Example : the Norton law

A very simple constitutive law : the viscoplastic Norton law. Let's dare the **equations** !

$$\begin{cases} \dot{\boldsymbol{\varepsilon}}^v = \dot{p}\,\boldsymbol{n} \\ \dot{p} = \left[\dfrac{\sigma_{eq}}{K}\right]^n \\ \boldsymbol{\sigma} = \boldsymbol{H}\left(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^v\right) \end{cases}$$

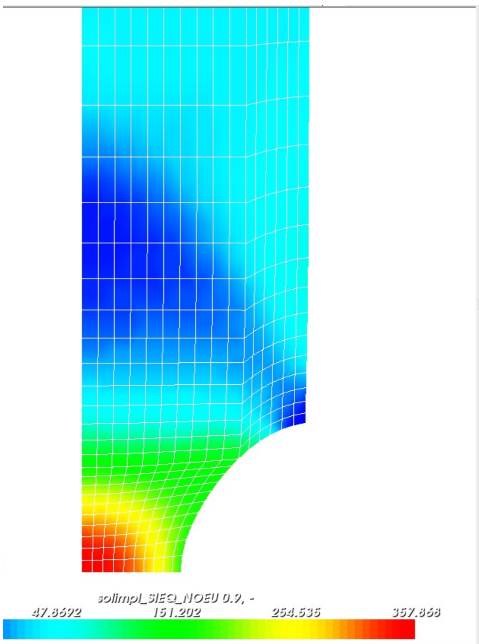
The coefficients to be defined are : $n > 0$ et $\dfrac{1}{K}$

$\boldsymbol{H}$ is the elasticity operator, σ_{eq} the Von Mises norm, $\boldsymbol{n}$ the normal vector to the loading surface.

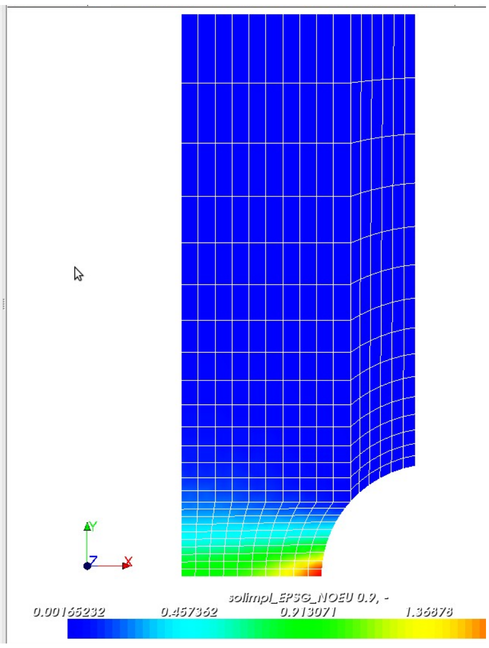
Let's dare now the **fortran** ! In addition to the classic routine of material coefficients reading, simply insert the following routine (called par LCDVIN) :

```
       SUBROUTINE NORTON(NVI,VINI,COEFT,NMAT,SIGI,DEPS,DVIN,IRET)
        IMPLICIT NONE
C ==================================================================
C      MODELE VISCOPLASTIQUE DE NORTON
C ==================================================================
C      DERIVEES DE L ENSEMBLE DES VARIABLES INTERNES DU MODELE
C       IN  NVI      :  NOMBRE DE VARIABLES INTERNES
C           VINI     :  VARIABLES INTERNES A T
C           COEFT    :  COEFFICIENTS MATERIAU INELASTIQUE A T
C           NMAT     :  DIMENSION MAXI DE COEFT
C           SIGP     :  CONTRAINTES A L'INSTANT COURANT, AVEC SQRT(2)
C           DEPS     :  INCREMENT DE DEFORMATIONS, AVEC SQRT(2)
C       OUT:
C           DVIN     :  DERIVEES DES VARIABLES INTERNES A T
C           IRET     :  CODE RETOUR =0 SI OK, =1 SI PB
C     ----------------------------------------------------------------
      INTEGER IRET,ITENS,NDI,NMAT,NVI,NDT
      REAL*8 COEFT(NMAT),VINI(NVI),DVIN(NVI),SMX(6),SIGI(6)
      REAL*8 DP,N,UNSURK,GRJ2V,EPSI,R8MIEM,LCNRTS,DEPS(6)
C     ----------------------------------------------------------------
      IRET=0
C     INITIALISATION DES DERIVEES DES VARIABLES INTERNES A ZERO
      CALL R8INIR(7,0.D0,DVIN,1)
C --  COEFFICIENTS MATERIAU
      N      = COEFT(1)
      UNSURK = COEFT(2)
C     ZERO NUMERIQUE ABSOLU
      EPSI=R8MIEM()
C------------ CALCUL DU TENSEUR DEVIATORIQUE DES CONTRAINTES ---
      CALL LCDEVI(SIGI , SMX )
C------------CALCUL DU DEUXIEME INVARIANT DE CONTRAINTE  -------
      GRJ2V  = LCNRTS(SMX )
C------ EQUATION DONNANT LA DERIVEE DE LA DEF VISCO PLAST
      IF (GRJ2V .GT. EPSI) THEN
         DP=(GRJ2V*UNSURK)**N
C        INUTILE DE CALCULER DES DEFORMATIONS PLASTIQUES MINUSCULES
         IF (DP .GT. 1.D-10) THEN
            DO 12 ITENS=1,6
               DVIN(ITENS)=1.5D0*DP*SMX(ITENS)/GRJ2V
   12       CONTINUE
            DVIN(7)=DP
         ENDIF
      ENDIF
      END
```

Creep test with the Norton law



Hydrostatic stress on the deformed geometry



deformation epyy on the initial geometry

Comparison of performances :

Algo	CPU time	Nb time steps	Nb iterations
Norton Implicit	83s	101	382
Norton Explicit (1/3 of calculation)	162s	1061	2275
Lemaitre implicit	23s	101	330

The implicit resolution is much more efficient than the explicit resolution (which does not lead beyond half of total time). The results are identical to the reference calculation (Lemaitre law), as shown by the maximum strain variation with time: