Modelization of the cables of prestressing

Summarized

to improve strength of certain structures of Civil engineer, one uses prestressed concrete: for that, the concrete is compressed using cables of prestressed out of steel. In *Code_Aster*, it is possible to do calculations of such structures: the cables of prestressing are modelled by elements of bar with two nodes, which are then kinematically dependant on the volume elements or of plate which constitute structural the concrete part. To carry out this computation, there exist three command specific to these cables of prestressing, `DEFI_CABLE_BP` which makes it possible to define geometrically the cable and the conditions of setting in tension, `AFFE_CHAR_MECA`, operand `RELA_CINE_BP`, which makes it possible to transform the information calculated by `DEFI_CABLE_BP` into loading for structure, and `CALC_PRECONT` which allows the application of prestressing on the structure.

Principal specificities of the modelization are the following ones:

- the profile of tension along a cable can be calculated (I) either according to regulation BPEL 91 [bib1] by taking account of the retreat of anchorage, of the loss by rectilinear and curvilinear friction, of the relaxation of the cables, the creep and the shrinking of the concrete (II) or according to regulation ETC-C while holding of the retreat of anchorage, of the loss by friction and relaxation of the cables. In all the cases, connection cables/concrete is supposed to be perfect, with the image of the sheaths injected by a coulis
- it is possible to define a zone of anchorage (instead of a point of anchorage) in order to attenuate the singularities of stresses due to the application of the tension on only one node of the cable (effect of the modelization),
- the behavior of the cables is elastoplastic, thermal thermal expansion being able to be taken into account.
- thanks to operator `CALC_PRECONT`, one can simulate the phasage of the setting in tension of the cables and the setting in tension can be done in several time step in the event of appearance of non-linearities. Lastly, the final tension in the cable is strictly equal to the tension prescribed by the BPEL.
- the cables being modelled by of the finite elements, their stiffness remains active throughout the analyses.

Operators DEFI_CABLE_BP and CALC_PRECONT compatible with all are element types finished mechanical voluminal and the shell elements (DKT, Q4GG) for the description of the concrete medium crossed by the cables of prestressed and the setting in tension.

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Certain structures of civil engineer are made up not only of concrete and passive reinforcements out of steel, but also of cables of prestressings. The analysis of these structures by the finite element method then requires to integrate not only the geometrical and material characteristics of these cables but also their initial tension.
Operator DEFI_CABLE_BP was designed according to the regulations of the regulation BPEL 91 which makes it possible to define the contractual tension of way. The mechanisms taken into account by this operator are then the following:

- the setting in tension of a cable by one or two ends,
- the loss of tension due to the frictions developed along the rectilinear and curvilinear ways,
- the loss of tension due to the retreat of anchorage,
- the loss of tension due to the relaxation of the cable,
- the loss of tension due to the shrinking and the creep of the concrete.

It is also possible to use the regulations of the ETC-C to define the tension. In this case, the mechanisms taken into account are the following:

- the setting in tension of a cable by one or two ends,
- the loss of tension due to frictions and on-line losses,
- the loss of tension due to the retreat of anchorage,
- the loss of tension due to the relaxation of the cable.

The cables are modelled by elements bars with two nodes, which implies to adopt a layout approached in the case of the layouts in curve. This can be made with more close to reality without major restriction (the nodes of cables must be inside the volume of the concrete elements) taking into consideration element mesh to the concrete. Structural the concrete part can be modelled thanks to all type of voluminal elements 2D and 3D or with the elements plates DKT. Operator DEFI_CABLE_BP has the possibility and of the creating kinematical conditions between the nodes of elements bars elements 2D or 3D which do not coincide in space. This has the advantage of simplifying the creation of the mesh and of leaving free choice to the user in term of provision of the elements and their number. So connection cables of prestressed/concrete is of perfect type, without possibility of relative sliding. The operator also allows to define a cone of diffusion of the stresses around the anchorages in order to limit to it the stress concentrations much higher than reality and which are due to the modelization.
The second principal function of operator DEFI_CABLE_BP is to evaluate the profile of the tension along the cables of prestressed by considering the technological aspects of their implementation. At the time of the installation of the cables, prestressing is obtained thanks to the hydraulic actuating cylinders placed at one or two ends of the cables. The profile of tension along a cable is affected by friction (rectilinear and/or curvilinear), by the strain of the surrounding concrete, the retreat of the anchorages at the ends of the cables and by the relaxation of steels.
This tension can then be taken into account like an initial stress state during the resolution of the problem complete finite element. The problem, it is that in this case, under the effect of the tension of the cable, the concrete group and cable are compressed involving a reduction in the tension of the cable. To avoid this problem and to have exactly the tension prescribed by the BPEL or the ETC-C in structure in equilibrium, the tension must be applied by the means of macro-command CALC_PRECONT. In more thanks to this method, it is possible to make phasage on the setting in tension of the cables or to impose the loading in several time step, which can be interesting if the behavior of the concrete becomes nonlinear as of the phase of setting in tension of the cables.

2 Operator DEFI_CABLE_BP

2.1 Evaluating of the characteristics of the layout of the cables

2.1.1 cubic Interpolation by spline

We present here the method used to obtain a geometrical interpolation of the cables, which is essential to compute: precisely the curvilinear abscisse and the angle α used in the formulas of loss of prestressing.
One starts by building an interpolation of the trajectory of the cable (in fact an interpolation of two projections of the trajectory in the two planes Oxy and Oxz), then from these interpolations, one estimates the curvilinear abscisse, and the angular deviation cumulated, according to the formulas:

$$s(x)=\int_0^x \sqrt{1+y'^2(x)+z'^2(x)}\,dx \qquad \text{éq 2.1-1}$$

$$\alpha(x)=\int_0^x \frac{\sqrt{y''^2(x)+z''^2(x)+\left[y''(x)z'(x)-y'(x)z''(x)\right]^2}}{1+y'^2(x)+z'^2(x)}\,dx \qquad \text{éq 2.1-2}$$

In order to preserve the topology of the cable (and in particular the scheduling of the nodes which compose it) operator DEFI_CABLE_BP works from meshes and of mesh groups, (rather than of nodes and nodes groups), in order to be able to calculate the quantities while following the sequence of the nodes along the cable.
The interpolation used for the computation of prestressing in the concrete will be a cubic Spline interpolation carried out in parallel on the three spatial coordinates according to the curvilinear abscisse. The coordinates of the nodes of the cable are the "real" coordinates, i.e. the coordinates defined by the mesh of the cable.

