Modelizations THHM. General information and algorithms

Summarized

moduli THM of *Code_Aster* are those which by means of treat the equations of the mechanics of the continuums the theory of the porous environments possibly unsaturated and by considering that the mechanical, thermal and hydraulic phenomena are completely coupled. We present here the balance equations, or conservation equations solved by these moduli. We give a definition of the generalized stresses and generalized strains, allowing to define way rather general what is a constitutive law THM - at least what the moduli considered consider thus - and allowing to treat the nonlinear equations displayed in the frame of the algorithms of operator STAT_NON_LINE.

Contents

1 Introduction

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We present here the balance equations, or conservation equations solved by these moduli. We give a definition of the generalized stresses and generalized strains, allowing to define way rather general what is a constitutive law `THM` - at least what the moduli considered consider thus - and allowing to treat the nonlinear equations displayed in the frame of the algorithms of operator `STAT_NON_LINE`.
Constitutive laws `THM` strictly speaking are not developed in this document, but in the document [R7.01.11].
Chemical phenomena (transformations of the components, reactions producing of components etc…), just as the radiological phenomena are not taken into account at this stage of the development of *Code_Aster*. The mechanical, hydraulic and thermal phenomena are taken into account or not according to the behavior called upon by the user in command `STAT_NON_LINE`, according to the following nomenclature:

Modelization	**Phenomena taken into account**
`KIT_HM`	Mechanics, hydraulics with a Mechanical,
`hydraulic`	unknown pressure KIT_HHM with two unknown pressures
`KIT_THH`	Thermal, hydraulics with two unknown pressures
`KIT_THM`	Thermal, mechanics, hydraulics with a Thermal,
`mechanical`	, hydraulic unknown pressure KIT_THHM with two unknown pressures

the document present describes the models of conservation for the most general case said `THHM`. The simpler cases are obtained starting from the general case by simply cancelling the quantity absent.

2 Presentation of problem: Assumptions, Notations

In this chapter, one mainly sticks to show the porous environment and its characteristics.

2.1 Description of the porous environment

the porous environment considered is a volume made up of a more or less homogeneous solid matrix, more or less coherent (very coherent in the case of the concrete, little in the case of sand). Between the solid elements, one finds pores. One distinguishes the closed pores which do not exchange anything with their neighbors and the connected pores in which the exchanges are numerous. When one speaks about porosity, it is many connected pores about which one speaks. Inside these pores are with more the two components present at more under two phases. The system is regarded as closed.

2.2 Notations

the quantities associated with a component c present under a phase p are noted X_c^p. The index of the component c can vary from 1 with 2 and that of the phase also. These components can be (and will be subscripted like such where necessary):

w for liquid water,
ad the dissolved air,
as the dry air,
vp the steam.

The porous environment at current time is noted Ω , its border $\partial\Omega$, and it is noted $\Omega_0, \partial\Omega_0$ at initial time.

$\mathbf{n}$ indicate the norm in a point of $\partial\Omega$, image of the norm $\mathbf{n_0}$ with $\partial\Omega_0$. We will note $d(\partial\Omega)$ (respectively $d(\partial\Omega_0)$) the surface element of $\partial\Omega$ (respectively $\partial\Omega_0$).

The medium is defined by:

- parameters (vector position $\mathbf{x}$, time t),
- variables (displacements, pressures, temperature),
- quantities intrinsic (forced and mass strains, contributions, heat, enthali, flux hydraulic, thermal…).

For the solid phase, one makes the assumption of small displacements.

The various notations are clarified hereafter.

2.2.1 Descriptive variables of the medium

These are the variables whose knowledge according to time and of the place make it possible to know the state of the medium completely. These variables break up into two categories:

- geometrical, variable
- variables of thermodynamic state.

2.2.1.1 Geometrical variables

In all that follows, one adopts a Lagrangian representation compared to the squelette (within the meaning of [bib1]) and the coordinated $\mathbf{x}=\mathbf{x}_s(t)$ are those of a material point attached to the squelette. All the spatial derivative operators are defined compared to these coordinates.

Displacements of the squelette are noted $\mathbf{u}(\mathbf{x},t)=\begin{pmatrix}u_x\\u_y\\u_z\end{pmatrix}$.

2.2.1.2 Thermodynamic variables of state

the thermodynamic variables are:

- pressures of the components: since we consider that there is with more the two components, there will be with more the two conservation equations of the mass, and thus by duality with more the two variables of pressure,
- the temperature of the medium $T(\mathbf{x},t)$.

2.2.1.3 Descriptive fields of the medium

the principal unknowns, who are also the nodal unknowns (noted $\mathbf{U}(\mathbf{x},t)$ in this document) are:

- 2 or 3 (according to the dimension of space) displacements $u_x(\mathbf{x},t), u_y(\mathbf{x},t), u_z(\mathbf{x},t)$ for modelizations `KIT_HM`, `KIT_HHM`, `KIT_THM`, `KIT_THHM`,
- the temperature $T(\mathbf{x},t)$ for modelizations `KIT_THH`, `KIT_THM`, `KIT_THHM`,
- two pressures $p_1(\mathbf{x},t), p_2(\mathbf{x},t)$ for modelizations `KIT_HHM`, `KIT_THH`, `KIT_THHM`,
- a pressure $p_1(\mathbf{x},t)$ for modelizations `KIT_HM`, `KIT_THM`.

2.2.2 Quantities

the balance equations are:

- conservation of the linear momentum for the mechanics,
- the conservation lots of fluid for the hydraulics,
- the conservation of energy for the thermal.

The balance equations utilize directly the generalized stresses. The generalized stresses are connected to the strains generalized by the constitutive laws. The generalized strains are calculated directly starting from the variables of state and their temporal spatial gradients.
The constitutive laws can use additional quantities, often arranged in the local variables. These quantities are not described in this document which strictly speaking does not treat constitutive laws.

2.2.2.1 Quantities characteristic of the heterogeneous medium

- eulerian porosity: φ .
  If one notes Ω_{φ} the part of volume Ω occupied by the vacuums in the current configuration, one a:

$$\varphi=\frac{\Omega_{\varphi}}{\Omega}$$ **éq 2.2.2.1 - 1**

  the definition of porosity φ is thus that of eulerian porosity.

- The saturation of the phase p : S^p .
  If one notes Ω^p the total volume occupied by the phase p , in the current configuration, one has by definition:

$$S^p=\frac{\Omega^p}{\Omega_j}$$ **éq 2.2.2.1 - 2**

  This saturation is thus finally a proportion varying enters 0 and 1 . In the equations of assessment, it is clear that it is the product of porosity by the saturation φS^p which will intervene. One can thus legitimately wonder why it is not that quantity which is taken as unknown. The response comes from what it is the saturation S^p which intervenes more simply in the constitutive laws.

- Eulerian density of the component c in the phase p : ρ_c^p .
  If one notes M_c^p the mass of the phase p of the component c , in volume Ω of the squelette in the current configuration, one has by definition:

$$M_c^p=\int_{\Omega^p}\rho_c^p\,d\,\Omega^p=\int_{\Omega_{\varphi}}\rho_c^p S^p\,d\,\Omega_{\varphi}=\int_{\Omega}\rho_c^p S^p\varphi\,d\,\Omega$$ **éq 2.2.2.1 - 3**

the density of the phase p is simply the sum of the densities of its components:

$$\rho^p=\sum_c \rho_c^p$$

- Lagrangian homogenized density: r .
  To time running, the mass of volume Ω , M_Ω is given by:

$$M_\Omega=\int_{\Omega_0} r\, d\,\Omega_0 \qquad \textbf{éq 2.2.2.1 - 4}$$

2.2.2.2 Mechanical magnitudes

- the tensor of the strains $\boldsymbol{\varepsilon}(\mathbf{u})(\mathbf{x},t)=\frac{1}{2}\left(\nabla\mathbf{u}^T+\nabla\mathbf{u}\right)$,
- the tensor of the stresses which are exerted on the porous environment: $\boldsymbol{\sigma}$.

This tensor breaks up into a tensor of the effective stresses plus a stress tensor of pressure $\boldsymbol{\sigma}=\boldsymbol{\sigma}'+\sigma_p\mathbf{I}$. $\boldsymbol{\sigma}'$ et σ_p are of the components generalized stresses. This cutting is finally rather arbitrary, but corresponds all the same to an assumption rather commonly allowed, at least for the mediums saturated with fluid.

2.2.2.3 Hydraulic quantities

- mass contributions in components m_c^p (unit: kilogram per cubic meter). They represent the mass of fluid brought between initial and current times. They belong to the generalized stresses.

$$m_c^p=J\,\rho_c^p\,\varphi\,S^p-\rho_c^{p_0}\varphi_0 S_0^p \qquad \textbf{éq 2.2.2.3 - 1}$$

the mass contributions make it possible to define the total density seen compared to the reference configuration: $r(\mathbf{x},t)=r_0+m_{lq}(\mathbf{x},t)+m_{vp}(\mathbf{x},t)+m_{as}(\mathbf{x},t)$, where r_0 the density homogenized in an initial state indicates.

- Hydraulic flux:

  $\mathbf{w}_c^p$ (unit: kilogram/second/square meter) of eulerian representation
  $\mathbf{M}_c^p$ (unit: kilogram/second/square meter) of Lagrangian representation

One notes $\mathbf{v}_c^p$ the velocity of the component c in the phase p , J the Jacobian of the material transformation and $\mathbf{v}_S=\frac{d\,\mathbf{u}}{dt}$ the velocity of the squelette. $\rho_{c0}^p,\varphi_0,S_0^p$ the densities, porosity indicate and the saturations at initial time. By definition:

$$\mathbf{w}_c^p=\rho_c^p\varphi\,S^p\left(\mathbf{v}_c^p-\mathbf{v}_s\right) \qquad \textbf{éq 2.2.2.3 - 2}$$

the Lagrangian form of $\mathbf{w}_c^p$ noted $\mathbf{M}_c^p$ is obtained while writing:

$$\mathbf{M}_c^p . \mathbf{n}_0 d\left(\partial \Omega_0\right) = \mathbf{w}_c^p . \mathbf{n} d\left(\partial \Omega\right)$$ **éq 2.2.2.3 - the 3**

variables $m_1, \mathbf{M}_1$ and $m_2, \mathbf{M}_2$ refer each one to a component **of conservative mass**.
One poses by principle:

$$m_1 = m_1^1 + m_1^2; \quad \mathbf{M}_1 = \mathbf{M}_1^1 + \mathbf{M}_1^2$$
$$m_2 = m_2^1 + m_2^2; \quad \mathbf{M}_2 = \mathbf{M}_2^1 + \mathbf{M}_2^2$$

What we will write:

$$m_{constituant} = \sum_{\substack{nb\ phase\ du \\ constituant}} m_{constituant}^{phase}$$
$$\mathbf{M}_{constituant} = \sum_{\substack{nb\ phase\ du \\ constituant}} \mathbf{M}_{constituant}^{phase}$$

In the applications, one could for example have:

- 2 components: air and water,
- 2 phases for water,
- 1 phase for the air.

One would have then: m_1^1 et $\mathbf{M}_1^1$: contribution of mass and liquid water flux
m_1^2 et $\mathbf{M}_1^2$: contribution of mass and vapor flux
m_2^1 et $\mathbf{M}_2^1$: contribution of mass and flux of dry air
m_2^2 et $\mathbf{M}_2^2$: non-existent

- pressures:

Since we consider that there can be two components other than solid, there are two conservation equations of the mass, and thus two associated multipliers, i.e. two pressures p_1 et p_2. No assumption is made on what these two pressures mean p_1 et p_2. That will depend on the constitutive laws and the way of writing them. For example one can choose:

$$p_1 = \text{pression capillaire } (\mathrm{p}(\text{gaz}) - \mathrm{p}(\text{liquide}))$$
$$p_2 = \text{pression de gaz}(\text{ vapeur + air})$$

2.2.2.4 Thermal quantities

- not convectée heat Q' (see further) (unit: Joule),
- mass enthali of the components h_c^{mp} (unit: Joule/Kelvin/kilogram),
- heat flux: $\mathbf{q}$ (unit: Square J/s/meter).

2.2.3 External data

- the mass force $\mathbf{F}^m$ (in practice gravity),
- heat sources Θ ,
- boundary conditions relating either to variables imposed, or on imposed flux.

2.3 Particulate derivatives, voluminal and mass densities

the description which we make of the medium is Lagrangian compared to the squelette. One will find in [bib1] a definition of the notion of squelette: “the matrix (left occluded solide+porosity) constituting the material part of the squelette and the connected porous space of ground volume in question constitute the material point of the squelette or particle of the squelette”.

Either a an unspecified field on Ω , or $\mathbf{x}_s(t)$ the punctual coordinate attached to the squelette that we follow in his motion and or $\mathbf{x}_{fl}(t)$ the punctual coordinate attached to the fluid. One notes $\dot{a}=\frac{d^s a}{dt}$ temporal derivative in the motion of the squelette:

$$\dot{a}=\frac{d^s a}{dt}=\lim_{\delta t\to 0}\frac{a\left(\mathbf{x}_s(t+\delta t),t+\delta t\right)-a\left(\mathbf{x}_s(t),t\right)}{\delta t}$$

$\dot{a}$ is called particulate and often noted derivative $\frac{da}{dt}$ (for example in [bib1]). We prefer to use a notation which recalls that the configuration used to locate a particle is that of the squelette compared to which a fluid particle has a relative velocity. For a fluid particle the location $\mathbf{x}_s(t)$ is unspecified, i.e. that the fluid particle which occupies the position $\mathbf{x}_s(t)$ at time t is not the same one as that which occupies the position $\mathbf{x}_S(t')$ at another time t'.

That is to say then $A=\int_\Omega a\,d\Omega$ a quantity related to one density *voluminal* a , which density is it even carried *partly by the solid matter constituents and the fluids*. Either $a^{m}{}^{p}_{c}$ mass *density* of a range by the liquid phase p of the component c and or a_s voluminal *density* of a related to the solid matter constituents. All these definitions finally amount writing:

$$A=\int_\Omega a d\,\Omega=A_s+A_{fl}=\int_\Omega a_s d\,\Omega+\int_\Omega a_{fl} d\,\Omega=\int_\Omega\left(a_s+\sum_{p,c}\rho_c^p jS^p a^{m}{}^{p}_{c}\right)d\,\Omega \qquad \textbf{éq 2.3-1}$$

While following [bib1], we note $\frac{d^{fl}A_{fl}}{dt}$ derivative of A_{fl} if we follow Ω in the motion of the fluid and $\frac{d^s A_s}{dt}$ derivative of A_s if we follow Ω in the motion of the squelette.

