Organization (S): *Manual*

EDF/IMA/MMN of data-processing Description
D8.01 Booklet: Presentation of documentation
Document graphic

D8.01.03 Charter for the realization of the mathematical formulas in the documentation of *the Code_Aster*

Summarized

After having identified the minimal general mathematical objects most commonly employed by the community of the mechanics developing in *Aster*,

$$\left(-j\omega^3\mathbf{M}-\omega^2\mathbf{M}+j\omega\mathbf{C}+\mathbf{K}\right)\mathbf{x} \;=\; \sum_{i=1}^{k}\mathrm{k}_i(\omega)_n^i\,\mathrm{e}^{j\varphi_i}\,.\mathrm{g}(P)$$

one exposes the instructions of striking of the mathematical formulas which allow on the one hand one returned paper and acceptable screen

$$\dot{\beta}(T)-\mathrm{div}\left(\lambda(T)\,\mathrm{grad}\,T\right) \;=\; \mathrm{f}(t)$$

and which, on the other hand, answers the criteria required in the international publications dealing with the mechanics of solid.

In documentation *Aster*, the mathematical formulas are developed under the Equation editor of *Microsoft* Word5 (version of “MathType Editor Equation” of Design Science Inc).

1 Forced spirit and

1.1 range imposed by the projection of the numerical documents *Aster* on a media

part of the instructions for the drafting of the formulas in the documents on the formalism *Aster*, was controlled by the concern to keep an acceptable esthetics and a legibility independently of the media and the basic police of the surrounding text.
In the current state of the art as regards physical representation of the formulas in the electronic documents, in the absence of DTD (Description of the Type of Document to formalism SGML), those are comparable to drawings. They thus do not undergo reformating according to the media of consultation (paper, cathode screens).
The e-book comprises as many external files of formulas (drawings). The contents of these files come to be displayed with the consultation of the book to the site which it must have in the text. The book comprises an array connecting the name of the file (the formula) and the position in the book.

1.2 Norms and recommendations *Aster*

They indicate the way typographically represent the types of the mathematical objects most frequently handled by the mechanics of solid. The principle is the use of typographical enrichments *Italic* and **Fats** to typify these objects.
The writer *Aster* will use of these recommendations which constitute an acceptable minimal representation by the community of the mechanics of solid developing in *Aster* . They:

- approach returned the Tex trainer,
- take as a starting point the necessary rules to publish in the following reviews:
  - Comp. Meth. Appl. Mech. Eng.
  - Int. J. Num. Meth. Eng.
  - ASME J. Appl. Mech.
  - Europ. J. Mech. A/Solids.
- take account of the possibilities and limitations of the Equation editor of *Microsoft* Word5.

What gives for example:

$$\left(-j\omega^3\mathbf{M}-\omega^2\mathbf{M}+j\omega\mathbf{C}+\mathbf{K}\right)\mathbf{x} \;=\; \sum_{i=1}^{k}\mathrm{k}_i\,(\omega)_n^i\,\mathrm{e}^{j\varphi_i}.\mathrm{g}(P)$$

(computation carried out by the operator `DYNA_LINE_HARM` [U4.54.02 §1])

$$\dot{\beta}(T)-\mathrm{div}\left(\lambda(T)\,\mathrm{grad}\,T\right) \;=\; \mathrm{f}(t)$$

(computation carried out by the operator `THER_NON_LINE` [U4.33.02 §1])

$$\sigma_{VM} \;=\; \sqrt{\sum_{i,j=1}^{2\text{ ou }3}\frac{2}{3}\left(\sigma_{ij}-\frac{1}{3}\mathrm{tr}(\boldsymbol{\sigma}).\delta_{ij}\right)^2}$$

(computation carried out by operand INVARIANT of procedure POST_RELEVE [U4.74.03]).

2 Typographical realization of the formulas in *Aster*

After having identified the mathematical objects selected, one enumerates enrichments which apply to it, the police to be used, the bodies, the relative positions of the elements which compose the formulas (indices, exponents, symbols of relations, etc…).

2.1 Enrichments and mathematical types of objects

the table hereafter summarizes on the objects selected, the basic typographical achievements that the writer *Aster* will employ as far as possible.