All the computations presented in the frame of operator DEFI_CABLE_BP are defined from the real geometry of structures and the real positions of the nodes. Computations of tension to the nodes will be carried out nodes in nodes, in the order given by the topology of the mesh, starting from the formulas quoted above [éq 2.1-1] and [éq 2.1-2].
The computation of the cumulated angular deviation and the curvilinear abscisse the precise computation of derivatives of the trajectory of the cable requires defined in the operator in a discrete way by the position of the nodes of the mesh of cable. The polynomials of Lagrange have instabilities, in particular for irregular meshes. Moreover, one significant number of points of discretization will lead to polynomials of high degrees. In addition a small uncertainty on the coefficients of interpolation will have as a consequence an important error on the results, in term of derivatives. By choosing a polynomial interpolation of small degree, one will obtain or not continuous null second derivative (according to the degree).
The interest of a cubic interpolation of Spline type is to obtain drifts second continuous and costs of computations of order n , if n is the number of points of the tabulated function to be interpolated, with polynomials of small degree. The principle of this method of interpolation is described exclusively in the case of a function of the form $x \rightarrow f(x)$.
It is supposed that one carries out an interpolation of the tabulated function, starting from the values of the function at the points of discretization $x_1, x_2, \ldots, x_n$, and his derivative second. One can thus build a polynomial of order 3, on each interval x_i, x_{i+1} , whose polynomial statement is the following one:

$$y=\frac{x_{j+1}-x}{x_{j+1}-x_j}\,y_j+\frac{x-x_j}{x_{j+1}-x_j}\,y_{j+1}+Cy_j''+Dy_{j+1}''$$

with:

$$C=\frac{1}{6}\left[\left(\frac{x_{j+1}-x}{x_{j+1}-x_j}\right)^3-\left(\frac{x_{j+1}-x}{x_{j+1}-x_j}\right)\right]\left(x_{j+1}-x_j\right)^2$$

$$D=\frac{1}{6}\left[\left(\frac{x-x_j}{x_{j+1}-x_j}\right)^3-\left(\frac{x-x_j}{x_{j+1}-x_j}\right)\right]\left(x_{j+1}-x_j\right)^2$$

One can check easily that:

$$y(x_j)=y_j \quad \text{et} \quad y''(x_j)=y_j''$$
$$y(x_{j+1})=y_{j+1} \quad \text{et} \quad y''(x_{j+1})=y_{j+1}''$$

It is then necessary to estimate the values of derivative second with the points of interpolation. By writing the equality of the interpolations on the intervals $[x_{i-1},x_i]$, and $[x_i,x_{i+1}]$ derivative of order one, at the point x_i, one obtains the following statement:

$$\frac{x_j-x_{j-1}}{6}y_{j-1}''+\frac{x_{j+1}-x_{j-1}}{3}y_j''+\frac{x_{j+1}-x_j}{6}y_{j+1}''=\frac{y_{j+1}-y_j}{x_{j+1}-x_j}-\frac{y_j-y_{j-1}}{x_j-x_{j-1}}$$

One obtains equations $(n-2)$ thus connecting the values of second derivative to the points of discretization $x_1,x_2,\dots,x_n$. By writing the boundary conditions in x_1 and x_n on the values of second derivative, one obtains a system (n,n) which one can determine in a single way the value of all derivatives, and to obtain the interpolation function thus. Two solutions arise then for the establishment of the boundary conditions:

- to arbitrarily fix the value of derivative second at the points x_1, and x_n, to zero for example,
- to allot the actual values of derivative second in these points, if this data is accessible.

One obtains a system of equations having for unknowns n second derivative of the tabulated function to interpolate. This linear system has the characteristic to be sort-diagonal, which means that the resolution is about $O(n)$. In practice the interpolation breaks up into two stages:

- the first consists in calculating the values estimated of derivative second with the points, operation which is carried out only once,
- the second consists in calculating, for a value given of x, the value of the interpolated function, operation which can be repeated as many times as one wishes it.

Tests carried out on the function sine, three periods, show that the results are strongly dependant amongst points, as well as distribution of the points of the curve to be interpolated, (result waited), but until even in delicate situations (few points and very irregular curve) the interpolation does not diverge. In other words, even if the correlation concerning the trajectory of the cable is not the very good (interpolation with very few points) interpolation is roughly located in a range close to the real trajectory. This case will not arise in practice, but makes it possible to check the stability of the method of interpolation.

For the problem that we consider here, one cannot always write the trajectory of the cable in the form $[y(x)],[z(x)]$, whenever this curve is not bijective, in particular when the projection of the trajectory in one of the two planes Oxy or Oxz cyclic or is closed (case of a circular concrete structure).

By taking an intermediate variable of the type $u=\int |x'|$, parameter always growing and of increase identical in absolute value to that in variable x , one can be brought back to statements $[y(u)]$ bijective functions of the variable u . The cubic interpolation Spline described above is then applicable to the function $y(u)$ (like with the function $z(u)$). In practice, that led however to problems of connections of tangent (angular points) at the points where the variable x changes meaning of variation, and to specific irregularities.

One describes the trajectory of the cable like a parametric curve. Knowing a set of points of the curve, the parameter most easily accessible is then the curvilinear abscisse. One writes the trajectory of the cable in the form $[x(p), y(p)]$, in the plane Oxy , (respectively $[x(p), y(p), z(p)]$ in a space with three dimensions).

