Constitutive law ENDO_ISOT_BETON

Summarized:

This documentation presents the theoretical writing and the numerical integration of constitutive law ENDO_ISOT_BETON which describes an asymmetrical local damage mechanism of the concretes, with effect of restoration of stiffness. In addition to the model local, the nonlocal formulation with regularized strain is also supported to control the phenomena of localization.

Contents

1 Introduction – Domaine d'application

constitutive law ENDO_ISOT_BETON aims at modelling most simply possible brittle elastic concrete behavior. It can be seen as an extension of the model ENDO_FRAGILE [R5.03.18] (with which it keeps a proximity of unquestionable formulation) for applications of Civil Engineering.

As for the model ENDO_FRAGILE, the material is isotropic. The stiffness can decrease, the loss of stiffness measured by an evolving scalar of 0 (operational material) to 1 (completely damaged material).
On the other hand, contrary to ENDO_FRAGILE, the loss of stiffness distinguishes the tension from compression, to privilege the damage in tension. Moreover this loss of stiffness can disappear by return in compression, it acts of the phenomenon of restoration of stiffness to the reclosing. It as should be noted as this damage model aims at describing the fracture of the concrete in tension; it is not thus adapted at all to the description of the nonlinear behavior of the concrete in compression. It thus supposes that the concrete remains in a moderate compactness.

Model ENDO_ISOT_BETON present of softening, which generally involves a loss of ellipticity of the equations of the problem and consequently a localization of the strains, from where a pathological dependence at the mesh. To mitigate this deficiency of the model, a nonlocal formulation must be adopted: for the model ENDO_ISOT_BETON, modelization GRAD_EPSI [R5.04.02], based on the regularization of the strain is usable. In this formulation, it should be noted that only the behavior models are faded compared to a classical local modelization; consequently, the stresses preserve their usual meaning.

Lastly, that one activates or not the nonlocal formulation, the softening character of the behavior also involves the appearance of instabilities, physics or parasites, which result in snap - backs on the total response and return the control of the essential loading in static. The control of the type PRED_ELAS [R5.03.80] then seems the mode of control of the loading more adapted.

2 Local constitutive law

2.1 theoretical Writing

If one seeks to take account of the effect of reclosing, it is necessary to pay a great attention to the continuity of the stresses according to the strains (what is an essential condition for a constitutive law in a computation software by finite elements), confer [bib1]. Indeed, if one models this effect in a too simplistic way, the constitutive law is likely great to present a discontinuous response.

To take account of the reclosing (i.e the transition between tension and compression), it is necessary to start by finely describing what one calls tension and compression, knowing that in tension (resp. compression) the crack will be considered “open” (resp. “closed”). A natural solution is to place itself in a clean reference of strain. In such a reference, the elastic free energy is written (and λ indicating μ the coefficients of Lamé): éq

$$\Phi(\varepsilon)\frac{\lambda}{2}(tr\,\varepsilon)^2+\mu\sum_i \varepsilon_i^2 \quad \textbf{2.1-1 One}$$

can then define:

a tension or voluminal compression, according to the sign of, $tr\,\varepsilon$
a tension or compression in each clean direction, according to the sign of. ε_i According to

the rather reasonable principle according to - in a case of tension ("open crack"), one corrects the elastic strain energy of a factor of damage; in a case of compression ("closed crack"), one keeps the statement of elastic strain energy -, the free energy endommageable is written: éq

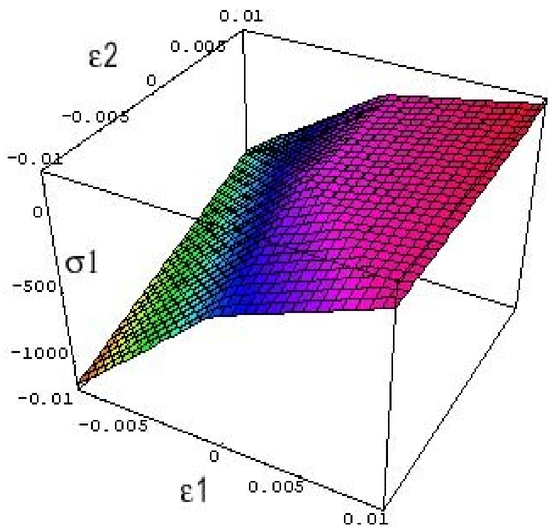
$$\Phi(\varepsilon, d)=\frac{\lambda}{2}(tr\,\varepsilon)^2\left(H(-tr\,\varepsilon)+\frac{1-d}{1+\gamma d}H(tr\,\varepsilon)\right)+\mu\sum_i \varepsilon_i^2\left(H(-\varepsilon_i)+\frac{1-d}{1+\gamma d}H(\varepsilon_i)\right)$$ **2.1-2 One**