We define then:

$$\frac{DA}{Dt}=\frac{d^s A_s}{dt}+\frac{d^{fl}A_{fl}}{dt}=\frac{d^s}{dt}\int_\Omega a_s d\,\Omega+\frac{d^{fl}}{dt}\int_\Omega\sum_{p,c}\rho_c^p\varphi S^p a^{m_c^p}d\,\Omega \qquad \textbf{éq 2.3-2}$$

the density $a^{m}{}^{p}_{c}$ is transported with a relative velocity from $\left(\mathbf{v}_c^p-\mathbf{v}_s\right)$ ratio with the squelette. Taking into account the definition of $\dot{a}=\frac{d^s a}{dt}$, and definition $\mathbf{w}_c^p=\rho_c^p\varphi S^p\left(\mathbf{v}_c^p-\mathbf{v}_s\right)$, one sees easily that the total derivative from A ratio with time is written finally:

$$\frac{DA}{Dt}=\int_{\Omega}\left(\dot{a}+\sum_{p,c}\mathrm{Div}\left(a^{m\,p}_{c}\mathbf{w}^{p}_{c}\right)\right)d\,\Omega$$ **éq 2.3-3**

Note::

Insofar as we made the assumption of small displacements of the squelette, $\dot{a}=\frac{d^s a}{dt}$ *can merge with partial derivative compared to time* $\frac{\partial a}{\partial t}$ *and* $\mathbf{v_s}$ *can be regarded as null. In the same way, in the continuation of the note we will confuse the Lagrangian and eulerian representations flux,* $\mathbf{M}^p_c$ *and* $\mathbf{w}^p_c$ *.*

3 Continuous equations

3.1 Mechanics: conservation of the linear momentum

We note $\boldsymbol{\sigma}$ the tensor of the stresses of Cauchy and $\mathbf{s}$ the second tensor (symmetric) of Piola-Kirchhoff.
We note $\mathbf{P}$ the gradient of the transformation $\mathbf{x}_0=\mathbf{x}_S(0)\rightarrow\mathbf{x}_S(\mathbf{x}_0,t)$.

$$\mathbf{P}=\frac{\partial\,\mathbf{x}_S(\mathbf{x}_0,t)}{\partial\,\mathbf{x}_0}$$

One a: $\mathbf{s}=\det\mathbf{P}.\mathbf{P}^{-1}.\boldsymbol{\sigma}.\mathbf{P}^{-T}$.

The balance equations mechanics are written in the configuration Ω_0 :

$$\mathrm{Div}_0(\mathbf{P}.\mathbf{s})+r\,\mathbf{F}^m=0$$

We noted Div_0 the operator of divergence compared to the variables of space $\mathbf{x}_0$ of the configuration Ω_0 .
Insofar as we make the assumption of small displacements and the small strains of the squelette, this equation can be approximate by:

$$\mathrm{Div}\,\boldsymbol{\sigma}+r\,\mathbf{F}^m=0$$ **éq 3.1-1**

We will further see we always adopt decomposition $\boldsymbol{\sigma}=\boldsymbol{\sigma}'+\sigma_p\mathbf{I}$, where $\boldsymbol{\sigma}'$ indicates the effective stress. It is thus with the load of the modulus of integration of the balance equations to make the sum: $\boldsymbol{\sigma}=\boldsymbol{\sigma}'+\sigma_p\mathbf{I}$.

3.2 Hydraulics: conservation of the mass

the eulerian writing of the conservation of the fluid mass for the component c is written:

$$\frac{d^{fl}}{dt}\int_{\Omega}\sum_{p}\rho^p_c\,\varphi\,S^p\,d\,\Omega=0$$

One can then apply [éq 2.3-1] while taking: $a_s=0$ and $a^{m\,p}_{c}=1$ [éq 2.3-3] will give:

$$\sum_p \frac{d^s \rho_c^p \varphi S^p}{dt} + \sum_p \mathrm{Div}\left(\mathbf{w}_c^p\right) = 0$$

By means of the definition of the mass contributions [éq 2.2.2.3 - 3], the definition of Lagrangian flux [éq 2.2.2.3 - 2] one finds the form Lagrangian of the conservation of the fluid mass:

$$\begin{cases} \dot{m}_1 + \mathrm{Div}_0\left(\mathbf{M}_1\right) = 0 \\ \dot{m}_2 + \mathrm{Div}_0\left(\mathbf{M}_2\right) = 0 \end{cases}$$ **éq 3.2-1**

3.3 Equation of energy

For the function thermodynamics, we adopt systematically a decomposition of the type [éq 2.3-1]. That corresponds to the fact that various energies have a whole a part carried by solid and a part carried by the fluids. The part carried by solid is characterized by one density voluminal whereas the parts carried by the fluid are characterized by mass densities, as we showed in the paragraph [§2.3].

Total internal energy: $$E = \int_\Omega \left(e_s + \sum_{p,c} \rho_c^p \varphi S^p e^{mp}_c \right) d\Omega$$ **éq 3.3.1**

total Entropy: $$S = \int_\Omega \left(s_s + \sum_{p,c} \rho_c^p \varphi S^p s^{mp}_c \right) d\Omega$$ **éq 3.3.2**

total Enthalpy $$H = \int_\Omega \left(h_s + \sum_{p,c} \rho_c^p \varphi S^p h^{mp}_c \right) d\Omega$$ **éq 3.3.3**

Free energy: $$\begin{cases} \Psi = E - T S \\ \Psi_s = e_s - T s_s \\ \Psi^{mp}_c = e^{mp}_c - T s^{mp}_c \end{cases}$$ **éq 3.3.4**

free Enthalpy: $$\begin{cases} G = H - T S \\ g_s = h_s - T s_s \\ g^{mp}_c = h^{mp}_c - T s^{mp}_c \end{cases}$$ **éq 3.3.5**

Lastly, by noting $\dot{Q}\left(\Omega\right)$ the rate of heat received by a volume Ω , one has by definition:

$$\dot{Q}\left(\Omega\right) = \int_{\partial\Omega} \mathbf{q}.\mathbf{n}\, d\Gamma + \int_\Omega \Theta\, d\Omega$$ **éq 3.3.6**

One recalls finally that the enthalpy of the fluids is calculated by the formula:

$$h = e + \frac{p}{\rho}$$ **éq 3.3.7**

3.3.1 the first principle

With the definitions given higher, it is written:

$$-\dot{e} - \sum_{p,c} \mathrm{Div}\left(h^{mp}_c \mathbf{M}_c^p\right) + \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}} + \sum_{p,c} \mathbf{M}_c^p . \mathbf{F}^m + \Theta - \mathrm{Div}\, q = 0$$ **éq 3.3.1-1**

This writing corresponds to the equation (22) chapter III-2-3 of [bib1], in which we neglected the terms of inertia. For the homogeneous mediums, it corresponds to the equation (31) of paragraph IV-3-2 of [bib3].

3.3.2 The second principle

Its form rather well-known is:

$$\dot{s}+\sum_{p,c}\mathrm{Div}\left(s^{mp}_{c}\mathbf{M}^{p}_{c}\right)+\mathrm{Div}\left(\frac{\mathbf{q}}{T}\right)-\frac{\Theta}{T}\geq 0$$ **éq the 3.3.2-1**

By means of classical thermodynamic considerations [bib1] related to the introduction of the free enthalpy [éq 3.3.5], one shows that one must necessarily have:

$$\boldsymbol{\sigma}-\frac{\partial\Psi}{\partial\boldsymbol{\varepsilon}}=0$$ **éq 3.3.2-2**

$$g^{mp}_{c}-\frac{\partial\Psi}{\partial m^{p}_{c}}=0$$ **éq 3.3.2-3**

$$s+\frac{\partial\Psi}{\partial T}=0$$ **éq 3.3.2-4**

3.3.3 Equation of energy

Rather often, one considers that, the transformations being reversible, the second principle provides finally an equality. Moreover, one replaces in [éq 3.3.2-1] the unknown temperature T by a value constant known as reference temperature. It is finally about a linearization of [éq 3.3.2-1] justified if the temperature variations are “small”. Let us note that the term of transport $\sum_{p,c}\mathrm{Div}\left(s^{mp}_{c}\mathbf{M}^{p}_{c}\right)$ complicates the processing of nonthe linearity due to the presence of the temperature in denominator of the other terms of [éq 3.3.2-1].

We work in enthalpy in order to overcome this difficulty. One leaves the equation of the first principle [éq 3.3.1-1] in which one injects the equations [éq 3.3.2-2], [éq 3.3.2-3], [éq 3.3.2-4], and the definition of the free enthalpy [éq 3.3-5] and one obtains:

$$T\,\dot{s}+\sum_{p,c}\left(h^{mp}_{c}\dot{m}^{p}_{c}-T\,s^{mp}_{c}\dot{m}^{p}_{c}\right)=-\sum_{p,c}\mathrm{Div}\left(h^{mp}_{c}\mathbf{M}^{p}_{c}\right)+\sum_{p,c}\mathbf{M}^{p}_{c}.\mathbf{F}^{m}+\Theta-\mathrm{Div}\,\mathbf{q}$$ **éq 3.3.3-1**

One poses then:

$$\delta Q'=T\,\delta s-T\sum_{p,c}s^{m^{p}_{c}}\delta m^{p}_{c}$$ **éq 3.3.3-2**

the quantity Q' has the dimension of an energy per unit of volume. It represents the heat received by the system in a transformation for which there are no contributions of heat per fluid entry having an enthalpy. Although $\delta Q'$ is not an exact differential, we take this quantity like variable of state.

Finally, the equation of energy selected has the following form:

$$\sum_{p,c} h^{m\,p}_{c}\dot{m}^{p}_{c}+\dot{Q}'+\sum_{p,c}\mathrm{Div}\left(h^{m\,p}_{c}\mathbf{M}^{p}_{c}\right)+\mathrm{Div}\,\mathbf{q}-\sum_{p,c}\mathbf{M}^{p}_{c}.\mathbf{F}^{m}=\Theta$$

éq 3.3.3-3

4 variational Writing of the balance equations

4.1 Mechanics

We note U_{ad} all the kinematically admissible fields of displacement, i.e. the elements of $\left(H^1(\Omega)\right)^3$ checking the boundary conditions in displacement on the part of $\partial\Omega$ supporting of such conditions [bib3].

The variational form of [éq 3.1-1] is:

$$\begin{cases}\boldsymbol{\sigma}=\boldsymbol{\sigma}'+\sigma_p\mathbf{I}\\ \int_\Omega \boldsymbol{\sigma}.\boldsymbol{\varepsilon}(\mathbf{v})=\int_\Omega r\,\mathbf{F}^m.\mathbf{v}\,d\Omega+\int_{\partial\Omega} f^{ext}\mathbf{v}\,d\Gamma \qquad \forall\mathbf{v}\in U_{ad}\end{cases}$$

éq 4.1-1

4.2 Hydraulics

We note P_{1ad} (resp. P_{2ad}) all the acceptable fields of pressure, i.e. elements of $H^1(\Omega)$ checking the boundary conditions in pressure P_1 (resp. P_2) on the part of $\partial\Omega$ supporting of such conditions [bib3]. The variational form of [éq 3.2-1] is:

$$\left.\begin{aligned}-\int_\Omega\left(\dot{m}^1_1+\dot{m}^2_1\right)\pi_1+\int_\Omega\left(\mathbf{M}^1_1+\mathbf{M}^2_1\right).\nabla\pi_1\,d\Omega=&\\ \int_{\partial\Omega}\left(\mathbf{M}^2_{1ext}+\mathbf{M}^2_{1ext}\right)\pi_1\,d\Gamma \qquad &\forall\pi_1\in P_{1ad}\\ -\int_\Omega\left(\dot{m}^1_2+\dot{m}^2_2\right)\pi_2+\int_\Omega\left(\mathbf{M}^1_2+\mathbf{M}^2_2\right).\nabla\pi_2\,d\Omega=&\\ \int_{\partial\Omega}\left(\mathbf{M}^2_{2ext}+\mathbf{M}^2_{2ext}\right)\pi_2\,d\Gamma \qquad &\forall\pi_2\in P_{2ad}\end{aligned}\right\}$$

éq 4.2-1

4.3 Thermal

We note T_{ad} all the acceptable fields of temperature, i.e. the elements of $H^1(\Omega)$ checking the boundary conditions in temperature on the part of $\partial\Omega$ supporting of such conditions. [bib3]. The variational form of [éq 3.3.3-3] is:

$$\begin{aligned}\int_\Omega\dot{Q}'\tau\,d\Omega+\sum_{p,c}\int_\Omega h^{mp}_{c}\dot{m}^{p}_{c}\tau\,d\Omega-\int_\Omega\left(\sum_{p,c}h^{mp}_{c}\mathbf{M}^p_c+\mathbf{q}\right).\nabla\tau\,d\Omega=&\\ \int_\Omega\left(\Theta+\sum_{p,c}\mathbf{M}^p_c.\mathbf{F}^m\right)\tau\,d\Omega-\int_{\partial\Omega}\left(\sum_{p,c}h^{mp}_{c}\mathbf{M}^p_{c\,ext}+\mathbf{q}_{ext}\right).\tau\,d\Gamma&\\ \forall\tau\in T_{ad}&\end{aligned}$$

éq 4.3-1

Let us note that, contrary to other presentations, and in particular [bib8] we did not inject the conservation equations of the mass, and we integrated by part the term of transport $\sum_{p,c} \mathrm{Div}\left(h_c^{mp}\mathbf{M}_c^p\right)$.

This last point has the advantage of not revealing of derivatives of a higher nature, and, contrary to revealing naturally relative boundary conditions at the entrance of heat related to hydraulic flux: $\sum_{p,c}\int_{\partial\Omega} h_c^{mp}\mathbf{M}_{c\,ext}^p.\tau\, d\Gamma$.

One will be able in makes consider that the heat flux conditions define directly:

$$\tilde{\mathbf{q}}_{ext}=h_c^{mp}\mathbf{M}_{cext}^p+\mathbf{q}_{ext}$$

5 Discretization in time

In this chapter, we are satisfied to take again the variational formulations in their applying a discretization compared to the time of the type sucked diagram. It is about a general method of integration of the differential equations [bib12] and [bib13].

