Behavior model GRANGER_FP for the clean creep of the Summarized

concrete:

This document the model presents clean creep of “Granger”, which is a way of modelling the clean creep of the concrete. One also details there the writing and the digital processing of the model.

Contents

1 Introduction

Into the frame of the studies of the long-term concrete structure behavior, a dominating share of the strains measured on structure relates to the differed strains which appear in the concrete during its life. They comprise the shrinkages with the young age, the shrinkage of desiccation, clean creep and the creep of desiccation.

The model presented here is dedicated to the modelization of the differed strain associated with clean creep. Clean creep is, in complement of the creep of desiccation, the share of creep of the concrete which one would observe during a test without exchange of water with outside. In experiments the concrete in clean creep presents a growing old viscous behavior. The strain of creep observed is proportional to the stress of loading, depends on the temperature and the hygroscopy. The longitudinal deflection is accompanied as in elasticity by a transverse strain by opposite sign.
The model selected is that proposed by L. Granger [bib1]. It is model of a viscoelastic type which takes into account the effect of the aging as well as the history of stress, temperature and the hygroscopy. It thus allows this fact of modelling the experimental facts quoted above.

One initially carries out a short recall on the linear viscoelastic models and one presents then the model itself like his numerical integration in *Code_Aster*.

In *Code_Aster*, 3 versions are available: `GRANGER_FP_V` the model complete, `GRANGER_FP` which does not take into account the effect of the aging and `GRANGER_FP_INDT`, which in more does not depend on the temperature.

2 Recall on behavior in creep of a linear viscoelastic material [bib3]

the classical curve of creep represents the evolution according to the time of the strain of a material subjected to a constant unidimensional stress σ . The strain of creep ε_{fl} is, in opposition to the instantaneous strain, the share of strain which evolves with time.
If a material has a linear viscoelastic behavior, then whatever σ the constant load applied as from the time of loading formulated t_c , the strain of creep (1D) can be written:

$$\varepsilon^{fl}(t)=f(t-t_c)\cdot\sigma \qquad 2\text{-}1$$

where $J(t,t_c)=f(t-t_c)$ is the creep function, increasing function of $(t-t_c)$ and null for $(t-t_c)$ negative.

2.1 Principle of superposition of Boltzmann

the relation [éq 2-1] is valid only for one constant loading. For a history of nonconstant loading one applies the principle of superposition of Boltzmann; the load history $\sigma(t)$ is broken up into increments of load:

$\sigma(t)=\sum_{i=0}^{n}\Delta\sigma_i\cdot H(t-t_i)$ where H is the function of Heavyside.

One can then write: $\varepsilon^{fl}(t)=\sum_{i=0}^{n} f(t-t_i)\cdot\Delta\sigma_i$

what uninterrupted gives:

$$\varepsilon^{fl}(t)=\int_{\tau=0}^{t} f(t-\tau)\frac{\partial \sigma}{\partial \tau} d\tau = f * \frac{\partial \sigma}{\partial t} = (f \otimes \sigma) \quad 2.1\text{-}1$$

where $*$ represents the product of convolution.

2.2 Model Kelvin in series

One can show that any linear viscoelastic body can be modelled by a series connection of models of Kelvin and that the creep function can then be put in the form

$$f(t)=\sum_{s=1}^{r} J_s.(1-\exp(-\frac{t}{\tau_s}))$$

τ_s and J_s are plus coefficients identified on the experimental curves of creep.

3 Presentation of the clean model of creep of Granger [bib1]

3.1 experimental Properties of the clean creep of the concrete in uniaxial loading

the clean creep tests on test-tube reveal the following properties:

- in a range of stress lower than 50% of the breaking strength, clean creep is proportional to the stress,
- the clean creep of a test-tube with hygroscopy h_{ext} is almost proportional to h_{ext} . The clean creep of a no-slump concrete is almost null and it is maximum for a concrete saturated with water,
- when the temperature T increases one has an acceleration of creep,
- clean creep is a strongly growing old phenomenon,
- a longitudinal deflection of creep is accompanied by a transverse strain of opposite sign (effect Fish).

One chooses to model the clean creep of the concrete with a linear viscoelastic model which will have moreover to take into account the dependence of creep with respect to the temperature and the hygroscopy.

3.2 Modelization by a series connection of models of Kelvin

One uses a series connection of models of Kelvin whose coefficients are identified from experimental curves of creep. It is shown in practice that one reproduces in a satisfactory way the curves of concrete creep with $r=8$ model in series.

