{\partial I_1}{\partial \sigma_{kl}} - 2\ s_{II} \frac{\partial s_{II}}{\partial \sigma_{kl}} \right)$$

$$- \frac{q_{II}^2 - (I_1\ X_{II})^2 - s_{II}^2}{s_{II}\ I_1\ X_{II}} \left(s_{II}\ X_{II} \frac{\partial I_1}{\partial \sigma_{kl}} + I_1\ X_{II} \frac{\partial s_{II}}{\partial \sigma_{kl}} \right)$$

$$= \frac{1}{s_{II}\ I_1\ X_{II}} \left[\left(q_{kl} - I_1\ X_{II}^2\ \delta_{kl} - s_{kl} \right) - \left(q_{II}^2 - \left(I_1\ X_{II} \right)^2 - s_{II}^2 \right) \left(s_{II}\ X_{II}\ \delta_{kl} + I_1\ X_{II} \frac{s_{kl}}{s_{II}} \right) \right]$$

$$\frac{\partial R_r}{\partial \sigma_{kl}} = -\frac{\mu}{I_1 + Q_{init}} \delta_{kl}$$

$$\frac{\partial \cos\left(\theta_s - \theta_q\right)}{\partial \sigma_{kl}} = -\sin\left(\theta_s - \theta_q\right) \left(\frac{\partial \theta_s}{\partial \sigma_{kl}} - \frac{\partial \theta_q}{\partial \sigma_{kl}} \right)$$

- of: $\frac{\partial G_{ij}^X}{\partial X_{kl}}$ Test

$$\frac{\partial G_{ij}^X}{\partial X_{kl}} = \frac{1}{b} \left(\frac{\partial Q_{ij}}{\partial X_{kl}} + \varphi \quad \frac{\partial X_{ij}}{\partial X_{kl}} \right) \left(I_1 + Q_{init} \right) \left(\frac{I_1 + Q_{init}}{3\ P_a} \right)^{-1,5}$$

$$= \frac{1}{b} \left(\frac{\partial Q_{ij}}{\partial q_{mn}} \frac{\partial q_{mn}}{\partial X_{kl}} + \varphi \quad \delta_{ik}\ \delta_{jl} \right) \left(I_1 + Q_{init} \right) \left(\frac{I_1 + Q_{init}}{3\ P_a} \right)^{-1,5}$$

$$= \frac{1}{b} \left(-I_1 \frac{\partial Q_{ij}}{\partial q_{mn}} \delta_{mk}\ \delta_{nl} + \varphi \quad \delta_{ik}\ \delta_{jl} \right) \left(I_1 + Q_{init} \right) \left(\frac{I_1 + Q_{init}}{3\ P_a} \right)^{-1,5}$$

4.4.3 of convergence

the convergence criterion remains $\frac{\|DY^{p+1}\|}{\|Y^{\mathbf{p+1}} - Y^0\|}$ RESI_INTE_RELA . `Integration`

4.5 of the mechanisms elastic nonlinear, plastic isotropic and plastic déviatoire In this case,

the new stress state, σ^+ checks: Taking into account

$$\sigma_{ij}^+ = \sigma_{ij}^- + D_{ijkl}\left(\sigma^+\right) \left(\Delta\ \varepsilon_{kl} - \Delta\ \varepsilon_{kl}^{ip} - \Delta\ \varepsilon_{kl}^{dp} \right)$$

what precedes, one from of deduced that the nonlinear system to solve is composed of: :

- $LE_{\mathbf{ij}}$ the elastic model state: :
  $$\sigma_{ij}^+ - \sigma_{ij}^- - D_{ijkl}\left(\sigma^+\right) \left(\Delta\ \varepsilon_{kl} + \frac{1}{3} \Delta\ \lambda^i\ \delta_{kl} - \Delta\ \lambda^d\ G_{kl}^d\left(\sigma^+,\ R^+,\ \boldsymbol{X}^+\right) \right) = 0$$
- LQ the model of hardening of the local variable: Q_{iso} : $Q_{iso}^+ - Q_{iso}^- - D\ \lambda^i G^{Q_{iso}}\left(Q_{iso}^+\right) = 0$
- LR the model of hardening of the variable: R : $R^+ - R^- - \Delta\ \lambda^d\ G^R\left(\sigma^+,\ R^+\right) = 0$
- $LX_{\mathbf{ij}}$ the model of hardening of the variable: X_{ij} : $X_{ij}^+ - X_{ij}^- - \Delta\ \lambda^d\ G_{ij}^X\left(\sigma^+,\ \boldsymbol{X}^+\right) = 0$
- FI the equation of the isotropic surface of load: : $-\frac{I_1^+ + Q_{init}}{3} + Q_{iso}^+ = 0$

- FD the equation of the surface of load déviatoire: As $q_{II}^{+}\, h\left(\theta_{q}^{+}\right)+R^{+}\left(I_1^{+}+Q_{init}\right)=0$

in the preceding paragraphs one solves by the method of Newton the system, $R(Y)=0$ where the unknown Y is given by and $Y=\left(\sigma_{ij}^{+},\ Q_{iso}^{+},\ R^{+},\ X_{ij}^{+},\ \Delta\ \lambda^{i},\ \Delta\ \lambda^{d}\right)$ where. $R=\left(LE_{ij},\ LQ,\ LR,\ LX_{ij},\ FI,\ FD\right)$ Initialization

4.5.1 and solution of test From

the state at time, t $\left(\sigma_{ij}^{-},\ Q_{iso}^{-},\ R^{-},\ X_{ij}^{-}\right)$ we seek a solution of test which brings us closer to the final solution. For that we solve the following system of equations: with

$$\begin{cases} f^{i}\left(s_{ij}^{-}+D_{ijkl}^{+}\left(De_{kl}+\frac{1}{3}D\lambda^{i}d_{kl}-D\lambda^{d}G_{kl}^{d}\right),Q_{iso}^{-}+D\lambda^{i}G^{Q_{iso}-}\right)=0 \\ f^{d}\left(s_{ij}^{-}+D_{ijkl}^{+}\left(De_{kl}+\frac{1}{3}D\lambda^{i}d_{kl}-D\lambda^{d}G_{kl}^{d}\right),R^{-}+D\lambda^{d}G^{R-},X_{ij}^{-}+D\lambda^{d}G_{ij}^{X-}\right)=0 \end{cases}$$

:

$D_{ijkl}^{-}=D_{ijkl}\left(\sigma^{-}\right)$ $G_{kl}^{d-}=G_{kl}^{d}\left(\sigma^{-},\ R^{-},\boldsymbol{X}^{-}\right)$, $G^{Q_{iso}-}=G^{Q_{iso}}\left(Q_{iso}^{-}\right)$ $G^{R-}=G^{R}\left(\sigma^{-},\ R^{-}\right)$ and $G_{ij}^{X-}=G_{ij}^{X}\left(\sigma^{-},\boldsymbol{X}^{-}\right)$ where the unknowns are the plastic multipliers and $\Delta\lambda^{i}$, $\Delta\lambda^{d}$ by only one iteration of Newton, i.e. finally that we have: that is to say

$$\begin{aligned} \frac{\partial f^{i}}{\partial\Delta\lambda^{i}}_{|\Delta\lambda^{i}=0,\ \Delta\lambda^{d}=0}\Delta\lambda^{i}+\frac{\partial f^{i}}{\partial\Delta\lambda^{d}}_{|\Delta\lambda^{i}=0,\ \Delta\lambda^{d}=0}\Delta\lambda^{d}=-f^{i}{}_{|\Delta\lambda^{i}=0,\ \Delta\lambda^{d}=0} \\ \frac{\partial f^{d}}{\partial\Delta\lambda^{i}}_{|\Delta\lambda^{i}=0,\ \Delta\lambda^{d}=0}\Delta\lambda^{i}+\frac{\partial f^{d}}{\partial\Delta\lambda^{d}}_{|\Delta\lambda^{i}=0,\ \Delta\lambda^{d}=0}\Delta\lambda^{d}=-f^{d}{}_{|\Delta\lambda^{i}=0,\ \Delta\lambda^{d}=0} \end{aligned}$$

still: and

$$\Delta\ \lambda^{i}=\frac{\frac{\partial f^{i}}{\partial\Delta\ \lambda^{d}}f^{d}-\frac{\partial f^{d}}{\partial\Delta\ \lambda^{d}}f^{i}}{\frac{\partial f^{i}}{\partial\Delta\ \lambda^{i}}\frac{\partial f^{d}}{\partial\Delta\ \lambda^{d}}-\frac{\partial f^{i}}{\partial\Delta\ \lambda^{d}}\frac{\partial f^{d}}{\partial\Delta\ \lambda^{i}}} \quad \text{with} \quad \Delta\ \lambda^{d}=\frac{\frac{\partial f^{d}}{\partial\Delta\ \lambda^{i}}f^{i}-\frac{\partial f^{i}}{\partial\Delta\ \lambda^{i}}f^{d}}{\frac{\partial f^{i}}{\partial\Delta\ \lambda^{i}}\frac{\partial f^{d}}{\partial\Delta\ \lambda^{d}}-\frac{\partial f^{i}}{\partial\Delta\ \lambda^{d}}\frac{\partial f^{d}}{\partial\Delta\ \lambda^{i}}}$$

: It

$$\frac{\partial f^{i}}{\partial\Delta\ \lambda^{i}}=-\left(K^{-}+K^{p-}\right)$$

$$\frac{\partial f^{i}}{\partial\Delta\ \lambda^{d}}=K^{-}\ tr\left(G^{d-}\right)$$

$$\frac{\partial f^{d}}{\partial\Delta\ \lambda^{i}}=h\left(\theta_{q}\right)\frac{\partial q_{II}}{\partial\Delta\ \lambda^{i}}+q_{II}\frac{\partial h\left(\theta_{q}\right)}{\partial\Delta\ \lambda^{i}}+\left(I_1+Q_{init}\right)\frac{\partial R}{\partial\Delta\ \lambda^{i}}+R\frac{\partial I_1}{\partial\Delta\ \lambda^{i}}$$

$$\frac{\partial f^{d}}{\partial\Delta\ \lambda^{d}}=h\left(\theta_{q}\right)\frac{\partial q_{II}}{\partial\Delta\ \lambda^{d}}+q_{II}\frac{\partial h\left(\theta_{q}\right)}{\partial\Delta\ \lambda^{d}}+\left(I_1+Q_{init}\right)\frac{\partial R}{\partial\Delta\ \lambda^{d}}+R\frac{\partial I_1}{\partial\Delta\ \lambda^{d}}$$

is known that $\frac{\partial f^d}{\partial \Delta \lambda^d}$ is calculated in the same way that previously when only the plastic mechanism déviatoire was activated. In addition, one has, for the computation of and $\frac{\partial f^d}{\partial \Delta \lambda^i}$ when and $\Delta \lambda^i = 0$, $\Delta \lambda^d = 0$ following relations: Ultimately

$$\frac{\partial q_{ij}}{\partial \Delta \lambda^i} = \frac{1}{3} D^-_{ijkl} \delta_{kl} - 3 K^{e-} \left(\frac{1}{3} \delta_{ij} + X^-_{ij} \right)$$

$$\frac{\partial q_{II}}{\partial \Delta \lambda^i} = \frac{\partial q_{II}}{\partial q_{ij}} \frac{\partial q_{ij}}{\partial \Delta \lambda^i} = \frac{q_{ij}}{q_{II}} \left(\frac{1}{3} D^-_{ijkl} \delta_{kl} - 3 K^{e-} \left(\frac{1}{3} \delta_{ij} + X^-_{ij} \right) \right)$$

$$\frac{\partial h(\theta_q)}{\partial \Delta \lambda^i} = \frac{\partial h(\theta_q)}{\partial q_{ij}} \frac{\partial q_{ij}}{\partial \Delta \lambda^i}$$

$$\frac{\partial R}{\partial \Delta \lambda^i} = 0$$

$$\frac{\partial I_1}{\partial \Delta \lambda^i} = 3 K^-$$

, we take for the solution of test: , $Y^0 = \left(\sigma^0_{ij}, Q^0_{iso}, R^0, X^0_{ij}, \Delta \lambda^{i0}, \Delta \lambda^{d0} \right)$ with the following values: :

$\Delta \lambda^{i0}$ the value found according to the preceding formulation. :

$\Delta \lambda^{d0}$ the value found according to the preceding formulation. Iterations

$$\sigma^0_{ij} = \sigma^-_{ij} + D^-_{ijkl} \left(\Delta \varepsilon_{kl} + \frac{1}{3} \Delta \lambda^{i0} \delta_{kl} - \Delta \lambda^{d0} G^{d-}_{kl} \right)$$

$$Q^0_{iso} = Q^-_{iso} + \Delta \lambda^{i0} G^{Q_{iso}-}$$

$$R^0 = R^- + \Delta \lambda^{d0} G^{R-}$$

$$X^0_{ij} = X^-_{ij} + \Delta \lambda^{d0} G^{X-}_{ij}$$

4.5.2 of Newton

$\frac{DR}{DY}$ is given here by: where

$$\frac{DR}{DY}=\begin{bmatrix} \frac{\partial LE_{ij}}{\partial \sigma_{kl}} & \frac{\partial LE_{ij}}{\partial Q_{iso}} & \frac{\partial LE_{ij}}{\partial R} & \frac{\partial LE_{ij}}{\partial X_{kl}} & \frac{\partial LE_{ij}}{\partial \Delta \lambda^i} & \frac{\partial LE_{ij}}{\partial \Delta \lambda^d} \\ \frac{\partial LQ}{\partial \sigma_{kl}} & \frac{\partial LQ}{\partial Q_{iso}} & \frac{\partial LQ}{\partial R} & \frac{\partial LQ}{\partial X_{kl}} & \frac{\partial LQ}{\partial \Delta \lambda^i} & \frac{\partial LQ}{\partial \Delta \lambda^d} \\ \frac{\partial LR}{\partial \sigma_{kl}} & \frac{\partial LR}{\partial Q_{iso}} & \frac{\partial LR}{\partial R} & \frac{\partial LR}{\partial X_{kl}} & \frac{\partial LR}{\partial \Delta \lambda^i} & \frac{\partial LR}{\partial \Delta \lambda^d} \\ \frac{\partial LX_{ij}}{\partial \sigma_{kl}} & \frac{\partial LX_{ij}}{\partial Q_{iso}} & \frac{\partial LX_{ij}}{\partial R} & \frac{\partial LX_{ij}}{\partial X_{kl}} & \frac{\partial LX_{ij}}{\partial \Delta \lambda^i} & \frac{\partial LX_{ij}}{\partial \Delta \lambda^d} \\ \frac{\partial FI}{\partial \sigma_{kl}} & \frac{\partial FI}{\partial Q_{iso}} & \frac{\partial FI}{\partial R} & \frac{\partial FI}{\partial X_{kl}} & \frac{\partial FI}{\partial \Delta \lambda^i} & \frac{\partial FI}{\partial \Delta \lambda^d} \\ \frac{\partial FD}{\partial \sigma_{kl}} & \frac{\partial FD}{\partial Q_{iso}} & \frac{\partial FD}{\partial R} & \frac{\partial FD}{\partial X_{kl}} & \frac{\partial FD}{\partial \Delta \lambda^i} & \frac{\partial FD}{\partial \Delta \lambda^d} \end{bmatrix}$$

the new terms are null:

$$\frac{\partial LQ}{\partial R}=0 \quad \frac{\partial LQ}{\partial X_{kl}}=0 \quad \frac{\partial LQ}{\partial \Delta \lambda^d}=0 \quad \frac{\partial LR}{\partial Q_{iso}}=0 \quad \frac{\partial LR}{\partial \Delta \lambda^i}=0 \quad \frac{\partial LX_{ij}}{\partial Q_{iso}}=0$$

$$\frac{\partial LX_{ij}}{\partial \Delta \lambda^i}=0 \quad \frac{\partial FI}{\partial R}=0 \quad \frac{\partial FI}{\partial X_{kl}}=0 \quad \frac{\partial FI}{\partial \Delta \lambda^d}=0 \ , \ \frac{\partial FD}{\partial Q_{iso}}=0 \text{ and } \frac{\partial FD}{\partial \Delta \lambda^i}=0$$

where the already definite terms remain unchanged, except for which $\frac{\partial LE_{ij}}{\partial \sigma_{kl}}$ becomes: test

$$\frac{\partial LE_{ij}}{\partial \sigma_{kl}}=\delta_{ik}\,\delta_{jl}-D^{lineaire}_{ijmn}\left(\Delta \varepsilon_{mn}+\frac{1}{3}\Delta \lambda^i \delta_{mn}-\Delta \lambda^d G^d_{mn}\right)\frac{n}{3 P_a}\left(\frac{I_1+Q_{init}}{3 P_a}\right)^{n-1}\delta_{kl}+D_{ijmn}\Delta \lambda^d \frac{\partial G^d_{mn}}{\partial \sigma_{kl}}$$