Type of object	*Romanian*	Ital	**Fatty**	Maig	**Police**
Number		X		X	scalar
Times Variable	X			X	Times or Symbol (1)
usual Function		X		X	Times (2)
Function with scalar value		X		X	Times or Symbol
Function with vectorial or tensorial values		X	X		Times or Symbol (3)
Tensor, Matrix, vector (dimension 2 and more)		X	X		Times or Symbol (3)
Spaces scalars or vectors		X		X	DESCARTES (4)
Spaces functions		X		X	*MonotypeCorsiva* (5)
Text		X		X	Geneva (6)

1) If a Greek capital letter is employed for a scalar variable then to always strike it as a Romain.
2) The Equation editor of Word5 can recognize the name of about forty usual functions like: det, lim, cos, Im etc…
3) For the Symbol police, **the Fat** appears on the screen but not clearly with the printing. Example: $\boldsymbol{\sigma}$ (fatty), σ (not fat).
4) Body of realities € , the complexes ç, the integers ı. One can have difficulty of printing police DESCARTES when it is employed in the Equation editor. The printer replaces characters DESCARTES by a blank. Unknown remedy for the date of publication of this document. To address itself to the Person in charge of Documentation *Aster*.
5) For example: ($\mathcal{F}$), (here Body 18) to note a space of functions, ($\mathcal{P}$) a problem, ($\mathcal{S}$) a system.
6) According to MacOS and the versions of Word5 and the Equation editor one has it is possible that Geneva in a “text” of formula left on the printer in `Courier`. To then prefer Helvetica which does not present this disadvantage.

Attention

> *It results from 4 and 5 that the operating systems MacOS of the writers Aster will have to be rigged by this police.*

2.2 Examples for the Dim.

functions of spaces	Writing of physical	the Examples application
$¤ \to ¤$	$f(x)=b \equiv f$	$E(T)$ Modulates YOUNG function of the geometrical
$¤^n \to ¤$	$f(T)=b \equiv f$	$g(s) = y$
$¤^n \to ¤^m$	$f(T)=V=f$	$K(s)$ temperature Stiffness
$¤ \to ¤^m$	$f(a)=T=f$	$A(T)$ Elasticity function of the temperature

2.3 Body of the components of the formulas

Elements of the formula	Body	Examples
normal Terms (*)	12 Pt	$\sum_{p=1}^{(1+B)^2} X_{n_k}^{kp}$
Exhibitors and indices	9 Pt	$\sum_{p=1}^{1+B^2} X_{n_k}^{kp}$
Symbols	18 Pt	$\sum_{p=1}^{(1+B)^2} X_{n_k}^{kp}$
Under symbols	12 Pt	$X^{\Sigma n_k}$

(*) If one uses *MonotypeCorsiva* for a normal term, to prefer the body 14 Pt.

That is to say the adjustment following in the heading **Définir...** of the menu **Taille** of the Editor of Mathematical formulas

Normale	12pt
Indice/Exposant	9pt
Sous-indice/Exposant	7pt
Symbole	18pt
Sous-symbole	12pt

2.4 relative Positions of the elements of a formula

It is necessary to understand by there, the relative position of the indices and exhibitors compared to the term which they affect and the relative position of the lines of equations or the lines and columns of matrixes. One takes the values by default of the equation editor of Microsoft Word5 expressed hereafter in % of the body of the symbols.

That is to say the adjustment following in the heading **Espacement...** of the menu **Format** of the Editor of mathematical formulas

Espacement ligne	150%
Espacement lignes matrice	150%
Espacement colonnes	100%
Hauteur de l'exposant	44,53%
Hauteur de l'indice	25%
Hauteur limite	25%

2.5 Style sheet for the formulas

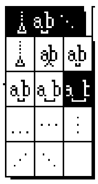
Heading **Définir...** of the menu **Style** of the Editor of mathematical formulas

Style	Police	Format de caractère Gras	Format de caractère Italique
Texte	Geneva	☐	☐
Fonction	Times	☐	☐
Variable	Times	☐	☒
Minuscule grecque	Symbol	☐	☒
Majuscule grecque	Symbol	☐	☐
Symbole	Symbol	☐	☐
Vecteur-Matrice	Times	☒	☐
Nombre	Times	☐	☐

OK / Annuler / Aide

2.6 spaces on both sides of sign =

One recommends to insulate the sign well = while having blanks on both sides of sign sufficiently. **Goal**: to make quite readable the two members of the equations. One recommends to add to affected spacing by default automatically by the Equation editor after the sign of relation = a blank of a quadratin.



