Constitutive law LABORD_1D into uniaxial for the Summarized

concrete:

This note presents the essence of the model written by Christian the BORDERIE (CLB) for the simulation of the behavior of concrete material under cyclic loading. Although this elastic model endommageable with unelastic strains is designed in 3D only its uniaxial writing is presented.

One gives some indications on the parameter setting of the model.

The use of this model relates to the modelization of beams and reinforced concrete columns, by means of the multifibre elements beams (PMF) where the behavior of each fiber is described by a model 1D: either of the concrete, or of the steel of reinforcement.

This model is adapted as well for static analyzes as dynamic (for example of seismic computations).

Contents

1 General information

This constitutive law of the concrete is based on the damage mechanics ([bib2] and [bib3]). This model developed with the L.M.T. Cachan by C.Laborderie [bib1], is adapted to the description of the behavior generated by the creation of microscopic cracks (lowering of the stiffness) and bound operation, during cycles, with their reclosing (unilaterality).

The main features for this model are the following ones: use

- of two scalar variables of damage (one in tension, and D_1 the other in compression,), taken D_2
- into account of the unelastic strains (or permanent), dependant on ε_{an} the damage, management
- of the opening of cracks and their closing by introducing a progressive restoration of the stiffness to closing (function). This $F(\sigma)$ model

, designed to carry out structure nonlinear simulations out of concrete or reinforced concrete, is particularly adapted to the seismic analyses thanks to its capacities of degradation of the elasticity moduli and unrecoverable deformations which confer aptitudes to him to dissipate energy during cycles of loading. In addition, the variations of the stiffness of the material allow a good appreciation of the eigenfrequencies of structure as well in elastic mode as nonlinear. The model

2 of uniaxial behavior uniaxial

2.1 Formulation Elaborate

in the frame of the thermodynamics of the irreversible processes, this model leans on the formulation of the free enthalpy of Gibbs from whom the constitutive laws are deduced. The constitutive law which results from it is schematically described in the figure hereafter, it uses the following notations: and the

σ^+ σ^-	forced uniaxial, respectively "positive" and "negative": $\sigma^+=Max(\sigma,0)$, $\sigma^-=Min(\sigma,0)$
D_1 D_2	variables of damage, respectively of tension and compression, controlled by rates of energy restitution, initial
E_0	Young's modulus, of
A_1 A_2	the positive constants: parameters characteristic of the material, of dimension reverses stresses,
B_1 B_2	constants without dimension: parameters characteristic of the material (,) $B_1>1$, $B_2>1$ of
β_1 β_2	the constants: parameters characteristic of the material (,), $\beta_1>0$ $\beta_2<0$
Y_{01} Y_{02}	initial thresholds of damage in compression and tensile stresses (positive), the stress
$\sigma_f>0$	of total reclosing of cracks in compression, the function
$F(\sigma)$	which makes it possible to manage the effects of the opening and reclosing of cracks. These

parameters must be identified starting from the characteristics material of the concrete (for example thresholds in tension, in σ_{ft} compression…) σ_{fc} : to see it [§ 2.2]. For an ordinary concrete, one adopts $\beta_1=1\ MPa$, $\beta_2=-40\ MPa$ according to $\sigma_f=3\ MPa$ [bib1]. The parameter must β_2 be selected lower than the limit of strength in compression of the concrete, such as: to see $\beta_2<-\sigma_{fc}$ [§ 2.2]. In

its general formulation, this model is three-dimensional but its use in the multifibre beam elements is classically unidimensional in the layers which compose it (assumption of Navier-Bernoulli), cf [bib5]. The uniaxial writing of the model is the following one, cf [bib1]: Density

- of energy complementary (or free enthalpy): éq

$$F^*(\sigma, D_1, D_2; z_1, z_2) = \frac{(\sigma^+)^2}{2E_0(1-D_1)} + \frac{(\sigma^-)^2}{2E_0(1-D_2)} + \frac{\beta_1 D_1 . f(\sigma)}{E_0(1-D_1)} + \frac{\beta_2 D_2 . \sigma}{E_0(1-D_2)} + G(z_1, z_2) \quad 2.1\text{-}1$$

indicates

$G(z_1, z_2)$ a function of hardening of the thresholds of damage. This density of energy makes it possible to couple the evolutions in tension and compression with the courses of the cycles. Relation

- uniaxial stress-strain: éq

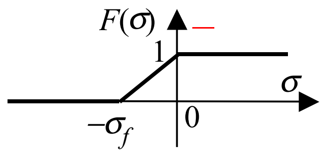
$$\varepsilon = \frac{\partial F^*}{\partial \sigma} \Rightarrow \varepsilon = \frac{\sigma^+}{E_0(1-D_1)} + \frac{\sigma^-}{E_0(1-D_2)} + \frac{\beta_1 D_1 . F(\sigma)}{E_0(1-D_1)} + \frac{\beta_2 D_2}{E_0(1-D_2)} \quad 2.1\text{-}2$$

the last two terms constitute the unelastic strain of the material. In practice, one will use the opposite relation rather, for example in compression: éq

$$\sigma^- = E_0(1-D_2) . \varepsilon - \beta_2 D_2 \quad 2.1\text{-}3$$ Function