4.5.3 of convergence

the convergence criterion remains □ $\frac{\|DY^{p+1}\|}{\|Y^{p+1}-Y^0\|}$ RESI_INTE_RELA `Procedure`

4.6 of relaxation based on an estimate of the norms on the surface of load déviatoire When

the plastic mechanism déviatoire intervenes, a procedure of relaxation inside the iterations of Newton is taken into account. This one makes it possible to avoid certain problems of oscillation in the computation of the solution which Y^{p+1} lead finally to nonthe convergence of numerical integration. Thus

, with the iteration, $p+1$ instead of bringing up to date the unknown by Y^{p+1} a complete increment one $\delta \ Y^{p+1}$

$$Y^{p+1}=Y^p+\delta \quad Y^{p+1}$$

poses and

$$Y^{p+1}_m=Y^p+\rho_m\,\delta \quad Y^{p+1}$$

one seeks, by carrying out a loop on under-iterations, m to determine an optimal value of the scalar. ρ_m This value is required by considering the rotation of the norm, in the déviatoire plane, on surface, f^d during under-iterations. This norm, noted, $\tilde{n}_m$ is expressed starting from the stresses contained in the term by Y_m^{p+1} From

$$\tilde{\boldsymbol{n}}_{\boldsymbol{m}}=2\ h\left(\theta\ _q\right)^5\ q_{II}\ \frac{\partial\ f^d}{\partial\ q_{ij}}=\left(2+\gamma\quad\cos\left(3\ \theta\ _q\right)\right)\ q_{ij}+\frac{\sqrt{6}\ \gamma}{q_{II}}\ \frac{\partial\det(q)}{\partial q_{ij}}$$

the initial value, $\rho\ _0=1.0$ the process set up consists of the following stages: computation

- of the norms and $\tilde{\boldsymbol{n}}_{\boldsymbol{m-1}}$ computation $\tilde{\boldsymbol{n}}_{\boldsymbol{m}}$
- of the swing angle between φ_m these norms: test $\cos\ \varphi\ _m=\frac{\tilde{\boldsymbol{n}}_{m-1}:\tilde{\boldsymbol{n}}_m}{\sqrt{\tilde{\boldsymbol{n}}_{m-1}}\ \sqrt{\tilde{\boldsymbol{n}}_m}}$
- on the evolution: $\cos\ \varphi\ _m$ so

  then $\cos\ \varphi\ _m\leq TOLROT$ and $\rho\ _{m+1}=DECREL\ \rho\ _m$ if not $m=m+1$

  end of the under-iterations and Recutting $Y^{p+1}=Y_m^{p+1}$

4.7 of time step As

for most behavior models, it was introduced for the model CJS the possibility of redécouper locally (with Gauss points) time step in order to facilitate numerical integration. This possibility is managed by operand ITER_INTE_PAS `of` key word CONVERGENCE `of` operator STAT_NON_LINE . `If` itepas , the value of ITER_INTE_PAS , `is worth` 0,1 or -1 there is no recutting (note: 0 are the value by default). If itepas `is` positive recutting is automatic, if it is negative recutting is taken into account only in the event of nonconvergence with the time step initial one.

Recutting consists in realizing, after the phase of elastic prediction, the integration of the plastic mechanisms brought into plays with an increment of strain whose components correspond to the components of the increment of strain initial divided by the absolute value of itepas . `Various`

4.8 remarks Computation

4.8.1 of the term $\cos\left(\theta\ _s-\theta\ _q\right)$

the term appears $\cos\left(\theta\ _s-\theta\ _q\right)$ in the statement of. $\varphi\ _o$ We adopted for his computation the same method as that used with the ECL. I.e. we determine the angles and $\theta\ _s$ of $\theta\ _q$ the way which follows: and

$\theta\ _s=\frac{1}{3}\ \text{Arctan}\left(\frac{\sqrt{1\text{-}\cos^2\left(3\theta\ _s\right)}}{\cos\left(3\theta\ _s\right)}\right)$ then $\theta\ _q=\frac{1}{3}\ \text{Arctan}\left(\frac{\sqrt{1\text{-}\cos^2\left(3\ \theta_q\right)}}{\cos\left(3\ \theta_q\right)}\right)$

we take the cosine of the difference. These

statements of and $\theta\ _s$ are also useful $\theta\ _q$ for computation of: with

$$\frac{\partial\cos\left(\theta\ _s-\theta\ _q\right)}{\partial\sigma\ _{kl}}=-\sin\left(\theta\ _s-\theta\ _q\right)\left(\frac{\partial\theta\ _s}{\partial\sigma\ _{kl}}-\frac{\partial\theta\ _q}{\partial\sigma\ _{kl}}\right)$$

Computation $\frac{\partial\theta\ _s}{\partial\sigma\ _{kl}}=-\frac{1}{3}\sqrt{1-\cos^2(3\ \theta_s)}\frac{\sqrt{54}}{q_{II}^3}\left(t_{kl}-3\frac{\det(q)}{q_{II}^2}q_{kl}\right)$

4.8.2 of R_r

the radius of fracture introduced into the model CJS3 is given by the formula In fact

$$R_r = R_c + \mu \ln\left(\frac{3\ p_c}{I_1 + Q_{init}}\right)$$

, when, $\frac{I_1 + Q_{init}}{3} > p_c$ one must block with R_r the value of. R_c The field of dilatence disappears and it is not admitted that can R_r decrease in on this side. R_c Consequently, one introduces, instead of the preceding formulation, the following statement Tension

$$R_r = R_c + \mu \max\left[0, \ln\left(\frac{3\ p_c}{I_1 + Q_{init}}\right)\right]$$

4.8.3 Non-cohesive

, the field of tension which corresponds to positive stresses is inadmissible for the soils. From the point of view of the integration of model CJS, when the state of the stresses tends towards the top of the cone of the surface of load, the numerical risk to rock in this prohibited field increases. However when that one projects oneself or when one makes a prediction in a point of this field, numerical computation leads either to result erroneous, or with a fatal error. Indeed, the tension appears numerically by a value of positive I_1 . This value poses then problem at the time to evaluate certain quantities like; $\left(\frac{I_1^+ + Q_{init}}{3\ P_a}\right)^{-1.5}$ in addition it would generate from a theoretical point of view a negative value q_{II} according to the equation of the surface of load déviatoire. Such

a phenomenon was detected on several levels: in a particular way in the elastic prediction with the model CJS1, and in a general way in the local iterations of Newton utilizing the mechanism déviatoire. The same answer was given in order to free itself from this pathology: it is a question of virtually projecting the stresses in the elastic domain on the hydrostatic axis while posing: One

$$\sigma_{11} = \sigma_{22} = \sigma_{33} = -1\ kPa$$
$$\sigma_{12} = \sigma_{13} = \sigma_{23} = 0$$

thus repositions the stress state in the field of compression while moving away little from the inadmissible initial prediction considered, and by hoping that the structure considerations will make it possible total computation to converge. Moreover

local variables do not evolve and one supposes being returned in the elastic domain tangent

5 Operator

the tangent operator called by option RIGI `_MECA_TANG corresponds` to the tangent operator deduced from the problem of velocity and calculated starting from the results known at the moment T.