notices that the free energy is continuous with each regime change. It is even continuously differentiable compared to the strains, since it is sum of differentiable functions (the function is $x^2 H(x)$ differentiable) and the continuity of partial derivatives at the points and $tr\,\varepsilon=0$ is $\varepsilon_i=0$ immediate. The stresses then are clarified (by knowing that they will be everywhere continuous functions of the strains). As in elasticity, the clean reference of the stresses coincides with the clean reference of the strains, result shown in appendix. One

writes the stresses in the clean reference: éq

$$\sigma_{ii}=\lambda(tr\,\varepsilon)\left(H(-tr\,\varepsilon)+\frac{1-d}{1+\gamma d}H(tr\,\varepsilon)\right)+2\mu\varepsilon_{ii}\left(H(-\varepsilon_{ii})+\frac{1-d}{1+\gamma d}H(\varepsilon_{ii})\right)$$ **2.1-3 Pennies**

this form, the continuity of the stresses with respect to the strains is clear. The figure opposite watch the stress in σ_{11} the plane with $(\varepsilon_{1,}\varepsilon_2)$ constant damage (case 2D, plane strain). The effect of the reclosing as well as the continuity of the stresses are quite visible. Appear



2-a: illustration of continuity.

The thermodynamic force associated F^d with the local variable of damage is written: éq

$$F^d=-\frac{\partial \Phi}{\partial d}=\frac{1+\gamma}{(1+\gamma d)^2}\left(\frac{\lambda}{2}(tr\,\varepsilon)^2 H(tr\,\varepsilon)+\mu\sum_i \varepsilon_i^2 H(\varepsilon_i)\right)$$ **2.1-4 It**

remains to define the evolution of the damage. The diagram selected is that of the generalized standard models. A criterion should be defined, that one takes in the form: éq

$$f(F^d)=F^d(\varepsilon, d)-k$$ **2.1-5 where**

K defines the threshold of damage. In order to take into account, on the level of the evolution of the damage, the effect of containment, the threshold depends k on the strain state, in the form: éq

$$k=k_0-k_1(tr\,\varepsilon)H(-tr\,\varepsilon)$$ **2.1-6**

One

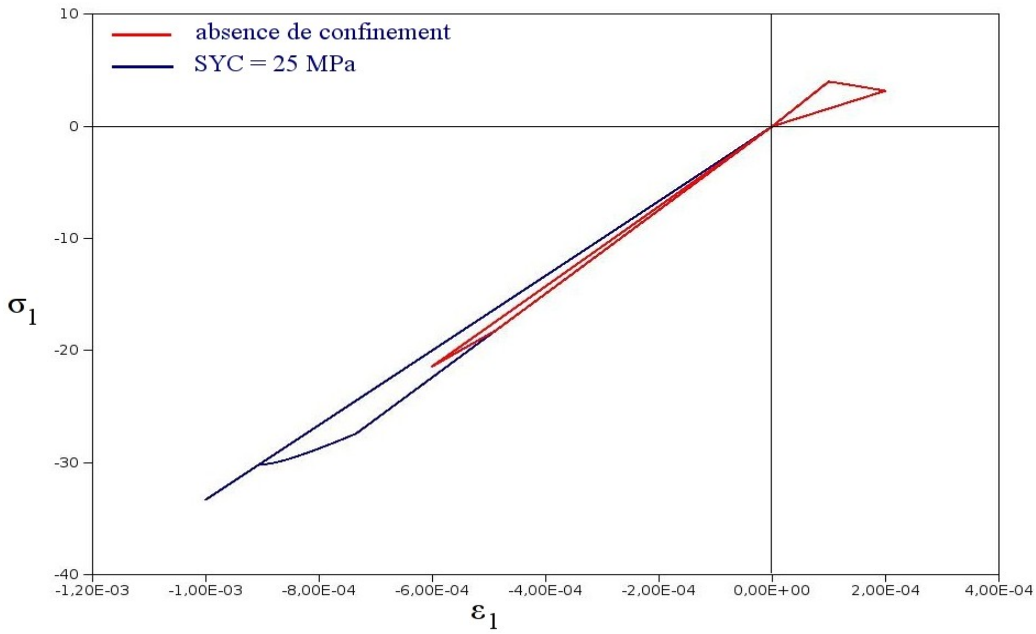
is compelled to remain in the field: éq

$$f(F^d)\leq 0$$ **2.1-7**

the evolution of the variable of damage is then determined by the conditions of Kuhn-Tucker: éq

$$\begin{cases}\dot{d}=0\ pour\ f<0\\ \dot{d}\geq 0\ pour\ f=0\end{cases}$$ **2.1-8 For**

a uniaxial request, the resulting curve is shown in the figure 2-b, for two values of containment (see 2.3.2.2 paragraph). In compression, the behavior remains roughly linear here, which represents the behavior of the material only up to 3-4 times the value of strength (uniaxial) to the tension. SYT It is clear that the use of the model is then indicated as much as one remains in this limit. In the figure 2-b, the test-tube is initially charged in tension and sudden thus a damage. Then, it is discharged and charged after in compression in the same direction. The resumption of stiffness in compression (crack reclosing) will thus be noticed. Appear