- of crack reclosing: Appear

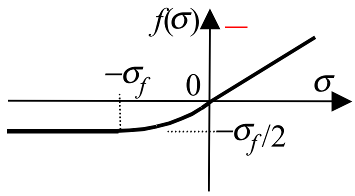
$$\begin{cases} si\ \sigma \geq 0 & F(\sigma)=1 \\ si\ 0 \geq \sigma \geq -\sigma_f & F(\sigma)=1+\sigma/\sigma_f \\ si\ \sigma \leq -\sigma_f & F(\sigma)=0 \end{cases}$$



2.1-a. Function. is F

f the primitive of the function i.e. F : Appear

$$\begin{cases} si\ \sigma \geq 0 & f(\sigma)=\sigma \\ si\ 0 \geq \sigma \geq -\sigma_f & f(\sigma)=\sigma(1+\sigma/(2\sigma_f)) \\ si\ \sigma \leq -\sigma_f & f(\sigma)=-\sigma_f/2 \end{cases}$$



2.1-b. Function. f

- Rates of energy restitution, variables associated with the local variables and D_1 , and D_2 classically obtained by derivative of the free enthalpy of Gibbs compared to the variables of damage, are written: éq

$$Y_i = \frac{\partial F^*}{\partial D_i} \Rightarrow \begin{cases} Y_1 = \frac{(\sigma^+)^2}{2E_0(1-D_1)^2} + \frac{\beta_1 . f(\sigma)}{E_0(1-D_1)^2} \\ Y_2 = \frac{(\sigma^-)^2}{2E_0(1-D_2)^2} + \frac{\beta_2 . \sigma}{E_0(1-D_2)^2} \end{cases} \quad 2.1\text{-}4$$ In practice

, one will use the following equivalent relation rather, for example in compression: éq

$$Y_2=\frac{1}{2E_0}\left(\left(E_{0.}\varepsilon+\beta_2\right)^2-\frac{\beta_2^2}{\left(1-D_2\right)^2}\right)$$ 2.1-5 Laws

- of evolution of the variables of damage: the rates of refund and Y_1 are Y_2 the pilot variables of the damage. One must note that and Y_1 are not Y_2 always positive. If one of them is negative, $Y_1<0$ the corresponding damage does not evolve: , cf $\dot{D}_i=0$ [bib1]. Thus

, the initial thresholds are: and

$Y_1-Y_{01}=0$ (and $Y_2-Y_{02}=0$ Y_{01} are Y_{02} parameters resulting from experimental data, to see hereafter) And the

evolutions beyond these thresholds, if, $Y_i>Y_{0i}$ are integrated directly according to [bib1], without passing by the solution of differential equations: éq

$$D_1=1-\frac{1}{1+[A_1(Y_1-Y_{01})]^{B_1}}\quad si\ \ \dot{Y}_1>0$$
$$D_2=1-\frac{1}{1+[A_2(Y_2-Y_{02})]^{B_2}}\quad si\ \ \dot{Y}_2>0$$
2.1-6 In practice

, one will use the opposite relation rather, for example in compression: éq

$$Y_2=Y_{02}+\frac{1}{A_2}\left(\frac{D_2}{1-D_2}\right)^{1/B_2}$$ 2.1-7 which

is an increasing function of. D_2 The variables of damage and D_1 check D_2 : . $0\le D_1\le 1$ Remarks $0\le D_2\le 1$

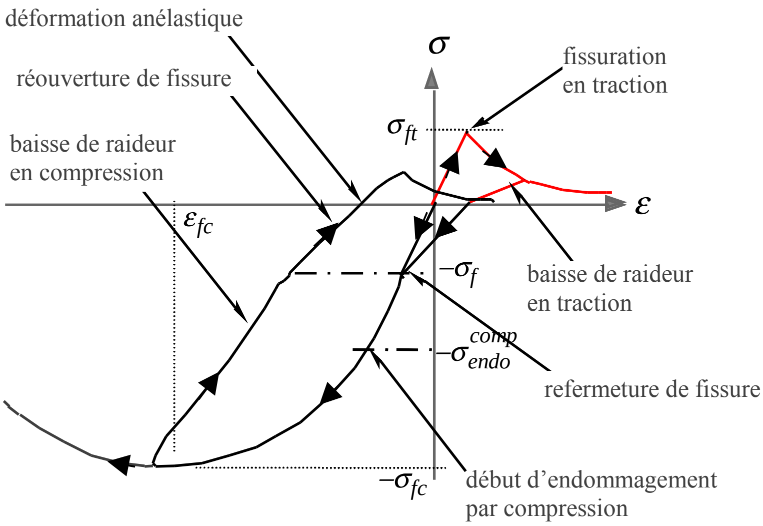
2.2 on the parameters of the model the constitutive law

uniaxial of Borderie has an elastic parameter and 9 E_0 unelastic parameters: $Y_{01}>0$ $Y_{02}>0$ $\beta_1>0$ $\beta_2<0$ $A_1>0$ $A_2>0$ $B_1>1$, $B_2>1$ that $\sigma_f>0$ one must associate with physical values located on a way of loading of traction and compression of the concrete. The figure