The following creep function is thus used:

$$J(t,t_c)=\sum_{s=1}^{8} J_s.\left(1-\exp\left[-\frac{t-t_c}{\tau_s}\right]\right) \quad 3.2\text{-}1$$

In practice it is very difficult to determine at the same time formulated J_s and τ_s as soon as the number of series of Kelvin exceeds 2. One thus makes generally a choice a priori on τ_s , $\tau_s=\tau_1 \cdot 10^{s-1}$ and one determines then by linear regression formulated J_s .

The statement [éq 3.2-1] is the basic creep function of the model. One shows below how the taking into account of the effect of the temperature, the hygroscopy and the aging is integrated in the model final.

3.3 Effect of the temperature

to take account of the effect of the temperature on the kinetics of creep, one defines a "equivalent time" $t_{eq}(t)$ which will replace time t in the model.

$$t_{eq}(t)=\int_{s=t_c}^{t}\exp\left(-\frac{U_c}{R}\left(\frac{1}{T(s)}-\frac{1}{T_{ref}}\right)\right)ds \qquad 3.3\text{-}1$$

Note:

The temperature and the term of activation of the model of Arrhenius $\frac{U_c}{R}$ *are expressed in degrees* K *.*

To model the effect of the temperature thus T *exploits only the kinetics of creep. For really utilizing* T *on the amplitude of the phenomenon of creep, in particular on the level of the value ad infinitum of the creep function,* T *is also introduced into the statement of J like a multiplicative function of the coefficients of creep such as:*

$$J(t,t_c,T)=\frac{T-(T_{ref}-45)}{45}\cdot\sum_{s=1}^{r}J_s\cdot\left(1-\exp-\left[\frac{t_{eq}-t_c}{\tau_s}\right]\right) \qquad 3.3\text{-}2$$

T_{ref} *is the reference temperature. It is chosen by the user. It is generally taken equalizes with* $20\,°C$ *. In the continuation of the document* T_{ref} *will be taken equalizes with* $20\,°C$ *.*

For the version independent of the temperature, one has simply $t_{eq}(t)=t$ *and* $J(t,t_c,T)=\sum_{s=1}^{r}J_s\cdot\left(1-\exp-\left[\frac{t_{eq}-t_c}{\tau_s}\right]\right)$ *.*

3.4 Effect of the hygroscopy

In the model, h is also introduced like a multiplicative parameter of the coefficients of creep so that:

$$J(t,t_c,T,h)=h\cdot\frac{T-248}{45}\cdot\sum_{s=1}^{r}J_s\cdot\left(1-\exp-\left[\frac{t_{eq}-t_c}{\tau_s}\right]\right) \qquad 3.4\text{-}1$$

Note::

It is variable drying noted C *that one has at the conclusion of computation Code_Aster of drying and it is the isothermal curve of sorption-desorption which makes it possible to pass from the variable* C *with the hygroscopy of the ambient conditions* h *. That is to say* C *the isothermal curve of desorption formulated:* $C=C(h)$ *and* $h=C^{-1}(C)$ *. The curve* $h=C^{-1}(C)$ *must be indicated by the user.*

3.5 Effect of the aging

For a growing old viscoelastic material, the creep function varies for two different times of loading. The aging is associated with the hydration with the young age and other phenomena like polymerization for the old concrete. The effect of the aging is modelled by multiplying the coefficients of creep by a function of aging $k\left(t_c\right)$ depend on the time of loading. The selected modelization to take into account the aging associated with the hydration is that of the CEB [bib2]:

$k(t_c)=\dfrac{28^{0.2}+0.1}{t_c^{0.2}+0.1}$, t_c is expressed in days.

To reveal a sensitivity of the phenomenon of aging compared to the temperature one also defines a time of equivalent loading $tc_{eq}(t_c)$ which replaces t_c in the function of aging.

$$tc_{eq}(t_c)=\int_{s=t_0}^{t_c} \exp\left(-\frac{U_v}{R}\left(\frac{1}{T(s)}-\frac{1}{T_{ref}}\right)\right)ds$$

t_0 : corresponds to the age of the concrete to the young age, it is generally taken equal to 28 days

t_c : the time or age of loading expressed in days

Note:

T and $\dfrac{U_v}{R}$ are in degrees K ,

for the old concrete it would be necessary to use another equivalent time and another function of aging,

if one does not take into account the aging, one has simply $k(t_c)=1$.