2-b: response with a uniaxial request. It

is noted that in damaging uniaxial pure load (), $\dot{d} \geqslant 0$ the thermodynamic force is expressed: , $F^d(d) = -E \frac{\varepsilon^2 . \xi_{,d}(d)}{2}$ having noted. $\xi(d) = \frac{1-d}{1+\gamma d}$ The condition of coherence is written then: éq

$$\dot{f} = -E \varepsilon \dot{\varepsilon} \xi_{,d} - E \frac{\varepsilon^2 \dot{d} \xi_{,dd}}{2} = 0$$ **2.1-9 From where**

: and $\dot{d} = -\frac{2 \dot{\varepsilon} \xi_{,d}}{\varepsilon \xi_{,dd}}$ thus the model of damaging uniaxial pure load membrane is: éq

$$\dot{\sigma} = E\left(\xi_{,d} \dot{d} \varepsilon + \xi_{\dot{\varepsilon}}\right) = E \dot{\varepsilon}\left(\xi - 2 \frac{\xi_{,d}^2}{\xi_{,dd}}\right) = \frac{-E}{\gamma} \dot{\varepsilon}$$ **2.1-10 what**

makes it possible to interpret the role of the parameter: γ the slope being constant, which is the justification of the algebraic form of the function. ξ Note:

From

a formal point of view, the generalized standard materials are characterized by a potential of dissipation function positively homogeneous of degree 1, transformed of Legendre-Fenchel of the indicating function of the field of reversibility, which is thus worth here: éq

$$\Delta(\dot{d}) = \sup_{F^d / f(F^d) \leq 0} F^d \dot{d} = k \dot{d} + I_{IR^+}(\dot{d})$$ **2.1-11 One**

will note the presence of an indicating function relating to, $\dot{d}$ which ensures that the damage is increasing. It

still remains to take into account the fact that the damage is raised by 1. From an intuitive point of view, that seems easy. To keep a writing completely compatible with the generalized standard formalism, it is enough to introduce an indicating function of the acceptable field into the statement of the free energy: éq

$$\Phi(\varepsilon, d) = \frac{\lambda}{2}(tr\,\varepsilon)^2\left(H(-tr\,\varepsilon) + \frac{1-d}{1+\gamma d} H(tr\,\varepsilon)\right) + \mu \sum_i \varepsilon_i^2\left(H(-\varepsilon_i) + \frac{1-d}{1+\gamma d} H(\varepsilon_i)\right) + I_{]-\infty;1]}(d)$$ **2.1-12**

the introduction of this indicating function prevents the damage from exceeding 1, indeed, for $d=1$, $F^d = \frac{-\partial f}{\partial d} = -\infty$ and the damage does not evolve any more. Taken

2.2 into account of the shrinkage and the temperature

the constitutive law takes into account a possible shrinkage of desiccation, a possible endogenous shrinkage and a possible thermal strain. The strain of which ε it is question in this document being then the “elastic strain”. $\tilde{\varepsilon} = \varepsilon - \varepsilon^{th} - \varepsilon^{rd} - \varepsilon^{re}$ On the other hand
, materials parameters in question in the next paragraph are regarded as constants (in particular, they cannot depend on the temperature, in the current level of development) Identification

2.3 of the parameters

the parameters of the constitutive law are 4 or 5 (see following paragraphs). They are classically provided in operator DEFI _MATERIAU. Elastic

2.3.1 parameters They

are simplest: it is the two classical parameters, Young's modulus and Poisson's ratio, provided under key word ELAS or ELAS_FO of DEFI _MATERIAU. Parameters

2.3.2 of damage According to whether

the user wants to use the dependence of the threshold with containment or not, it is necessary to provide 2 or 3 parameters to control the damage model. Use

2.3.2.1 without dependence with containment In this case,

one considers that the parameter is k_1 null. It should be noted that the compactness of the concrete must remain moderate so that the model remains valid (compressive stress about some times the stress with the peak of tension, in absolute value).
The user must inform, under key word BETON_ECRO_LINE of DEFI _MATERIAU, the values of: SYT

- : limit of stress tensile simple, D_SIGM_EPSI
- : slope of the curved post-peak in tension.

The value of SYC is then calculated automatically to have k1=0 and take its minimal value, that is to say: Use

$$SYC = SYT\sqrt{\frac{1+\nu-2\,\nu}{2\,\nu^2}}$$

2.3.2.2 with dependence with containment In this case,

the dependence with containment makes it possible the concrete to keep a realistic behavior in compression until the order of magnitude of appearance of nonthe linearity in compression, given by SYC , cf below (classically, a compressive stress of about ten times the stress with the peak of tension, in absolute value).