θ is a numerical parameter understood enters 0 and 1. For the linear differential equations (what is not our case…) this diagram is unconditionally stable for $\theta\geq 1/2$, it is of order 1 for $\theta\neq 1/2$ and of order 2 for $\theta=1/2$. Nevertheless, it can be preferable to use a value different from $1/2$, and this for parasitic reasons of oscillations [bib12].

The subscripted quantities by $+$ are the quantities at the end of time step, and those subscripted by $-$ are those of the beginning of time step. One notes:

$$\Delta t=t^+-t^-$$
$$a^\theta=\theta a^+ +\left(1-\theta\right)a^- \qquad \forall a$$

5.1 Mechanics

$$\begin{cases}\boldsymbol{\sigma}^+=\boldsymbol{\sigma}'^+ +\sigma_p^+\mathbf{I}\\ \int_\Omega \boldsymbol{\sigma}^+.\boldsymbol{\varepsilon}\left(\mathbf{v}\right)d\Omega=\int_\Omega r^+\mathbf{F}^{\mathbf{m}+}.\mathbf{v}\,d\Omega+\int_{\partial\Omega}\mathbf{f}^{\mathbf{ext}+}.v\,d\Gamma \qquad \forall\,\mathbf{v}\in U_{ad}\end{cases} \quad \text{éq 5.1-1}$$

5.2 Hydraulics

$$\left.\begin{array}{l}-\int_\Omega\left(m_1^{1^+}+m_1^{2^+}\right)\pi_1 d\Omega+\theta\Delta t\int_\Omega\left(\mathbf{M}_1^{1^+}+\mathbf{M}_1^{2^+}\right).\nabla\pi_1 d\Omega=\\ \quad -\int_\Omega\left(m_1^{1^-}+m_1^{2^-}\right)\pi_1 d\Omega-\left(1-\theta\right)\delta t\int_\Omega\left(\mathbf{M}_1^{1^-}+\mathbf{M}_1^{2^-}\right).\nabla\pi_1 d\Omega\\ \quad\quad +\delta t\int_{\partial\Omega}\left(\mathbf{M}_{1\,\mathbf{ext}}^{1^\theta}+\mathbf{M}_{1\,\mathbf{ext}}^{2^\theta}\right).\pi_1 d\Gamma \qquad \forall\pi_1\in P_{1ad}\\ -\int_\Omega\left(m_2^{1^+}+m_2^{2^+}\right)\pi_2 d\Omega+\theta\Delta t\int_\Omega\left(M_2^{1^+}+\mathbf{M}_2^{2^+}\right).\nabla\pi_2 d\Omega=\\ \quad -\int_\Omega\left(m_2^{1^-}+m_2^{2^-}\right)\pi_2 d\Omega-\left(1-\theta\right)\delta t\int_\Omega\left(\mathbf{M}_2^{1^-}+\mathbf{M}_2^{2^-}\right).\nabla\pi_2 d\Omega\\ \quad\quad +\Delta t\int_{\partial\Omega}\left(\mathbf{M}_{2\,\mathbf{ext}}^{1^\theta}+\mathbf{M}_{2\,\mathbf{ext}}^{2^\theta}\right).\pi_2 d\Gamma \qquad \forall\pi_2\in P_{2ad}\end{array}\right\} \quad \text{éq 5.2-1}$$

5.3 Thermal

$$\left.\begin{array}{c}\int_{\Omega}\left(Q'^{+}-Q'^{-}\right)\tau\,d\Omega-\theta\Delta t\int_{\Omega}\left(\sum_{p,c}h_c^{m\,p+}\mathbf{M}_c^{p+}+\mathbf{q}^{+}\right)\nabla\tau\,d\Omega\\ -\left(1-\theta\right)\Delta t\int_{\Omega}\left(\sum_{p,c}h_c^{m\,p-}\mathbf{M}_c^{p-}+\mathbf{q}^{-}\right)\nabla\tau\,d\Omega+\theta\int_{\Omega}\left(\sum_{p,c}h_c^{m\,p+}\left(m_c^{p+}-m_c^{p-}\right)\right)\tau\,d\Omega\\ +\left(1-\tau\right)\int_{\Omega}h_c^{m\,p-}\left(m_c^{p+}-m_c^{p-}\right)\tau\,d\Omega=\\ +\tau\Delta t\int_{\Omega}\sum_{p,c}\mathbf{M}_c^{p+}.\mathbf{F}^{m}\tau\,d\Omega+\left(1-\theta\right)\Delta t\int_{\Omega}\sum_{p,c}\mathbf{M}_c^{p-}.\mathbf{F}^{m}\tau\,d\Omega\\ +\Delta t\int_{\Omega}\Theta^{\theta}\tau\,d\Omega-\Delta t\int_{\partial\Omega}\left(\sum_{p,c}h_c^{m\,p^{\theta}}\mathbf{M}_{c\,ext}^{p^{\theta}}+\mathbf{q}_{ext}^{\theta}\right).\tau\,d\Gamma\,\forall\tau\in T_{ad}\end{array}\right\} \quad \textbf{éq 5.3-1}$$

One can again consider that the heat flux conditions define directly:

$$\tilde{\mathbf{q}}_{ext}^{\theta}=\sum_{p,c}h_c^{m\,p^{\theta}}\mathbf{M}_{c\,ext}^{p^{\theta}}+\mathbf{q}_{ext}^{\theta}$$

6 Principle of the virtual wors, strains and stresses generalized, constitutive laws

6.1 Forced and strains generalized

While referring to the variational formulations [éq 4.1-1], [éq 4.2-1] and [éq 4.3-1], it appears that one can choose:

For the generalized stresses:

$$\Sigma=\left\{\begin{array}{l}\boldsymbol{\sigma}',\boldsymbol{\sigma}_p;\\ m_1^1,\mathbf{M}_1^1,h_1^{m1};m_1^2,\mathbf{M}_1^2,h_1^{m2};\\ m_2^1,\mathbf{M}_2^1,h_2^{m1};m_2^2,\mathbf{M}_2^2,h_2^{m2};\\ Q',\mathbf{q}\end{array}\right\} \quad \textbf{éq 6.1-1}$$

For the generalized strains:

$$E=\left\{\mathbf{u},\varepsilon\left(\mathbf{u}\right);p_1,\nabla p_1;p_2,\nabla p_2;T,\nabla T\right\} \quad \textbf{éq 6.1-2}$$

One notices the fact that the generalized strains contain displacements. That is due at the end $\int_{\Omega}r\,\mathbf{F}^{m}.\mathbf{v}$ of the variational formulation of the conservation equation of the linear momentum [éq 4.1-1], which term and the couples finally the generalized stresses displacements because of [éq 6.3.4-1]. The generalized strains contain the pressure and the temperature because the associated equations are parabolic.

6.2 Principle of the virtual wors

all the nonlinear equations to solve can be put in the form:

$$\mathbf{R}\left(\mathbf{U}\right)=\mathbf{L}^{meca} \quad \textbf{éq 6.2-1}$$

where $\mathbf{U}$ indicates generalized displacements, i.e.: $\mathbf{U}=\{\mathbf{u}_x, \mathbf{u}_y, \mathbf{u}_z, p_1, p_2, T\}$ in the most general case. The internal forces $\mathbf{R}$ are expressed from a principle of virtual wors generalized. In the case of the mechanics of the continuums "classical", i.e. when there is no other constituting that the solid, one is used defining the internal forces by:

$\mathbf{w}^T.\mathbf{R}=\int_\Omega \boldsymbol{\varepsilon}(\mathbf{w}).\boldsymbol{\sigma}\, d\Omega \quad \forall \mathbf{w}$, kinematically admissible field of displacement.

In this formulation, the strain field $\boldsymbol{\varepsilon}$ depends only on the field of displacement and its spatial derivatives, possibly in a nonlinear way if one takes into account finished strains. One writes symbolically:

$$\mathbf{R}=\mathbf{Q}^T\boldsymbol{\sigma}$$

The constitutive law connects the stresses $\boldsymbol{\sigma}$ to the strains $\boldsymbol{\varepsilon}$.
In the frame of the theory of the porous environments developed here, we try as much as possible to bring us closer to this formulation by introducing generalized $\boldsymbol{\Sigma}$ generalized stresses and strains $\mathbf{E}$;
The generalized strains depend only on the field of generalized displacements $\mathbf{U}$ and its spatial derivatives. The operator $\mathbf{U}\rightarrow\mathbf{E}(\mathbf{U})$ is an operator of derivative compared to the field of coordinates.
The constitutive law makes it possible to calculate $\boldsymbol{\Sigma}$ according to $\mathbf{E}$.

On the other hand, we cannot write directly $\mathbf{W}^T.\mathbf{R}=\int_\Omega \mathbf{E}(\mathbf{W}).\boldsymbol{\Sigma}\, d\Omega$, for the following reasons:

- the equations which we treat are evolutionary equations in time and the derived compared to time from the quantities intervene,
- the equations are nonlinear because of terms of transport related to the eulerian representation of the fluids: these nonlinear terms appear only in the equation of thermal,
- the choice of the unknowns makes that the nonlinear terms of transport intervene in the generalized stresses. That is to say a term of transport in the equation [éq 4.3-1] $\int_\Omega \left(\sum_{p,c} h_c^{mp}\,\mathbf{M}_c^p+\mathbf{q}\right).\nabla\tau\, d\Omega$, since one took as principal unknowns for the hydraulics the pressures, the quantities $\mathbf{M}_c^p$ related to the velocities of the fluids belong to the generalized stresses, just as the enthali h_c^{mp} , and the term of transport given in example is linear in strain generalized and quadratic in generalized stress. This made a difference with the formulation of the theory of the classical continuums where the terms of large deformation are quadratic quantities of the strains.

For all these reasons, we introduce a field noted $\overline{\boldsymbol{\Sigma}}$ such as:

$$\mathbf{R}=\mathbf{Q}^T\overline{\boldsymbol{\Sigma}} \qquad \text{éq 6.2-2}$$

$\mathbf{Q}^T$ is defined by:

$$\mathbf{W}^T.\mathbf{R}=\int_\Omega \mathbf{E}(\mathbf{W}).\overline{\boldsymbol{\Sigma}}\, d\Omega$$
$$\forall\mathbf{W} \quad \text{champ de déplacement généralisé cinématiquement admissible} \qquad \text{éq 6.2-3}$$

One sees easily and very classically that $\mathbf{Q}^T$ is transposed of the operator $\mathbf{Q}$ such as:

$$\mathbf{E}=\mathbf{QU} \qquad \text{éq 6.1-4}$$

the field $\overline{\boldsymbol{\Sigma}}$ is a function linear of $\dot{\boldsymbol{\Sigma}}$ and nonlinear of $\boldsymbol{\Sigma}$:

$$\overline{\boldsymbol{\Sigma}}=\overline{\boldsymbol{\Sigma}}(\dot{\boldsymbol{\Sigma}},\boldsymbol{\Sigma}) \qquad \text{éq 6.1-5}$$

After discretization in time, $\overline{\boldsymbol{\Sigma}^+}$ will become a nonlinear function of $\boldsymbol{\Sigma}^+$ and $\boldsymbol{\Sigma}^-$:

$$\overline{\boldsymbol{\Sigma}^{+}}=\overline{\boldsymbol{\Sigma}^{+}}\left(\boldsymbol{\Sigma}^{+},\boldsymbol{\Sigma}^{-}\right)$$ **éq 6.1-6**

Let us note finally that for algorithmic reasons (inter alia), one needs to know derivative of the internal forces compared to generalized displacements:

$$\frac{\partial \mathbf{R}}{\partial \mathbf{U}}=\frac{\partial \mathbf{R}}{\partial \overline{\boldsymbol{\Sigma}}}\frac{\partial \overline{\boldsymbol{\Sigma}}}{\partial \boldsymbol{\Sigma}}\frac{\partial \boldsymbol{\Sigma}}{\partial \mathbf{E}}\frac{\partial \mathbf{E}}{\partial \mathbf{U}}=\mathbf{Q}^{T}\frac{\partial \overline{\boldsymbol{\Sigma}}}{\partial \boldsymbol{\Sigma}}\frac{\partial \boldsymbol{\Sigma}}{\partial \boldsymbol{\Sigma}}\mathbf{Q}$$

It is clear that $\frac{\partial \overline{\boldsymbol{\Sigma}}}{\partial \boldsymbol{\Sigma}}$ depends only on the form of the balance equations.

6.3 Constitutive laws

a constitutive law will be simply defined like an unspecified relation between generalized generalized stresses and strains. The local variables are defined like fields necessary to the computation of the stresses, whose evolution is given by the constitutive laws, but which do not intervene directly in the balance equations.

Moreover, we consider that the constitutive laws are written in incremental form and that they are local. By noting $\boldsymbol{\chi}$ the local variables, a constitutive law is thus a relation:

$$\boldsymbol{\Sigma},\boldsymbol{\chi},\dot{\mathbf{E}}\rightarrow\dot{\boldsymbol{\Sigma}},\dot{\boldsymbol{\chi}}$$

After discretization in time, the constitutive law becomes a relation:

$$\boldsymbol{\Sigma}^{-},\boldsymbol{\chi}^{-},\mathbf{E}^{-},\mathbf{E}^{+}\rightarrow\boldsymbol{\Sigma}^{+},\boldsymbol{\chi}^{+}$$

The constitutive law will have to also provide the only term which in the statement $\frac{\partial \mathbf{R}}{\partial \mathbf{U}}=\mathbf{Q}^{T}\frac{\partial \overline{\boldsymbol{\Sigma}}}{\partial \boldsymbol{\Sigma}}\frac{\partial \boldsymbol{\Sigma}}{\partial \mathbf{E}}\mathbf{Q}$ depends on it, namely $\frac{\partial \boldsymbol{\Sigma}}{\partial \mathbf{E}}$. Finally a behavior model is a relation:

$$\boldsymbol{\Sigma}^{-},\boldsymbol{\chi}^{-},\mathbf{E}^{-},\mathbf{E}^{+}\rightarrow\boldsymbol{\Sigma}^{+},\boldsymbol{\chi}^{+},\frac{\partial \boldsymbol{\Sigma}^{+}}{\partial \mathbf{E}^{+}}$$ **éq 6.3-1**

In the following paragraphs, we specify certain aspects of the constitutive laws by distinguishing the mechanical, hydraulic and thermal contributions.