2.7 Texts in the formulas

If the author wishes to accompany his formula by a text (what is disadvised) for, for example, to clarify certain terms, this text will be in Geneva 10 Romain nonfatty Style “Text” of the style sheet of the Equation editor (with the reserves expressed in [§2.1]). In this case, the group formulates + text forms only one graphic block.

$$\left(-j\omega^3\mathbf{M}-\omega^2\mathbf{M}+j\omega\mathbf{C}+\mathbf{K}\right)\mathbf{x} \;=\; \sum_{i=1}^{k}\mathrm{k}_i\,(\omega)_n^i\,\mathrm{e}^{j\varphi_i}\,.\mathrm{g}(P)$$

$$\text{où}\;\;\mathbf{C} = \text{ Matrice d'Amortissement}$$

2.8 Formulas except text and in text

the typography of the terms of formulas integrated in a paragraph is the same one as in the formula it even. An example is given in [§3.6].

3 Recommendations and advice

3.1 Notations author --> reader

At the top of document the writer will expose his notations, mainly in what they differ or supplement the recommendations *Aster*. He will take care to choose a symbolism present in the Equation editor of Word.

3.2 Notations author --> typist

the writer will indicate on her manuscript, by a code with him the instructions of enrichment of the terms of her mathematical formulas.

3.3 The “transposed” sign

Transposed of a matrix or a vector (and opposite of matrix) as follows:
$\mathbf{M}^T,\mathbf{M}^{-1},\mathbf{M}^{-T},\mathbf{x}^T$. Modal mass for the mode i : $\mathbf{u}_i^T\mathbf{M}\mathbf{u}_i$

3.4 Tiny Greek

In the Symbol police one will prefer the tiny phi φ with ϕ to avoid confusions

3.5 Functions and variables

not to confuse the function and his realization for a given value of his variable.

To always indicate what depend the functions the first time that the function appears. Example:

$$\mathrm{g}(\sigma,\alpha) \quad = \quad \sqrt{(\sigma-\frac{1}{3}(\mathrm{tr}\,\boldsymbol{\sigma})\mathbf{Id})^2}-\sigma_y(\alpha) \text{ (Plasticity criterion)}$$

3.6 Derived

To indicate **where** the derivatives are taken, at least during their first appearance. The following formalism is recommended:

that is to say the function $\mathrm{g}(\sigma,\alpha)$, its partial derivative compared to σ for $\sigma=\tau$ and $\alpha=\beta$ are written:

$$\left.\frac{\partial \mathrm{g}}{\partial \sigma}\right|_{(\tau,\beta)}$$

or this one

$$\sigma_{ij,j}+\mathrm{f}_i \quad = \quad 0$$

for a balance equation.

3.7 Convention of the indices repeated

In a indicielle notation, one will use the convention of EINSTEIN known as **"of the repeated indices"**. This convention, makes it possible to reduce the writing and to be freed from employment from the symbol from summation $\sum$.

Principle: an index repeated twice, once in top, once in bottom, or more simply twice in bottom, indicates automatically a summation (1,…, N).

Example: $\mathbf{v} = \sum_{i=1}^{n} v^i \mathbf{e}_i = \mathbf{v}^i \mathbf{e}_i$

$\mathbf{v}$, vector
v^i , components
$\mathbf{e}_i$, basic vector

$$\mathrm{tr}\,\boldsymbol{\sigma} = \sigma_k^k = \sigma_1^1 + \sigma_2^2 + \sigma_3^3$$

$\mathrm{tr}\,\boldsymbol{\sigma}$ = trace du tenseur $\boldsymbol{\sigma}$

$$= \mathrm{Id}.\boldsymbol{\sigma} = \sigma_{ij}\delta^{ij} = \sigma_k^k$$

$$\sigma.\varepsilon = \sum_{i=1}^{3}\sum_{j=1}^{3}\sigma^{ij}.\varepsilon_{ij} = \sigma^{ij}.\varepsilon_{ij}$$ or more simply $\sigma_{ij}.\varepsilon_{ij}$.