[Figure 2.2-a] represents a cycle of traction and compression on a concrete element. The chock of the parameters of the model must be done overall on such a representation of the response in traction and compression of the material. One can identify several accessible characteristic physical parameters from an experimental curve : the threshold

- σ_{ft} of damage in monotonous tension since a virgin state of the concrete, which is also the peak of the traction diagram; the stress
- σ_f of total reclosing of cracks in compression (noted in absolute value by convention); the threshold
- σ_{endo}^{comp} of damage in monotonous compression (noted in absolute value by convention); strength
- σ_{fc} in monotonous compression of the concrete (noted in absolute value by convention), i.e. the peak of the curve in compression;

- $\varepsilon_{fc}<0$ the défo rmation corresponding to this strength in monotonous compression of the concrete (one must have); $\varepsilon_{fc}<-\sigma_{fc}/E_0$
- $\varepsilon_{res}<0$ the strain corresponding to the discharge supplements since the peak of compression. Appear



2.2-a. Response in traction and compression of the concrete model of Borderie. Way

2.2.1 of uniaxial load monotonous in tension In practice

. $Y_{02}\gg Y_{01}$ The 1st threshold will thus be reached before the second; one will have thus. It $\dot{D}_2=0$ is possible to connect the threshold of damage in monotonous σ_{ft} tension, since a virgin state of the concrete, to the parameters of the model: éq

$$Y_{01}=\frac{\sigma_{ft}^2}{2E_0}+\frac{\beta_1\sigma_{ft}}{E_0}\quad\Leftrightarrow\quad\sigma_{ft}=-\beta_1+\sqrt{\beta_1^2+2E_0Y_{01}}\qquad 2.2\text{-}1$$

the equation of coherence in load for the 1st threshold leads to: Thus

$$\dot{D}_1\cdot\left(\frac{E_0A_1^{-1}}{B_1}\left(\frac{D_1}{1-D_1}\right)^{1/B_1-1}-\frac{\sigma^+\left(\sigma^++2\beta_1\right)}{1-D_1}\right)=\dot{\sigma}^+\cdot\left(\sigma^++\beta_1\right)$$

for the incipient damage (and $D_j=0$), and $\dot{D}_1>0$, one $\sigma^+\left(\sigma^++2\beta_1\right)=2E_0Y_{01}$ a: One can

$$\dot{\varepsilon}=\frac{-\beta_1^2\dot{\sigma}^+}{E_0\sigma^+\left(\sigma^++2\beta_1\right)}=\frac{-\beta_1^2\dot{\sigma}^+}{2E_0^2.Y_{01}}$$

thus draw the value from by identifying $\beta_1>0$ the initial slope post-peak in tension by (negative E_{pp} a priori) by the statement: éq

$$\beta_1=-\sigma_{ft}E_0\cdot\frac{1+\sqrt{1-E_{pp}/E_0}}{E_{pp}}\qquad 2.2\text{-}2$$

In practice

, exploits $\beta_1 > 0$ the width of the peak in tension and the extent of the unelastic strains post-peak, which can have only one weak influence on the mechanical response of studied reinforced concrete structure. According to [bib6], one will be able to adopt: (in

$$\beta_1 = 0{,}5 + 0{,}35\,\sigma_{ft} \qquad)\text{ éq } MPa \qquad \text{2.2-3 what}$$

then makes it possible to determine with the equation [2.2 - 1]. The parameter β_1 is dimensioned by the Young's modulus: if E_0 this modulus changes between two different concretes, but that one wishes to keep the same shape of curve it will be enough to modify the parameter in β_1 the ratio of the moduli. As

$A_1 > 0$ intervenes only for adimensionnaliser which Y_1 itself is controlled by the Young's modulus of the concrete E_0 considered, one will be able to vary because A_1 reverse of, without E_0 changing the evolution of the damage. Way

2.2.2 of uniaxial load monotonous in compression According to

[bib6], one will be able to adopt for the stress of total reclosing of cracks in compression (noted in absolute value by convention): éq

$$\sigma_f = 0{,}10\,\sigma_{fc} \qquad \text{2.2-4 Let us consider}$$

a way on the field. With $0 \geq \sigma \geq -\sigma_f$ the appearance of L " damage in monotonous compression (noted σ_{endo}^{comp} in absolute value) since a virgin state of the concrete (), to see $D_2 = 0$ [Figure 2.2-a], the threshold is reached and one has, cf equation [2.1-4]: éq

$$Y_{02} = \frac{\left(\sigma_{endo}^{comp}\right)^2}{2\,E_0} - \frac{\beta_2\,\sigma_{endo}^{comp}}{E_0} \iff \sigma_{endo}^{comp} = \beta_2 + \sqrt{\beta_2^2 + 2\,Y_{02}.\,E_0} \;>\; 0 \qquad \text{2.2-5 is}$$

also the linear equation in and Y_{02} : éq β_2

$$E_0\,Y_{02} + \sigma_{endo}^{comp}\,\beta_2 = \frac{1}{2}\left(\sigma_{endo}^{comp}\right)^2 \qquad \text{2.2-6 In practice}$$