The creep function, which will be the final creep function of the model, is written then:

$$J(t,t_c,T,h)=h\cdot\frac{T-248}{45}\cdot k(tc_{eq})\cdot\sum_{s=1}^{n} J_s.\left(1-\exp-\left[\frac{t_{eq}-t_c}{\tau_s}\right]\right) \quad 3.5\text{-}1$$

3.6 Modelization 3D

the classical assumption consists in supposing the existence of a Poisson's ratio of creep constant and equal to the elastic Poisson's ratio, that is to say $\nu_f=0.2$. From where for σ, T, h constant:

$$\varepsilon^{fl}(t)=J(t,t_c,T,h)\cdot[(1+\nu_f)\sigma-\nu_f tr(\sigma)I]$$

and thus:

$$\tilde{\varepsilon}^{\text{fl}}(t)=J\left(t,t_c,T,h\right).\left(1+\nu_f\right)\tilde{\sigma}$$
$$\text{tr}\left(\varepsilon^{\text{fl}}(t)\right)=J\left(t,t_c,T,h\right).\left(1-2\nu_f\right)\text{tr}\,\sigma$$

3.7 formulated on the stress, the temperature and the hygroscopy (1D)

to simplify the demonstration, one takes in this part like creep function of the components of the series of Kelvin, without catch into account of the effect of the aging, nor of equivalent time parameterizing the temperature, that is to say:

$$J(t,t_c,T,h)=h\cdot\frac{T-248}{45}\cdot J_s.\left(1-\exp-\left[\frac{t-t_c}{\tau_s}\right]\right)$$

the strain of then being written creep formulated: $\varepsilon^{fl}=\sigma\cdot h\cdot\frac{T-248}{45}\cdot J_s.\left(1-\exp-\left[\frac{t-t_c}{\tau_s}\right]\right)$.

It is pointed out that this writing of the strain of creep is valid for σ , T and h constant (in this case the model is equivalent in fact to take a Young modulus decreasing according to time).

For a load history, of nonconstant temperature and hygroscopy one applies the principle of superposition of Boltzmann.

Let us suppose that for a given volume element, one knows at time t_n the quantities $(\varepsilon_n^{\mathrm{fl}},\sigma_n,T_n,h_n)$. At time t_{n+1} the quantities will be $(\varepsilon_{n+1}^{\mathrm{fl}},\sigma_{n+1},T_{n+1},h_{n+1})$.

For $t_n<t<t_{n+1}$ formulated proposes to calculate the strain of creep in the following way:

$$\varepsilon_{n+1}^{\mathrm{fl}}(\mathrm{t})=\varepsilon_n^{\mathrm{fl}}(\mathrm{t})-\sigma_n J(t,t_n,T_n,h_n)+\sigma_{n+1}J(t,t_n,T_{n+1},h_{n+1})$$

i.e.:

$$\varepsilon^{fl_{n+1}}(t)=\varepsilon^{fl_n}(t)+\sigma_{n+1}\cdot\frac{T_{n+1}-248}{45}\cdot h_{n+1}\cdot J_s\left(1-\exp-\left[\frac{t-t_n}{\tau_s}\right]\right)$$
$$-\sigma_n\cdot\frac{T_n-248}{45}\cdot h_n\cdot J_s\left(1-\exp\left[\frac{t-t_n}{\tau_s}\right]\right)$$

the superposition formulated is thus considered not only on the stress but also on the temperature and the hygroscopy which are treated mathematically in the same way. From where:

$$\varepsilon^{fl}(t)=\sum_{i=0}^{n}J_s.\left(1-\exp-\left[\frac{t-t_i}{\tau_s}\right]\right)\Delta\left(\sigma\cdot\frac{T-248}{45}\cdot h\right)_i$$

formulated has then in integral writing, the strain of creep of a component *S* of the series of Kelvin:

$$\varepsilon_s^{fl}(t)=\int_{\tau=t_0}^{t}J_s.\left(1-\exp-\left[\frac{t-\tau}{\tau_s}\right]\right)d\left(\sigma\cdot\frac{T-248}{45}\cdot h\right) \qquad 3.7\text{-}1$$

4 Behavior models *Code_Aster*

One introduces into *the Code_Aster* three behavior models associated with clean creep:

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GRANGER_FP_V
GRANGER_FP
GRANGER_FP_INDT
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the first takes account of the group of the effects (forced, temperature, hygroscopy and aging), the second does not take account of the phenomenon of aging and the last takes account neither of the aging nor of the effect of the temperature. They are available in modelization 2D, 3D and plane stresses.