The user must inform, under key word BETON_ECRO_LINE of DEFI _MATERIAU, the values of: SYT

- : limit of stress tensile simple, SYC
- : limit of the stress compressive simple, D_SIGM_EPSI
- : slope of the curved post-peak in tension.

The figure 2-b watch the behavior with two values of containment. The increase in parameter SYC has like effect to prolong the linear behavior of the material. Transition

2.3.2.3 of the values “model” “user” to the values For

information, one obtains the values of, γ and k_0 possibly (if k_1 the user informed SYC) by the following formulas, cf [éq 2.1-10]: Numerical integration

$$\gamma = \frac{-E}{\mathrm{D\_SIGM\_EPSI}}$$

$$k_0=(SYT)^2\left(\frac{1+\gamma}{2E}\right)\left(\frac{1+\nu-2\nu^2}{1+\nu}\right)$$
$$k_1=SYC\frac{(1+\gamma)\nu^2}{(1+\nu)(1-2\nu)}-k_0\frac{E}{(1-2\nu)(SYC)}$$

2.4 Two

points are with being regulated before establishing the model: the first relates to the evaluating of the damage; the second consists in calculating the tangent matrix, computation made a little more delicate than usually by the transition in a clean reference of strain. One
places oneself here in the frame of the implicit integration of the constitutive laws. The dependence of the criterion according to containment [éq 2.1-6] is taken into account in explicit form, i.e the threshold k is entirely determined by the strain state of the preceding step, this to simplify the integration of the model. Evaluating

2.4.1 of the damage As

one will see it, a simple scalar equation makes it possible to obtain the damage, which makes it possible to avoid a recourse to the iterative methods. One notes
the damage d^- with the preceding step and the evaluating d^+ of the damage to the current step with the current iteration which will be the damage with the current step when convergence is reached. Simplest to evaluate the damage of the current iteration is to suppose that one reaches the criterion at current time, which results in: éq 2.4

$$f(F^d)=0 \;\Rightarrow\; \frac{1+\nu}{(1+\gamma d)^2}\left(\frac{\lambda}{2}(tr\,\varepsilon)^2 H(tr\,\varepsilon)+\mu\sum_i \varepsilon_i^2 H(\varepsilon_i)\right)=k$$ **.1-1 what**

gives: éq 2.4

$$d^{test}=\frac{1}{\gamma}\left(\sqrt{\frac{1+\gamma}{k}\left(\frac{\lambda}{2}(tr\,\varepsilon)^2 H(tr\,\varepsilon)+\mu\sum_i \varepsilon_i^2 H(\varepsilon_i)\right)}-1\right)$$ **.1-2 3 cases**

arise: : that

- $d^{test}\leq d^-$ wants to say that at time running, the criterion is not reached, one concludes from it that: $d^+=d^-$
- $d^-\leq d^{test}\leq 1$ the criterion is thus reached, the condition of coherence implies: $d^+=d^{test}$
- $d^{test}\geq 1$ the material is then ruined in this point, from where. Computation $d^+=1$

2.4.2 of the tangent matrix the tangent

matrix is the sum of two terms, the first expressing the forced relation/strain with constant damage, the second resulting from the condition. Indeed $f=0$, one can write: éq 2.4

$$\frac{\partial\sigma_{ij}}{\partial\varepsilon_{kl}}=\left\{\frac{\partial\sigma_{ij}}{\partial\varepsilon_{kl}}\right\}_{d=c^{te}}+\frac{\partial\sigma_{ij}}{\partial d}\left\{\frac{\partial d}{\partial\varepsilon_{kl}}\right\}_{f=0}$$ **.2-1 If**

the user asks for computation with tangent matrix (cf documentation of STAT_NON_LINE , [U4.51.03]), the constitutive law provides the statement given by [éq 2.4.2-1]. On the other hand, if the user asks for computation with the matrix of discharge, the constitutive law provides the secant matrix, i.e. the first term of the member of right of [éq 2.4.2-1]. Stamp