The cumulated rope p discretized at the tabulated points of the function which one interpolates $P^1, P^{2,} \ldots, P^n$ calculates in the following way:

$p(1)=0$ at the point P^1 ,
$p(k)=p(k-1)$ + distance $(P^{k-1} P^k)$ to the point P^k

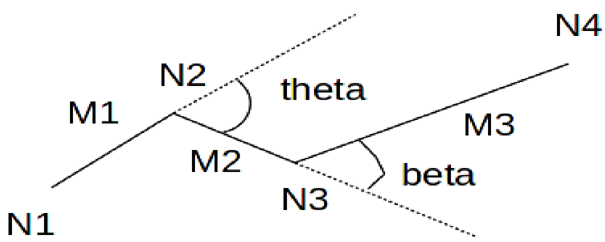
One thus has two curves defined by a set of couple [X (I), p (I)] and [there (I), p (I)] which one can directly apply the cubic Spline interpolation presented before, and which makes it possible to be freed from the difficulties encountered previously. The interpolation is made for the two coordinates, (or three coordinates, in dimension 3), independently one of the other.

2.1.2 Method without interpolation

It is possible to simply calculate the curvilinear abscisse and the angle α without making interpolation. This method is obviously less precise but it is very robust. Moreover, more the mesh is fine, plus its accuracy increases. However the problems involved in the interpolation by spline precisely occur when the mesh is too fine compared to the irregularities which it contains.
This method without interpolation is used when the interpolation by spline failed (see 8).

The computation curvilinear abscisses is very simple. It consists in adding the length of meshes of cables.

Computation of the cumulated angular deviation:



The following example is enough to describe the computation of the angular deviation by this method. Meshes M_1
, M_2 and M_3 constitute a cable. N_1 is the first node of the cable, the value of its cumulated angular deviation is $\alpha=0$. The angle $theta$ is the angular deviation between meshes M_1 and M_2 .
In any point of $]N_1 N_2[$ the cumulated angular deviation is always null because the tangent vector with the curve in these points is $\overrightarrow{N_1 N_2}$. In any point of $]N_2 N_3[$, the cumulated angular deviation is equal to $theta$ because the tangent vector each one of these points is $\overrightarrow{N_2 N_3}$ ($theta$ is the angle

between $\overrightarrow{N_1 N_2}$ and $\overrightarrow{N_2 N_3}$). It was decided, that the tangent vector with the curve in N_2 is the average of $\overrightarrow{N_1 N_2}$ and $\overrightarrow{N_2 N_3}$. What gives that the angular deviation cumulated in N_2 is $\alpha=\frac{theta}{2}$.

With same logic, the angular deviation cumulated in N_3 is $\alpha=theta+\frac{beta}{2}$, and $theta+beta$ in N_4 .

2.1.3 Control interpolation by spline

In order to control if the interpolations by spline for the three coordinates of space are correct, one calculates the number of changes of variation of derivative first and the number of changes of sign of derivative second. If the number of changes of sign is smaller than the number of changes of variation (+ a whole constant fixed at 10), it is considered that the interpolation is of good quality. In the contrary case, one passes to the method without interpolation.

2.2 Determination of the profile of tension in the cable according to BPEL 91

2.2.1 Formulates general

operator `DEFI_CABLE_BP` makes it possible to calculate the tension $F(s)$ along the curvilinear abscisse S of the cable. This one is given starting from the rules of the BPEL 91 [bib1].
All in all, one leads to the following formulation:

$$F(s)=\tilde{F}(s)-\left\{x_{flu}\times F_0+x_{ret}\times F_0+r(j)\times\frac{5}{100}\times\rho_{1000}\left[\frac{\tilde{F}(s)}{S_a\times\sigma_y}-\mu_0\right]\times\tilde{F}(s)\right\} \quad \text{éq 2.2.1-1}$$

where s the curvilinear abscisse along the cable indicates. The parameters introduced into this statement are:

- F_0 initial tension,
- x_{flu} standard rate of loss of tension by creep of the concrete, compared to the initial tension,
- x_{ret} standard rate of loss of tension by shrinking of the concrete, compared to the initial tension,
- ρ_{1000} relaxation of steel at 1000 hours, expressed in %,
- S_a area of the cross-section of the cable,
- σ_y stress yield stress of steel,
- μ_0 adimensional coefficient of relaxation of prestressed steel.

In this formula, F_0 the initial tension with the anchorages indicates (before retreat), $\tilde{F}(s)$ represents the tension after the taking into account of the losses by friction and by retreat of anchorage, $x_{flu}\times F_0$ the loss of tension by creep of the concrete represents, $x_{ret}\times F_0$ the loss of tension by shrinking of the concrete, $r(j)\times\frac{5}{100}\times\rho_{1000}\left[\frac{\tilde{F}(s)}{S_a\times s_y}-m_0\right]\times\tilde{F}(s)$ the losses by relaxation of steels.

Note:

The introduction into these elements of losses of tension is optional. Thus, if one plans to do a calculation of creep and/or shrinking of the concrete by means of a suitable model with

STAT_NON_LINE , one should not introduce these elements into the losses calculated by DEFI_CABLE_BP.

The evaluating of the losses requires the knowledge of the curvilinear abscisse s and the cumulated angular deviation α calculated as from derivatives first and second of the trajectory of the cable. The computation precise of these derivatives an interpolation between the points of transition of the cable requires. This interpolation is carried out using Splines, better than the polynomials of Lagrange which have instabilities, in particular for irregular meshes (cf preceding paragraph).
In what follows each mechanism intervening in the computation of the tension is detailed.