The tangent operator called by option FULL `_MECA should` correspond to the tangent operator with the discretized problem in an implicit way. Actually, we did not carry out this computation. We take then, when option FULL `_MECA` is retained, the tangent operator deduced from the problem of velocity and calculated starting from the results known at time t+dt. We

detail below the tangent operator deduced from the problem of it velocity according to or of the concerned mechanisms. Tangent

5.1 operator of the nonlinear elastic mechanism We

have simply the following nonlinear elastic relation: from where

$$\dot{\sigma}_{ij}=D_{ijkl}(\sigma)\,\dot{\varepsilon}_{kl}=\left(\frac{I_1+Q_{init}}{3\,P_a}\right)^n D_{ijkl}^{lineaire}\,\dot{\varepsilon}_{kl}$$

immediately the tangent operator: Tangent

$$H_{ijkl}^{elas.nl}=\left(\frac{I_1+Q_{init}}{3\,P_a}\right)^n D_{ijkl}^{lineaire}$$

5.2 operator of the mechanisms isotropic elastic and plastic In this case,

we have the following relation: it

$$\dot{\sigma}_{ij}=D_{ijkl}(\sigma)\left(\dot{\varepsilon}_{kl}-\dot{\varepsilon}_{kl}^{ip}\right)=D_{ijkl}(\sigma)\left(\dot{\varepsilon}_{kl}+\frac{1}{3}\dot{\lambda}^{i}\,\delta_{kl}\right)$$

comes: By $\dot{I}_1=3\,K\left(\dot{\varepsilon}_v+\dot{\lambda}^i\right)$

taking account of this relation and the model of hardening of, Q_{iso} the condition becomes $\dot{f}^i=0$: that is to say

$$\dot{f}^i=-\frac{\dot{I}_1}{3}+\dot{Q}_{iso}=-K\left(\dot{\varepsilon}_v+\dot{\lambda}^i\right)-K^p\,\dot{\lambda}^i=0$$

: While $\dot{\lambda}^i=-\frac{K}{K+K^p}\,\dot{\varepsilon}_v$

deferring this result in the statement of, $\dot{\sigma}_{ij}$ one finds: from where

$$\dot{\sigma}_{ij}=D_{ijkl}\left(\dot{\varepsilon}_{kl}-\frac{1}{3}\,\frac{K}{K+K^p}\,\dot{\varepsilon}_{mm}\,\delta_{kl}\right)=\left(D_{ijkl}-\frac{1}{3}\,\frac{K}{K+K^p}\,D_{ijmn}\,\delta_{mn}\,\delta_{kl}\right)\dot{\varepsilon}_{kl}$$

the tangent operator: One

$$H_{ijkl}^{ip}=D_{ijkl}-\frac{1}{3}\,\frac{K}{K+K^p}\,D_{ijmn}\,\delta_{mn}\,\delta_{kl}$$

can also write in matric form: where

$$H^{ip}=\left(\frac{I_1}{3\ P_a}\right)^n\begin{bmatrix}\lambda\ -\chi\ +2\ \mu & \lambda\ -\chi & \lambda\ -\chi & 0 & 0 & 0\\ \lambda\ -\chi & \lambda\ -\chi\ +2\ \mu & \lambda\ -\chi & 0 & 0 & 0\\ \lambda\ -\chi & \lambda\ -\chi & \lambda\ -\chi\ +2\ \mu & 0 & 0 & 0\\ 0 & 0 & 0 & 2\ \mu & 0 & 0\\ 0 & 0 & 0 & 0 & 2\ \mu & 0\\ 0 & 0 & 0 & 0 & 0 & 2\ \mu\end{bmatrix}$$

for this formula only and λ are μ the coefficient of Lamé and. $\chi\ =\dfrac{K_o^{e^2}}{K_o^e+K_o^p}$ Tangent

5.3 of the mechanisms elastic and plastic operator déviatoire

the condition $\dot{f}^d=0$ is written:

$$\dot{f}^d=\frac{\partial f^d}{\partial\sigma_{ij}}\dot{\sigma}_{ij}+\frac{\partial f^d}{\partial R}\dot{R}+\frac{\partial f^d}{\partial X_{ij}}\dot{X}_{ij}=\frac{\partial f^d}{\partial\sigma_{ij}}\dot{\sigma}_{ij}+\frac{\partial f^d}{\partial R}\dot{\lambda}^d G^R+\frac{\partial f^d}{\partial X_{ij}}\dot{\lambda}^d G^X_{ij}=0$$

The tensor being G^X purely déviatoire, the product $\dfrac{\partial f^d}{\partial X_{ij}}G^X_{ij}$ is reduced to:

$$\frac{\partial f^d}{\partial X_{ij}}G^X_{ij}=dev\left(\frac{\partial f^d}{\partial X_{ij}}\right)G^X_{ij}=-I_1\ Q_{ij}\ G^X_{ij}$$

The plastic multiplier can thus be put in the form: while

$$\dot{\lambda}^d=\frac{1}{H^{dev}}\frac{\partial f^d}{\partial\sigma_{ij}}\dot{\sigma}_{ij}$$

revealing the plastic modulus, H^{dev} given by:

$$H^{dev}=I_1^2\left(\frac{I_1+Q_{init}}{3\ P_a}\right)^{-1,5}\left[A\left(1-\frac{R}{R_m}\right)^2+\frac{1}{b}\ Q_{ij}\ \left(Q_{ij}+\varphi\quad X_{ij}\right)\right]$$

The relation stress-strains then makes it possible to write: what

$$\frac{\partial f^d}{\partial\sigma_{ij}}\dot{\sigma}_{ij}=\frac{\partial f^d}{\partial\sigma_{ij}}D_{ijkl}\left(\dot{\varepsilon}_{kl}-\dot{\lambda}^d G^d_{kl}\right)=\frac{\partial f^d}{\partial\sigma_{ij}}D_{ijkl}\ \dot{\varepsilon}_{kl}-\dot{\lambda}^d\frac{\partial f^d}{\partial\sigma_{ij}}D_{ijkl}\ G^d_{kl}$$

gives finally for the plastic multiplier: While

$$\dot{\lambda}^d=\frac{\dfrac{\partial f^d}{\partial\sigma_{ij}}D_{ijkl}\ \dot{\varepsilon}_{kl}}{H^{dev}+\dfrac{\partial f^d}{\partial\sigma_{ij}}D_{ijkl}\ G^d_{kl}}$$

deferring this result in the statement of, $\dot{\sigma}_{ij}$ one finds: from where

$$\dot{\sigma}_{ij}=D_{ijkl}\left(\dot{\varepsilon}_{kl}-\frac{\dfrac{\partial f^d}{\partial\sigma_{pq}}D_{pqmn}\ \dot{\varepsilon}_{mn}}{H^{dev}+\dfrac{\partial f^d}{\partial\sigma_{rs}}D_{rstu}\ G^d_{tu}}G^d_{kl}\right)$$

the tangent operator:

$$H^{dp}_{ijkl}=D_{ijkl}-D_{ijmn}\,G^{d}_{mn}\,\frac{\dfrac{\partial f^d}{\partial\sigma_{pq}}\,D_{pqkl}}{H^{dev}+\dfrac{\partial f^d}{\partial\sigma_{rs}}\,D_{rstu}\,G^{d}_{tu}}$$

The tangent operator thus obtained is not symmetric. However for time model CJS leans on of the finite elements which claims a symmetric operator. Ultimately, we retain not but H^{dp}_{ijkl} who $\tilde{H}^{dp}_{ijkl}$ is given by: with

$$\tilde{H}^{dp}_{ijkl}=\frac{H^{dp}_{ijkl}+H^{dp}_{klij}}{2}$$ and ij taken kl in tangent $(11,\,22,\,33,\,12,\,13,\,23)$

5.4 Operator of the mechanisms elastic, plastics isotropic and déviatoire One

must satisfy the two following conditions: and $\dot{f}^i=0$. $\dot{f}^d=0$ Taking into account the relation stress-strains which is written:

$$\dot{\sigma}_{ij}=D_{ijkl}\left(\dot{\varepsilon}_{kl}+\frac{1}{3}\dot{\lambda}^{i}\,\delta_{kl}-\dot{\lambda}^{d}\,G^{d}_{kl}\right)$$

the first condition gives: where

$$\dot{f}^i=-K\left(\dot{\varepsilon}_v+\dot{\lambda}^i-\dot{\lambda}^d\,G^d_v\right)-K^p\,\dot{\lambda}^i=0$$

one posed. $G^d_v=G^d_{kk}=tr\left(\boldsymbol{G}^d\right)$

The second condition led to: Thus

$$\frac{\partial f^d}{\partial\sigma_{ij}}\,D_{ijkl}\,\dot{\varepsilon}_{kl}+\frac{1}{3}\dot{\lambda}^i\,\frac{\partial f^d}{\partial\sigma_{ij}}\,D_{ijkl}\,\delta_{kl}-\dot{\lambda}^d\,\frac{\partial f^d}{\partial\sigma_{ij}}\,D_{ijkl}\,G^d_{kl}-H^{dev}\,\dot{\lambda}^d=0$$