2.4.2.1 tangent with constant damage As

we underlined previously, the computation of the tangent matrix is a little delicate because of writing of the model in the clean reference of strain. Thus, one easily knows the tangent matrix with constant damage in the clean reference of strain, but what one seeks is this same tangent matrix in the total reference. If

the damage does not evolve, in the clean reference of strain, the required matrix expresses a simple relation of degraded elasticity: éq 2.4.2.1

$$\left\{\frac{\delta\tilde{\sigma}_i}{\delta\tilde{\varepsilon}_j}\right\}_{d=C^{te}}=\lambda\left(H(-tr\,\varepsilon)+\frac{1-d}{1+\gamma d}H(tr\,\varepsilon)\right)+2\mu\delta_{ij}\left(H(-\varepsilon_j)+\frac{1-d}{1+\gamma d}H(\varepsilon_j)\right)$$ **- 1 It is now necessary**

to express the transition of the total reference to the clean reference of strains, at least if the eigenvalues of strain are different. The tangent matrix being necessary only to the algorithms of numerical resolution **(diagram** of Newton), one will be allowed, during computation of the tangent matrix (and only **in** this case) to disturb possible identical eigenvalues numerically (in order to make them distinct). One will notice in particular that allows, null damage, to find the elastic stiffness matrix. One notes

with a tilde the tensors in the clean reference of strain (which, one points out it, is also the clean reference of stresses). By definition, by noting the eigenvector U_i associated with the i-ème eigenvalue, the matrix basic change, one $Q=(U_1 U_2 U_3)$ a: If

$$\sigma=Q\tilde{\sigma}Q^T \Rightarrow \delta\sigma_{ij}=Q_{\mathrm{im}}Q_{\mathrm{jm}}\delta\tilde{\sigma}_m+\delta Q_{\mathrm{im}}Q_{\mathrm{jm}}\tilde{\sigma}_m+Q_{\mathrm{im}}\delta Q_{\mathrm{jm}}\tilde{\sigma}_m$$

the eigenvalues of strain are distinct, the evolution of the eigenvectors and eigenvalues is given by (cf previously [§2]): for

$$\dot{U}_j.U_k=\frac{\dot{\tilde{\varepsilon}}_{jk}}{\tilde{\varepsilon}_j-\tilde{\varepsilon}_k} \text{ éq 2.4.2.1 } j\neq k$$ **- 2 for**

$$\dot{\tilde{\varepsilon}}_i=\dot{\tilde{\varepsilon}}_{ii} \text{ éq 2.4.2.1 } j\neq k$$ **- 3 One from of**

deduced easily: éq δQ 2.4.2.1

$$\delta Q_{ij}=\sum_{k\neq j}\frac{\delta\tilde{\varepsilon}_{jk}}{\tilde{\varepsilon}_j-\tilde{\varepsilon}_k}(U_k)_i=\sum_{k\neq j}\frac{\delta\tilde{\varepsilon}_{jk}}{\tilde{\varepsilon}_j-\tilde{\varepsilon}_k}Q_{ik}$$ **- 4 By means of**

then (the last statement being used only to obtain a clearly symmetric matrix): One thus

$$\delta\tilde{\varepsilon}_{ij}=Q_{ki}Q_{lj}\varepsilon_{kl}=\frac{1}{2}(Q_{ki}Q_{lj}+Q_{li}Q_{kj})\varepsilon_{kl}$$

obtains: éq 2.4.2.1

$$\delta \sigma_{ij} = \sum_{m,n} Q_{im} Q_{jm} \left\{ \frac{\partial \tilde{\sigma}_m}{\partial \tilde{\varepsilon}_n} \right\} \delta\tilde{\varepsilon}_n + \sum_m \delta Q_{im} Q_{jm} \tilde{\sigma}_m + Q_{im} \delta Q_{jm} \tilde{\sigma}_m$$

$$= \sum_{m,n,k,l} Q_{im} Q_{jm} Q_{kn} Q_{ln} \left\{ \frac{\partial \tilde{\sigma}_m}{\partial \tilde{\varepsilon}_n} \right\} \delta \varepsilon_{kl} + \frac{1}{2} \sum_{\substack{k,l \\ n\neq m}} \frac{Q_{in} Q_{km} Q_{ln} Q_{jm}}{\tilde{\varepsilon}_m - \tilde{\varepsilon}_n} \tilde{\sigma}_m \delta \varepsilon_{kl}$$

$$+ \frac{1}{2} \sum_{\substack{k,l \\ n\neq m}} \frac{Q_{im} Q_{jn} Q_{km} Q_{ln}}{\tilde{\varepsilon}_m - \tilde{\varepsilon}_n} \tilde{\sigma}_m \delta \varepsilon_{kl} + \frac{1}{2} \sum_{\substack{k,l \\ n\neq m}} \frac{Q_{in} Q_{lm} Q_{kn} Q_{jm}}{\tilde{\varepsilon}_m - \tilde{\varepsilon}_n} \tilde{\sigma}_m \delta \varepsilon_{kl} + \frac{1}{2} \sum_{\substack{k,l \\ n\neq m}} \frac{Q_{im} Q_{jn} Q_{lm} Q_{kn}}{\tilde{\varepsilon}_m - \tilde{\varepsilon}_n} \tilde{\sigma}_m \delta \varepsilon_{kl}$$