2.2.2 Loss of tension by friction

We start by calculating the tension along the cable by taking account of the losses per contact between the cable and the concrete: $F_c(s)=F_0\exp(-f\alpha-\varphi)$
where α indicates the angular deviation cumulated and the introduced parameters are:

- f coefficient of kinetic friction of the cable on the partly curved concrete, in rad^{-1} ,
- φ coefficient of kinetic friction per unit of length, in m^{-1} ,
- F_0 tension applied to one or the two ends of the cable.
-

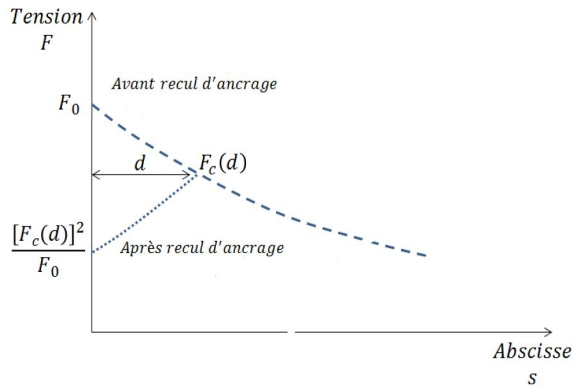
2.2.3 Loss of tension by retreat of anchorage

To take into account the retreat of anchorage, one makes the following reasoning:
the tension along the cable is affected by the retreat of anchorage at a distance d which one calculates by solving a problem with two unknowns: the function $F^*(s)$ which represents the force after retreat of the anchorage and the scalar d :

$$\Delta=\frac{1}{E_a S_a}\int_0^d [F(s)-F^*(s)]ds ,$$

$F(s)$ is worth $F_0 e^{(-f\alpha-\varphi s)}$

Δ is the value of the retreat of the anchorage (it is a data)



$F^*(s)$, the force after retreat of the anchorage, is given from the formula [bib1]:

$$[F(s).F^*(s)]=[F(d)]^2 ,$$

The length d will be given in an iterative way thanks to the preceding integral. Other authors use different relations such as:

$$[F(s)-F(d)]=[F(d)-F^*(s)]$$

For the computation of d , three typical cases can arise:

1) This loss by retreat of the anchorage is localised in the zone of the anchorage. If the cable is curved, and the sufficiently short length of the cable, it can happen that d is larger than the length of the cable. In this case, the loss of prestressing due to the retreat of the anchorage applies everywhere. It is necessary to calculate the surface ranging between the two curves

$F(s)$ and $F^*(s)$, which must be equal to $E_a S_a \Delta$, and which thus makes it possible to calculate $F^*(s)$.

2) If a tension is applied to each of the two ends of the cable, let us call $F_1(s)$ the distribution of calculated initial tension as if the tension were applied only to the first anchorage, and $F_2(s)$ the distribution of initial tension calculated as if the tension were applied only to the second anchorage. The value which must be retained in any point of the cable as initial tension is $F(s)=Max(F_1(s),F_2(s))$.
3) Lastly, if *D* is larger than the length of the cable, and when a tension is applied to each of the two ends of the cable (superposition of the two preceding cases), one must apply the following procedure:
   - computation of $F_1(s)$ calculated initial tension as if the tension were applied only to the first anchorage and by taking account of the retreat of anchorage (as in typical case 1),
   - computation of $F_2(s)$ calculated initial tension as if the tension were applied only to the second anchorage and by taking account of the retreat of anchorage (as in typical case 1),
   - computation of $F(s)=Min\left(F_1(s),F_2(s)\right)$.

2.2.4 Strains differed from steel

the loss by relaxation of steel, for an infinite time, is expressed in the following way:

$$r(j)\times\frac{5}{100}\times\rho_{1000}\left[\frac{\tilde{F}(s)}{S_a\times\sigma_y}-\mu_0\right]\times\tilde{F}(s)$$

(ρ_{1000} relaxation with 1000 hour in %; μ_0 the coefficient of relaxation of prestressed steel and σ_y the guaranteed value of the maximum loading to the fracture of the cable).
This relation expresses the loss by relaxation of the cables for an infinite time. The BPEL 91 proposes the following formula: $r(j)=\frac{j}{j+9.r_m^0}$ where j the age of the work in days indicates and r_m^0 a radius characteristic obtained by submitting the report of the section of structure out of concrete m^2 , by the perimeter of the section (in meters) of concrete.

2.2.5 Loss of tension by instantaneous strains of the concrete

the instantaneous losses are not taken into account in the formula [éq 2.2.1-1] used in *Code_Aster*. What the BPEL calls loss of instantaneous tension is in fact the loss of tension induced in cables already posed by the installation of a new group of cables. To model this phenomenon, it is necessary to represent the phasage of setting in prestressed in computation Code_Aster, i.e. not to tighten all the cables at the same time but in a successive way by connecting `the CALC_PRECONT` (see test SSNV164).

2.3 Determination of the profile of tension in the cable according to the ETC-C

2.3.1 Formulates general

operator `DEFI_CABLE_BP` makes it possible to calculate the tension $F(s)$ along the curvilinear abscisse S of the cable according to the rules of the ETC-C [bib4].

The theoretical formula is the following one:

$$F(s)=F_0-\Delta F_\mu-\Delta F_{anc}-\Delta F_{el}-\Delta F_r-\Delta \epsilon_{cs}-\Delta \epsilon_{cc}$$

where:

- F_0 is the initial tension applied to the cable
- ΔF_μ are the losses of tension by friction,
- ΔF_{anc} are the losses of tension due to the retreat of anchorage,
- ΔF_{el} are the losses of tension due to the elastic strain of the concrete,
- ΔF_r are the losses of tension due to the relaxation of steels,
- ΔF_{cs} are the losses of tension due to the shrinking of the concrete,
- ΔF_{cc} the losses of tension due to the creep of the concrete.

The losses due to the elastic strain are estimated according to the ETC-C at:

$$\Delta F_{el}(s)=\frac{A_p E_p \Delta \sigma_c(x)}{2E}$$

with E Young modulus of the concrete, A_p and E_p the section and Young the modulus of steel, and $\Delta \sigma_c(x)$ the stress induced in the concrete by prestressing.
They can be estimated by simulating the phasage of the setting in prestressed thanks to operator CALC_PRECONT. These losses are not taken into account in operator DEFI_CABLE_BP.

The losses of tension due to the shrinking of the concrete ΔF_{cc} and the creep of the concrete ΔF_{cs}, can be obtained by imposing an equivalent strain field after the setting in tension of the cables. Still, they are thus not taken into account in operator DEFI_CABLE_BP.