6.3.1 Mechanical constitutive law

6.3.1.1 general Writing

the local variables are noted $\boldsymbol{\chi}$. A constitutive law of mechanics, in frame `THM` is written:

$$\begin{cases}\boldsymbol{\sigma}^{+}=\boldsymbol{\sigma}^{+}\left(\boldsymbol{\varepsilon}^{+},p_1^{+},p_2^{+},T^{+};\boldsymbol{\varepsilon}^{-},p_1^{-},p_2^{-},T^{-},\boldsymbol{\sigma}^{-},\boldsymbol{\chi}^{-}\right)\\ \boldsymbol{\chi}^{+}=\boldsymbol{\chi}^{+}\left(\boldsymbol{\varepsilon}^{+},p_1^{+},p_2^{+},T^{+};\boldsymbol{\varepsilon}^{-},p_1^{-},p_2^{-},T^{-},\boldsymbol{\sigma}^{-},\boldsymbol{\chi}^{-}\right)\end{cases}$$ **éq 6.3.1.1 - 1**

6.3.1.2 Case of the effective stresses

In the case of the assumption of the effective stresses, one has decomposition: $\boldsymbol{\sigma}=\boldsymbol{\sigma}'+\sigma_p\mathbf{I}$ where $\boldsymbol{\sigma}'$ is the tensor of the effective stresses and σ_p is a scalar.

The local variables are separate in two parts: mechanical local variables $\boldsymbol{\chi}_\sigma$ and the hydraulic local variables $\boldsymbol{\chi}_H$. The mechanical constitutive law is divided then into two models, **whose first can be an already existing model in the usual frame of thermomechanics.**

$$\begin{cases} \boldsymbol{\sigma}'^{+}=\boldsymbol{\sigma}'^{+}\left(\boldsymbol{\varepsilon}^{+},T^{+};\boldsymbol{\varepsilon}^{-},T^{-},\boldsymbol{\sigma}'^{-},\boldsymbol{\chi}'^{-}_{\sigma}\right) \\ \boldsymbol{\chi}'^{+}_{\sigma}=\boldsymbol{\sigma}'^{+}_{\sigma}\left(\boldsymbol{\varepsilon}^{+},T^{+};\boldsymbol{\varepsilon}^{-},T^{-},\boldsymbol{\sigma}'^{-},\boldsymbol{\chi}'^{-}_{\sigma}\right) \end{cases}$$ **éq 6.3.1.2 - 1**

$$\begin{cases} \boldsymbol{\sigma}^{+}_{p}=\boldsymbol{\sigma}^{+}_{p}\left(p_1^{+},p_2^{+};p_1^{-},p_2^{-},\boldsymbol{\chi}^{-}_{H}\right) \\ \boldsymbol{\chi}^{+}_{H}=\boldsymbol{\chi}^{+}_{H}\left(p_1^{+},p_2^{+};p_1^{-},p_2^{-},\boldsymbol{\chi}^{-}_{H}\right) \end{cases}$$ **éq 6.3.1.2 - the 2**

dependences indicated by the equations [éq 6.3.1.2 - 1] and [éq 6.3.1.2 - 2] do not have a theoretical justification a priori. It is simply a question of showing the most general possible dependences from the point of view of the data-processing programming. One notices in this decomposition that the dependence compared to the thermal was left in the effective stresses; typically, it is thought that the models on the effective stresses are written as in classical thermomechanics:

$$\boldsymbol{\sigma}'^{+}=\boldsymbol{\sigma}'^{+}\left(\boldsymbol{\varepsilon}^{+}-\alpha^{+}T^{+}\mathbf{I},\boldsymbol{\varepsilon}^{-}-\alpha^{-}T^{-}\mathbf{I},\boldsymbol{\sigma}'^{-},\boldsymbol{\chi}^{-}_{\sigma}\right)$$

6.3.1.3 Choice of the stresses

Because of rather frequent use of the assumption of the effective stresses, one decides that the vector of the stresses for the mechanical part contains in all the cases the tensor of the effective stresses $\boldsymbol{\sigma}'$ and the scalar σ_p . In the general case where the assumption of the effective stresses is not retained, one will have simply: $\sigma_p=0$. It is thus with the load of the modulus of integration of the balance equations (and not of the constitutive laws) to make the sum: $\boldsymbol{\sigma}^{+}=\boldsymbol{\sigma}'^{+}+\sigma_p^{+}\mathbf{I}$.

6.3.2 Hydraulics

the hydraulic constitutive law provides the following relations:

$$\left.\begin{cases} m_c^{p+}=m_c^{p+}\left(\boldsymbol{\varepsilon}^{+},p_{1}^{+},{}_{2}^{+},T^{+};\boldsymbol{\varepsilon}^{-},p_{1}^{-},{}_{2}^{-},T^{-},m_c^{p-},\mathbf{M}_c^{p-},\boldsymbol{\chi}_H^{-}\right) \\ \mathbf{M}_c^{p+}=\mathbf{M}_c^{p+}\begin{pmatrix} \boldsymbol{\varepsilon}^{+},p_1^{+},\nabla p_1^{+},p_2^{+},\nabla p_2^{+},T^{+},\nabla T^{+}; \\ \boldsymbol{\varepsilon}^{-},p_1^{-},\nabla p_1^{-},p_2^{-},\nabla p_2^{-}; \\ T^{-},\nabla T^{-},\mathbf{M}_c^{p-},\boldsymbol{\chi}_H^{-};\mathbf{F}^{m+} \end{pmatrix} \end{cases}\right\} \begin{matrix}\forall c \\ \forall p\end{matrix}$$
$$\boldsymbol{\chi}_H^{+}=\boldsymbol{\chi}_H^{+}\left(\boldsymbol{\varepsilon}^{+},p_{1}^{+},{}_{2}^{+},T^{+};\boldsymbol{\varepsilon}^{-},p_{1}^{-},{}_{2}^{-},T^{-},m_c^{p-},\boldsymbol{\chi}_H^{-}\right)$$ **éq 6.3.2-1**

One notices that the gravity field is a data of the hydraulic constitutive law because the evolution of the flux vector follows relations of the type: $\mathbf{M}=\lambda_H\rho^{fl}\left[-\nabla P+\rho^{fl}\mathbf{F}^m\right]$.

6.3.3 Thermal constitutive law

the constitutive laws give:

$$\begin{cases} Q'^{+}=Q'^{+}\left(\boldsymbol{\varepsilon}^{+},p_1^{+},p_2^{+},T^{+};\boldsymbol{\varepsilon}^{-},p_1^{-},p_2^{-},T^{-},Q'^{-}\right) \\ h_c^{mp+}=h_c^{mp+}\left(\boldsymbol{\varepsilon}^{+},p_1^{+},p_2^{+},T^{+};\boldsymbol{\varepsilon}^{-},p_1^{-},p_2^{-},T^{-},h_c^{mp-}\right) \qquad \forall c \text{ et } \forall p \\ \mathbf{q}^{+}=\mathbf{q}^{+}\left(\boldsymbol{\varepsilon}^{+},p_1^{+},p_2^{+},T^{+},\nabla T^{+};\boldsymbol{\varepsilon}^{-},p_1^{-},p_2^{-},T^{-},\nabla T^{-},\mathbf{q}^{-}\right) \\ \chi_T^{+}=\chi_T^{+}\left(\boldsymbol{\varepsilon}^{+},p_1^{+},p_2^{+},T^{+},\nabla T^{+};\boldsymbol{\varepsilon}^{-},p_1^{-},p_2^{-},T^{-},\nabla T^{-},\boldsymbol{\chi}_T^{-}\right) \end{cases}$$ **éq 6.3.3-1**

Let us note that we introduced possible local variables related to the thermal.

6.3.4 Density homogenized

By definition, the homogenized density, which intervenes in the balance equation of the mechanics [éq 3.1-1] is given by:

$$r^{+}=r_0+m_1^{1+}+m_1^{2+}+m_2^{1+}+m_2^{2+}$$ **éq 6.3.4-1**

This equation is not a constitutive law, but she belongs to the conservation equations. She is integrated in the modulus of computation of the balance equations, the moduli of computation of the constitutive laws not having to treat it.

6.3.5 Operator of mass

This operator is appealable by option MASS_MECA for all the elements concerned with the modelizations containing of the hydromechanics in 3D and in D_PLAN, that is to say: "THM", "THHM", "HHM", "HH2M", "THH2M". It thus makes it possible to treat a dynamic analysis. It understands only terms associated with the degrees of freedom in translation, that is to say respectively: " DX" , "DY" in 2D, and "DX" , "DY" , "DZ" in 3D.

These terms are evaluated in each element before assembly starting from the shape functions of the isoparametric elements $\mathbf{N}$ described in the document [R3.01.00] according to the statement:

$$M_{ij}^{e}=\int_{\Omega_e}\rho\,\mathbf{N_i}\mathbf{N_j}\,d\,\Omega$$

The preceding statement of the command contains the initial homogenized ρ density (corresponding r_0 with in the paragraph [§6.3.4]) entered behind the operand RHO of key word THM_DIFFU DEFI_MATERIAU. Indeed, one judges for the current dynamic applications of short time (of the standard seismic loading) that the modification of the density is sufficiently weak, cf [bib15]. Thus one avoids an update of the mass matrix.

The elementary terms M_{ij}^{e} are supplemented during their assembly by null terms associated with the additional degrees of freedom specific to the coupled modelization. He during results from it their expansion in the assembled mass matrix a shift from degrees of freedom related to possible presence of the components " PRESS1" , "PRESS2" , and " TEMPE" in the modelization.

7 Algorithm of Algorithm

7.1 nonlinear resolution of resolution of the balance equations

In the general case of the modelization (variable coefficients, desaturation, convection) the variational problem presented above [éq 4.1-1] with [éq 4.3-1] is nonlinear compared to the fields of displacement, pressure and temperature. After discretization by finite elements, one obtains a nonlinear matric system. The matrix of the tangent operator contains moreover one treated asymmetric term as tel. One uses in all the cases of modelization nonlinear solver STAT_NON_LINE of *Code_Aster* resting on a method of Newton-Raphson, described in [bib5]. The vectorial functional calculus is introduced:

$$\mathbf{F}(\mathbf{U})=\mathbf{R}(\mathbf{U})-\mathbf{L}^{meca}$$ **éq 7.1-1**

the associated tangent operator is noted: $D\mathbf{F}=\frac{\partial \mathbf{F}}{\partial \mathbf{U}}$.

For the moduli THM, objects of this note, the operator $\mathbf{L}^{meca}$ does not depend on generalized displacements. All the terms depending on generalized displacements were introduced into $\mathbf{R}$, and it is precisely for this reason that displacements are found in the generalized strains. Let us note on this subject the very particular processing of the term $\int_{\Omega} r\,\mathbf{F}^m.\mathbf{v}$ of the equation [éq 4.1-1].

According to [éq 6.3.4-1] $\int_{\Omega} r\,\mathbf{F}^m.\mathbf{v}\,d\,\Omega=\int_{\Omega}\left(r_0+m_1^{1+}+m_1^{2+}+m_2^{1+}+m_2^{2+}\right)\mathbf{F}^m.\mathbf{v}\,d\,\Omega$.

We chose to divide this term into two:
The term $\int_{\Omega} r_0\,\mathbf{F}^m.\mathbf{v}\,d\,\Omega$ is a contribution to $\mathbf{L}^{meca}$ if the user informed operand PESANTEUR of the loading used (definite by the command AFFE_CHAR_MECA), whereas the term $\int_{\Omega}\left(m_1^{1+}+m_1^{2+}+m_2^{1+}+m_2^{2+}\right)\mathbf{F}^m.\mathbf{v}\,d\,\Omega$, which depends on the generalized stresses is a contribution to $\mathbf{R}$.

7.2 Transition of the nodal values to the values with Gauss points

As in all the codes of finite elements, the terms are calculated by loop on the elements and buckles on the points of gauss. By noting $\mathbf{R}_g^{el}$ and the $D\mathbf{F}_g^{el}$ values at the Gauss point g of the élémentdes el nodal forces and the tangent operator, and w_g^{el} the weight of integration related to this Gauss point, one a:

$$\mathbf{R}(\mathbf{U})=\sum_{el}\sum_{g} w_g^{el}\,\mathbf{R}_g^{el}(\mathbf{U})$$ **éq 7.2-1**

$$D\mathbf{F}(\mathbf{U})=\sum_{el}\sum_{g} w_g^{el}\,D\mathbf{F}_g^{el}(\mathbf{U})$$ **éq 7.2-2**

then Notes $\mathbf{U}^{el}$ the vector of the nodal unknowns on a finite element el . One can thus have:

$$\text{par exemple } \mathbf{U}^{el}=\left.\begin{matrix} u\\ v\\ w\\ p_1\\ p_2\\ T \end{matrix}\right\}\text{noeud 1}\quad \left.\begin{matrix} u\\ v\\ w\\ p_1\\ p_2\\ T \end{matrix}\right\}\text{noeud 2}\quad \left.\begin{matrix} u\\ v\\ w\\ p_1\\ p_2\\ T \end{matrix}\right\}\text{noeud 3}.$$

Let us note also $\mathbf{E}_g^{el}$ the vector of the strains generalized at the Gauss point g of the element el and $\boldsymbol{\Sigma}_g^{el}$ the stress vector generalized for the Gauss point g of the element el. In the most complete case one has as follows:

$$\mathbf{E}_g^{el}=\left\{\begin{matrix} \mathbf{u}\\ \boldsymbol{\varepsilon}(u)\\ p_1\\ \nabla p_1\\ p_2\\ \nabla p_2\\ T\\ \nabla T \end{matrix}\right\}_g^{el};\boldsymbol{\Sigma}_g^{el}=\left\{\begin{matrix} \boldsymbol{\sigma}'\\ \boldsymbol{\sigma}_p\\ m_1^1\\ \mathbf{M}_1^1\\ h^{m_1^1}\\ m_1^2\\ \mathbf{M}_1^2\\ h^{m_1^2}\\ m_2^1\\ \mathbf{M}_2^1\\ h^{m_2^1}\\ m_2^2\\ \mathbf{M}_2^2\\ h^{m_2^2}\\ Q'\\ \mathbf{q} \end{matrix}\right\}_g^{el}$$

The shape functions of the finite elements make it possible to then calculate the transition matrix $\mathbf{Q}_g^{el}$ of the nodal unknowns to the strains generalized with Gauss points defined by:

$$\mathbf{E}_g^{el}=\mathbf{Q}_g^{el}.\mathbf{U}_g^{el}$$ **éq 7.2-3**

7.3 Vectors and matrixes according to the options

the presentations of the two following paragraphs are made in the most general case where one has an equation of mechanics, two equations of hydraulics and an equation of thermal. The indices g and el from now on are omitted, but it is clear that what is described applies to each Gauss point of each element.