- 5 the tangent

matrix with constant damage is thus written: éq 2.4.2.1

$$A_{ijkl} = \frac{\partial \sigma_{ij}}{\partial \varepsilon_{kl}} = \sum_{m,n} Q_{im} Q_{jm} Q_{kn} Q_{ln} \left\{ \frac{\delta \tilde{\sigma}_m}{\delta \tilde{\varepsilon}_n} \right\}_{d=C^{te}} + \frac{1}{2} \sum_{m,n:n\neq m} \left(\frac{(Q_{km} Q_{ln} + Q_{lm} Q_{kn})(Q_{in} Q_{jm} Q_{jn} Q_{im})}{\tilde{\varepsilon}_n - \tilde{\varepsilon}_m} \right) \tilde{\sigma}_m$$

- 6 Term

2.4.2.2 of the tangent matrix due to the evolution of the damage the statement

to be evaluated is written: éq 2.4.2.2

$$\frac{\partial \sigma_{ij}}{\partial d} \left\{ \frac{\partial d}{\partial \varepsilon_{kl}} \right\}_{f=0}$$

- 1 One writes

the equation [éq 2.4.1-1] in the form: éq 2.4.2.2

$$\frac{1+\gamma}{(1+\gamma d)^2} [W(\varepsilon)] = k$$

- 2 with

: . While $W(\varepsilon) = \frac{\lambda}{2} (tr\,\varepsilon)^2 H(tr\,\varepsilon) + \mu \sum_i \varepsilon_i^2 H(\varepsilon_i)$ differentiating this statement, it comes: éq 2.4.2.2

$$-\frac{2\gamma(1+\gamma)}{(1+\gamma d)^3} W(\varepsilon) \delta d + \frac{1+\gamma}{(1+\gamma d)^2} \sigma^{el} \cdot \delta \varepsilon = 0$$

- 3 with

: One uses $\sigma^{el} = \frac{\partial W}{\partial \varepsilon}$

then the following equality: éq 2.4.2.2

$$\frac{\partial \sigma}{\partial d} = -\frac{1+\gamma}{(1+\gamma d)^2} \sigma^{el}$$

- 4 One concludes

: éq 2.4.2.2

$$\left\{\frac{\partial \sigma_{ij}}{\partial \varepsilon_{kl}}\right\}_{f=0} = -\frac{1+\gamma}{2\gamma(1+\gamma d)W(\varepsilon)}\sigma_{ij}^{el}\sigma_{kl}^{el}$$ **- 5 Cases of**

2.4.3 the material completely damaged In the case of

the completely damaged material, $d=1$ the stiffness of the material point can be cancelled. That poses problem for the stress by no means; on the other hand, that can actuate null pivots in the stiffness matrix. To mitigate this difficulty, one allows oneself to define a minimal stiffness, for the tangent matrix or the matrix of discharge. This minimal stiffness does not affect the value of the damage (which can reach 1) or the stress (which can reach 0).

To preserve a reasonable conditioning of the stiffness matrix, the minimal stiffness is taken with 10^{-5} stiffness initale. An indicator specifies χ the behavior during time step running: : no

1) $\chi=0$ evolution of the damage during the step: evolution
2) $\chi=1$ of the damage during the step: saturated
3) $\chi=2$ damage. Description $d=1$

2.5 of the local variables The model

has two local variables: : damage

1) $VI(1)$: indicator d
2) $VI(2)$ Formulation χ

3 with regularized strain Formulation

3.1 the approach

with regularized strain [R5.04.02] also makes it possible to control the phenomena of localization and for this reason seems an alternative to the formulation with gradient of damage. But unlike the latter, this formulation has the advantage of resorting to the standard algorithms for the nonlinear problems. Indeed, the only difference compared to the local constitutive law lies in the data of two strains instead of one, the local strain which intervene ε in the relation stress-strain and the regularized strain which controls $\bar{\varepsilon}$ the evolution of the damage. This one results from the local strain by resolution of the system of equations with partial derivatives according to: éq 3.1

$$\begin{cases} \bar{\varepsilon} - L_b^2 \Delta \bar{\varepsilon} = 0 & \text{dans la structure} \\ \nabla \bar{\varepsilon} \cdot \boldsymbol{n} = 0 & \text{sur le bord de normale } \boldsymbol{n} \end{cases}$$ **- 1 where**

the characteristic length is again L_b indicated under key word LONG_CARA `of DEFI _ MATERIAU. Finally`, the behavior model is written in the following way, the equation (2-3) remains identical, while the equation [éq 2.1-5] takes into account the regularized strain: éq 3.1

$$\sigma_{ij} = \lambda (tr\, \varepsilon) \left(H(-tr\, \varepsilon) + \frac{1-d}{1+\gamma d} H(tr\, \varepsilon) \right) + 2 \mu \varepsilon_{ij} \left(H(-\varepsilon_{ij}) + \frac{1-d}{1+\gamma d} H(\varepsilon_{ij}) \right)$$ **- 2 éq 3.1**