3.8 Greek indices and Latin indices

One advises the use the index Greek (α β, etc…) for a path in the interval {1, 2} and the Latin indices (i j k, etc…) in the interval {1, 2,3}.

3.9 Alignment and balance of the equations

To adopt a provision such as the similar terms are on the same balance.

$$\sigma_{\alpha\beta} = \mathbf{A}_{\alpha\beta\gamma\delta}\left(\mathbf{E}_{\alpha\beta}(\boldsymbol{U}) - z_3\ \mathbf{K}_{\gamma\delta}\left(\mathbf{U}_3^0\right)\right) + \mathbf{A}_{\alpha\beta ij}\left(\mathbf{E}_{\mu\nu}(\boldsymbol{U})\cdot\varepsilon_{ij}^Z\left(\chi^{\mu\nu}\right) + \mathbf{K}_{\mu\nu}\left(\mathbf{U}_3^0\right)\cdot\varepsilon_{ij}^2\left(\xi^{\mu\nu}\right)\right.$$
$$\left.+\varepsilon_{ij}^Z\left(\mathbf{U}_{dil}^2\right) - \alpha_{kl}\left(T^0 - T^{réf}\right)\ \delta_{ik}\delta_{jl}\right) + o(\eta)$$

$$\sigma_{33} = \mathbf{A}_{33\gamma\delta}\left(\mathbf{E}_{\gamma\delta}(\boldsymbol{U}) - z_3\cdot\mathbf{K}_{\gamma\delta}\left(\mathbf{U}_3^0\right)\right) + \mathbf{A}_{33ij}\left(\mathbf{E}_{\mu\nu}(\boldsymbol{U})\cdot\varepsilon_{ij}^Z\left(\chi^{\mu\nu}\right) + \mathbf{K}_{\mu\nu}\left(\mathbf{U}_3^0\right)\cdot\varepsilon_{ij}^Z\left(\xi^{\mu\nu}\right)\right.$$
$$\left.+\varepsilon_{ij}^Z\left(\mathbf{U}_{dil}^2\right) - \alpha_{kl}\left(T^0 - T^{réf}\right)\ \delta_{ik}\delta_{jl}\right) + o(\eta)$$

4 Examples

These examples are extracted from the isotropic form of thermoelasticity.

$$\sigma_{ij}^{D} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}$$

$$\sigma_{VM}^{éq} = \sqrt{\frac{3}{2}\sigma^{D}.\sigma^{D}} = \sqrt{\frac{3\sigma.\sigma - (\operatorname{tr}\sigma)^2}{2}} = \sqrt{\frac{1}{2}\sum_{I,J}(\sigma_I - \sigma_J)^2}$$

$$\sigma^{D}.\sigma^{D} = \frac{2}{3}\left(\sigma_{VM}^{éq}\right)^2$$

4.1 Thermodynamic potential, density of free energy 3D

$$F(\varepsilon,T) = \frac{1}{2}\lambda(\operatorname{tr}\varepsilon)^2 + \mu\varepsilon_{ij}.\varepsilon_{ij} - 3K\alpha\left(T - T^{réf}\right)\operatorname{tr}\varepsilon - \frac{1}{2}\frac{C}{T}\left(T - T^{réf}\right)^2$$

$$F(\varepsilon,T) = \frac{K}{2}(\operatorname{tr}\varepsilon)^2 + \mu\varepsilon_{ij}^{D}.\varepsilon_{ij}^{D} - 3K\alpha\left(T - T^{réf}\right)\operatorname{tr}\varepsilon - \frac{1}{2}\frac{C}{T}\left(T - T^{réf}\right)^2$$

Stability: positive definite potential:

$$\mu > 0 \ ; \ 3K = 3\lambda + 2\mu > 0 \ \Leftrightarrow \ E > 0 \ ; \ -1 > \nu > 0{,}5$$