, the plastic multipliers and $\dot{\lambda}^i$ $\dot{\lambda}^d$ are obtained by solving the system: that is to say

$$\begin{cases}-\left(K+K^p\right)\dot{\lambda}^i+K\,G^d_v\,\dot{\lambda}^d=K\,\dot{\varepsilon}_v\\[2mm] -\dfrac{1}{3}\,\dfrac{\partial f^d}{\partial\sigma_{ij}}\,D_{ijkl}\,\delta_{kl}\,\dot{\lambda}^i+\left(\dfrac{\partial f^d}{\partial\sigma_{ij}}\,D_{ijkl}\,G^d_{kl}+H^{dev}\right)\dot{\lambda}^d=\dfrac{\partial f^d}{\partial\sigma_{ij}}\,D_{ijkl}\,\dot{\varepsilon}_{kl}\end{cases}$$

: These

$$\dot{\lambda}^i=\frac{K\,G^d_v\,\dfrac{\partial f^d}{\partial\sigma_{ij}}\,D_{ijkl}\,\dot{\varepsilon}_{kl}-\left(\dfrac{\partial f^d}{\partial\sigma_{mn}}\,D_{mnpq}\,G^d_{pq}+H^{dev}\right)K\,\dot{\varepsilon}_v}{\left(K+K^p\right)\left(\dfrac{\partial f^d}{\partial\sigma_{rs}}\,D_{rstu}\,G^d_{tu}+H^{dev}\right)-\dfrac{1}{3}\,K\,G^d_v\,\dfrac{\partial f^d}{\partial\sigma_{vw}}\,D_{wvxy}\,\delta_{xy}}$$

$$\dot{\lambda}^d=\frac{\left(K+K^p\right)\dfrac{\partial f^d}{\partial\sigma_{ij}}\,D_{ijkl}\,\dot{\varepsilon}_{kl}-\dfrac{1}{3}K\,\dfrac{\partial f^d}{\partial\sigma_{mn}}\,D_{mnpq}\,\delta_{pq}\,\dot{\varepsilon}_v}{\left(K+K^p\right)\left(\dfrac{\partial f^d}{\partial\sigma_{rs}}\,D_{rstu}\,G^d_{tu}+H^{dev}\right)-\dfrac{1}{3}\,K\,G^d_v\,\dfrac{\partial f^d}{\partial\sigma_{vw}}\,D_{vwxy}\,\delta_{xy}}$$

statements are still written: and

$\dot{\lambda}^{i}=T_{1_{kl}}\,\dot{\varepsilon}_{kl}$ where $\dot{\lambda}^{d}=T_{2_{kl}}\,\dot{\varepsilon}_{kl}$

the tensors and $\boldsymbol{T_1}$ $\boldsymbol{T_2}$ are given by: By

$$T_{1_{kl}}=\frac{K\;G_v^d\;\frac{\partial f^d}{\partial\sigma_{ij}}\;D_{ijkl}-\left(\frac{\partial f^d}{\partial\sigma_{mn}}\;D_{mnpq}\;G_{pq}^d+H^{dev}\right)K\;\delta_{kl}}{\left(K+K^p\right)\left(\frac{\partial f^d}{\partial\sigma_{rs}}\;D_{rstu}\;G_{tu}^d+H^{dev}\right)-\frac{1}{3}\;K\;G_v^d\;\frac{\partial f^d}{\partial\sigma_{vw}}\;D_{vwxy}\;\delta_{xy}}$$

$$T_{2_{kl}}=\frac{\left(K+K^p\right)\frac{\partial f^d}{\partial\sigma_{ij}}\;D_{ijkl}-\frac{1}{3}K\;\frac{\partial f^d}{\partial\sigma_{mn}}\;D_{mnpq}\;\delta_{pq}\;\delta_{kl}}{\left(K+K^p\right)\left(\frac{\partial f^d}{\partial\sigma_{rs}}\;D_{rstu}\;G_{tu}^d+H^{dev}\right)-\frac{1}{3}\;K\;G_v^d\;\frac{\partial f^d}{\partial\sigma_{vw}}\;D_{vwxy}\;\delta_{xy}}$$

deferring the statements and $\dot{\lambda}^{i}$ of $\dot{\lambda}^{d}$ in the formula of, $\dot{\sigma}_{ij}$ one finds: from where

$$\dot{\sigma}_{ij}=D_{ijkl}\left(\dot{\varepsilon}_{kl}+\frac{1}{3}\;T_{1_{nm}}\;\dot{\varepsilon}_{nm}\;\delta_{kl}-T_{2_{pq}}\;\dot{\varepsilon}_{pq}\;G_{kl}^d\right)$$

the tangent operator: This

$$H_{ijkl}^{idp}=D_{ijkl}+\frac{1}{3}\;D_{ijmn}\;\delta_{mn}\;T_{1_{kl}}-D_{ijpq}\;G_{pq}^d\;T_{2_{kl}}$$

tangent operator not being symmetric, we retain not but H_{ijkl}^{idp} who $\tilde{H}_{ijkl}^{idp}$ is given by: with

$\tilde{H}_{ijkl}^{idp}=\frac{H_{ijkl}^{idp}+H_{klij}^{idp}}{2}$ and ij taken kl in Sources $(11,\;22,\;33,\;12,\;13,\;23)$

6 Aster Lists

6.1 routines modified and added Only

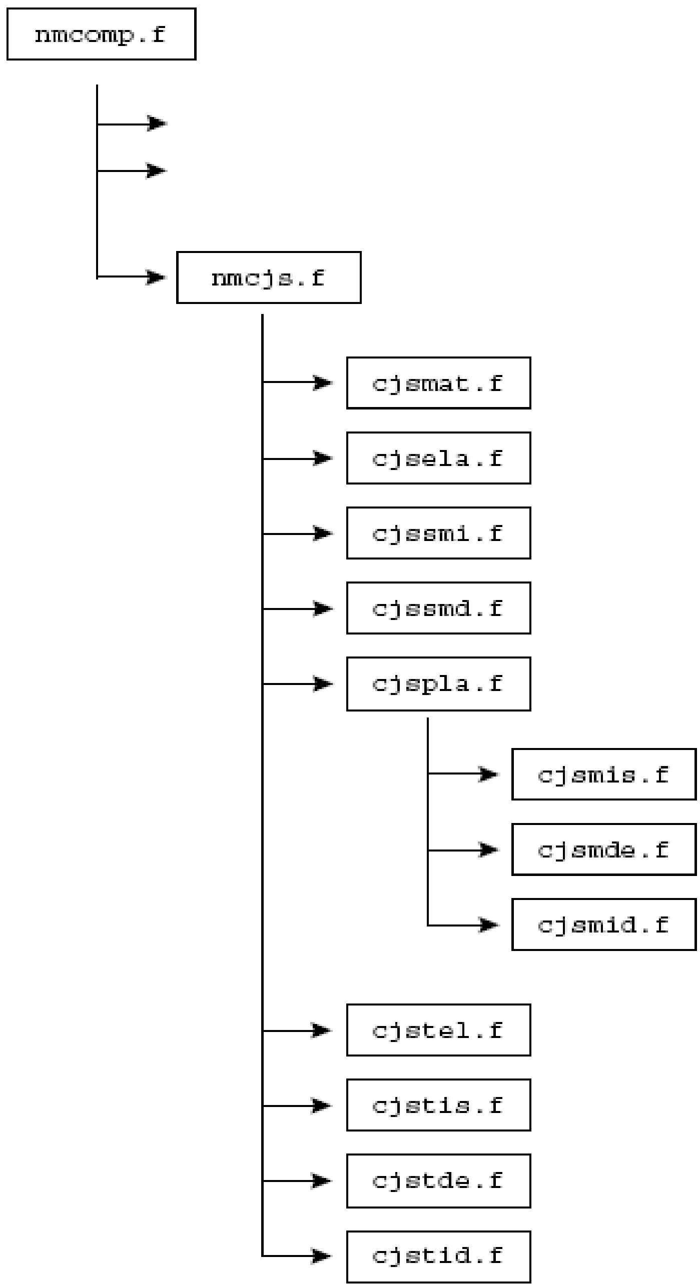
the routine nmcomp.f was modified. It makes it possible to call, when behavior CJS is chosen, the routine nmcjs.f, `starting point` of the integration of the model. All

routines FORTRAN developed in the frame of the integration of model CJS in the Code_Aster *are the following* : cjsc3q.f, cjsci

```
1.f, cjsdtd.f    , cjsela.f,       cjside.f, cjsiid.f    , cjsjde.f       ,
cjsjid.f, cjsjis.f     , cjsmat.f
, cjsmde.f, cjsmid.f       , cjsmis.f       , cjsnor.f,       cjspla.f,
cjsqco.f     , cjsqij.f,       cjssmd.f, cjssmi.f
, cjst.f, cjstde.f     , cjstel.f,  cjstid.f, cjstis.f     , lcdete.f,
    nmcjs.f, cjsinp.f     , cjsncn.f, cjsncv.f
, cjsnvi.f       , cjsqq.f.       Top-level flowchart             of
the principal
routines      principal              routines     FORTRAN       for
the integration
```