, one will have to take: ($Y_2(-\sigma_f) = \sigma_f \dfrac{\sigma_f - 2\,\beta_2}{2\,E_0} \;\leq\; Y_{02}$ the damage in compression begins only after closing from cracks). In this

point of appearance of the damage, one $\dot{D}_2 > 0$ has, following the equations [2.1-3] and [2.1-5]: and éq

$$\dot{\sigma} = E_0\,\dot{\varepsilon} - \left(\beta_2 - \sigma_{endo}^{comp}\right)\dot{D}_2 \qquad \dot{Y}_2 = \dot{\varepsilon}\left(\beta_2 - \sigma_{endo}^{comp}\right) - \frac{\beta_2^2\,\dot{D}_2}{E_0} \qquad \text{2.2-7 In}$$

, $D_2 = 0$ the equation [2.1-7] produced rates and $\dot{Y}_2$ non $\dot{D}_2$ definite. Indeed one from of deduced: éq

$$\dot{Y}_2=\frac{\dot{D}_2}{A_2 B_2 D_2^2}\left(\frac{D_2}{1-D_2}\right)^{(1+B_2)/B_2}$$ 2.2-8 As soon as

a value is taken, small $D_2>0$ in front of, one 1 can express: éq

$$\dot{Y}_2\approx\frac{\dot{D}_2}{A_2 B_2}\left(D_2\right)^{(1-B_2)/B_2}$$ 2.2-9 Then

, using the equation [2.2-7], one obtains: éq

$$\dot{D}_2\approx\dot{\varepsilon}\left(\beta_2-\sigma_{endo}^{comp}\right)\left(\frac{\beta_2^2}{E_0}+\frac{\left(D_2\right)^{(1-B_2)/B_2}}{A_2 B_2}\right)^{-1}$$ 2.2-10 And

the slope of the curve of compression to the appearance of the damage is expressed: éq

$$\dot{\sigma}\approx\dot{\varepsilon}\left(E_0-\left(\beta_2-\sigma_{endo}^{comp}\right)^2\left(\frac{\beta_2^2}{E_0}+\frac{\left(D_2\right)^{(1-B_2)/B_2}}{A_2 B_2}\right)^{-1}\right)\approx E_0\,\dot{\varepsilon}$$ 2.2-11 Thus

with the appearance of the damage (en) $D_2=0$ the slope of the curve in compression remains continuous, property directly due to the choices operated for the formulation of the model. One cannot thus use this slope for the identification of the parameters. With the peak

of stress (strength in monotonous compression of the concrete), by noting the level of damage reached $\hat{D}_2$ in monotonous compression, one has according to the equation [2.1-3] (noted σ_{fc} in absolute value): éq

$$\varepsilon_{fc}=\frac{\beta_2\hat{D}_2-\sigma_{fc}}{E_0\left(1-\hat{D}_2\right)}\;\Leftrightarrow\;\hat{D}_2=\frac{\sigma_{fc}+E_0\varepsilon_{fc}}{\beta_2+E_0\varepsilon_{fc}}\;\Leftrightarrow\;1-\hat{D}_2=\frac{\beta_2-\sigma_{fc}}{\beta_2+E_0\varepsilon_{fc}}$$ 2.2-12 Moreover

, with the peak, one a:, therefore $\dot{\sigma}=0$ according to the equation [2.1-3]: éq

$$E_0\,\dot{\varepsilon}\left(1-\hat{D}_2\right)=\dot{D}_2\left(E_0\,\varepsilon_{fc}+\beta_2\right)\;<\;0$$ 2.2-13 In addition

, with the peak, one has according to the equation [2.1-5]: éq

$$\dot{Y}_2=\dot{\varepsilon}\left(E_0\,\varepsilon_{fc}+\beta_2\right)-\frac{\beta_2^2\,\dot{D}_2}{E_0\left(1-\hat{D}_2\right)^3}$$ 2.2-14 and by means of

the equations [2.2-12] and [2.2-13] to eliminate and $\dot{\varepsilon}$, one $\hat{D}_2$ obtains: éq

$$\dot{Y}_2=\frac{\sigma_{fc}\,\dot{D}_2}{E_0}\frac{\left(E_0\,\varepsilon_{fc}+\beta_2\right)^3\left(\sigma_{fc}-2\,\beta_2\right)}{\left(\beta_2-\sigma_{fc}\right)^3}$$ 2.2-15 Then

one expresses rate with the peak $\dot{Y}_2$ using the equation [2.1-7]: éq

$$\dot{Y}_2=\frac{\dot{D}_2}{A_2 B_2 \hat{D}_2^{\,2}}\left(\frac{\hat{D}_2}{1-\hat{D}_2}\right)^{(1+B_2)/B_2}$$ 2.2-16 and by means of

the equation [2.2-12] to eliminate, one $\hat{D}_2$ obtains: éq

$$\dot{Y}_2=\frac{\dot{D}_2}{A_2 B_2}\left(\frac{\sigma_{fc}+E_0\varepsilon_{fc}}{\beta_2-\sigma_{fc}}\right)^{(1+B_2)/B_2}\left(\frac{\beta_2+E_0\varepsilon_{fc}}{\sigma_{fc}+E_0\varepsilon_{fc}}\right)^2$$ 2.2-17 Consequently