7.3.1 Residue or nodal force: options RAPH_MECA and FULL_MECA

One distributes the terms of the variational formulation according to the following principle:
If $\mathbf{E}_g^{*el}$ indicates a virtual strain field, $\mathbf{E}_g^{*el}=\left(\mathbf{v},\boldsymbol{\varepsilon}(\mathbf{v}),\pi_1,\nabla\pi_1,\pi_2,\nabla\pi_2,\tau,\nabla\tau\right)$ calculated from a displacement vector nodal virtual $\mathbf{U}^{*el}$, one can define: $\mathbf{E}_g^{*el\,T}.\boldsymbol{\Sigma}_g^{el}(\mathbf{U})=\overline{\Sigma}_1\mathbf{v}+\overline{\Sigma}_2(\mathbf{v})+\overline{\Sigma}_3\pi_1+\overline{\Sigma}_4\nabla\pi_1+\overline{\Sigma}_5\pi_2+\overline{\Sigma}_6\nabla\pi_2+\overline{\Sigma}_7\tau+\overline{\Sigma}_8\nabla\tau$. The discrete variational formulations then are taken again [éq 5.1-1], [éq 5.2-1], [éq 5.3-1], and one replaces there the integrals $\int_\Omega f\,d\Omega$ by $\sum_{el}\sum_g w_g^{el}f_g^{el}$ for all the integral ones f. One distinguishes the terms multiplying respectively $\mathbf{v}$ $\boldsymbol{\varepsilon}(\mathbf{v})$ π_1 $\nabla\pi_1$ π_2 $\nabla\pi_2$, τ and $\nabla\tau$, and one finds:

Index	$\overline{\Sigma}$	associated with
1	$-\left(m_1^{1+}+m_1^{2+}+m_2^{1+}+m_2^{2+}\right)\mathbf{F}^{m+}$	$\mathbf{v}$
2	$\boldsymbol{\sigma}'^{+}+\sigma_p^{+}\mathbf{I}$	$\boldsymbol{\varepsilon}(\mathbf{v})$
3	$-m_1^{1+}-m_1^{2+}+m_1^{1-}+m_1^{2-}$	π_1
4	$\theta\Delta t\left(\mathbf{M}_1^{1+}+\mathbf{M}_1^{2+}\right)+(1-\theta)\Delta t\left(\mathbf{M}_1^{1-}+\mathbf{M}_1^{2-}\right)$	$\nabla\pi_1$
5	$-m_2^{1+}-m_2^{2+}+m_2^{1-}+m_2^{2-}$	π_2
6	$\theta\Delta t\left(\mathbf{M}_2^{1+}+\mathbf{M}_2^{2+}\right)+(1-\theta)\Delta t\left(\mathbf{M}_2^{1-}+\mathbf{M}_2^{2-}\right)$	$\nabla\pi_2$
7	$-Q'^{+}+Q'^{-}$ $-\left(\theta h_1^{m1+}+(1-\theta)h_1^{m1-}\right)\left(m_1^{1+}-m_1^{1-}\right)-\left(\theta h_1^{m2+}+(1-\theta)h_1^{m2-}\right)\left(m_1^{2+}-m_1^{2-}\right)$ $-\left(\theta h_2^{m1+}+(1-\theta)h_2^{m1-}\right)\left(m_2^{1+}-m_2^{1-}\right)-\left(\theta h_2^{m2+}+(1-\theta)h_2^{m2-}\right)\left(m_2^{2+}-m_2^{2-}\right)$ $+\Delta t\,\theta\left(\mathbf{M}_1^{1+}+\mathbf{M}_1^{2+}+\mathbf{M}_2^{1+}+\mathbf{M}_2^{2+}\right).\mathbf{F}^m+\Delta t(1-\theta)\left(\mathbf{M}_1^{1-}+\mathbf{M}_1^{2-}+\mathbf{M}_2^{1-}+\mathbf{M}_2^{2-}\right).\mathbf{F}^m$	τ
8	$+\theta\Delta t\left(h_1^{m1+}\mathbf{M}_1^{1+}+h_1^{m2+}\mathbf{M}_1^{2+}+h_2^{m1+}\mathbf{M}_2^{1+}+h_2^{m2+}\mathbf{M}_2^{2+}+\mathbf{q}^{+}\right)+$ $+(1-\theta)\Delta t\left(h_1^{m1-}\mathbf{M}_1^{1-}+h_1^{m2-}\mathbf{M}_1^{2-}+h_2^{m1-}\mathbf{M}_2^{1-}+h_2^{m2-}\mathbf{M}_2^{2-}+\mathbf{q}^{-}\right)$	$\nabla\tau$

Note::

In the first term $\overline{\boldsymbol{\Sigma}}_1$ the term does not appear $-r_0\mathbf{F}^m$ because it is put in the loading external $\mathbf{L}^{meca}$ and calculated by the computation option of the loading external of gravity.

By means of the definition [éq 7.2-1] of $\mathbf{R}_g^{el}$, one a:

$\mathbf{U}^{*el^T}.\mathbf{R}_g^{el}=\mathbf{E}_g^{*el^T}.\boldsymbol{\Sigma}_g^{el}$, which still gives us:

$$\mathbf{R}_g^{el}=\mathbf{Q}_g^{el^T}.\overline{\boldsymbol{\Sigma}}_g^{el}$$

This last equality is only the local form on the level of a Gauss point of [éq 6.2-2].

7.3.2 Tangent operator: options FULL_MECA, RIGI_MECA_TANG

In what follows, if $\mathbf{X}$ indicates a vector of components X^i and $\mathbf{Y}$ a vector of components Y^j, $\left[\frac{\partial X}{\partial Y}\right]$ will indicate a matrix whose occupying element line i and the column j are $\frac{\partial X^i}{\partial Y^j}$.

To compute: the tangent operator, one will calculate the following quantities:

$[\mathbf{DRDE}]=$

DR1U	DR1E	DR1P1	DR1GP1	DR1P2	DR1GP2	DR1T	DR1GT
DR2U	DR2E	DR2P1	DR2GP1	DR2P2	DR2GP2	DR2T	DR2GT
DR3U	DR3E	DR3P1	DR3GP1	DR3P2	DR3GP2	DR3T	DR3GT
DR4U	DR4E	DR4P1	DR4GP1	DR4P2	DR4GP2	DR4T	DR4GT
DR5U	DR5E	DR5P1	DR5GP1	DR5P2	DR5GP2	DR5T	DR5GT
DR6U	DR6E	DR6P1	DR6GP1	DR6P2	DR6GP2	DR6T	DR6GT
DR7U	DR7E	DR7P1	DR7GP1	DR7P2	DR7GP2	DR7T	DR7GT
DR8U	DR8E	DR8P1	DR8GP1	DR8P2	DR8GP2	DR8T	DR8GT

Where one noted:

$$DRiU=\frac{\partial \mathbf{F}_i}{\partial \mathbf{u}} \qquad DRiGP1=\frac{\partial \mathbf{F}_i}{\partial \nabla p_1}$$

$$DRiE=\frac{\partial \mathbf{F}_i}{\partial \boldsymbol{\varepsilon}} \qquad DRiGP2=\frac{\partial \mathbf{F}_i}{\partial \nabla p_2}$$

$$DRiP1=\frac{\partial \mathbf{F}_i}{\partial p_1} \qquad DRiT=\frac{\partial \mathbf{F}_i}{\partial T}$$

$$DRiP2=\frac{\partial \mathbf{F}_i}{\partial p_2} \qquad DRiDT=\frac{\partial \mathbf{F}_i}{\partial \nabla T}$$

To do these calculations one considers that the constitutive laws provide, for the corresponding options, all the following derivatives:

$$\mathbf{D}\boldsymbol{\Sigma}\,\mathbf{DE}=\begin{bmatrix}
\frac{\partial \boldsymbol{\sigma}'}{\mathbf{u}} & \frac{\partial \boldsymbol{\sigma}'}{\partial \boldsymbol{\varepsilon}} & \frac{\partial \boldsymbol{\sigma}'}{\partial p_1} & \frac{\partial \boldsymbol{\sigma}'}{\partial \nabla p_1} & \frac{\partial \boldsymbol{\sigma}'}{\partial p_2} & \frac{\partial \boldsymbol{\sigma}'}{\partial \nabla p_2} & \frac{\partial \boldsymbol{\sigma}'}{\partial T} & \frac{\partial \boldsymbol{\sigma}'}{\partial \nabla T} \\
\frac{\partial \sigma_p}{\partial \mathbf{u}} & \frac{\partial \sigma_p}{\partial \boldsymbol{\varepsilon}} & \frac{\partial \sigma_p}{\partial p_1} & \frac{\partial \sigma_p}{\partial \nabla p_1} & \frac{\partial \sigma_p}{\partial p_2} & \frac{\partial \sigma_p}{\partial \nabla p_2} & \frac{\partial \sigma_p}{\partial T} & \frac{\partial \sigma_p}{\partial \nabla T} \\
\frac{\partial m_1^1}{\partial \mathbf{u}} & \frac{\partial m_1^1}{\partial \boldsymbol{\varepsilon}} & \frac{\partial m_1^1}{\partial p_1} & \frac{\partial m_1^1}{\partial \nabla p_1} & \frac{\partial m_1^1}{\partial p_2} & \frac{\partial m_1^1}{\partial \nabla p_2} & \frac{\partial m_1^1}{\partial T} & \frac{\partial m_1^1}{\partial \nabla T} \\
\frac{\partial \mathbf{M}_1^1}{\partial \mathbf{u}} & \frac{\partial \mathbf{M}_1^1}{\partial \boldsymbol{\varepsilon}} & \frac{\partial \mathbf{M}_1^1}{\partial p_1} & \frac{\partial \mathbf{M}_1^1}{\partial \nabla p_1} & \frac{\partial \mathbf{M}_1^1}{\partial p_2} & \frac{\partial \mathbf{M}_1^1}{\partial \nabla p_2} & \frac{\partial \mathbf{M}_1^1}{\partial T} & \frac{\partial \mathbf{M}_1^1}{\partial \nabla T} \\
\frac{\partial h^{m_1^1}}{\partial \mathbf{u}} & \frac{\partial h^{m_1^1}}{\partial \boldsymbol{\varepsilon}} & \frac{\partial h^{m_1^1}}{\partial p_1} & \frac{\partial h^{m_1^1}}{\partial \nabla p_1} & \frac{\partial h^{m_1^1}}{\partial p_2} & \frac{\partial h^{m_1^1}}{\partial \nabla p_2} & \frac{\partial h^{m_1^1}}{\partial T} & \frac{\partial h^{m_1^1}}{\partial \nabla T} \\
\frac{\partial m_1^2}{\partial \mathbf{u}} & \frac{\partial m_1^2}{\partial \boldsymbol{\varepsilon}} & \frac{\partial m_1^2}{\partial p_1} & \frac{\partial m_1^2}{\partial \nabla p_1} & \frac{\partial m_1^2}{\partial p_2} & \frac{\partial m_1^2}{\partial \nabla p_2} & \frac{\partial m_1^2}{\partial T} & \frac{\partial m_1^2}{\partial \nabla T} \\
\frac{\partial \mathbf{M}_1^2}{\partial \mathbf{u}} & \frac{\partial \mathbf{M}_1^2}{\partial \boldsymbol{\varepsilon}} & \frac{\partial \mathbf{M}_1^2}{\partial p_1} & \frac{\partial \mathbf{M}_1^2}{\partial \nabla p_1} & \frac{\partial \mathbf{M}_1^2}{\partial p_2} & \frac{\partial \mathbf{M}_1^2}{\partial \nabla p_2} & \frac{\partial \mathbf{M}_1^2}{\partial T} & \frac{\partial \mathbf{M}_1^2}{\partial \nabla T} \\
\frac{\partial h^{m_1^2}}{\partial \mathbf{u}} & \frac{\partial h^{m_1^2}}{\partial \boldsymbol{\varepsilon}} & \frac{\partial h^{m_1^2}}{\partial p_1} & \frac{\partial h^{m_1^2}}{\partial \nabla p_1} & \frac{\partial h^{m_1^2}}{\partial p_2} & \frac{\partial h^{m_1^2}}{\partial \nabla p_2} & \frac{\partial h^{m_1^2}}{\partial T} & \frac{\partial h^{m_1^2}}{\partial \nabla T} \\
\frac{\partial m_2^1}{\partial \mathbf{u}} & \frac{\partial m_2^1}{\partial \boldsymbol{\varepsilon}} & \frac{\partial m_2^1}{\partial p_1} & \frac{\partial m_2^1}{\partial \nabla p_1} & \frac{\partial m_2^1}{\partial p_2} & \frac{\partial m_2^1}{\partial \nabla p_2} & \frac{\partial m_2^1}{\partial T} & \frac{\partial m_2^1}{\partial \nabla T} \\
\frac{\partial \mathbf{M}_2^1}{\partial \mathbf{u}} & \frac{\partial \mathbf{M}_2^1}{\partial \boldsymbol{\varepsilon}} & \frac{\partial \mathbf{M}_2^1}{\partial p_1} & \frac{\partial \mathbf{M}_2^1}{\partial \nabla p_1} & \frac{\partial \mathbf{M}_2^1}{\partial p_2} & \frac{\partial \mathbf{M}_2^1}{\partial \nabla p_2} & \frac{\partial \mathbf{M}_2^1}{\partial T} & \frac{\partial \mathbf{M}_2^1}{\partial \nabla T} \\
\frac{\partial h^{m_2^1}}{\partial \mathbf{u}} & \frac{\partial h^{m_2^1}}{\partial \boldsymbol{\varepsilon}} & \frac{\partial h^{m_2^1}}{\partial p_1} & \frac{\partial h^{m_2^1}}{\partial \nabla p_1} & \frac{\partial h^{m_2^1}}{\partial p_2} & \frac{\partial h^{m_2^1}}{\partial \nabla p_2} & \frac{\partial h^{m_2^1}}{\partial T} & \frac{\partial h^{m_2^1}}{\partial \nabla T} \\
\frac{\partial m_2^2}{\partial \mathbf{u}} & \frac{\partial m_2^2}{\partial \boldsymbol{\varepsilon}} & \frac{\partial m_2^2}{\partial p_1} & \frac{\partial m_2^2}{\partial \nabla p_1} & \frac{\partial m_2^2}{\partial p_2} & \frac{\partial m_2^2}{\partial \nabla p_2} & \frac{\partial m_2^2}{\partial T} & \frac{\partial m_2^2}{\partial \nabla T} \\
\frac{\partial \mathbf{M}_2^2}{\partial \mathbf{u}} & \frac{\partial \mathbf{M}_2^2}{\partial \boldsymbol{\varepsilon}} & \frac{\partial \mathbf{M}_2^2}{\partial p_1} & \frac{\partial \mathbf{M}_2^2}{\partial \nabla p_1} & \frac{\partial \mathbf{M}_2^2}{\partial p_2} & \frac{\partial \mathbf{M}_2^2}{\partial \nabla p_2} & \frac{\partial \mathbf{M}_2^2}{\partial T} & \frac{\partial \mathbf{M}_2^2}{\partial \nabla T} \\
\frac{\partial h^{m_2^2}}{\partial \mathbf{u}} & \frac{\partial h^{m_2^2}}{\partial \boldsymbol{\varepsilon}} & \frac{\partial h^{m_2^2}}{\partial p_1} & \frac{\partial h^{m_2^2}}{\partial \nabla p_1} & \frac{\partial h^{m_2^2}}{\partial p_2} & \frac{\partial h^{m_2^2}}{\partial \nabla p_2} & \frac{\partial h^{m_2^2}}{\partial T} & \frac{\partial h^{m_2^2}}{\partial \nabla T} \\
\frac{\partial Q'}{\partial \mathbf{u}} & \frac{\partial Q'}{\partial \boldsymbol{\varepsilon}} & \frac{\partial Q'}{\partial p_1} & \frac{\partial Q'}{\partial \nabla p_1} & \frac{\partial Q'}{\partial p_2} & \frac{\partial Q'}{\partial \nabla p_2} & \frac{\partial Q'}{\partial T} & \frac{\partial Q'}{\partial \nabla T} \\
\frac{\partial \mathbf{q}}{\partial \mathbf{u}} & \frac{\partial \mathbf{q}}{\partial \boldsymbol{\varepsilon}} & \frac{\partial \mathbf{q}}{\partial p_1} & \frac{\partial \mathbf{q}}{\partial \nabla p_1} & \frac{\partial \mathbf{q}}{\partial p_2} & \frac{\partial \mathbf{q}}{\partial \nabla p_2} & \frac{\partial \mathbf{q}}{\partial T} & \frac{\partial \mathbf{q}}{\partial \nabla T}
\end{bmatrix}$$