6.2 of model CJS are connected in the following way

: Details of the features of developed routines FORTRAN Routine: Objective CJSC3Q: computation



6.3 of Variables of entry and output: INSIG: STRESSES

6.3.1 X: VARIABLES

HAMMER-HARDENED MOVIES PA: ATMOSPHERIC $\cos(3\ \theta_q)$

PRESS (MATERIAL CHARACTERISTIC) OUTQ

```
:      DEV.  (SIG) - TRACE (
SIG        ) *X  QII: SQRT (QIJ*QIJ)
 COS        3TQ  : SQRT (54) *DET (Q)/(QII ** 3) Routine

: CJSCI              1  Purpose: resolution
of       the equation by
the secant method    , for
```

6.3.2 the nonlinear elastic behavior

Variables of entry $I_1^+ - I_1^- - 3\ K_o^e \left(\frac{I_1^+}{3\ P_a} \right)^n tr(\Delta\varepsilon) = 0$ and output: INCRIT:

CONVERGENCE CRITERIA MATER: COEFFICIENTS MATERIAU

A T+DT LIFO: INCREMENT OF DEFORMATION

```
SIGD  : STRESS A T OUTI1
: TRACE SIG T+DT LEAFLET A: LOGICAL
VARIABLE  INDICATING the TENSION
Routine  : Objective CJSDTD
    : computation of derivative
of the tensor compared to Variables of entry
```

6.3.3 and output

: INMOD:

MODELISATION Q: TENSOR (6 COMPONENTS t^d) OUTDTDDQ q

: TENSOR RESULTAT (6 COMPONENTS)

```
Routine            : Objective CJSELA
             : nonlinear elastic design
    of  the Variable stresses of entry
```

6.3.4 and output :

INMOD: MODELISATION

CRIT: CONVERGENCE CRITERIA MATERF

: COEFFICIENTS MATERIAU A T+DT SIGD

```
: STRESS     A T LIFO
:   INCREMENT  OF DEFORMATION
OUTSIGF:  STRESS A T+DT Organization
of   CJSELA  computation  of
the first   invariant  of the I1
stresses       to t+   dt  : call of CJSCI
```

1 computation of the coefficients

- of the elastic matrix and assembly of `the matrix` computation
  •of `the increment`
- of the stresses and the stresses with t+dt: call of LCPRMV and LCSOVE
- Routine: Objective CJSIDE: for the integration of the plastic mechanism
  •déviatoire , computation `D`

6.3.5 “a solution `D`”

test in order to

start the local iterations of Newton then. Variables of entry and output: INMOD: MODELISATION MATER: COEFFICIENTS MATERIAU A T+DT

EPSD: DEFORMATION A T+DT LIFO :

```
INCREMENT              OF DEFORMATION
       YD    :  VARIABLES A T = (SIGD, VIND,
       LAMB)      VARGD  : TENSOR OF
       THE LOI      D YIELDING  DEV.
       OUTDY         : SOLUTION D TEST Organization of
CJSIDE           computation  of the elastic operator, computation of
models     of       hardening  and
```

, computation of the flow `model`

- of the plastic mechanism déviatoire
- , computation of the threshold, G^R its $\boldsymbol{G}^X$ derivative
- and the plastic multiplier, computation of the solution of $\boldsymbol{G}^d$ test
- Routine: Objective f^d CJSIID: for $\frac{\partial f^d}{\partial \Delta \ \lambda^d}$ the simultaneous integration $\Delta \ \lambda^d$ of
- the plastic mechanisms isotropic

6.3.6 and déviatoire ,

computation of

a solution of test in order to start the local iterations of Newton then. Variables of entry and output: INMOD: MODELISATION MATER: COEFFICIENTS MATERIAU A T+DT

EPSD: DEFORMATION A T+DT LIFO :

```
INCREMENT              OF DEFORMATION
       YD    :  VARIABLES A T = (SIGD, VIND,
       LAMB)      VARGD  : TENSOR OF
       THE LOI      D YIELDING  DEV.
       OUTDY         : SOLUTION D TEST Organization of
CJSIID           computation  of the elastic operator, computation of
models     of       hardening  and
```

, computation of the flow model

- of the plastic mechanism déviatoire
- , computation of the thresholds and G^R , $\boldsymbol{G^X}$
- their plastic derivatives, and, and multipliers and, $\boldsymbol{G^d}$ computation
- of the solution f^i of f^d test Routine: Objective $\frac{\partial f^i}{\partial \Delta \lambda^i}$ $\frac{\partial f^i}{\partial \Delta \lambda^d}$ $\frac{\partial f^d}{\partial \Delta \lambda^i}$ CJSJDE $\frac{\partial f^d}{\partial \Delta \lambda^d}$: computation of and for the resolution $\Delta \lambda^i$ $\Delta \lambda^d$
- of (plastic mechanism déviatoire

6.3.7) Variable `of`

entry and

output $DRDY$: INMOD R : MODELISATION MATER $\frac{DR}{DY}\left(Y^p\right) DY^{p+1}=-R\left(Y^p\right)$: COEFFICIENTS MATERIAU A T+DT

EPSD: DEFORMATION A T LIFO:
```
INCREMENT
             OF  DEFORMATION
       YD:    VARIABLES  A T = (SIGD, VIND, LAMBDD
       ) YF      : VARIABLES A T
       +DT =     (SIGF  , VINF, LAMBDF) VARGD
       : TENSOR        OF THE LOI D YIELDING
       DEV.       OUTR  : SECOND MEMBER SIGNE: SIGNE OF
S: DEPSDP             DRDY: JACOBIAN Organization of CJSJDE computation
of              the elastic operator
          ,  computation of models of
       hardening      and
```

, computation of the flow model

- of the plastic mechanism déviatoire
- , computation of multiple G^R intermediate $\boldsymbol{G^X}$
- derivatives computation of the terms, computation of the components $\boldsymbol{G^d}$
- of and assembly of and Routine:
- Objective CJSJID: $\frac{\partial G^R}{\partial \sigma_{ij}}$ computation $\frac{\partial G^R}{\partial R}$ $\frac{\partial G^X_{mn}}{\partial \sigma_{ij}}$ of $\frac{\partial G^X_{mn}}{\partial X_{ij}}$ $\frac{\partial G^d_{mn}}{\partial \sigma_{ij}}$ and $\frac{\partial G^d_{mn}}{\partial R}$ for $\frac{\partial G^d_{mn}}{\partial X_{ij}}$
- the resolution of (plastic $DRDY$ R
- mechanisms $DRDY$ isotropic R

6.3.8 and déviatoire

) Variable

of entry $DRDY$ and R output: INMOD: MODELISATION $\frac{DR}{DY}\left(Y^p\right) \; DY^{p+1}=-R\left(Y^p\right)$ MATER: COEFFICIENTS MATERIAU A T+DT

EPSD: DEFORMATION A T LIFO: `INCREMENT`

```
          OF  DEFORMATION
      YD:    VARIABLES  A T = (SIGD, VIND, LAMBDD
      ) YF      : VARIABLES A T
      +DT =     (SIGF  , VINF, LAMBDF) VARGD
      : TENSOR        OF THE LOI D YIELDING
      DEV.        OUTR  : SECOND MEMBER SIGNE: SIGNE OF
S: DEPSDP            DRDY: JACOBIAN Organization of CJSJID computation
of              the elastic operator
          ,  computation of models of
      hardening      , and
```