The various parameters of the model are indicated in DEFI_MATERIAU, under key word GRANGER_FP and V_GRANGER_FP.
Are well informed under the key word GRANGER_FP, of which the use is common to three behavior models GRANGER_FP_INDT, GRANGER_FP and GRANGER_FP_V, the characteristic materials following:

	GRANGER_FP:
2×8 constant characteristics of the creep function J_i, τ_i,	J1 : J_1 formulated : τ_1 … J8 : J_8 formulated : τ_8
the curve of sorption-desorption formulated giving h according to variable drying C	FONC_DESORP : *C-1formula* (C)
the constant of energy of activation for time-temperature equivalence.	QSR_K: $\frac{U_c}{R}$

parameter QSR_K formulated is not necessary for model GRANGER_FP_INDT, it is ignored so well informed.

If one uses the growing old behavior model then one informs in more the key word V_GRANGER_FP under which the characteristics associated with the aging are indicated, namely energy of activation for the computation of the time of loading are equivalent and the function of aging $k(tc_{eq})$.

V_GRANGER_FP:

QSR_VEIL : $\frac{U_v}{R}$

formulated : $k(tc_{eq})$

5 of the model Discretization

5.1 formulated (1D)

Pose $S=\sigma\cdot T'\cdot h$ $\qquad T'=\frac{T-248}{45}$

the statement [éq 3.7-1] is thus written:

$$\varepsilon_s^{fl}(t)=\int_{\tau=t_0}^{t} J_s.(1-\exp-\left[\frac{t-\tau}{\tau_s}\right]).\frac{\partial S}{\partial\tau}.d\tau$$

the discretization in time is such as for formulated $t\in[t_{n-1,}t_n]$ formulated considers a linear evolution of S (decomposition of $S(t)$ in linear functions per piece). One has then:

$$\varepsilon_s^{fl}(t_n)=\sum_{i=1}^{n}\frac{\Delta S_i}{\Delta t_i}.\int_{\tau=t_{i-1}}^{t_i}J_s.\left(1-\exp-\left[\frac{t_n-\tau}{\tau_s}\right]\right)d\tau$$

$$\varepsilon_s^{fl}(t_n)=\sum_{i=1}^{n}\frac{\Delta S_i}{\Delta t_i}\cdot J_s\cdot\Delta t_i-\sum_{i=1}^{n}\left(\frac{\Delta S_i}{\Delta t_i}\right)\cdot J_s\cdot\tau_s\cdot\left(\exp-\left[\frac{t_n-t_i}{\tau_s}\right]-\exp-\left[\frac{t_n-t_{i-1}}{\tau_s}\right]\right)$$

$$\varepsilon_s^{fl}(t_n)=J_s\cdot\sum_{i=1}^{n}\Delta S_i-\sum_{i=1}^{n}\Delta S_i\cdot J_s\cdot\frac{\tau_s}{\Delta t_i}\cdot\left(\exp-\left[\frac{t_n-t_i}{\tau_s}\right]\right)\left(1-\exp-\frac{\Delta t_i}{\tau_s}\right)$$

Note: **formulated:**

Notation $\Delta X_i=X_i-X_{i-1}$.

Now let us consider the 8 models of Kelvin in series one a: $\varepsilon^{fl}(t_n)=\sum_{s=1}^{8}\varepsilon_s^{fl}(t_n)=\sum_s\varepsilon_s^{fl}(t_n)$

One can then break up the strain of creep [éq 5.1-1] on the basis formulated $\left\{1;\exp\left(-\frac{t}{\tau_s}\right)\right\}$ and carry out a recurrence on the coefficients of this base. According to [éq 5.1-1] one has with t_n :

$$\varepsilon_s^{fl}(t_n)=\underbrace{J_s\cdot\sum_{i=1}^{n}\Delta S_i}_{A_n^0}-\underbrace{\sum_{i=1}^{n}\Delta S_i\cdot J_s\cdot\frac{\tau_s}{\Delta t_i}\cdot\left(\exp-\left[\frac{t_n-t_i}{\tau_s}\right]\right)\left(1-\exp-\frac{\Delta t_i}{\tau_s}\right)}_{A_n^s}=J_S\cdot A_n^0-A_n^S$$

one t_{n+1} A can also write:

$$\varepsilon_s^{fl}(t_{n+1})=J_s\cdot\sum_{i=1}^{n}\Delta S_i-\sum_{i=1}^{n}\Delta S_i\cdot\frac{1}{\Delta t_i}\cdot J_s\cdot\tau_s\cdot\left(\exp-\left[\frac{t_{n+1}-t_i}{\tau_s}\right]\right)\left(1-\exp-\frac{\Delta t_i}{\tau_s}\right)$$
$$+\Delta S_{n+1}\cdot J_s-\Delta S_{n+1}\cdot\frac{\tau_s}{\Delta t_{n+1}}\cdot J_s\cdot\left(1-\exp-\frac{\Delta t_{n+1}}{\tau_s}\right)$$