Note:

In these statements, the derivatives all compared to $\mathbf{u}$ *are null, but we keep the writing taking into account the definition of the matrixes* $\mathbf{Q}_g^{el}$ *which we adopted.*

The call to the constitutive laws will provide the pieces of the matrix $D\Sigma\mathbf{DE}$ according to the equations present:

$$[\mathbf{DMECDE}]=\begin{bmatrix}\frac{\partial\boldsymbol{\sigma}'}{\partial\boldsymbol{\varepsilon}}\\\frac{\partial\sigma_p}{\partial\varepsilon}\end{bmatrix};[\mathbf{DMECP1}]=\begin{bmatrix}\frac{\partial\boldsymbol{\sigma}'}{\partial p_1}&\frac{\partial\boldsymbol{\sigma}'}{\partial\nabla p_1}\\\frac{\partial\sigma_p}{\partial p_1}&\frac{\partial\sigma_p}{\partial\nabla p_1}\end{bmatrix};[\mathbf{DMECP}2]=\begin{bmatrix}\frac{\partial\boldsymbol{\sigma}'}{\partial p_2}&\frac{\partial\boldsymbol{\sigma}'}{\partial\nabla p_2}\\\frac{\partial\sigma_p}{\partial p_2}&\frac{\partial\sigma_p}{\partial\nabla p_2}\end{bmatrix}[\mathbf{DMECDT}]=\begin{bmatrix}\frac{\partial\boldsymbol{\sigma}}{\partial T}&\frac{\partial\boldsymbol{\sigma}}{\partial\nabla T}\\\frac{\partial\sigma_p}{\partial T}&\frac{\partial\sigma_p}{\partial\nabla T}\end{bmatrix}$$

$$[\mathbf{DP11DE}]=\begin{bmatrix}\frac{\partial m_1^1}{\partial\boldsymbol{\varepsilon}}\\\frac{\partial\mathbf{M}_1^1}{\partial\boldsymbol{\varepsilon}}\\\frac{\partial h^{m_1^1}}{\partial\boldsymbol{\varepsilon}}\end{bmatrix};[\mathbf{DP11P1}]=\begin{bmatrix}\frac{\partial m_1^1}{\partial p_1}&\frac{\partial m_1^1}{\partial\nabla p_1}\\\frac{\partial\mathbf{M}_1^1}{\partial p_1}&\frac{\partial\mathbf{M}_1^1}{\partial\nabla p_1}\\\frac{\partial h^{m_1^1}}{\partial p_1}&\frac{\partial h^{m_1^1}}{\partial\nabla p_1}\end{bmatrix};[\mathbf{DP11P2}]=\begin{bmatrix}\frac{\partial m_1^1}{\partial p_2}&\frac{\partial m_1^1}{\partial\nabla p_2}\\\frac{\partial\mathbf{M}_1^1}{\partial p_2}&\frac{\partial\mathbf{M}_1^1}{\partial\nabla p_2}\\\frac{\partial h^{m_1^1}}{\partial p_2}&\frac{\partial h^{m_1^1}}{\partial\nabla p_2}\end{bmatrix}[\mathbf{DP11DT}]=\begin{bmatrix}\frac{\partial m_1^1}{\partial T}&\frac{\partial m_1^1}{\partial\nabla T}\\\frac{\partial\mathbf{M}_1^1}{\partial T}&\frac{\partial\mathbf{M}_1^1}{\partial\nabla T}\\\frac{\partial h^{m_1^1}}{\partial T}&\frac{\partial h^{m_1^1}}{\partial\nabla T}\end{bmatrix}$$

$$[\mathbf{DP12DE}]=\begin{bmatrix}\frac{\partial m_1^2}{\partial\boldsymbol{\varepsilon}}\\\frac{\partial\mathbf{M}_1^2}{\partial\boldsymbol{\varepsilon}}\\\frac{\partial h^{m_1^2}}{\partial\boldsymbol{\varepsilon}}\end{bmatrix};[\mathbf{DP12P1}]=\begin{bmatrix}\frac{\partial m_1^2}{\partial p_1}&\frac{\partial m_1^2}{\partial\nabla p_1}\\\frac{\partial\mathbf{M}_1^2}{\partial p_1}&\frac{\partial M_1^2}{\partial\nabla p_1}\\\frac{\partial h^{m_1^2}}{\partial p_1}&\frac{\partial h^{m_1^2}}{\partial\nabla p_1}\end{bmatrix};[\mathbf{DP12P2}]=\begin{bmatrix}\frac{\partial m_1^2}{\partial p_2}&\frac{\partial m_1^2}{\partial\nabla p_2}\\\frac{\partial\mathbf{M}_1^2}{\partial p_2}&\frac{\partial\mathbf{M}_1^2}{\partial\nabla p_2}\\\frac{\partial h^{m_1^2}}{\partial p_2}&\frac{\partial h^{m_1^2}}{\partial\nabla p_2}\end{bmatrix}[\mathbf{DP12DT}]=\begin{bmatrix}\frac{\partial m_1^2}{\partial T}&\frac{\partial m_1^2}{\partial\nabla T}\\\frac{\partial\mathbf{M}_1^2}{\partial T}&\frac{\partial\mathbf{M}_1^2}{\partial\nabla T}\\\frac{\partial h^{m_1^2}}{\partial T}&\frac{\partial h^{m_1^2}}{\partial\nabla T}\end{bmatrix}$$

$$[\mathbf{DP21DE}]=\begin{bmatrix}\frac{\partial m_2^1}{\partial\boldsymbol{\varepsilon}}\\\frac{\partial\mathbf{M}_2^1}{\partial\boldsymbol{\varepsilon}}\\\frac{\partial h^{m_2^1}}{\partial\boldsymbol{\varepsilon}}\end{bmatrix};[\mathbf{DP21P1}]=\begin{bmatrix}\frac{\partial m_2^1}{\partial p_1}&\frac{\partial m_2^1}{\partial\nabla p_1}\\\frac{\partial\mathbf{M}_2^1}{\partial p_1}&\frac{\partial\mathbf{M}_2^1}{\partial\nabla p_1}\\\frac{\partial h^{m_2^1}}{\partial p_1}&\frac{\partial h^{m_2^1}}{\partial\nabla p_1}\end{bmatrix};[\mathbf{DP21P2}]=\begin{bmatrix}\frac{\partial m_2^1}{\partial p_2}&\frac{\partial m_2^1}{\partial\nabla p_2}\\\frac{\partial\mathbf{M}_2^1}{\partial p_2}&\frac{\partial\mathbf{M}_2^1}{\partial\nabla p_2}\\\frac{\partial h^{m_2^1}}{\partial p_2}&\frac{\partial h^{m_2^1}}{\partial\nabla p_2}\end{bmatrix}[\mathbf{DP21DT}]=\begin{bmatrix}\frac{\partial m_2^1}{\partial T}&\frac{\partial m_2^1}{\partial\nabla T}\\\frac{\partial\mathbf{M}_2^1}{\partial T}&\frac{\partial\mathbf{M}_2^1}{\partial\nabla T}\\\frac{\partial h^{m_2^1}}{\partial T}&\frac{\partial h^{m_2^1}}{\partial\nabla T}\end{bmatrix}$$

$$[\mathbf{DP22DE}]=\begin{bmatrix}\frac{\partial m_2^2}{\partial\boldsymbol{\varepsilon}}\\\frac{\partial\mathbf{M}_2^2}{\partial\boldsymbol{\varepsilon}}\\\frac{\partial h^{m_2^2}}{\partial\boldsymbol{\varepsilon}}\end{bmatrix};[\mathbf{DP22P1}]=\begin{bmatrix}\frac{\partial m_2^2}{\partial p_1}&\frac{\partial m_2^2}{\partial\nabla p_1}\\\frac{\partial\mathbf{M}_2^2}{\partial p_1}&\frac{\partial\mathbf{M}_2^2}{\partial\nabla p_1}\\\frac{\partial h^{m_2^2}}{\partial p_1}&\frac{\partial h^{m_2^2}}{\partial\nabla p_1}\end{bmatrix};[\mathbf{DP22P2}]=\begin{bmatrix}\frac{\partial m_2^2}{\partial p_2}&\frac{\partial m_2^2}{\partial\nabla p_2}\\\frac{\partial\mathbf{M}_2^2}{\partial p_2}&\frac{\partial\mathbf{M}_2^2}{\partial\nabla p_2}\\\frac{\partial h^{m_2^2}}{\partial p_2}&\frac{\partial h^{m_2^2}}{\partial\nabla p_2}\end{bmatrix}[\mathbf{DP22DT}]=\begin{bmatrix}\frac{\partial m_2^2}{\partial T}&\frac{\partial m_2^2}{\partial\nabla T}\\\frac{\partial\mathbf{M}_2^2}{\partial T}&\frac{\partial\mathbf{M}_2^2}{\partial\nabla T}\\\frac{\partial h^{m_2^2}}{\partial T}&\frac{\partial h^{m_2^2}}{\partial\nabla T}\end{bmatrix}$$

$$[\mathbf{DTDE}]=\begin{bmatrix}\frac{\partial Q'}{\partial\boldsymbol{\varepsilon}}\\\frac{\partial\mathbf{q}}{\partial\boldsymbol{\varepsilon}}\end{bmatrix};[\mathbf{DTDP1}]=\begin{bmatrix}\frac{\partial Q'}{\partial p_1}&\frac{\partial Q'}{\partial\nabla p_1}\\\frac{\partial\mathbf{q}}{\partial p_1}&\frac{\partial\mathbf{q}}{\partial\nabla p_1}\end{bmatrix};[\mathbf{DTDP2}]=\begin{bmatrix}\frac{\partial Q'}{\partial p_2}&\frac{\partial Q'}{\partial\nabla p_2}\\\frac{\partial\mathbf{q}}{\partial p_2}&\frac{\partial\mathbf{q}}{\partial\nabla p_2}\end{bmatrix}[\mathbf{DTDT}]=\begin{bmatrix}\frac{\partial Q'}{\partial T}&\frac{\partial Q'}{\partial T}\\\frac{\partial\mathbf{q}}{\partial T}&\frac{\partial\mathbf{q}}{\partial T}\end{bmatrix}$$