$$f(F^d) = F^d(\bar{\varepsilon}, d) - k$$ **- 3 Integration**

3.2 of the constitutive law One of

the advanced advantages for the nonlocal formulation with regularized strain is the little of modifications which it involves in the construction of the constitutive law. Indeed, the integration of the local variables is completely controlled by the regularized strain. $\bar{\varepsilon}$ The integration method

is exactly that described in the paragraph [§2.4.1], on condition that replacing the strain by the strain regularized in the equations. For

the computation of the tangent matrix, the statements are the same ones as those given to the paragraph [§2.4.2], certain statements of the strain are to be replaced by the regularized strain (when the strain concerns the criterion), while others do not change (when the strain concerns the relation stress-strain). Local variables

3.3 They

is the same local variables as for the local law: damage

$VI(1)$, indicator d
$VI(2)$. Control χ

4 by elastic prediction the control

of the type PRED_ELAS `controls` the intensity of the loading to satisfy a certain equation related to the value with the function threshold during f^{el} the elastic test. Consequently, only the points where the damage is not saturated will be taken into account. The algorithm which deals with this mode of control, cf [R5.03.80], requires the resolution of each one of these Gauss points of the following scalar equation in which is $\Delta\tau$ a data and the unknown η : éq 4

$$\tilde{f}^{el}(\eta)=\Delta\tau$$ **- 1 the method**

used to version 8 was the following one: the function provides $\tilde{f}^{el}$ the value of the function threshold during an elastic test when the field of displacement breaks up in the following way according to the scalar parameter: éq η 4

$$u=u_0+\eta u_1$$ **- 2 where**

and u_0 u_1 is given. Thanks to the linearity in small strains of the operators strain (computation of the strains starting from displacements) and regularized strain, one also obtains following decompositions: and éq

$$\varepsilon=\varepsilon_0+\eta\varepsilon_1 \;\; 4 \;\; \bar{\varepsilon}=\bar{\varepsilon}_0+\eta\bar{\varepsilon}_1$$ **- 3 the function**

presenting f^{el} the good property to be convex, the equation [éq 4-1] presents zero, one or two solutions, which are required as follows: Determination

1) amongst solutions by study at the boundaries and possibly $\pm\infty$ (if the value at the two boundaries is each time positive) determination so present f^{el} a negative minimum; Determination
2) of a framing of each solution from the study preceding Determination
3) of the solution (for a convex function knowing the framing, this search is simple and fast) Since

version 9, for more ease of use, the parameter corresponds $\Delta\tau$ to the increment of damage which one seeks to obtain for at least a point of structure. One

then does not seek any more one parameter of control which makes η time step leave the criterion a value $\Delta\tau$ with the damage resulting from preceding (cf Eq 4-1), but a parameter which brings back η for us on the criterion with a damage increased by: This $\Delta\tau$ coefficient

$$\tilde{f}^{el}(\eta, d^-)=\Delta\tau \;\;\Rightarrow\;\; \tilde{f}^{el}(\eta, d^- + \Delta\tau)=0$$

is calculated $\Delta\tau$ in the following way: where

$$\Delta\tau=\frac{\Delta t}{\mathrm{COEF\_MULT}}$$

corresponds Δt to the increment of time defined in the list of times of computation and COEF_MULT `is` the coefficient specified by the key word COEF_MULT `of` the option CONTROL `in` operator STAT_NON_LINE `[U4.51` .03]. Bibliography

5 P.B.

1 BADEL: Contributions to the reinforced concrete structure computational simulation. Thesis of the University Paris VI, 2001. Functionalities

6 and checking This document

relates to constitutive law ENDO_ISOT_BETON (key word COMP_INCR of STAT_NON_LINE) and its associated material ENDO_ISOT_BETON (command DEFI_ MATERIAU). This

constitutive law is checked by the cases following tests: COMP005

Cd Test	of the behavior. Simulation in a material point. Not documented	SSLA103
E Computation	of the shrinkage of desiccation and the endogenous shrinkage on a cylinder [V3.06	.103] SSNS106
Damage	of a plane plate under requests varied with constitutive law GLRC_DM [V6.05	.106] SSNS108
Simulation	of test SAFE by progressive push [V6.05	.108] SSNV149
Test	of ENDO_ISOT_BETON [V6.04	.149] SSNV169
Coupling	creep – damage [V6.04	.169] WTNV121
Damping	of the concrete with a damage model [V7.31	.121] Description