is:

$$\varepsilon_s^{fl}(t_{n+1})=J_s\cdot\sum_{i=1}^{n+1}\Delta S_i-\sum_{i=1}^{n}\Delta S_i\cdot\frac{1}{\Delta t_i}\cdot J_s\cdot\tau_s\cdot\left(\exp-\frac{\Delta t_{n+1}}{\tau_s}\right)\cdot\left(\exp-\left[\frac{t_n-t_i}{\tau_s}\right]\right)\left(1-\exp-\frac{\Delta t_i}{\tau_s}\right)$$
$$-\Delta S_{n+1}\cdot\frac{\tau_s}{\Delta t_{n+1}}\cdot J_s\cdot\left(1-\exp-\frac{\Delta t_{n+1}}{\tau_s}\right)$$

formulated can thus write:

$$\varepsilon_S^{\text{fl}}(t_{n+1}) = J_{S\cdot} A_n^0 + \Delta S_{n+1} \cdot J_S - A_n^S \cdot \exp\left(\frac{-\Delta t_{n+1}}{\tau_S}\right) - \Delta \frac{S_{n+1} \cdot \tau_S}{\Delta t_{n+1}} \cdot J_S \cdot \left(1 - \exp - \frac{\Delta t_{n+1}}{\tau_S}\right)$$

Pose $J = \sum_{s=1}^{8} J_s$, one has then:

$$\varepsilon^{fl}(t_n) = J \cdot A_n^0 - \sum_{s=1}^{8} A_n^s \quad \text{et} \quad \varepsilon^{fl}(t_{n+1}) = J \cdot A_{n+1}^0 - \sum_{s=1}^{8} A_{n+1}^s$$

with

$$\begin{cases} A_{n+1}^0 = A_n^0 + \Delta S_{n+1} \\ A_{n+1}^S = A_n^S \cdot \exp\left(\frac{-\Delta t_{n+1}}{\tau_s}\right) + \Delta S_{n+1} \cdot \frac{\tau_S}{\Delta t_{n+1}} \cdot J_S \cdot \left(1 - \exp - \frac{\Delta t_{n+1}}{\tau_S}\right) \end{cases}$$

more precisely, if:

one takes into account equivalent time for the temperature and the aging,
during time step parameter T is evaluated in the middle of time step for the computation of equivalent times t_{eq} and tc_{eq} , his linear evolution being supposed during this time step,

then one a:

$$\varepsilon^{fl}(t_{n+1}) = J \cdot A_{n+1}^0 - \sum_{s=1}^{8} A_{n+1}^s$$

with

$$t_{eq}(t_{n+1}) - t_{eq}(t_n) = dt_{eq}(t_{n+1}) = \exp\left(-\frac{U_c}{R}\left(\frac{1}{T_{n+1/2}} - \frac{1}{T_{ref}}\right)\right) \Delta t_{n+1}$$

$$tc_{eq}(t_{n+1}) - tc_{eq}(t_n) = dtc_{eq}(t_{n+1}) = \exp\left(-\frac{U_v}{R}\left(\frac{1}{T_{n+1/2}} - \frac{1}{293}\right)\right) \Delta t_{n+1}$$

$$A_{n+1}^0 = A_n^0 + k(tc_{\text{éq}}(t_{n+1/2})) \cdot \Delta S_{n+1}$$

$$A_{n+1}^S = A_n^S \cdot \exp\left(\frac{-\Delta t_{\text{eq}_{n+1}}}{\tau_S}\right) + \Delta S_{n+1} \cdot \frac{\tau_S}{\Delta t_{n+1}} \cdot J_S \cdot k(tc_{\text{éq}}(t_{n+1/2})) \cdot \left(1 - \exp - \frac{\Delta t_{\text{eq}_{n+1}}}{\tau_S}\right)$$

Note:

If one does not take account of the aging k *then* $A_{n+1}^0 = \sum_{i=1}^{n+1} \Delta S_i = S_{n+1}$,

one noted $X_{n+1/2} = \frac{X_{n+1} + X_n}{2}$, one noted $\Delta t_{eq_{n+1}} = \Delta t_{eq}(t_{n+1})$.

To have ε_{fl} at time t_{n+1} , one should not store that A_0 and the A_s the time step preceding one, are 9 variables. In 3D A_0 and the A_s are tensors. One will associate then with the two behavior models clean creep (9x6) local variables corresponding to the components of the tensors A . They characterize the advance of creep.