With final, the formula established in DEFI_CABLE_BP is the following one:

$$F(s)=F_0-\Delta F_\mu-\Delta F_{anc}-\Delta F_r \qquad . 2.3.1\text{-}1$$

the 3 types of loss are detailed in the paragraphs below.

2.3.2 Losses of tension by friction

In accordance with the ETC-C, the losses by friction are estimated by the following formula:

$$F_c(s)=F_0\left(1-e^{-\mu(\alpha+k s)}\right) \qquad \text{[éq. 2.3.2-1]}$$

where α indicates the angular deviation cumulated and the introduced parameters are:

- μ coefficient of kinetic friction of the cable on the concrete E
- k formulates the loss ratio in $[m^{-1}]$
- F_0 tension applied to one or the two ends of the cable.
-

2.3.3 Losses of tension by retreat of anchorage

the formula is identical to the BPEL. To refer to the §2.2.32.2.3.

2.3.4 Losses due to the relaxation of steel

the formula given by the ETC-C is the following one:

$$\Delta F_r(s)=0{,}8\times 0{,}66\,\rho_{1000}\,.\exp^{9{,}1\tilde{F}(s)/F_{prg}}.\left(\frac{nh}{1000}\right)^{0{,}75(1-F(s))/F_{prg}}.10^{-5}\tilde{F}(s) \qquad \text{éq 2 3.4-1}$$

where:

- s indicate the curvilinear abscisse along the cable.
- ρ_{1000} relaxation of steel at 1000 hours, expressed in %,
- F_{prg} formula with fracture in steel,
- nh the number of hours after the setting in prestressing corresponding to the date or the losses by relaxation of steel are calculated.

In this formula, $\tilde{F}(s)$ represents the tension after the taking into account of the losses by friction and retreat of anchorage like normally after taking into account of the elastic losses.

Two computation options are proposed corresponding to choice `TYPE_RELAX=' ETCC_DIRECT'` or `TYPE_RELAX=' ETCC_DIRECT'`.

If the user chooses option `TYPE_RELAX=' ETCC_DIRECT'`, then the tension used to compute: the loss due to the relaxation of steels does not take into account the elastic losses but only the losses by friction and retreat of anchorage.

If the user chooses option `TYPE_RELAX=' ETCC_REPRISE'`, then the tension used to compute: the loss due to the relaxation of steels takes the 3 types of losses of prestressed into account. This tension must be provided by the user (key word `TENSION_CT` under `DEFI_CABLE`). It will have been obtained during the first computation that one can qualify state with “short-term” by modelling the losses, by friction, retreat of anchorage and the elastic losses by modelization of the phasage (cf test SSNV229B for example of implementation).

2.4 Determination of the kinematic relations between steel and concrete

Since the nodes of the mesh of cable do not coincide inevitably with the nodes of the concrete mesh, it is necessary to define kinematic relations modelling perfect adhesion between the cables and the concrete.
The following paragraphs describe in the order the spatial geometrical considerations making it possible to define the notion of vicinity between the nodes of cable elements and concrete, then the method of calculating of the coefficients of the kinematic relations.

2.4.1 Definition of the nodes close

The computation to the coefficients of the kinematic relations requires to determine the nodes "close" to each node of the mesh of the cable. The diagram which follows symbolizes a node cables and a mesh concrete:

The mesh defined by the nodes $1,2,3,4$ contains the node cables. The close nodes are thus the tops $1,2,3,4$. If the node cable is located inside an element at p nodes $P_1, P_2, \ldots, P_n$, then the nodes $P_1, P_2, \ldots, P_n$ are called "nodes close" to the node cables.
One treats in the same way, the shell elements without eccentring, and the solid elements. The computation of the eccentring of each node of the mesh cable is necessary for the computation of the coefficients of the kinematic relations.

In the case of shell elements, when the node cable is characterized by a non-zero eccentring, one defines the nodes close as all to the top nodes of the element which contains the projection of the node cables in the tangent plane with the mesh concrete. If the node cables (or its projection in the tangent plane with the mesh concrete) belongs to a border of an element, in fact the tops of this border form all the close nodes.

2.4.2 Computation of the coefficients of the kinematic relations

In all descriptions which follow the quantities are systematically expressed in the total reference of the mesh. Kinematical connections are thus expressed according to the degrees of freedom expressed in this base. The norms and vectors rotation are expressed in the total reference, except explicit contrary mention.
In the modelization finite elements of the structure cable-concrete, the displacement of a material point of concrete structure can be expressed easily using the shape functions of the element or of the mesh concrete whose tops form the close nodes, according to displacements of the nodes close to the discretization "concrete". In the same way, a quantity or a displacement of a point of the cable, (or of its projection on the tangent level of the mesh concrete) is identical to the value of this quantity at the material point of concrete structure which occupies this same position (perfect connection between the concrete and steel), and is thus expressed according to the value of this same quantity at the tops of the element, using the shape functions.
If (x, y, z) are the coordinates of the node cables, or those of its projection, and the $N_1, N_2, \ldots, N_n$ shape functions associated with the nodes concrete $P_1, P_2, \ldots, P_n$ tops with an element with the mesh concrete (or tops of a border of an element of the mesh concrete), and the (x_i, y_i, z_i) coordinated node i , then the interpolation of a variable u on the element is written:

$$u(x, y, z)=\sum_{i=1}^{n} N_i(x, y, z).u(x_i, y_i, z_i)=\sum_{i=1}^{n} N_i(x, y, z).u_i$$

u being able to be a coordinate, or any other nodal data.
Kinematical connections make it possible to express the identity of displacement between the node of the mesh cables, and the material point concrete which occupies the same position. This corresponds to the assumption of a perfect connection between the concrete and the cable.