4.2 Complementary potential, density of enthalpy free 3D

$$F^{*}(\sigma,T) = \frac{-\nu}{2E}(\operatorname{tr}\sigma)^2 + \frac{1+\nu}{2E}\sigma_{ij}.\sigma_{ij} + \frac{\alpha}{2}\left(T - T^{réf}\right)\operatorname{tr}\sigma + \frac{1}{2}\frac{C}{T}\left(T - T^{réf}\right)^2$$

$$F^{*}(\sigma,T) = \frac{1}{18K}(\operatorname{tr}\sigma)^2 + \frac{1}{4\mu}\sigma_{ij}^{D}.\sigma_{ij}^{D} + \frac{\alpha}{2}\left(T - T^{réf}\right)\operatorname{tr}\sigma + \frac{1}{2}\frac{C}{T}\left(T - T^{réf}\right)^2$$

4.3 Coefficients of elastic stiffness 3D

$$\left.\frac{\partial F}{\partial \varepsilon_{ij}}\right|_{(\varepsilon,T)} = \sigma^{ij} = \lambda^{ijkl}\varepsilon_{kl} + \left(T - T^{réf}\right)D^{ij} = \left(\lambda\delta^{ij}\delta^{kl} + 2\mu\delta^{ik}\delta^{jl}\right)\varepsilon_{kl} - 3K\alpha\left(T - T^{réf}\right)\delta^{ij}$$

4.4 Relations stress-strains 3D

$$\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij} - 3K\alpha\left(T - T^{réf}\right)\delta_{ij}$$

$$\sigma_{ij} = \frac{E}{1+\nu}\left(\varepsilon_{ij} + \frac{\nu}{1-2\nu}\operatorname{tr}\varepsilon\delta_{ij}\right) - \frac{\alpha E}{1-2\nu}\left(T - T^{réf}\right)\delta_{ij}$$

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{12}\\ \sigma_{23}\\ \sigma_{31}\end{bmatrix} = \begin{bmatrix}\lambda+2\mu & \lambda & \lambda & 0 & 0 & 0\\ \lambda & \lambda+2\mu & \lambda & 0 & 0 & 0\\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0\\ 0 & 0 & 0 & 2\mu & 0 & 0\\ 0 & 0 & 0 & 0 & 2\mu & 0\\ 0 & 0 & 0 & 0 & 0 & 2\mu\end{bmatrix} . \begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ \varepsilon_{12}\\ \varepsilon_{23}\\ \varepsilon_{31}\end{bmatrix} - 3\alpha K\left(T - T^{réf}\right)\begin{bmatrix}1\\ 1\\ 1\\ 0\\ 0\\ 0\end{bmatrix}$$

4.5 Relations strain-forced 3D

$$\varepsilon_{ij} \quad = \quad \frac{-\nu}{E}\sigma_{kk}\delta_{ij} + \frac{1+\nu}{E}\sigma_{ij} + \alpha\left(T - T^{réf}\right)\delta_{ij}$$

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{12} \\ \varepsilon_{23} \\ \varepsilon_{31} \end{bmatrix} = \frac{1}{E}\begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1+\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 1+\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 1+\nu \end{bmatrix} . \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{bmatrix} + \alpha\left(T - T^{réf}\right)\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

4.6 elastic Plane stresses 2D

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 1-\nu \end{bmatrix} . \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{bmatrix} - \frac{\alpha E}{1-\nu}\left(T - T^{réf}\right)\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{aligned} \sigma^{\alpha\beta} &= \lambda_{COPL}^{\alpha\beta\gamma\delta}\varepsilon_{\gamma\delta} + \left(T - T^{réf}\right)D_{COPL}^{\alpha\beta} \\ &= \frac{E}{1-\nu^2}\left(\nu\delta^{\alpha\beta}\delta^{\gamma\delta} + \frac{1-\nu}{2}\left(\delta^{\beta\gamma}\delta^{\alpha\delta} + \delta^{\beta\delta}\delta^{\alpha\gamma}\right)\right)\varepsilon_{\gamma\delta} - \frac{\alpha E}{1-\nu}\left(T - T^{réf}\right)\delta^{\alpha\beta} \end{aligned}$$

4.7 Potential complementary 2D

$$F*_{DEPL}(\sigma) \quad = \quad \frac{1-\nu^2}{2E}\left(\mathrm{tr}_{2D}\,\sigma\right)^2 + \frac{1+\nu}{E}\left(\sigma_{12}^2 - \sigma_{11}.\sigma_{22}\right)$$

intentionally white Page.