7 of the versions of the document Index

document Version	Aster Author	(S) Organization (S) Description	of the modifications B 7.4
P	. Badel	EDF-R &D/AMA initial	Text C 8.5
P	. Badel	EDF-R &D/AMA Correction	of sign page 10: it missed a sign – with the second member of the 2.4.2.2 equation - 4 like in the following equation 2.4.2.2 - 5 D 9.4
V	. Godard	EDF-R &D/AMA Modification	of control by elastic prediction. 10.3
	F.Voldoi re	EDF-R &D/AMA Addition	of a transition page 5 explaining the slope in uniaxial load. Demonstration

Annexe 1 of the clean reference of stress the term

in trace in energy does not pose problem: it is invariant by any change of reference. Remain the term in. Notation $\sum_i \varepsilon_i^2 \left(H(-\varepsilon_i) + \frac{1-d}{1+\gamma d} H(\varepsilon_i) \right)$

: one writes with an index (for example) it ε_i - ème i eigenvalue of a tensor which is written (by clarifying its two **indices**). If ε_{kl}

the eigenvalues all of the strain are distinct, it is shown whereas, with $\dot{\varepsilon}_i = \dot{\varepsilon}_{ii}$ the components $\dot{\varepsilon}_{kl}$ of in $\dot{\varepsilon}$ the fixed reference coinciding with the clean reference of strain at time considered (in this reference one thus has). Indeed $\varepsilon_{kl} = \varepsilon_k \delta_{kl}$
, let us write the strains in the form: While differentiating

$$\varepsilon = \sum_i \varepsilon_i U_i \otimes U_i$$

this statement, it comes: By means of

$$\dot{\varepsilon} = \sum_i \dot{\varepsilon}_i U_i \otimes U_i + \varepsilon_i \dot{U}_i \otimes U_i + \varepsilon_i U_i \otimes \dot{U}_i$$

the fact that the eigenvectors are orthonormal: one obtains

$$U_i \cdot U_j = \delta_{ij} \Rightarrow \dot{U}_i \cdot U_j + U_i \cdot \dot{U}_j = 0$$

the variations of the eigenvalues and the eigenvectors: and for

$$\dot{\varepsilon}_i = \dot{\varepsilon}_{ii} \quad \dot{U}_j \cdot U_k = \frac{\dot{\varepsilon}_{jk}}{\varepsilon_j - \varepsilon_k} \text{ This } j \neq k$$

is obviously valid only if the eigenvalues are distinct (as one can clearly see it on the statement of the variations of the eigenvectors). That comes owing to the fact that the eigenvectors are not continuous functions of the elements of the matrix. If

two eigenvalues of strains are equal (and apart from the very particular case where they are also null), they are either positive, or negative. Let us take the case where they are positive (the other case lends itself to a demonstration in any similar point). Energy concerning these two eigenvalues is written then: ($\sum_{i=2}^{3} \varepsilon_i^2$ the two equal eigenvalues are considered to have indices 2 and 3). By differentiating this statement, one obtains: by noting

$2 \sum_{i=2}^{3} \varepsilon_i d\varepsilon_i + 2\varepsilon \sum_{i=2}^{3} d\varepsilon_i$ the common ε eigenvalue. By invariance

of the trace of a matrix, here the restriction of the strain on the clean plane considered, one obtains: , whatever the

$\sum_{i=2}^{3} d\varepsilon_i = \sum_{i=2}^{3} d\varepsilon_{ii}$ evolution which underwent the clean reference at this time. For

the remaining eigenvalue (distinct from both others and index 1 with the selected notations), one a:. By $d\varepsilon_1 = d\varepsilon_{11}$ gathering

these statements, one obtains: In conclusion

$$d\left(\sum_{i=1}^{3}\varepsilon_i^2 H(\varepsilon_i)\right)=d\left(\varepsilon_1^2 H(\varepsilon_1)\right)+d\left(\sum_{i=1}^{3}\varepsilon_i^2 H(\varepsilon_i)\right)=2\sum_{i=1}^{3}\varepsilon_{ii} H(\varepsilon_{ii}) d\,\varepsilon_{ii}$$

, that the eigenvalues are distinct or not, one obtains: with

$d\left(\sum_i \varepsilon_i^2 H(\varepsilon_i)\right)=2\sum_i \varepsilon_{ii} H(\varepsilon_{ii})\, d\,\varepsilon_{ii}$ the adopted notations. This reasoning

spreads easily with the case of three equal eigenvalues. The differential

of energy to constant damage is written then: On this

$$d\,\Phi(\varepsilon,d)\big|_{d=C^{te}}= \lambda(tr\,\varepsilon)d(tr\,\varepsilon)\left(H(-tr\,\varepsilon)+\frac{1-d}{1+\gamma d}H(tr\,\varepsilon)\right)+ 2\mu\sum_i \varepsilon_{ii}\,d\,\varepsilon_{ii}\left(H(-\varepsilon_{ii})+\frac{1-d}{1-\gamma d}H(\varepsilon_{ii})\right)$$

statement, it is observed well that the clean reference of strain is also reference clean of stress.