2.4.2.1 Case where the concrete is modelled by of the finite elements massive

By taking again the preceding notations and by considering dx^c, dy^c, dz^c displacements of the node cables, and the dx_j^b, dy_j^b, dz_j^b displacements of the nodes j ($j=1,n$) of concrete structure close to the node of the cable we obtain the following relations:

$$\begin{cases} dx^c=\sum_{i=1}^{n} N_i(x^c, y^c, z^c)\, dx_{i^b} \\ dy^c=\sum_{i=1}^{n} N_i(x^c, y^c, z^c)\, dy_{i^b} \\ dz^c=\sum_{i=1}^{n} N_i(x^c, y^c, z^c)\, dz_{i^b} \end{cases}$$

n being the number of nodes of the element concrete neighbors of the node of the cable, or that of one of its borders. For each node of the cable, one obtains 3 kinematic relations between displacements of the nodes of the two meshes cables and concrete.

2.4.2.2 Case where the concrete is modelled by of the finite elements plate

Either P_0^c the initial position of a point of cable in the not deformed geometry and or P^c the position of this same point after strain. Let us call P_0^p the projection of P_0^c on the surface of the average average of the concrete shell not deformed and P^p the projection of P^c on the surface of the average average of the concrete shell deformed. That is to say $\vec{n}_0$ the norm with the average plane of the shell of concrete in P_0^p and $\vec{n}$ that in P^p.

P_O^p is given by: $\begin{Bmatrix} x_0^p \\ y_0^p \\ z_0^p \end{Bmatrix} = \begin{Bmatrix} x_0^c \\ y_0^c \\ z_0^c \end{Bmatrix} - \left[\begin{Bmatrix} x_0^c - x_0^b \\ y_0^c - y_0^b \\ z_0^p - y_0^b \end{Bmatrix} \cdot \begin{Bmatrix} n_{0x} \\ n_{0y} \\ n_{0z} \end{Bmatrix} \right] \cdot \begin{Bmatrix} n_{0x} \\ n_{0y} \\ n_{0z} \end{Bmatrix}$

P^p is given by: $\begin{Bmatrix} x^p \\ y^p \\ z^p \end{Bmatrix} = \begin{Bmatrix} x^c \\ y^c \\ z^c \end{Bmatrix} - \left[\begin{Bmatrix} x^c - x^b \\ y^c - y^b \\ z^p - y^b \end{Bmatrix} \cdot \begin{Bmatrix} n\,x \\ n\,y \\ n\,z \end{Bmatrix} \right] \cdot \begin{Bmatrix} n\,x \\ n\,y \\ n\,z \end{Bmatrix}$

The point P_O^p belongs to a mesh of concrete plate whose nodes are noted $P_1^b, P_2^b \text{et} P_3^b$.
One defines the eccentring of the cable compared to the concrete shell as the distance $e=\|\overrightarrow{P_0^p P_0^c}\|$ and one makes the assumption that this eccentring does not vary when the structure becomes deformed: $e=\|\overrightarrow{P_0^p P_0^c}\|=\|\overrightarrow{P^p P^c}\|$
One introduces displacements of the points of the cable and his projection:

$$\overrightarrow{P_0^c P^c}=\begin{cases} d\,x^c \\ d\,y^c \\ d\,z^c \end{cases} \qquad \overrightarrow{P_0^p P^p}=\begin{cases} dx^p=\sum_{i=1}^{n} N_i(x_0^p, y_0^p, z_0^p)\,dx_{i^b} \\ dy^p=\sum_{i=1}^{n} N_i(x_0^p, y_0^p, z_0^p)\,dy_{i^b} \\ dz^p=\sum_{i=1}^{n} N_i(x_0^p, y_0^p, z_0^p)\,dz_{i^b} \end{cases}$$

One and the introduces the vector "rotation $\vec{\theta}$ " of the plate at P^p the point degrees of freedom of rotation of the nodes of the plate: $\vec{\theta}=\begin{cases} drx^b=\sum_{i=1}^{n} N_i(x_0^p, y_0^p, z_0^p)\,drx_{i^b} \\ dry^b=\sum_{i=1}^{n} N_i(x_0^p, y_0^p, z_0^p)\,dry_{i^b} \\ drz^b=\sum_{i=1}^{n} N_i(x_0^p, y_0^p, z_0^p)\,drz_{i^b} \end{cases}$

By definition of $\vec{\theta}$, one a: $\vec{n}-\vec{n}_0=\vec{\theta}\wedge\vec{n}_0$
One can then write:

$$\overrightarrow{P_0^p P_0^c}=e\,\vec{n}_0$$
$$\overrightarrow{P^p P^c}=e\,\vec{n}$$

By withdrawing these two equations, by taking account of the definition of $\vec{\theta}$ one finds:

$$\begin{cases} dx^c-dx^p=e.\left(dry^p.n_{0z}-drz^p.n_{0y}\right) \\ dy^c-dy^p=e.\left(drz^p.n_{0x}-drx^p.n_{0z}\right) \\ dz^c-dz^p=e.\left(drx^p.n_{0y}-dry^p.n_{0x}\right) \end{cases}$$

By injecting into this last equation the shape functions, one has finally:

$$\begin{cases} dx^c-\left(\sum_{i=1}^{n} N_i(x^p, y^p, z^p)\,dx_i^b\right)=e.\left(\left(\sum_{i=1}^{n} N_i(x^p, y^p, z^p)\,dry_i^b\right).n_{0z}-\left(\sum_{i=1}^{n} N_i(x^p, y^p, z^p)\,drz_i^b\right).n_{0y}\right) \\ dy^c-\left(\sum_{i=1}^{n} N_i(x^p, y^p, z^p)\,dx_i^b\right)=e.\left(\left(\sum_{i=1}^{n} N_i(x^p, y^p, z^p)\,drz_i^b\right).n_{0x}-\left(\sum_{i=1}^{n} N_i(x^p, y^p, z^p)\,drx_i^b\right).n_{0z}\right) \\ dz^c-\left(\sum_{i=1}^{n} N_i(x^p, y^p, z^p)\,dx_i^b\right)=e.\left(\left(\sum_{i=1}^{n} N_i(x^p, y^p, z^p)\,drx_i^b\right).n_{0y}-\left(\sum_{i=1}^{n} N_i(x^p, y^p, z^p)\,dry_i^b\right).n_{0x}\right) \end{cases}$$