The writing in increment of strain, nearer to the programming gives as for it:

$$\Delta\varepsilon_s^{fl}(t_{n+1}) = \varepsilon_s^{fl}(t_{n+1}) - \varepsilon_s^{fl}(t_n) = A_n^s\left(1-\exp(-\frac{\Delta t_{eq_{n+1}}}{\tau_s})\right)$$

$$+\Delta S_{n+1}\cdot k(tc_{eq}(t_{n+1/2}))\cdot J_s\cdot\left(1-\frac{\tau_s}{\Delta t_{n+1}}\cdot(1-\exp-\frac{\Delta t_{eq_{n+1}}}{\tau_s})\right)$$

5.2 of the behavior model formulated

Is the increment of strain $\Delta\varepsilon = \frac{1}{2}\left(\nabla(\Delta u) + \nabla^T(\Delta u)\right)$.

If account is taken, in the partition of strain, of the thermal strain, the strains associated with the endogenous shrinkage and the shrinkage with desiccation, then:

$$\Delta\varepsilon = \Delta\varepsilon^e + \Delta\varepsilon^{fl} + \Delta\varepsilon^{th} + \Delta\varepsilon^{ret\,end} + \Delta\varepsilon^{ret\,dés}$$

where:

$\varepsilon^e = H\boldsymbol{\sigma}$: elastic strain

$\varepsilon^{\mathrm{th}} = \alpha(T - T_{\mathrm{ref}})\boldsymbol{I_d}$: thermal strain

$\varepsilon^{\mathrm{retend}} = -\beta\xi\boldsymbol{I_d}$: endogenous strain of shrinkage

$\varepsilon^{\mathrm{retdes}} = \kappa(C_{\mathrm{ref}} - C)\boldsymbol{I_d}$: strain of shrinkage of desiccation

with:

ξ : hydration,

C : water concentration.

T_{ref} C_{ref} : temperature and drying of reference

H, α, β, κ : characteristic materials

Note:

In the continuation of the document, one will note $\Delta\varepsilon^A = \Delta\varepsilon^{th} + \Delta\varepsilon^{ret\,end} + \Delta\varepsilon^{ret\,des}$.

$$\tilde{\sigma} = 2\mu\,\tilde{\varepsilon}^e = \frac{2\mu}{2\mu^-}\tilde{\sigma}^- + 2\mu\Delta\,\tilde{\varepsilon} - 2\mu\Delta\,\tilde{\varepsilon}^{fl}$$

and

$$\tilde{\varepsilon}^{fl}(t_{n+1}) = (1+\nu_f)\left(J\cdot\tilde{A}_{n+1}^0 - \sum_{s=1}^{8}\tilde{A}_{n+1}^s\right)$$

it results that:

$$\tilde{\sigma}\,[1+(2\mu)(1+\nu_f)(h\cdot T^{'}\cdot k(t\,c_{eq_{n+1/2}}))\cdot\sum_s J_S(1-(\frac{\tau_S}{\Delta t_{n+1}})(1-\exp(-\frac{\Delta t_{\text{eq}}(t_{n+1})}{\tau_S})))]=\frac{2\mu}{2\mu^-}\tilde{\sigma}^-+2\mu\Delta\tilde{\varepsilon}$$

$$-(2\mu)(1+\nu_f)[\sum_S \tilde{A}_n^s(1-\exp-(\frac{\Delta t_{\text{eq}}(t_{n+1})}{\tau_s}))-(\tilde{\sigma}^-\cdot h^-\cdot T^{'-})\cdot k(tc_{\text{eq}_{n+1/2}})\cdot\sum_s J_s\cdot(1-(\frac{\tau_s}{\Delta t_{n+1}})(1-\exp(\frac{\Delta t_{\text{eq}}(t_{n+1})}{\tau_s})))]$$

In the same way:

$$\text{tr}(\sigma)=3\text{K}\,\text{tr}(\varepsilon^{th})=\frac{3\text{K}}{3\text{K}^-}\text{tr}(\sigma^-)+3\text{K}\,\text{tr}(\Delta\varepsilon^e)-3\text{K}\,\text{tr}(\Delta\varepsilon^{fl})-3\text{K}\,\text{tr}(\Delta\varepsilon^A)$$

and

$$\text{tr}(\varepsilon^{fl})=(1-2\nu_f)\left(J\cdot A_n^0-\sum_{s=1}^{8}A_n^s\right)$$

from where:

$$\text{tr}(\sigma)\left[1+3\text{K}(1-2\nu_f)(h\cdot T^{'}\cdot k(t\,c_{\text{eq}_{n+1/2}}))\cdot\sum_s J_s(1-\frac{\tau_s}{\Delta t_{n+1}}(1-\exp(\frac{-\Delta t_{\text{eq}}(t_{n+1})}{\tau_s})))\right]=\frac{3\text{K}}{3\text{K}^-}\text{tr}(\sigma^-)$$