In addition, by deriving the statement from the residue compared to the stresses, one defines:

$$\mathbf{D}\overline{\boldsymbol{\Sigma}}\,\mathbf{D}\boldsymbol{\Sigma}=\begin{bmatrix}
\frac{\partial\overline{\Sigma}_1}{\partial\boldsymbol{\sigma}'} & \frac{\partial\overline{\Sigma}_1}{\partial\sigma_p} & \frac{\partial\overline{\Sigma}_1}{\partial m_1^1} & \frac{\partial\overline{\Sigma}_1}{\partial\mathbf{M}_1^1} & \frac{\partial\overline{\Sigma}_1}{\partial h^{m_1^1}} & \frac{\partial\overline{\Sigma}_1}{\partial m_1^2} & \frac{\partial\overline{\Sigma}_1}{\partial\mathbf{M}_1^2} & \frac{\partial\overline{\Sigma}_1}{\partial h^{m_1^2}} & \frac{\partial\overline{\Sigma}_1}{\partial m_2^1} & \frac{\partial\overline{\Sigma}_1}{\partial\mathbf{M}_2^1} & \frac{\partial\overline{\Sigma}_1}{\partial h^{m_2^1}} & \frac{\partial\overline{\Sigma}_1}{\partial m_2^2} & \frac{\partial\overline{\Sigma}_1}{\partial\mathbf{M}_2^2} & \frac{\partial\overline{\Sigma}_1}{\partial h^{m_2^2}} & \frac{\partial\overline{\Sigma}_1}{\partial Q'} & \frac{\partial\overline{\Sigma}_1}{\partial\mathbf{q}} \\
\frac{\partial\overline{\Sigma}_2}{\partial\boldsymbol{\sigma}'} & \frac{\partial\overline{\Sigma}_2}{\partial\sigma_p} & \frac{\partial\overline{\Sigma}_2}{\partial m_1^1} & \frac{\partial\overline{\Sigma}_2}{\partial\mathbf{M}_1^1} & \frac{\partial\overline{\Sigma}_2}{\partial h^{m_1^1}} & \frac{\partial\overline{\Sigma}_2}{\partial m_1^2} & \frac{\partial\overline{\Sigma}_2}{\partial\mathbf{M}_1^2} & \frac{\partial\overline{\Sigma}_2}{\partial h^{m_1^2}} & \frac{\partial\overline{\Sigma}_2}{\partial m_2^1} & \frac{\partial\overline{\Sigma}_2}{\partial\mathbf{M}_2^1} & \frac{\partial\overline{\Sigma}_2}{\partial h^{m_2^1}} & \frac{\partial\overline{\Sigma}_2}{\partial m_2^2} & \frac{\partial\overline{\Sigma}_2}{\partial\mathbf{M}_2^2} & \frac{\partial\overline{\Sigma}_2}{\partial h^{m_2^2}} & \frac{\partial\overline{\Sigma}_2}{\partial Q'} & \frac{\partial\overline{\Sigma}_2}{\partial\mathbf{q}} \\
\frac{\partial\overline{\Sigma}_3}{\partial\boldsymbol{\sigma}'} & \frac{\partial\overline{\Sigma}_3}{\partial\sigma_p} & \frac{\partial\overline{\Sigma}_3}{\partial m_1^1} & \frac{\partial\overline{\Sigma}_3}{\partial\mathbf{M}_1^1} & \frac{\partial\overline{\Sigma}_3}{\partial h^{m_1^1}} & \frac{\partial\overline{\Sigma}_3}{\partial m_1^2} & \frac{\partial\overline{\Sigma}_3}{\partial\mathbf{M}_1^2} & \frac{\partial\overline{\Sigma}_3}{\partial h^{m_1^2}} & \frac{\partial\overline{\Sigma}_3}{\partial m_2^1} & \frac{\partial\overline{\Sigma}_3}{\partial\mathbf{M}_2^1} & \frac{\partial\overline{\Sigma}_3}{\partial h^{m_2^1}} & \frac{\partial\overline{\Sigma}_3}{\partial m_2^2} & \frac{\partial\overline{\Sigma}_3}{\partial\mathbf{M}_2^2} & \frac{\partial\overline{\Sigma}_3}{\partial h^{m_2^2}} & \frac{\partial\overline{\Sigma}_3}{\partial Q'} & \frac{\partial\overline{\Sigma}_3}{\partial\mathbf{q}} \\
\frac{\partial\overline{\Sigma}_4}{\partial\boldsymbol{\sigma}'} & \frac{\partial\overline{\Sigma}_4}{\partial\sigma_p} & \frac{\partial\overline{\Sigma}_4}{\partial m_1^1} & \frac{\partial\overline{\Sigma}_4}{\partial\mathbf{M}_1^1} & \frac{\partial\overline{\Sigma}_4}{\partial h^{m_1^1}} & \frac{\partial\overline{\Sigma}_4}{\partial m_1^2} & \frac{\partial\overline{\Sigma}_4}{\partial\mathbf{M}_1^2} & \frac{\partial\overline{\Sigma}_4}{\partial h^{m_1^2}} & \frac{\partial\overline{\Sigma}_4}{\partial m_2^1} & \frac{\partial\overline{\Sigma}_4}{\partial\mathbf{M}_2^1} & \frac{\partial\overline{\Sigma}_4}{\partial h^{m_2^1}} & \frac{\partial\overline{\Sigma}_4}{\partial m_2^2} & \frac{\partial\overline{\Sigma}_4}{\partial\mathbf{M}_2^2} & \frac{\partial\overline{\Sigma}_4}{\partial h^{m_2^2}} & \frac{\partial\overline{\Sigma}_4}{\partial Q'} & \frac{\partial\overline{\Sigma}_4}{\partial\mathbf{q}} \\
\frac{\partial\overline{\Sigma}_5}{\partial\boldsymbol{\sigma}'} & \frac{\partial\overline{\Sigma}_5}{\partial\sigma_p} & \frac{\partial\overline{\Sigma}_5}{\partial m_1^1} & \frac{\partial\overline{\Sigma}_5}{\partial\mathbf{M}_1^1} & \frac{\partial\overline{\Sigma}_5}{\partial h^{m_1^1}} & \frac{\partial\overline{\Sigma}_5}{\partial m_1^2} & \frac{\partial\overline{\Sigma}_5}{\partial\mathbf{M}_1^2} & \frac{\partial\overline{\Sigma}_5}{\partial h^{m_1^2}} & \frac{\partial\overline{\Sigma}_5}{\partial m_2^1} & \frac{\partial\overline{\Sigma}_5}{\partial\mathbf{M}_2^1} & \frac{\partial\overline{\Sigma}_5}{\partial h^{m_2^1}} & \frac{\partial\overline{\Sigma}_5}{\partial m_2^2} & \frac{\partial\overline{\Sigma}_5}{\partial\mathbf{M}_2^2} & \frac{\partial\overline{\Sigma}_5}{\partial h^{m_2^2}} & \frac{\partial\overline{\Sigma}_5}{\partial Q'} & \frac{\partial\overline{\Sigma}_5}{\partial\mathbf{q}} \\
\frac{\partial\overline{\Sigma}_6}{\partial\boldsymbol{\sigma}'} & \frac{\partial\overline{\Sigma}_6}{\partial\sigma_p} & \frac{\partial\overline{\Sigma}_6}{\partial m_1^1} & \frac{\partial\overline{\Sigma}_6}{\partial\mathbf{M}_1^1} & \frac{\partial\overline{\Sigma}_6}{\partial h^{m_1^1}} & \frac{\partial\overline{\Sigma}_6}{\partial m_1^2} & \frac{\partial\overline{\Sigma}_6}{\partial\mathbf{M}_1^2} & \frac{\partial\overline{\Sigma}_6}{\partial h^{m_1^2}} & \frac{\partial\overline{\Sigma}_6}{\partial m_2^1} & \frac{\partial\overline{\Sigma}_6}{\partial\mathbf{M}_2^1} & \frac{\partial\overline{\Sigma}_6}{\partial h^{m_2^1}} & \frac{\partial\overline{\Sigma}_6}{\partial m_2^2} & \frac{\partial\overline{\Sigma}_6}{\partial\mathbf{M}_2^2} & \frac{\partial\overline{\Sigma}_6}{\partial h^{m_2^2}} & \frac{\partial\overline{\Sigma}_6}{\partial Q'} & \frac{\partial\overline{\Sigma}_6}{\partial\mathbf{q}} \\
\frac{\partial\overline{\Sigma}_7}{\partial\boldsymbol{\sigma}'} & \frac{\partial\overline{\Sigma}_7}{\partial\sigma_p} & \frac{\partial\overline{\Sigma}_7}{\partial m_1^1} & \frac{\partial\overline{\Sigma}_7}{\partial\mathbf{M}_1^1} & \frac{\partial\overline{\Sigma}_7}{\partial h^{m_1^1}} & \frac{\partial\overline{\Sigma}_7}{\partial m_1^2} & \frac{\partial\overline{\Sigma}_7}{\partial\mathbf{M}_1^2} & \frac{\partial\overline{\Sigma}_7}{\partial h^{m_1^2}} & \frac{\partial\overline{\Sigma}_7}{\partial m_2^1} & \frac{\partial\overline{\Sigma}_7}{\partial\mathbf{M}_2^1} & \frac{\partial\overline{\Sigma}_7}{\partial h^{m_2^1}} & \frac{\partial\overline{\Sigma}_7}{\partial m_2^2} & \frac{\partial\overline{\Sigma}_7}{\partial\mathbf{M}_2^2} & \frac{\partial\overline{\Sigma}_7}{\partial h^{m_2^2}} & \frac{\partial\overline{\Sigma}_7}{\partial Q'} & \frac{\partial\overline{\Sigma}_7}{\partial\mathbf{q}} \\
\frac{\partial\overline{\Sigma}_8}{\partial\boldsymbol{\sigma}'} & \frac{\partial\overline{\Sigma}_8}{\partial\sigma_p} & \frac{\partial\overline{\Sigma}_8}{\partial m_1^1} & \frac{\partial\overline{\Sigma}_8}{\partial\mathbf{M}_1^1} & \frac{\partial\overline{\Sigma}_8}{\partial h^{m_1^1}} & \frac{\partial\overline{\Sigma}_8}{\partial m_1^2} & \frac{\partial\overline{\Sigma}_8}{\partial\mathbf{M}_1^2} & \frac{\partial\overline{\Sigma}_8}{\partial h^{m_1^2}} & \frac{\partial\overline{\Sigma}_8}{\partial m_2^1} & \frac{\partial\overline{\Sigma}_8}{\partial\mathbf{M}_2^1} & \frac{\partial\overline{\Sigma}_8}{\partial h^{m_2^1}} & \frac{\partial\overline{\Sigma}_8}{\partial m_2^2} & \frac{\partial\overline{\Sigma}_8}{\partial\mathbf{M}_2^2} & \frac{\partial\overline{\Sigma}_8}{\partial h^{m_2^2}} & \frac{\partial\overline{\Sigma}_8}{\partial Q'} & \frac{\partial\overline{\Sigma}_8}{\partial\mathbf{q}}
\end{bmatrix}$$

All these quantities not being inevitably calculated, one will note, for i 1 to 8:

$$[\mathbf{D}\overline{\Sigma}\,i\,\mathbf{D}\sigma]=\left[\frac{\partial\overline{\Sigma}_i}{\partial\boldsymbol{\sigma}'},\frac{\partial\overline{\Sigma}_i}{\partial\sigma_p}\right]\qquad[\mathbf{D}\overline{\Sigma}\,i\,\mathbf{DP}21]=\left[\frac{\partial\overline{\Sigma}_i}{\partial m_2^1},\frac{\partial\overline{\Sigma}_i}{\partial\mathbf{M}_2^1},\frac{\partial\overline{\Sigma}_i}{\partial h^{m_2^1}}\right]$$

$$[\mathbf{D}\overline{\Sigma}\,i\,\mathbf{DP}11]=\left[\frac{\partial\overline{\Sigma}_i}{\partial m_1^2},\frac{\partial\overline{\Sigma}_i}{\partial\mathbf{M}_1^1},\frac{\partial\overline{\Sigma}_i}{\partial h^{m_1^2}}\right]\qquad[\mathbf{D}\overline{\Sigma}\,i\,\mathbf{DP}22]=\left[\frac{\partial\overline{\Sigma}_i}{\partial m_2^2},\frac{\partial\overline{\Sigma}_i}{\partial\mathbf{M}_2^2},\frac{\partial\overline{\Sigma}_i}{\partial h^{m_2^2}}\right]$$

$$[\mathbf{D}\overline{\Sigma}\,i\,\mathbf{DP}12]=\left[\frac{\partial\overline{\Sigma}_i}{\partial m_1^2},\frac{\partial\overline{\Sigma}_i}{\partial\mathbf{M}_1^1},\frac{\partial\overline{\Sigma}_i}{\partial h^{m_1^2}}\right]\qquad[\mathbf{D}\overline{\Sigma}\,i\,\mathbf{DT}]=\left[\frac{\partial\overline{\Sigma}_i}{\partial Q'},\frac{\partial\overline{\Sigma}_i}{\partial\mathbf{q}}\right]$$

It is then clear that:

$$\mathbf{D}\overline{\boldsymbol{\Sigma}}\,\mathbf{DE}=\mathbf{D}\overline{\boldsymbol{\Sigma}}\,\mathbf{D}\boldsymbol{\Sigma}\,.\,\mathbf{D}\boldsymbol{\Sigma}\,\mathbf{DE}$$

And the contribution of the Gauss point to the tangent matrix $D\mathbf{F}_g^{el}$ is obtained by:

$$D\mathbf{F}_g^{el}=\mathbf{Q}_g^{el^T}.\mathbf{D}\overline{\Sigma}\,\mathbf{DE}.\mathbf{Q}_g^{el}$$

7.4 Total algorithm

the algorithm becomes then:

Initializations:

Computation of $\mathbf{L}^{meca^+}$ (option `CHAR_MECA`)

Computation of $D\mathbf{F}^-$ (option `RIGI_MECA_TANG`)

Computation from $\Delta\mathbf{U}_0$: $D\mathbf{F}^-.\Delta\mathbf{U}_0=\mathbf{L}^{meca^+}-\mathbf{L}^{meca^-}$

Iterations of equilibrium of Newton
Buckles elements el
Boucle Gauss points G
computation $\mathbf{Q}_g^{el}$
computation $\mathbf{E}_g^{el^-}=\mathbf{Q}_g^{el}.\mathbf{U}^{el^-}$ and $\mathbf{E}_g^{el^+}=\mathbf{Q}_g^{el}.\mathbf{U}^{el^+}$
computation of: $\boldsymbol{\Sigma}_{gn}^{el^+},\boldsymbol{\alpha}_g^{el^+},\frac{\partial\boldsymbol{\Sigma}_{gn}^{el^+}}{\partial\mathbf{E}_{gn}^{el^+}}$ (according to option) from $\mathbf{E}_g^{el^-},\boldsymbol{\Sigma}_g^{el^-},\boldsymbol{\alpha}_g^{el^-},\mathbf{E}_{gn}^{el^+}$
computation of $\overline{\boldsymbol{\Sigma}}_{gn}^{el^+}$ from $\boldsymbol{\Sigma}_{gn}^{el^+}$; $\mathbf{R}_{gn}^{el^+}=\mathbf{Q}_g^{el^T}.\overline{\boldsymbol{\Sigma}}_{gn}^{el^+}$
computation of $\frac{\partial\overline{\boldsymbol{\Sigma}}_{gn}^{el^+}}{\partial\boldsymbol{\Sigma}_{gn}^{el^+}}$ from $\boldsymbol{\Sigma}_{gn}^{el^+}$; $D\,\mathbf{F}_{gn}^{el^+}=\mathbf{Q}_g^{el^T}.\frac{\partial\overline{\boldsymbol{\Sigma}}_{gn}^{el^+}}{\partial\boldsymbol{\Sigma}_{gn}^{el^+}}.\frac{\partial\boldsymbol{\Sigma}_{gn}^{el^+}}{\partial\mathbf{E}_{gn}^{el^+}}.\mathbf{Q}_g^{el}$ (according to option)
Computation from $\delta\mathbf{U}_{n+1}$:

$$D\,\mathbf{F}_n^+.\delta\,\mathbf{U}_{n+1}=-\mathbf{R}_n^++\mathbf{L}^{meca^+}$$