2.4.2.3 Case where the node of the cable projects on a node of the mesh concrete

the distance between the projection P_0^p of the node cables P_0^c and a node concrete P_i^b is given by:

$$d=\|P_0^P P_i^b\|=\left\|\begin{pmatrix}x^c-x_i^b\\ y^c-y_i^b\\ z^c-z_i^b\end{pmatrix}-\left[\begin{pmatrix}x^c-x_i^b\\ y^c-y_i^b\\ z^c-z_i^b\end{pmatrix}.\vec{n}_0\right].\vec{n}_0\right\|$$

If it happens that this distance is null (in practice lower than 10-5), it is that the node cable is projected at the top of a concrete mesh, and then the kinematic relations are simplified:

$$\begin{cases} dx^c-dx_i^p=e.\left(dry_i^p.n_{0z}-drz_i^p.n_{0y}\right)\\ dy^c-dy_i^p=e.\left(drz_i^p.n_{0x}-drx_i^p.n_{0z}\right)\\ dz^c-dz_i^p=e.\left(drx_i^p.n_{0y}-dry_i^p.n_{0x}\right)\end{cases}$$

These relations are the general relations in which: $N_j(x^p,y^p,z^p)=0$ if $j\neq i$.

2.5 Processing of the zones of end of the cable

The modelization of a cable of prestressed such as it is made in *Code_Aster* consists in representing the group cables, sheath of transition, and all the parts of anchorage, only thanks to one continuation of elements of bar. The restrain between the cable elements and the concrete medium is ensured by kinematical conditions on the degrees of freedom of each node of the cable, and those of the crossed elements concrete.

When the setting in tension of the cable is applied, it is observed that the reactions generated at the ends of the cables on the concrete create levels of stresses much higher than reality, and cause the damage of the concrete. As example, in certain studies, one could observe compressive stresses of more than $200\,MPa$, which largely exceeds the experimental value observed ($40\,MPa$). In reality, this phenomenon is not observed thanks to the installation of a cone of diffusion of stress (see drawing below) which distributes the force of prestressed on a large surface of the concrete. In the case of the model with the finite elements, this surface does not exist, since the force is directly taken again by a node.

Real situation Models EF without cone

This way of modelization has several disadvantages:

- the concentration of this force crushes the concrete,
- the spatial discretization of the model changes the results.

To cure this problem, key word `CONE` of operator `DEFI_CABLE_BP` makes it possible to distribute this force of prestressed either on a node, but on all the nodes contained in a volume (all the nodes of this volume are dependant between them to form a rigid solid) delimited by a cylinder of radius R and length L , representing the equivalent of the zone of influence of the cone of blooming of an anchorage (see figure below).

The identification and the creation of the kinematic relations between the nodes of the concrete and the cable are made in an automatic way DEFI_CABLE_BP `by the command` , where the new data R and L will be with providing by the user.

2.6 Note: computation of the tension of the cable as a mechanical loading

We made the choice the model to leave the cable elements in mechanical support of computation by finite elements (linear or not). So there is not computation step of equivalent force to defer to the nodes of the mesh. One is simply satisfied to say that the cables of prestressing have a non-zero stress state initial. This stress state is that deduced from the tension as calculated by DEFI_CABLE_BP.

For reasons of simplicity, the data-processing object created by the operator DEFI_CABLE_BP is an array memorizing of the values to the nodes of the cable. Then let us consider two related elements of the cable:

$e1$ tops $N1$ and $N2$, and
$e2$ of top $N2$ and $N3$.

We suppose that l_1 and S_1 are the length and the section of an element $e1$ and that l_2 and S_2 are the length and the section of the element $e2$.

DEFI_CABLE_BP will calculate with the node $N2$ a tension T_{N_2} defined by:

$$T_{N_2}=\frac{1}{2}\left(\frac{\int_{e_1} T(s)\,ds}{l_1}+\frac{\int_{e_2} T(s)\,ds}{l_2}\right)$$

Conversely, for computation finite element, operator STAT_NON_LINE will consider that the initial stress in the element $e1$ is $\sigma_0^{e1}=\frac{T_{N_1}+T_{N_2}}{2S_1}$

Note::

It will be always considered that the constitutive law of the cable is of incremental type.

3 Macro-command CALC_PRECONT

3.1 Why a macro-command for the setting in tension?

- It is possible to transform the tension in the cables calculated by DEFI_CABLE_BP into a loading directly taken into account by STAT_NON_LINE thanks to command AFFE_CHAR_MECA operand RELA_CINE_BP (SIGM_BPEL=' OUI'). In this case, the tension is taken into account like an initial stress state during the resolution of the complete problem finite elements.

The resolution of the problem makes it possible to reach a state of equilibrium between the cable of prestressed and the rest of structure after instantaneous strain. Indeed, under the action of the tension of the cable, the group cables (S) and concrete will compress initial position compared to the (cable in tension, mesh not deformed). The length of the cable will thus decrease, and the initial tension also, consequently, will decrease. One thus obtains a final state with a tension in the cable different from the tension calculated initially. It is then essential to increase proportionally the tension applied *in situ* to the level them anchorages to take account of this loss.

The use of macro-command CALC_PRECONT makes it possible to avoid this phase of correction, by obtaining the state of equilibrium of structure with a tension in the cables equal to the lawful tension. In addition because of adopted method, it allows besides applying the tension in several time step, which can be interesting in the event of plasticization or of damage of the concrete. It makes it possible moreover to tighten the cables in a nonsimultaneous way and thus in a way closer to reality of the building sites.

To profit from these advantages, the loading is applied in the form of an external loading and not like an initial state, which allows the progressive loading of structure. In addition, to avoid the loss of tension in the cable, the idea is not to make act the stiffness of the cables during the phase of setting in tension (cf [bib3]).
The various stages carried out by the macro-command are here detailed.