, computation of `the flow model`

- of the plastic mechanism
- déviatoire, computation of multiple $G^{Q_{iso}}$ G^R intermediate $\boldsymbol{G}^{\boldsymbol{X}}$
- derivatives computation of the terms, computation of the components $\boldsymbol{G}^{\boldsymbol{d}}$
- of and assembly of and Routine
- : Objective CJSJIS $\frac{\partial G^{Q_{iso}}}{\partial Q_{iso}}$: $\frac{\partial G^R}{\partial \sigma_{ij}}$ computation $\frac{\partial G^R}{\partial R}$ $\frac{\partial G^X_{mn}}{\partial \sigma_{ij}}$ of $\frac{\partial G^X_{mn}}{\partial X_{ij}}$ $\frac{\partial G^d_{mn}}{\partial \sigma_{ij}}$ and $\frac{\partial G^d_{mn}}{\partial R}$ for $\frac{\partial G^d_{mn}}{\partial X_{ij}}$
- the resolution of (isotropic $DRDY$ R
- plastic mechanism $DRDY$ R

6.3.9) Variable `of entry`

and

output: INMOD $DRDY$ R : MODELISATION MATER $\frac{DR}{DY}\left(Y^p\right) \; DY^{p+1}=-R\left(Y^p\right)$: COEFFICIENTS MATERIAU A T+DT

LIFO: INCREMENT OF DEFORMATION `YD`

```
:       VARIABLES       A T = (SIGD
      , VIND    ,  LAMBDD) YF: VARIABLES A T
      +DT =     (SIGF  , VINF, LAMBDF) OUTR:
       SECOND        MEMBER DRDY: JACOBIAN Organization
      of        CJSJIS computation of the elastic operator
,       computation         of the components
      of     and  assembly
```

of and Routine :

- Objective CJSMAT: recovery of
- data materials, amongst $DRDY$ R

- components $DRDY$ of R

6.3.10 the fields,

local variables

and of selected level CJS. Variables of entry and output: INIMAT: ADDRESSES MATERIAU CODE MOD: TYPE OF MODELISATION

TEMPF: TEMPERATURE A T+DT OUTMATERF

```
:   COEFFICIENTS  MATERIAU A T+DT
NDT    :  TOTAL NB OF COMPONENTS
TENSORS   NDI: NB OF DIRECT
COMPONENTS  TENSORS NVI: NB OF
LOCAL VARIABLES     NIVCJS: NIVEAU 1,2
OR 3    OF  LOI CJS Organization of CJSMAT
recovery     amongst components
of  the fields and local variables
```

according to

- the selected modelization, recovery of data materials, recognition of level CJS chosen according to the parameters
- given. Routine: Objective CJSMDE
- : elastoplastic computation of the stresses with the plastic mechanism

6.3.11 deviatoire `activated`

: resolution

by the method of Newton of Variables of entry and output: INMOD: MODELISATION CRIT: CONVERGENCE CRITERIA MATER $R(Y)=0$

: COEFFICIENTS MATERIAU A T+DT NVI

```
:       NB      OF  LOCAL VARIABLES
             EPSD  : STRAINS A T
       LIFO:    INCREMENT  OF DEFORMATION SIGD
       : STRESS       A T VIND: LOCAL VARIABLES
             A T STOPNC
       : ARRET      IN THE EVENT OF NO CONVERGENCE
       VARSIGF      : STRESS
       A T+DT     VINF  : LOCAL VARIABLES
       A T+   DT  NOCONV: NO the CONVERGENCE Organization
            of  CJSMDE initialization
            from  the state with T computation
       of   a solution  of test with
```

CJSIDE buckles `on`

- the iterations of Newton YD incrementing
- computation of and: CJSJDE resolution `of the system`
- by the method of Gauss: MTGAUS

  •actualization $YF=YD+DY$

  •of the solution $DRDY$ R test `of up to date`

  •convergence put of the stresses and local variables

  1) Routine: Objective CJSMID DY

  •: elastoplastic computation
- of the stresses with the plastic mechanisms

6.3.12 isotropic and deviatoire

activated

: resolution by the method of Newton of Variables of entry and output: INMOD: MODELISATION CRIT: CONVERGENCE CRITERIA MATER $R(Y)=0$

: COEFFICIENTS MATERIAU A T+DT NVI

```
:       NB      OF  LOCAL VARIABLES
             EPSD  : STRAINS A T
        LIFO:    INCREMENT  OF DEFORMATION SIGD
        : STRESS       A T VIND: LOCAL VARIABLES
              A T STOPNC
        : ARRET      IN THE EVENT OF NO CONVERGENCE
        VARSIGF      : STRESS
        A T+DT     VINF  : LOCAL VARIABLES
        A T+   DT  NOCONV: NO the CONVERGENCE Organization
             of  CJSMID initialization
             from  the state with T computation
        of   a solution  of test with
```

CJSIID buckles on

- the iterations of Newton YD incrementing
- computation of and: CJSJID resolution of the system
- by the method of Gauss: MTGAUS
  •actualization $YF=YD+DY$
  •of the solution $DRDY$ R test of up to date
  •convergence put of the stresses and local variables
  1) Routine: Objective CJSMIS DY
  •: elastoplastic computation
- of the stresses with the activated isotropic

6.3.13 plastic mechanism

: resolution

by the method of Newton of Variables of entry and output: INMOD: MODELISATION CRIT: CONVERGENCE CRITERIA MATER $R(Y)=0$

: COEFFICIENTS MATERIAU A T+DT LIFO

```
        : INCREMENT       OF DEFORMATION
             SIGD  : STRESS A T VIND
        :    LOCAL VARIABLES  A T STOPNC
        : ARRET      IN THE EVENT OF NO CONVERGENCE
        VARSIGF      : STRESS
        A T+DT     VINF  : LOCAL VARIABLES
        A T+   DT  NOCONV: NO the CONVERGENCE Organization
             of  CJSMIS initialization
             from  the elastic prediction
        buckles    on the iterations
```

of Newton incrementing

- computation of and YD : CJSJIS resolution of the system
- by the method of Gauss: MTGAUS
  •actualization $YF=YD+DY$
  •of the solution $DRDY$ R test of up to date
  1) convergence put of the stresses and local variables

•Routine: Objective CJSNOR DY
•: computation of a vector
- parallel with Variables of entry and output
-

6.3.14 : INMATER : MATERIAU

SIG

: STRESSES X: KINEMATICAL $\frac{\partial f^d}{\partial q_{ij}}$

LOCAL VARIABLES OUTNOR: ESTIMATE OF

```
THE DITRECTION          OF
       THE NORM     WITH SURFACE
            DEVIATOIRE   IN LE PLANE DEVIATOIRE
PERPENDICULAR       TO TRISECTING LE THE NOR VECTEUR (1:
                 NDT) N IS NOT NORM SA NORM NOR EAST (NDT+1) Routine
                 : Objective CJSPLA: elastoplastic
                 computation of the stresses. Variables
                 of entry and
```

6.3.15 output: INMOD

: MODELISATION
CRIT: CONVERGENCE CRITERIA MATER

: COEFFICIENTS MATERIAU A T+DT SEUILI

```
:    ISO
LOADING FUNCTION   . CALCULEE  AVEC PREDICT ELAS
SEUILD  :  LOADING FUNCTION DEV. CALCULEE
AVEC  PREDICT ELAS NVI: LOCAL VARIABLE EPSD MANY
: STRAINS  A T LIFO: INCREMENT OF DEFORMATION SIGD
    :  STRESS A T VIND: LOCAL VARIABLES
    A T VARSIGF
: STRESS    A T+DT (IN - > ELAS,
OUT -   > PLASTI  ) OUTVINF
: LOCAL VARIABLES    A T+DT MECANI
:   MECHANISM  (S) ACTIVATES (S) Organization of CJSPLA assumption
on    the plastic mechanisms
activated  according to the values
```

of the thresholds and calculated

- from the elastic prediction, processing of the possible recutting of time step f^i f^d saves elastic prediction, elastoplastic
- computation, isotropic plastic mechanism
- : Plastic CJSMIS mechanism déviatoire
- : CJSMDE isotropic plastic
  •mechanisms and déviatoire simultaneously
  •: CJSMID computation of the thresholds from
  •the stresses with t+dt call of CJSSMI and CJSSMD if (assumption
- of an isotropic and positive mechanism) or (assumption
  1) of a mechanism déviatoire
     1) and positive): return to elastoplastic computation f^d with plastic mechanisms isotropic and déviatoire f^i simultaneously, if not end of routine Routine: Objective CJSQCO: utility routine of CJS allowing the computation
  1) of standard quantities