, one draws the nonlinear equation in β_2: A_2 éq B_2

$$\frac{\left(E_0\varepsilon_{fc}+\beta_2\right)\left(\sigma_{fc}-2\beta_2\right)}{\left(\beta_2-\sigma_{fc}\right)}=\frac{E_0}{A_2 B_2 \sigma_{fc}}\left(\frac{\sigma_{fc}+E_0\varepsilon_{fc}}{\beta_2-\sigma_{fc}}\right)^{(1-B_2)/B_2}$$ 2.2-18 which

is not very practical to solve starting from the values of and σ_{fc} . We ε_{fc}

interest maintaining in the défo rmation corresponding $\varepsilon_{res}<0$ to the complete discharge since the peak of compression, which remains elastic. One has in this point, by means of the equation [2.2-12]: éq

$$\varepsilon_{res}=\frac{\beta_2\left(\sigma_{fc}+E_0\varepsilon_{fc}\right)}{E_0\left(\beta_2-\sigma_{fc}\right)} \Leftrightarrow \beta_2=\frac{E_0\varepsilon_{res}\sigma_{fc}}{E_0\left(\varepsilon_{res}-\varepsilon_{fc}\right)-\sigma_{fc}}=\frac{E_0\varepsilon_{res}\sigma_{fc}}{E_0\hat{D}_2\left(\varepsilon_{res}-\varepsilon_{fc}\right)}$$ 2.2-19 what

gives the value of. β_2 The parameter

$\beta_2<0$ is dimensioned by the Young's modulus and controls E_0 the extent of the unelastic strains, as well as the value of stress of peak. One $|\sigma_{fc}|$ can

note numerically for example, that to lower cause to drop β_2 , but $|\sigma_{fc}|$ also the value of the strain reached $|\varepsilon_{fc}|$ with the threshold of strength in compression. One also

notes that the value of is B_2 sensitive to that of, itself ε_{res} very sensitive to that of the damage reached $\hat{D}_2$ with the peak: éq

$$\varepsilon_{res}=\frac{\sigma_{fc}}{E_0(1-\hat{D}_2)}+\varepsilon_{fc}$$ 2.2-20 In general

, one considers in the literature, cf [bib7], that the elastic loss of stiffness is still very weak when the peak is reached and that it increases rather in the phase of posterior descent to the peak. By means of

the equation [2.2-6], one obtains directly: éq Y_{02}

$$Y_{02}=\frac{\sigma_{endo}^{comp}}{2E_0}\left(\sigma_{endo}^{comp}-2\frac{E_0\varepsilon_{res}\sigma_{fc}}{E_0\left(\varepsilon_{res}-\varepsilon_{fc}\right)}\right)$$ 2.2-21 To increase

made Y_{02} increase the value of strength in compression; moreover $|\sigma_{fc}|$, then the linear part of the phase of compression is more important, while $|\varepsilon_{fc}|$ is not modified [feeding-bottle 6]. If one

does not have experimental values giving, one ε_{fc} will be able to adopt the frequently allowed value of (model $-0{,}2\ \%$ parabola-rectangle, model of P.Faessel [feeding-bottle 8]…). If one

does not have experimental values giving, one ε_{res} will be able to adopt the frequently allowed value of, knowing $\varepsilon_{fc} - \sigma_{fc}/(0{,}95\,E_0)$ that the damage is $\hat{D}_2$ still weak (estimated at 5%) with the peak of the curve of compression, cf for example [bib7]. As

N" A_2 intervenes that for adimensionnaliser, which Y_2 itself is controlled by the Young's modulus, one E_0 will be able to vary because A_2 reverse of, without E_0 changing L" evolution of the damage. The response in compression is very sensitive to the parameter: to increase A_2 strongly A_2 made drop and $|\sigma_{fc}|$. To lower $|\varepsilon_{fc}|$
the value of increases B_2 the value of and lowers $|\varepsilon_{fc}|$: $|\sigma_{fc}|$ ductility increases. These

parameters thus have essential consequences on the analysis of the failure of reinforced concrete structure (by attack of the limit in compressive stresses like in strain). One can

exploit benchmark SSNL120 of Code_Aster *which* deals with the problem of the concrete material point under uniaxial cyclic loading, cf [bib4], to help itself to identify the parameters. Caution:

In severe

case loading, the peak of strength in compression can be met, and even exceeded, which results in a softening of the material. We noticed that in certain cases (according to the set of parameters of the model) of the phenomena of oscillations can appear if this stress had suddenly gone up in on this side. It $-\sigma_f$ is advisable
to relativize this problem since the strains usually met for a concrete under compression are seldom also strong. Summarized

2.2.3 for the identification of the parameters

the experimental data rather easy to obtain and necessary for the parameter setting are: the Young's modulus

- E_0 ; the threshold
- σ_{ft} of damage in monotonous tension since a virgin state of the concrete, which is also the peak of the traction diagram; the stress
- $\sigma_f > 0$ of total reclosing of cracks in compression (noted in absolute value by convention); the threshold
- σ_{endo}^{comp} of damage in monotonous compression (noted in absolute value by convention); strength
- σ_{fc} in monotonous compression of the concrete (noted in absolute value by convention), i.e. the peak of the curve in compression;
- $\varepsilon_{fc} < 0$ the défo rmation corresponding to this strength in monotonous compression of the concrete (one must have); $\varepsilon_{fc} < -\sigma_{fc}/E_0$
- $\varepsilon_{res} < 0$ the strain corresponding to the discharge supplements since the peak of compression; and possibly
- (negative E_{pp} a priori), the initial slope post-peak in tension. One draws

the values from them from the parameters of constitutive law LABORD_1D : ; ;

- E_0 one $\sigma_f > 0$ will be able to choose, according to the equation [2.2 - 4]: ; (in $\sigma_f = 0{,}10\,\sigma_{fc}$
- $\beta_1 = 0{,}5 + 0{,}35\,\sigma_{ft}$) according to MPa [bib6], or according to $\beta_1 = -\sigma_{ft} E_0 \cdot \dfrac{1+\sqrt{1-E_{pp}/E_0}}{E_{pp}}$ the

  equation [2.2-2] if the slope post-peak is had; according to E_{pp}
- $Y_{01} = \sigma_{ft} \dfrac{\sigma_{ft} + 2\beta_1}{2E_0}$ the equation [2.2-1]; according to
- $\beta_2 = \dfrac{E_0 \varepsilon_{res} \sigma_{fc}}{E_0\left(\varepsilon_{res} - \varepsilon_{fc}\right) - \sigma_{fc}}$ the equation [2.2-19]; according to
- $Y_{02} = \dfrac{\sigma_{endo}^{comp}}{2E_0}\left(\sigma_{endo}^{comp} - 2\dfrac{E_0 \varepsilon_{res} \sigma_{fc}}{E_0\left(\varepsilon_{res} - \varepsilon_{fc}\right)}\right)$ the equation [2.2-21]. One

does not have a direct analytical relation allowing to exploit the only data and σ_{fc} . one σ_{ft} will have to thus check with the set of parameters LABORD_1D `the desired` compromise. For

the parameters, $A_1 > 0$ and A_2 B_1 , B_2 the following approach will be adopted:

- the parameters without dimension, exploit $B_1 > 1$ $B_2 > 1$ the evolution of the thresholds; they are more difficult to fix: one will be able to take the values used in the references; to lower
- the value of increases $B_2 > 1$ the value of and lowers $|\varepsilon_{fc}|$: $|\sigma_{fc}|$ ductility increases; one will be able
- to vary and A_1 (dimension A_2 reverses stresses) because reverse of: they E_0 intervene only for adimensionnaliser and Y_1 , without Y_2 changing the evolution of the damage; to increase strongly A_2 made drop and $|\sigma_{fc}|$; one $|\varepsilon_{fc}|$ will be able however to use the equation [2.2 - 18] to obtain from A_2 the other parameters (or to check the coherence of its value). Numerical

3 resolution The model

expresses the strain according to the stress and is not invertible analytically. The problem

consists in finding, on the basis of a converged state of index for j a total deflection, ε to find the new state converged for an increment of total deflection. Data $\Delta\varepsilon$

3.1 and unknowns

the facts of the case are:

constants: initial	E_0	elasticity modulus, parameters
	β_1 β_2	of non-elasticity, thresholds
	Y_{01} Y_{02}	initial
	A_1 , A_2 parameters B_1 B_2	of evolution of the variables of damage forced
	σ_f	of crack reclosing
historical variables: , damages	D_1 D_2	with the preceding step, thresholds

	$Z_1 \quad Z_2$	with the preceding step the loading
: total	$\varepsilon+\Delta\varepsilon$	deflection with the current step

the unknowns of the problem are:

new variables: , damages	$D_1 \quad D_2$	with the current step, thresholds
	$Z_1 \quad Z_2$	with the current step the response
: stress	σ	modulates
	E_t	tangent Resolution

3.2 One distinguishes

3 cases according to the stress at the end of time step: case

- 1: , only $\sigma\geq 0$ D_1 can evolve, case
- 2: , and $0\geq\sigma\geq-\sigma_f$ D_1 D_2 do not evolve (by making the assumption only), case $Y_2(-\sigma_f)<Y_{02}$
- 3: , only $\sigma\leq-\sigma_f$ D_2 can evolve. In practice