3.1.1 Stage 1: computation of the equivalent nodal forces

This stage consists in transforming the internal tensions of the cables calculated by `DEFI_CABLE_BP` into an external loading. For that, one carries out a first `STAT_NON_LINE` only on the cables which one wishes to put in prestressing, with the following loading:

- cable clamped
- the tension given by `DEFI_CABLE_BP`

Appears 3.1.1-a: Loading at stage 1

One calculates the nodal forces on the cable. One recovers these forces thanks to `CREA_CHAMP`. And one builds the vector associated loading F .

3.1.2 Stage 2: application of prestressing to the concrete

the following stage consists in applying prestressing to concrete structure, without making take part the stiffness of the cable. For that, one supposes for this computation that the Young modulus of steel is null. One can choose time step to apply the loading of prestressed in only one or several time step if the concrete is damaged.
The loading is thus the following:

- blocking of motions of rigid bodies for the concrete,
- the nodal forces resulting from the first computation on the cable,
- kinematical connections between the cable and the concrete.

Appear 3.1.2-a: Loading at stage 2

3.1.3 Stage 3: swing of the external forces in internal forces

Before continuing computation in a traditional way, it is necessary of retransformer the external forces which made it possible to deform concrete structure in internal forces. This operation is done without modification on displacements and the forced of the group of structure, since the equilibrium was reached at stage 2: it is about a simple artifice to be able to continue computation. The loading is thus the following:

- blocking of motions of rigid bodies for the concrete,
- connections kinematical between the cable and the concrete,
- tension in the cables.

Appear 3.1.3-a: Loading at stage 3

4 Procedure of modelization

4.1 various stages: standard case

to manage to model a prestressed concrete structure the procedure to be followed is the following one:

- to model the concrete elements (DKT, Q4GG, 2D or 3D),
- to model the cables of prestressed by elements bars with two nodes (BAR),
- to allot to the elements bars the mechanical characteristics of the cables of prestressing,
- thanks to operator DEFI_CABLE_BP compute kinematical data (kinematic relations between the nodes of the cable and those of the concrete elements) and statics (profile of tension along the cables),
- to define the kinematical data as mechanical loading,
- to call upon operator CALC_PRECONT,
- to solve the problem with operator STAT_NON_LINE and the by integrating only the kinematical data loadings other than prestressing.

For more practical information, to refer to the document [U2.03.06].

4.2 Typical case

So for a reason where the other, the user does not wish to use macro-command CALC_PRECONT it is possible to adopt the following procedure:

- to model the concrete elements,
- to model the cables of prestressed by elements bars with two nodes (BAR),
- to allot to the elements bars the mechanical characteristics of the cables of prestressing,
- thanks to operator DEFI_CABLE_BP compute kinematical data (kinematic relations between the nodes of the cable and those of the concrete elements) and statics (profile of tension along the cables),
- to apply these kinematical and static data like a mechanical loading,
- to solve the problem with operator STAT_NON_LINE by integrating all the loadings.

At the conclusion of this computation, it is necessary to determine the coefficients of correction to apply to the initial tensions applied to the cables (on the level of the declaration of operator DEFI_CABLE_BP) making it possible to compensate for the loss by instantaneous strain of structure.

Once the command file modified by these coefficients of correction, the modelization of the cables of prestressing is accomplished.

Attention, in the case of sequence of STAT_NON_LINE, it is appropriate starting from the second call, to include in the loading only the kinematic relations and not the tension in the cables, under penalty of adding this tension, with each computation.

4.3 Precautions of use and remarks

It is recommended to limit the recourse to a large number of kinematic relations under penalty of weighing down the computing time. However, when a node of the elements of bar constituting the cables coincides topologically with a node concrete, there is no addition of kinematic relation.

If one carries out a first STAT_NON_LINE before putting in tension in the cables, it is preferable to disable the cables, either by not taking them into account in the model, or in their affecting a tension constantly null (constitutive law SANS), and by including in the loading the kinematic relations binding the cable to the concrete.
If one carries out a phasage of the setting in prestressing, it is necessary to think of including the kinematic relations in the loading for the cables already tended at the preceding stages.

5 Features and checking

the commands evoked in this document are checked by the cases following tests:

CALC_PRECONT		
SSLV115	[V3.04.115]	Element of prestressed concrete in compression and gravity
SSNV164	[V6.04.164]	Put in tension of cables of prestressed in a beam 3D
SSLS137	[V3.03.137]	Plates prestressed concrete with excentré cable in bending

DEFI_CABLE_BP		
SSLV115	[V3.04.115]	Element of prestressed concrete in compression and gravity
SSNV164	[V6.04.164]	Put in tension of cables of prestressed in a beam 3D
SSNP108	[V6.03.108]	Element of prestressed concrete in compression
SSNP109	[V6.03.109]	Cables of prestressing excentré in a concrete straight beam
SSNV137	[V6.04.137]	Cables of prestressed in a concrete straight beam
SSNV229	[V6.04.229]	Validation of formulas ETCC in DEFI_CABLE_BP
ZZZZ111	[V1.01.111]	Validation of operator DEFI_CABLE_BP

6 Bibliography

1) Rules BPEL 91, technical Rules of design and computation of the works and prestressed concrete constructions following the method of the limiting states. CSTB, ISBN 2-86891-214-1.

2) [R3.07.03] “Shell elements DKT, DST, DKTG and Manual Q4g” of Reference Aster.

3) S. GHAVAMIAN, E. LORENTZ: Improvement of the functionalities of the taking into account of prestressing in *Code_Aster*, CR – AMA 2002-01

7 Description of the versions of the document

Version Aster	**Author (S) Organization (S)**	**Description of modifications**
6	*S. MICHEL-PONNELLE, A. ASSIRE (EDF-R&D/AMA)*	initial Text
7	*S. MICHEL-PONNELLE, A. ASSIRE (EDF-R&D/AMA)*	
11	*S. MICHEL-PONNELLE, A. ASSIRE (EDF-R&D/AMA)*	Shells usable with `CALC_PRECONT` Addition of formulas ETC-C for computation of tension