6.3.16 listed below

Variable
of entry and output : INGAMMA: MATERIAL PARAMETER SIG: STRESSES X: VARIABLES

HAMMER-HARDENED MOVIES PREF: PRESS REF
FOR STANDARDIZATION EPSSIG: EPSILON
FOR NULLITY
I1 DEVIATOR: TRACE TENSOR
OF STRESSES OUTS: DEV. (SIG)
SOFTWARE FIRM: SQRT (S: S) SIIREL: SII/PREF COS3TS
: LODE (SIG) HTS: FONCTION H (TETHA_S
) DETS : DETERMINANT
OF S Q: Q (
SIG-X) QII : SQRT (
Q: Q) QIIREL : QII/PREF
COS 3TQ HTQ: FONCTION
H (TETHA _Q) DETQ: DETERMINANT
OF Q Routine
: Objective CJSQIJ
: computation of
the tensor
Variables of entry and
output : IN: DIMENSION

6.3.17 OF S , X, Q

S: I1
DEVIATOR: FIRST q_{ij}

INV. X: CENTER OF THE SURFACE OF CHARGE
DEVIATOIRE OUTQ: TENSOR
RESULTAT Routine
: Objective CJSSMD
: computation of the threshold of the plastic mechanism
déviatoire
. Variables of

6.3.18 entry and output

: INSIG
: STRESS WINE: LOCAL VARIABLES OUTSEUILD:

SEUIL ELASTICITY OF MECHANISM DEVIATOIRE
Routine : Objective
CJSSMI : computation of the threshold
of the isotropic plastic mechanism. Variables of entry

6.3.19 and output :

INSIG:
STRESS WINE: LOCAL VARIABLES OUTSEUILI:

SEUIL ELASTICITY OF the ISOTROPIC MECHANISM

```
Routine            : Objective CJST
           : computation of =. Variables
      of  entry and output: INS: MATRICE
```

6.3.20 OUTT: T (IN

VECTORIAL FORM

AVEC t RAC $\dfrac{\partial \det s}{\partial s}$

2) Routine: Objective CJSTDE: computation
of the tangent
matrix for the plastic mechanism déviatoire

6.3.21 Variable of entry

and
output: INMOD: MODELISATION MATER: COEFFICIENTS MATERIAU NVI

: NB OF LOCAL VARIABLES EPS: STRAINS
SIG: STRESSES
WINE : LOCAL VARIABLES
OUTDSDESY : MATRICE TANGENTE
SYMETRISEE Organization
of CJSTDE computation
of the elastic operator
, computation of models of hardening and

, computation of the flow model

- of the plastic mechanism déviatoire
- , computation of intermediate terms G^R $\boldsymbol{G}^X$
- computation of the tangent matrix symmetrization of the tangent matrix $\boldsymbol{G}^d$
- Routine: Objective CJSTEL:
- computation of the tangent matrix
- for the elastic mechanism Variables

6.3.22 of entry and

output:
INMOD: MODELISATION MATER: COEFFICIENTS MATERIAU SIG

: STRESSES OUTHOOK: ELASTIC OPERATOR
STIFFNESS Organization
of CJSTEL computation of
the elastic operator
Routine : Objective CJSTID: computation

of the tangent matrix for

- the plastic mechanisms isotropic

6.3.23 and déviatoire

Variable D
“entered and of output: INMOD: MODELISATION MATER: COEFFICIENTS MATERIAU NVI

: NB OF LOCAL VARIABLES EPS: STRAINS

```
          SIG: STRESSES
       WINE  : LOCAL VARIABLES
    OUTDSDESY      : MATRICE TANGENTE
    SYMETRISEE      Organization
    of      CJSTEL computation
    of     L”  elastic operator
,      computation  of  models of hardening and
```

, computation of the flow model

- of the plastic mechanism déviatoire
- , computation of intermediate terms G^R G^X
- computation of the tangent matrix symmetrization of the tangent matrix G^d
- Routine: Objective CJSTIS:
- computation of the tangent matrix
- for the isotropic plastic mechanism

6.3.24 Variables D “entered

and of output
: INMOD: MODELISATION MATER: COEFFICIENTS MATERIAU SIG

: STRESSES WINE: LOCAL VARIABLES

```
    OUTDSDE      : MATRICE
    TANGENTE    Organization of CJSTEL
    computation      of the tangent
    matrix      Routine: Objective
    LCDETE     : computation D”
```

a matrix determining 3

- 3 Variables of entry and

6.3.25 output: INA:

MATRICE OUTLCDETE
: DETERMINANT Routine: Objective NMCJS

: realization of the integration

```
          of  model
CJS    : computation  of the stresses
```

6.3.26 with t+dt and/or

the tangent
matrix, according to the selected computation option. Variables of entry and output: INTYPMOD TYPE OF MODELISATION IMAT BEHAVIOR ADDRESSES MATERIAU

CODE COMP OF L ELEMENT CRIT

```
CRITERES       URGENT BUILDINGS INSTAM
```

```
        T    INSTAP URGENT T+DT TEMPM
        TEMPERATURE    A T TEMPF TEMPERATURE
            A T+DT TREF  TOTAL

        REFERENCE TEMPERATURE  EPSD
        DEFORMATION   TOTAL A T LIFO
        INCREMENT   OF DEFORMATION
        SIGD    FORCED A T VIND LOCAL VARIABLES
            A T + INDICATING
        STATE    T OPT COMPUTATION OPTION A TO MAKE
        OUTSIGF    FORCED
        A T+    DT VINF LOCAL VARIABLES    A T+DT + INDICATING
        STATE     T+DT DSDE MATRICE OF
BEHAVIOR          TANGENT A T+DT
        OR T    Organization of NMCJS recovery of data
        materials    , amongst components of the fields,
```

local variables

- and of selected level CJS: call of CJSMAT blocking of local variables according to selected level CJS computation of the stresses with
  •t+dt elastic
- prediction: Isotropic CJSELA computation of the thresholds of
- the mechanisms and déviatoire: CJSSMI
  •and CJSSMD if one of the thresholds
  •is exceeded, elastoplastic computation: CJSPLA computation of the tangent matrix
  •according to the concerned mechanism elastic: Isotropic plastic
- CJSTEL: Plastic CJSTIS déviatoire: Isotropic plastic CJSTDE

1) and déviatoire
2) : CJSTID Bibliography Mr. MALEKI
3) , B. CAMBOU, P. DUBUJET , "hierarchical
4) Modelization of the behavior of

7 the soils", to appear

[1] . B. CAMBOU, K. JAFARI, "Models behavior of the non-cohesive soils", Honest rev. Géotech. N

[2] 44, p.p 43-55, 1988. K. ELAMRANI, "Contribution to the validation of model CJS for the granular materials", Thesis of

[3] Doctorate of the Central School of Lyon, 1992. Functionalities and checking This document relates to constitutive law CJS (key word COMP_INCR

8 of STAT_NON_LINE) and its associated

material CJS (command DEFI_MATERIAU). This constitutive law is checked by the cases following tests : SSNV 135 triaxial Compression test drained

with the model CJS (level 1) [V6.04.135] SSNV136 triaxial Compression test drained

with	the model CJS (level 2) [V6.04.136] SSNV 154 triaxial	Compression test drained
with	the model CJS (level 3) [V6.04.154] SSNV 155 triaxial	Compression test drained
on	a turned sample of an angle of − PI/6 compared to	
axis X with	the model CJS (level 2) [V6.04.155] WTNV100 triaxial Compression test not drained with the model CJS (level 1) [V7.31 .100] Description	of the versions
of the document	Version Aster Author (S) Organization (S) Description of	the modifications

9 6,4 C. CHAVANT, pH. AUBERT EDF-R&

D/AMA EDF - SAY	/CNEPE initial Text	
6.4		