, one does not know the final stress before to have calculated the damage. to L " contributes of the relation stress-strain, one rewrites the three cases according to: case ε

- 1: , case $\varepsilon\geq\varepsilon_1$
- 2: , case $\varepsilon_1>\varepsilon>\varepsilon_2$
- 3: , only $\sigma\leq-\sigma_f$ D_2 can evolve, with

and $\varepsilon_1=\dfrac{\beta_1 D_1}{E_0\left(1-D_1\right)}+\dfrac{\beta_2 D_2}{E_0\left(1-D_2\right)}$. One $\varepsilon_2=\dfrac{\beta_2 D_2-\sigma_f}{E_0(1-D_2)}$ thus

carries out a test on the value of the strain of end of time step, fixed during iterations, while the thresholds depend on L" damage, and can evolve during iterations. The iterations proceed as follows: from

- the damages at the beginning of time step, one calculates the limits and ε_1^0 , with ε_2^0
- the strain at the end of time step one makes $\varepsilon+\Delta\varepsilon$ the test to determine case 1,2 or 3, according to
- the case one calculates the evolution of the damage ([§3.2.1], [§3.2.2] or [§3.2.3]), one calculates
- the new limits and ε_1^k , one ε_2^k remakes
- the tests of case, always with the strain, if $\varepsilon+\Delta\varepsilon$
- the case changed, one calculates the new evolution of the damages and one starts again, if not one leaves with the brought up to date variables calculated in [§3.2.1], [§3.2.2] or [§3.2.3]. Case

3.2.1 1: tension In

this case alone can D_1 evolve. One calculates the rate of energy restitution which can be written according to the strain: éq

$$Y_1=\frac{1}{2E_0}\left[\left(E_0\varepsilon+\beta_1-\frac{\beta_2 D_2}{1-D_2}\right)^2-\left(\frac{\beta_1}{1-D_1}\right)^2\right]$$ 3.2.1-1 If

$Y_1<Z_1$ the damage D_1 does not evolve, if not it is necessary to solve the system of 2 equations to 2 unknowns according to: éq

$$\begin{cases} Y_1 - \frac{1}{2E_0}\left[\left(E_0\varepsilon+\beta_1-\frac{\beta_2 D_2}{1-D_2}\right)^2-\left(\frac{\beta_1}{1-D_1}\right)^2\right]=0 \\ D_1-1+\frac{1}{1+[A_1(Y_1-Y_{01})]^{B_1}}=0 \end{cases}$$ 3.2.1-2 In

the current establishment, this resolution is made by dichotomy, on a function depending only on, is obtained D_1 by incorporating the first line of the system [éq 3.2.1-2] in the second. This dichotomy can be parameterized by key keys ITER_INTE_MAXI `and` RESI_INTE_RELA `of key word` COMP_INCR . `[U4.51.11]`. The first is the number of authorized divisions of the interval of search, the second is the accuracy desired for the resolution. Once

and D_1 found Y_1 , one can bring up to date the threshold and calculate $Z_1(t^+)=\max(Z_1(t^-), Y_1(t^+))$ the stress, cf [éq 2.1-2]: éq

$$\sigma=E_0.\varepsilon(1-D_1)-\beta_1 D_1-\frac{\beta_2 D_2(1-D_1)}{1-D_2}$$ 3.2.1-3 Cases

3.2.2 2: weak compression In

this case does not evolve D_1 . If the initial threshold for rate of energy restitution is sufficiently large so that then $Y_2(-\sigma_f)<Y_{02}$ D_2 does not evolve either because in case 2: . One $0>\sigma>-\sigma_f$ calculates

the value then of: éq $F(\sigma)$

$$F(\sigma)=\frac{E_0\varepsilon(1-D_2)-\beta_2 D_2+\sigma_f}{\sigma_f+\frac{\beta_1 D_1(1-D_2)}{1-D_1}}$$ 3.2.2-1 One from of

deduced the stress, cf [éq 2.1-2]: éq

$$\sigma=E_0.\varepsilon\left(1-D_2\right)-\beta_2 D_2-\frac{\beta_1 D_1\left(1-D_2\right).F\left(\sigma\right)}{1-D_1}$$ 3.2.2-2 Cases

3.2.3 3: compression beyond the reclosing of cracks In

this case alone can D_2 evolve. One calculates the rate of energy restitution which can be written according to the strain: éq

$$Y_2=\frac{1}{2E_0}\left[(E_0\varepsilon+\beta_2)^2-\left(\frac{\beta_2}{1-D_2}\right)^2\right]$$ 3.2.3-1 If