$$+3\text{K}\,\text{tr}(\Delta\varepsilon^e)-3\text{K}(1-2\nu_f)\left[\sum_s \text{tr}(A_n^s)(1-\exp(\frac{-\Delta t_{\text{eq}}^{(t_{n+1})}}{\tau_s}))-(\text{tr}\,\sigma^-\cdot h^-\cdot T^{'-})\cdot k(tc_{\text{eq}_{n+1/2}})\cdot\right.$$

$$\left.\sum_s J_s(1-\frac{\tau_s}{\Delta t_{n+1}}(1-\exp(\frac{-\Delta t_{\text{eq}}(t_{n+1})}{\tau_s})))\right]$$

$$-3\text{K}\,\text{tr}(\Delta\varepsilon^A)$$

formulated of deduced then σ since $\sigma_{ij}=\tilde{\sigma}_{ij}+\frac{1}{3}\text{tr}\,\sigma\delta_{ij}$

5.3 state formulated

the variables of state of the two behavior models are thus:

- σ : tensor of the stresses,
- ε : tensor of the strains,
- T : temperature,
- C : water concentration,
- ξ : hydration,
- A_s : tensors characteristic of the advance of creep, are 6×9 variable,
- tc_{eq} : time of equivalent loading, characteristic of the age of the concrete.

A_s and tc_{eq} are local variables of the constitutive laws, which thus comprise 55 local variables.

5.4 Stamp tangent

$$\frac{\partial \sigma}{\partial \varepsilon}=\frac{\partial \tilde{\sigma}}{\partial \varepsilon}+\frac{1}{3}\frac{\partial(\operatorname{tr}\sigma)}{\partial \varepsilon}I_d$$

$$\frac{\partial \tilde{\sigma}}{\partial \varepsilon}=\frac{\partial \tilde{\sigma}}{\partial \tilde{\varepsilon}}\frac{\partial \tilde{\varepsilon}}{\partial \varepsilon} \qquad \frac{\partial \tilde{\varepsilon}_{ij}}{\partial \varepsilon_{kl}}=\delta_{ik}\delta_{jl}-\frac{1}{3}\delta_{ij}\delta_{kl}$$

$$\frac{\partial(\operatorname{tr}\sigma)}{\partial \varepsilon}=\frac{\partial(\operatorname{tr}\sigma)}{\partial(\operatorname{tr}\varepsilon)}\frac{\partial(\operatorname{tr}\varepsilon)}{\partial \varepsilon} \qquad \frac{\partial(\operatorname{tr}\varepsilon)}{\partial \varepsilon_{ij}}=\delta_{ij}$$

Iteration of Newton:

$$\frac{\partial \tilde{\sigma}}{\partial \tilde{\varepsilon}}\left[1+2\mu(1+\nu_f).h\cdot T'\cdot k(tc_{eq_{n+1/2}})\cdot\sum_s J_s\left(1-(\frac{\tau_s}{\Delta t_{n+1}})(1-\exp(-\frac{\Delta t_{eq}(t_{n+1})}{\tau_s}))\right)\right]=2\mu I$$

with $I_{ijkl}=\delta_{ik}\delta_{jl}$

$$\frac{\partial(\operatorname{tr}\sigma)}{\partial(\operatorname{tr}\varepsilon)}\left[1+3K(1-2\nu_f).h\cdot T'\cdot k(tc_{eq_{n+1/2}})\cdot\sum_s J_s\left(1-(\frac{\tau_s}{\Delta t_{n+1}})(1-\exp(-\frac{\Delta t_{eq}(t_{n+1})}{\tau_s}))\right)\right]=3KI$$

Phase of prediction for time step $[t_n, t_{n+1}]$

Remark:

> *In 1D:* $\left(\frac{\partial \varepsilon_s^{fl}}{\partial t}\right)_{t_n}=\frac{A_s^-}{\tau_s}-J_s\cdot k(tc_{eq})\cdot\frac{\partial S}{\partial t}$.