Actualization :

$$\Delta\mathbf{U}_{n+1}=\Delta\mathbf{U}_n+\rho\,\delta\mathbf{U}_{n+1}$$

IF test convergence OK
fine Newton: time step according to
If not
$n=n+1$

8 option FORC_NODA

On the level of the continuous equations, option FORC_NODA corresponds to the computation of the operator $\mathbf{R}=\mathbf{Q}^T\overline{\boldsymbol{\Sigma}}$. At the discrete level, option FORC_NODA comes down to calculating the vector $\mathbf{R}_g^{el}=\mathbf{Q}_g^{el^T}.\overline{\boldsymbol{\Sigma}}_g^{el}$.
As we already noted that $\overline{\boldsymbol{\Sigma}}$ depends not only on $\boldsymbol{\Sigma}$, but also of $\dot{\boldsymbol{\Sigma}}$, one should not be astonished to see appearing time step Δt and the forced at the same time at time + and at time˜
the algorithm of Newton-Raphson of the command STAT_NON_LINE uses option FORC_NODA for the computation of the prediction at the beginning of each time step. It is not thus pain-killer to correctly calculate all the terms for this option, including those which depend on time step. We illustrate this question for a simple example corresponding to the only equation of the hydraulics.
That is to say a simplified version of the hydraulic equation:

$$-\int_\Omega\frac{dm}{dt}p^*d\,\Omega+\int_\Omega\mathbf{M}\nabla p^*d\,\Omega=F_{ext}\,p^*$$

After discretization in time:

$$-\int_\Omega\Delta mp^*d\,\Omega+\Delta t\int_\Omega\left(\theta\,\mathbf{M}^++\left(1-\theta\right)\mathbf{M}^-\right)\nabla p^*d\,\Omega=F_{ext}^\theta\,p^*$$

Revealing $\Delta\mathbf{M}=\mathbf{M}^+-\mathbf{M}^-$ and $\Delta\,p=p^+-p^-$, and writing a constitutive law: $\Delta m=N\,\Delta\,p$, one finds:

$$-\int_\Omega N\,\Delta\,pp^*d\,\Omega+\Delta\,tq\int_\Omega\Delta\mathbf{M}\nabla p^*d\,\Omega=F_{ext}^\theta\,p^*-\Delta t\int_\Omega\mathbf{M}^-\nabla p^*d\,\Omega$$

By definition the phase of prediction STAT_NON_LINE is written:

$$\mathbf{K}_0\Delta\,\mathbf{u}^0=F_{ext}^1-\mathbf{Q}^T\boldsymbol{\sigma}_0$$

It is then clear that one must take $\int_{\Omega} \mathbf{Q}^T \boldsymbol{\sigma}_0 p^* d\,\Omega = -\Delta t \int_{\Omega} \mathbf{M}^- \nabla p^* d\,\Omega$

9 spatial Discretization

finite elements `THM` of *Code_Aster* are mixed elements, in the sense that they have at the same time unknowns of displacements, pressures and temperatures. A choice of discretization where displacements, the pressures and the temperatures are interpolated with the same order of approximation led to oscillations, especially for choices of time step too small compared to the discretization spaces some One will consult on this subject for example [bib10]. This problem is also in keeping with the way calculate the matrix known as of mass, and one will be able to consult on this subject [bib14]. We give in addition in appendix, to illustrate our matter, the solution for the first time step of a problem of mono consolidation dimensional with an interpolation $P1P1$. It is seen that for small time step, it is very oscillating.
For this reason, quadratic elements `THM` are elements in $P2P1$, i.e. the interpolation of displacements is quadratic and that of the temperatures and pressures is linear. We nevertheless kept all the unknowns on all the nodes, including the nodes mediums, but we imposed in the computation of the stiffness matrixes that the pressure of a medium node of segment is equal to the half adds top nodes of the segment to which it belongs.
In addition, in the programming, we took account of the following property:
That is to say s a top node, w_s^1 its shape function as pertaining to a linear element (for example QUAD4), and w_s^2 its shape function as pertaining to a quadratic element (for example QUAD8). That is to say na the number of edges having s like end and w_{ma}^2 the shape function of the quadratic interpolation attached to a medium node of edge, there is then the relation:

$$w_s^1 = w_s^2 + \sum_{ma=1}^{na} \frac{w_{ma}^2}{2}$$

This said, and including in interpolation $P2P1$, of the conditions of nonoscillation exist on time step. [bib10] the relation gives:

$$\Delta t > \frac{\Delta x^2}{20\, C_v}$$

where C_v is the coefficient of consolidation: $C_v = \frac{kE(1-\nu)}{\rho_{lq}(1+\nu)(1-2\nu)}$, k being the permeability measured in m/s .

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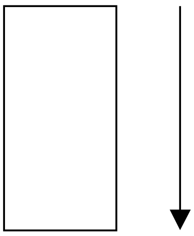
11 the versions of the document Version Aster

Author (S) Organization	**(S) Description of**	**the modifications 5 C.Chavant EDF**
5	R & D /AMA initial Text.	11 S.Granet EDF
11	R & D /AMA Modifications	: working; and statement of the conservation of the linear momentum in dynamics, § 6.3.5. Dimensional mono

Annexe 1 problem P1P1 One considers

a unidimensional problem of consolidation whose unknowns vary only according to the only variable of space. A rectangular x field
length, is filled D L “a porous material with coefficients of Lamé and, of coefficient λ μ of biot and modulus of b biot and hydraulic $N=\frac{1}{M}$ conductivity. The density λ_h of the fluid is noted. One notes ρ the stress σ , displacement σ_{xx} u in the direction, the strain x $\varepsilon=\frac{\partial u}{\partial x}$, the pressure, p L” contribution mass m out of fluid, the fluid flux $\mathbf{M}$. The boundary conditions

are: in: in:
for $x=0$ $\sigma=0\,;\,p=0$
the initial conditions $x=L$ $u=0\,;\,p=1$ $t>0$
are. The setting $t=0$ in equation $\sigma=u=p=0$



$x=0$; $\sigma=0$; $p=0$

$x=L$; $u=0$; $p=1$

gives: Balance mechanical
and linear elasticity: Conservation of $\sigma=(\lambda+2\mu)\epsilon-bp=0$
the mass: Model of Darcy $\frac{\partial m}{\partial t}+\frac{\partial \mathbf{M}}{\partial x}=0$
: Couplings: $\mathbf{M}=-\lambda_h\rho\frac{\partial p}{\partial x}$
We make $\frac{m}{\rho}=Np+b\epsilon$

a discretization in implicit time and we are interested **in computation of the first time step:** The system

of two equations is obtained: The mixed

$$\begin{cases}\sigma=(\lambda+2\mu)\epsilon(u)-bp=0\\ Np+b\epsilon-\lambda_h\Delta t\,\Delta\,p=0\end{cases}$$

variational formulation of this problem is: éq Year 1-1 Concerning

$$\begin{cases}\int_\Omega\left[(\lambda+2\mu)\epsilon(u)\epsilon(u^*)-bp\,\epsilon(u^*)\right]=0 & \forall u^*\\ \int_\Omega\left[Npp^*+b\epsilon(u)\,p^*+\lambda_h\Delta t\,\nabla p\,\nabla p^*\right]=0 & \forall p^*\end{cases}$$

the spatial discretization, we cut out the mediums in finite elements n . The nodes of the element are and. One i notes i and $i+1$ the displacement u_e^1 p_e^1 and the pressure of the first node of the element, and the displacement e u_e^2 p_e^2 and the pressure of its second node. It is supposed that one uses of the finite elements P1P1, i.e. that displacements as the pressures are interpolated linearly. The discretization of the first equation gives then: From where one

$$\sum_e \frac{u_e^{*2}-u_e^{*1}}{\Delta x}\left[(\lambda+2\mu)\frac{u_e^2-u_e^1}{\Delta x}-b\frac{p_e^1+p_e^2}{2}\right]=0$$

deduces: éq Year the 1-2

$$\left(u_e^2-u_e^1\right)=\frac{b\,\Delta x}{\lambda+2\mu}\frac{p_e^1+p_e^2}{2} \qquad \forall e$$

discretization

of the second equation gives: In there bearing [

$$\sum_e \frac{p_e^{*2}+p_e^{*1}}{2}\left[b\frac{u_e^2-u_e^1}{\Delta x}+N\frac{p_e^1+p_e^2}{2}\right]+\lambda_h \Delta t\sum_e \frac{p_e^{*2}-p_e^{*1}}{\Delta x}\frac{p_e^2-p_e^1}{\Delta x}=0$$

éq Year 1-2], one finds: éq Year 1-3 So

$$\sum_e \frac{p_e^{*2}+p_e^{*1}}{2}\left(N+\frac{b^2}{\lambda+2\mu}\right)\frac{p_e^1+p_e^2}{2}+\lambda_h\frac{\Delta t}{\Delta x^2}\sum_e\left(p_e^{*2}-p_e^{*1}\right)\left(p_e^2-p_e^1\right)=0$$

now,

we make tend time step towards zero to step of constant space, and [éq Year 1 $\lambda_h\frac{\Delta t}{\Delta x^2}<<\left(N+\frac{b^2}{\lambda+2\mu}\right)$ - 3] is reduced to: One introduces

$$\sum_e\left(p_e^{*2}+p_e^{*1}\right)\left(p_e^1+p_e^2\right)=0$$

a total classification of the nodes and unknowns of pressure: One can see that

$$p_j^1=p_j \ ; \quad p_j^2=p_{j+1}$$

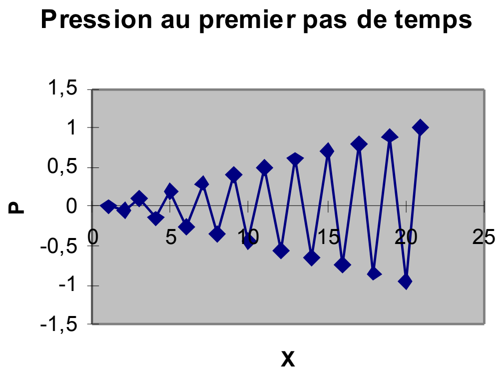
this set of relations gives finally: The solution of

$$\begin{cases} p_1=0 \\ p_i+2\,p_{i+1}+p_{i+2}=0 \qquad \forall i\in[1,n-1] \\ p_{n+1}=1 \end{cases}$$

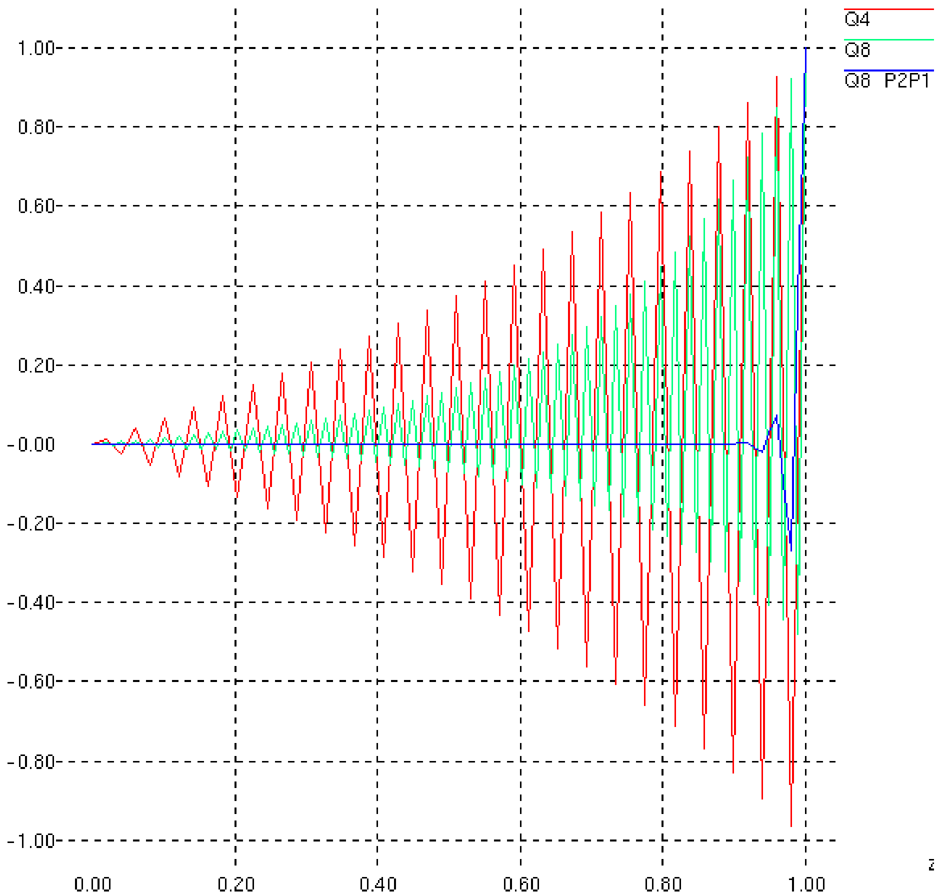
this continuation is: What gives

$$\begin{cases} p_{2j-1} = \dfrac{j-1}{n} \\ p_{2j} = -\dfrac{2j-1}{2n} \end{cases}$$

the distribution of following pressure: As an indication



, we give Ci below a comparison of results numerical obtained with elements, and. It is seen $P1P1$ $P2P2$ that $P2P1$



the element does not remove $P2P1$ the oscillation, but attenuates it appreciably.