$Y_2<Z_2$ the damage D_2 does not evolve, if not it is necessary to solve the system of 2 equations to 2 unknowns according to: éq

$$\begin{cases} Y_2 - \dfrac{1}{2E_0}\left[\left(E_0\varepsilon+\beta_2\right)^2 - \left(\dfrac{\beta_2}{1-D_2}\right)^2\right] = 0 \\ D_2 - 1 + \dfrac{1}{1+[A_2(Y_2-Y_{02})]^{B_2}} = 0 \end{cases}$$ 3.2.3-2 the resolution

is made same way as in the paragraph § 3.2.112

and D_2 found Y_2 , one can bring up to date the threshold and calculate $Z_2=\max(Z_{2,} Y_2)$ the stress: éq

$$\sigma = E_0 . \varepsilon(1-D_2) - \beta_2 D_2$$ 3.2.3-3 tangent

3.2.4 Modulus In

the actual position, the coherent tangent modulus was not calculated analytically. One calculates

the tangent modulus "numerical" to which one adds a percentage of the initial modulus to avoid certain classical problems of NON-convergence when one uses a tangent resolution with models of softening damage (and particularly here with discontinuity at the time of the reclosing of cracks). éq

$$E_t = \frac{\Delta\sigma}{\Delta\varepsilon} + \alpha E_0$$ 3.2.4-1 where

is $\Delta\sigma$ the variation of stress calculated during the increment of strain. One $\Delta\varepsilon$ chooses. Produced $\alpha=0.10$

3.3 local variables the behavior model

produces the following "local variables": Variable

$V1$	of Variable damage of tension D_1
$V2$	of damage of limiting compression D_2
$V3$	Value of limiting Y_1
$V4$	Value of tangent Y_2
$V5$	Modulus In the event of E_t

poursuite or resumption of a preceding computation, they are exploited, with the value of the stress, to initialize the state of the material. Establishment

4 in Code_Aster

the constants of the model are given in DEFI _MATERIAU [U4.43 .01] by the user: ELAS

```
: E LABORD_1D
: Y01, YO2, A1, A2, B1, B2, BETA1, BETA2, SIGF subroutine
```

NMCB 1D receives in its arguments constant the materials, the stress and the strain of the preceding increment (and $SIG0$), $EPSM$ the local variables (: damages $VAR0$, thresholds and tangent modulus) of the preceding increment and the increment of strain () of $DEPS$ the iteration in progress (after having deduced the thermal strain, having supposed the constant coefficient of thermal expansion α). It returns in return the stress (), $SIGF$ the tangent modulus () and the EF new local variables (). $VARF$ A test

is made to switch according to the case where one is: Subroutine

1) NMCB 13 draft cases 1 and 3. The system is solved by dichotomy. Subroutine
2) NMCB 2 draft case 2. After

computation of the new damage, a new test is carried out to check if one is always in the same case. If necessary a transition is remade. The computation

tangent modulus is carried out then: The value $EF=(SIGF-SIG0)/DEPS+CO*E0$

of is CO built-in arbitrarily to 0,10, cf [éq 3.2.4-1]. It is not currently modifiable by the user. It can

happen that the increment of strain is null (). In this case, $DEPS=0$ to avoid a division by zero, one takes for the value EF with the preceding increment. For this reason one must store it like local variable. Functionalities

5 and checking This document

relates to constitutive law LABORD_1D for the multifibre beams (command DEFI _COMPOR, key word MULTIFIBRE /RELATION) and its associated material LABORD_1D (command DEFI _MATERIAU). This

constitutive law is checked by the cases following tests: Seismic

SDNL 130	Response of a reinforced concrete beam (rectangular section) with nonlinear behavior [V5.02	.130] SSNL
119 static	Response of a reinforced concrete beam (rectangular section) with nonlinear behavior [V6.02	.119] SSNL
120 cyclic	Response of the constitutive law of the concrete (models of Borderie) [V6.02	.120] SSNS
106hij Damage	of a plane plate under requests varied with constitutive law GLRC_DM [V6.05	.106] Bibliography

6 Borderie

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4) .120] SSNL120 - Response *cyclic of the constitutive law of the concrete (models of Borderie). [R3.08*

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7 of the versions of the document Version

Aster Author	**(S) Organization (S) Description**	**of the modifications 8 F.**
VOLDOIR E	*, EDF-R&D/AMA, L.DAVENNE , ENSC/LMT, S.GHAVAMIAN , NECS. Initial*	text 10 F
. VOLDOIR E	*, EDF-R&D/AMA Corrections*	on the version Open-Office of documentation. 10.2
F. VOLDOIR E	*, EDF-R&D/AMA Complements*	on the identification of the parameters, § 2.2 10.4
F. VOLDOIR E	*, EDF-R&D/AMA Addition*	of statements close to the source, § 2.1. 11.1
F. VOLDOIRE	*, EDF-R&D/AMA Correction*	following file 16566 , cf § 3.2.1 and equation 3.2.1-2. 11.1
F. VOLDOIRE	*, EDF-R&D/AMA Addition*	of one bibliographical reference and comments on the identification of the parameters, § 2.2. 11.1
F. VOLDOIRE	*, EDF-R&D/AMA Addition*	of relations between the parameters of the model and the experimental values accessible on curves from compression tests and tension of the concrete.