Writing of velocity at time t_n :

$$\frac{\partial \tilde{\sigma}}{\partial t}\left[1+2\mu(\sum_s J_s\cdot k(tc_{\mathrm{eq}_n})\cdot T'^-\cdot h^-)\right]=2\mu\frac{\partial \tilde{\varepsilon}}{\partial t}-2\mu(1+\nu_f)$$
$$\left[\sum_s\left(\frac{\tilde{A}_s^-}{\tau_s}-J_s\cdot k(tc_{\mathrm{eq}_n})\cdot\tilde{\sigma}^- h^-\frac{dT'}{dt}-J_s\cdot k(tc_{eq_n})\cdot\tilde{\sigma}^- T'^-\frac{dh}{dt}\right)\right]$$

for the phase of prediction of time step formulated $[t_n, t_{n+1}]$:

$$\Delta\tilde{\sigma}\left[1+2\mu(1+\nu_f)(\sum_s J_s\cdot k(tc_{\mathrm{eq}_n})\cdot T'^-\cdot h^-)\right]=$$
$$2\mu\Delta\tilde{\varepsilon}-2\mu(1+\nu_f)\left[\sum_s\frac{\tilde{A}_s^-}{\tau_s}\cdot\Delta t-J_s\cdot k(tc_{\mathrm{eq}_n})\cdot\tilde{\sigma}^- h_-\Delta T'-J_s\cdot k(tc_{\mathrm{eq}_n})\cdot\tilde{\sigma}T'^-\Delta h\right]$$

formulated of velocity at time t_n :

$$\frac{\partial(tr\,\sigma)}{\partial t}\left[1+3\mathrm{K}(1-2\nu_f)\left(\sum_s J_s\cdot k(tc_{\mathrm{eq}})\cdot T^{'-}\cdot h^-\right)\right]=3\mathrm{K}\frac{\partial(tr\,\varepsilon)}{\partial t}$$
$$-3\mathrm{K}(1-2\nu_f)\left[\sum_s \frac{tr\,A_s^-}{\tau_s}-J_s\cdot k(tc_{\mathrm{eq}_n})\cdot(tr\,\sigma^-)h^{'-}\frac{dT^{'}}{dt}-J_s\cdot k(tc_{\mathrm{eq}_n})\cdot(tr\,\sigma^-)T^{'-}\frac{dh}{dt}\right]$$
$$-3\mathrm{K}(3\alpha\frac{dT}{dt})+3\mathrm{K}(3\beta\frac{d\xi}{dt})+3\mathrm{K}(3\kappa\frac{dC}{dt})$$

for the phase of prediction of time step formulated $[t_n, t_{n+1}]$:

$$\Delta(\mathrm{tr}\,\sigma)\left[1+3\mathrm{k}(1-2\nu_f)\left(\sum_s J_s\cdot k(tc_{eq_n})\cdot T^{'}\cdot h^-\right)\right]=3\mathrm{K}\,\Delta(\mathrm{tr}\,\varepsilon)$$
$$-3\mathrm{K}(1-2\nu_f)\left[\sum_s \frac{(tr\,\boldsymbol{A}_s^-)}{\tau_s}\cdot\Delta t-J_s\cdot k(tc_{\mathrm{eq}_n})\cdot(\mathrm{tr}\,\sigma^-)h^-\Delta T^{'}-J_s\cdot k(tc_{\mathrm{eq}_n})\cdot(\mathrm{tr}\,\sigma^-)T^{'-}\Delta h\right]$$
$$-3\mathrm{K}(3\alpha\Delta T)+3\mathrm{K}(3\beta\Delta\xi)+3\mathrm{K}(3\kappa\Delta C)$$

the creep thus introduced a specific term of second member at the time of the phase of prediction which in fact is neglected, without consequence on the results.

6 Bibliography

1) L. GRANGER: Behavior differed from the concrete in the enclosures of nuclear power plant: analyzes and modelization. Doctorate of the ENPC (February 1995).

2) CEB FIP Model (1990) General task group n°9, concrete Evaluating of the time behavior of.

3) J. LEMAITRE, J-L CHABOCHE: Mechanics of the solid materials. Dunod.

7 Functionalities and checking

This document relates to constitutive laws GRANGER_FP, GRANGER_FP_INDT, GRANGER_FP_V (key word BEHAVIOR of STAT_NON_LINE) and their associated materials GRANGER_FP, V_GRANGER_FP (command DEFI_MATERIAU).

These constitutive laws are respectively checked by the cases following tests:

GRANGER_FP	SSNP116	Coupling creep/cracking - uniaxial Tension	[V6.03.116]
GRANGER_FP_INDT	SSNV142	clean Creep test: model Granger	[V6.04.142]
GRANGER_FP_V	nothing		

8 Description of the versions of the document

Version Aster	**Author (S) Organization (S)**	**Description of the modifications**
		initial Text
7.4	S.Michel-Ponnelle EDF-R&D/AMA	
11.8	Marina Bottoni	Elimination of key word GRANGER_FP_INDT

	EDF-R&D/AMA	